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Theory and Methodology

Application of Grey theory and multiobjective programming towards airline network design

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Abstract

Airline network design encompasses decisions on an airline network shape and route flight frequencies. Related investigations handle the trade-off between enhancing passengers' service levels and lowering the airline's operating costs by applying deterministic optimization methods. In contrast with other conventional methods, Grey theory is a feasible mathematical device capable of forecasting airline traffic with minimum data and resolving problems containing uncertainty and indetermination. In the light of these developments, this study develops a series of models capable of forecasting airline city-pair passenger traffic, designing a network of airline routes and determining flight frequencies on individual routes by applying Grey theory and multiobjective programming. A case study demonstrates the feasibility of applying the proposed models. Results in this study not only confirm the practical nature of the proposed models, but also their ability to provide high flexibility in decision making for airlines. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

1.1. The airline network design problem

Airline network design, including how to determine a network's shape, flight frequencies and aircraft types on individual routes, is a prerequisite for an air carrier's operational planning such

as flight scheduling and crew assignment. Network design is heavily emphasized since the chosen network shape, flight frequency and aircraft type on individual routes directly influence the operating effectiveness of the air carrier and the quality of service provided to passengers. Designing an airline network is an extremely complex task, largely owing to the fact that transportation facilities must adhere to passenger demand. Thus, accurately forecasting the future passenger traffic on each route is of priority concern in the planning and design of an airline network. The extent to which the economic cycle

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influences air transportation demand is quite apparent. For instance, an economic recession obviously impacts air traffic and, under such a circumstance, the airline industry more slowly recovers than other manufacturing or service industries. Thus, the fluctuations surrounding air traffic may affect the accuracy of forecasted results. Actually, the number of available traffic observations has in general not been large due to a short accumulation time, particularly city-pair data (Horonjeff and McKelvey, 1994; Hsu and Wen, 1998). Therefore, accumulating a large number of data points with good statistical distribution to develop conventional statistical forecasting models is a challenging task. In addition to city-pair's traffic, the uncertainty surrounding other input data also complicates the design of the airline network. For instance, a situation frequently arises in which only approximate costs are known for certain routes.

Airline network design as an optimization of the network system originates from the perspective of either the air carrier or the passenger. Airlines strive to generate the lowest possible operating costs and achieve a higher load factor; while passengers concern themselves primarily with areas such as flight frequencies, nonstop flights, and minimum layover time (Teodorovic, 1988; Teodorovic et al., 1994). A tradeoff, although existing between satisfying the concerns of the airlines and those of the passengers, should be considered when designing the airline network and determining flight frequencies on specific routes.

Studies on designing the airline network and determining flight frequencies are in nature, focusing on different aspects such as the airline network structure or the airline hubbing problem (e.g., Kanafani, 1981; Kanafani and Ghobrial, 1985; Hall, 1989; Chou, 1990; Jaillet et al., 1996; etc.). Pertinent literature focuses largely on aircraft choice or flight frequency determination by deterministic programming (e.g., Kanafani and Ghobrial, 1982; Teodorovic, 1983; Teodorovic, 1986; Teodorovic and Krmar-Notic, 1989; etc.). Teodorovic et al. (1994) applied fuzzy set theory to design an airline network and determine flight frequencies on routes. Fuzzy set theory has been extensively applied to solve problems containing

uncertainty, subjective, ambiguity and indetermination. While assuming that airlines fly with a single type of aircraft, Teodorovic et al. (1994) addressed this problem using fuzzy logic and single-objective programming; however, they did not develop a model to forecast passenger traffic on individual city-pairs. In contrast to their applied methodologies, this study applies grey clustering on network shape design and grey time-series model to forecast airline city-pair's traffic. Multi-objective programming is also applied to determine flight frequency.

Similar to fuzzy theory, Grey theory is an effective mathematical means of resolving problems containing uncertainty and indetermination. Deng developed Grey theory in 1982. Although a majority of pertinent literature is in Chinese (owing to its origin), Deng (1988, 1989) discussed its principles and application areas. Related models have also been recently used in many applications (e.g., Deng et al., 1988; Sun, 1991; Chen and Tien, 1996; 1997; Tien and Chen, 1997, 1998; Huang and Moore, 1993; Huang et al., 1995; Hsu and Wen, 1998). This multidisciplinary and generic theory deals with systems characterized by poor information and/or for which information is lacking. Fields covered by Grey theory include systems analysis, data processing, modeling, prediction, as well as decision making and control.

In contrast to fuzzy logic, the steps of evaluation procedure for grey clustering are markedly less than those for fuzzy logic in cases involving a larger number of basic evaluated parameters. On the other hand, grey forecasting is an efficient means of forecasting airline traffic with a small amount of data. In addition, the fact that airlines are more interested in both enhancing passengers' service levels and lowering airlines' operating costs accounts for why multiobjective programming is preferred over single objective programming formulation in a network design problem. This study develops a series of models to design a network of airline routes and to determine flight frequencies and routing of individual routes by applying grey clustering and multiobjective programming. While not radically departing from the developments in previous studies on airline network design, the

basic goal of this paper is to show how to apply Grey theory and multiobjective programming towards a generic airline network problem. In the light of the above discussion, this study applies grey forecasting to forecast passenger traffic flows. These forecasted traffic flows for all origin–destination (O–D) are used as input parameters for network shape design and frequency determination. In addition, airline network shape is designed and formed into a network structure by applying Floyd algorithm and grey clustering. On the basis of the designed network shape (structure), this study also constructs multiobjective mathematical programming models to determine the optimal frequencies for flights on all of the routes forming the designed network.

1.2. Some basic concepts of Grey theory

In Grey theory, random variables are regarded as grey numbers, and a stochastic process is referred to as a grey process. A grey system is defined as a system containing information presented as grey numbers; and a grey decision is defined as a decision made within a grey system (Deng, 1985; Deng et al., 1988; Huang et al., 1995). Moreover, a grey system can often be characterized by a (time) series; and a stochastic process (i.e., grey process) is defined as a family of time series random variables (i.e., grey numbers) (Deng et al., 1988). Deng considers most of the existing systems to be a “generalized energy system,” and he emphasizes that nonnegative smooth discrete functions can be transformed into a sequence with the so-called grey exponential law (Deng, 1985; Deng et al., 1988) which is an approximate exponential law. In grey forecasting, accumulated generating operation (AGO) and inverse accumulated generating operation (IAGO) are the main methods which provide a manageable approach to treating disorganized evidence (Deng et al., 1988). The generated series can be used to build a grey forecasting model, which is developed by applying the grey exponential law (Deng et al., 1988). The following sections further define grey number and the whitening function; and briefly introduce the concepts and methods of grey clustering.

Definition 1.2.1. Let x denote a closed and bounded set of real numbers. A grey number, $\otimes x$, is defined as an interval with known upper and lower bounds but unknown distribution information for x (Deng, 1985; Deng et al., 1988; Huang and Moore, 1993; Huang et al., 1995):

$$\otimes x = [\underline{\otimes}x, \overline{\otimes}x] = [x' \in x \mid \underline{\otimes}x \leq x' \leq \overline{\otimes}x],$$

where $\underline{\otimes}x$ and $\overline{\otimes}x$ are the lower and upper bounds of $\otimes x$, respectively.

Definition 1.2.2. The whitened value $\tilde{\otimes}x$ of a grey number $\otimes x$, is defined as a deterministic value with its value lying between the lower and upper bounds of $\otimes x$ (Deng, 1985; Deng et al., 1988). It can be marked by:

$$\tilde{\otimes}x = x', \quad x' \in [\underline{\otimes}x, \overline{\otimes}x].$$

Whitening a grey number means specifying for it a deterministic value in its defined interval. Since the status of the given values of x' , which belong to a certain grey number $\otimes x$, is varied, a whitening function is used for describing the status of x' in $\otimes x$. The whitening function is the weight function which can be used to obtain the weight value (in the interval $[0, 1]$) of x' in $\otimes x$.

Grey clustering is based on whitening functions; and it is an approach applicable to decision making on uncertain decision rules. In a decision space, it involves a set of decision criteria (indices) ζ ($\zeta = 1, 2, \dots, m_\zeta$), a set of decision categories κ ($\kappa = 1, 2, \dots, m_\kappa$) and a set of decision units ι ($\iota = 1, 2, \dots, m_\iota$); where m_ζ , m_κ and m_ι are, respectively, the number of elements of these three sets. The following procedure describes how to perform grey clustering:

Step 1. Calculate index values for each of the indices for all decision units. Let x_{ζ} denote certain calculated values of indices ζ for decision units ι , such that there are $m_\iota \times m_\zeta$ calculated values associated with indices ζ and units ι . Grey clustering is an approach which determines the category of the ι th decision unit. When the rules for judging the decision category for a decision index are uncertain, we can use the whitening functions to describe these decision rules.

Step 2. Formulate the whitening functions. In grey clustering, the whitening function describing the level of the decision category κ for the index ζ denoted by $f_{\zeta}^{\kappa}(x)$ — is said to be a grey clustering function (Deng et al., 1988). This grey clustering function $f_{\zeta}^{\kappa}(x_{i\zeta})$ serves as a criterion for judging the category level among the calculated values $x_{i\zeta}$.

Step 3. Compute the decision weight parameters. For decision making, we set the weights of the indices, ω_{ζ} , to establish the relative importance of the decision indices, where $\sum_{\zeta=1}^{m_{\zeta}} \omega_{\zeta} = 1$. Let σ_i^{κ} denote the decision weight parameter of grey clustering associated with category κ and decision unit i . The decision weight parameter can be calculated by the weighted summation of whitening function values for all indices, i.e., $\sigma_i^{\kappa} = \sum_{\zeta} f_{\zeta}^{\kappa}(x_{i\zeta}) \omega_{\zeta}$.

Step 4. Classify and cluster. Grey clustering classifies the decision unit i into a certain category, by ranking the weight parameters of unit i and selecting the highest among all categories κ ; i.e., if $\sigma_i^{\kappa^*} = \max_{\kappa} \{\sigma_i^{\kappa}\}$, then classifying i into κ^* .

To summarize, Grey theory deals with systems characterized by poor information, uncertainty, multi-input and discrete data (grey systems). In grey systems, grey forecasting is applied for predicting the grey series. Grey clustering is used to treat the uncertain decision rules for grey decisions.

The rest of this paper is organized as follows. Section 2 defines the route length index, the total number of intermediate stops and the concentration index of traffic flow. These three indices are then used to determine the route candidates and select a set of routes to form the designed shape of network by applying grey clustering. Section 3 presents the grey forecasting models proposed herein to forecast an airline's city-pair traffic and estimate the upper and lower limits to reflect the extents of variations in future trends. These forecasted traffic values of individual routes are employed as input data of network shape design and flight frequency determination. Grey clustering and multiobjective programming approach are then employed to obtain a group of designed networks coordinated with different forecasted values. Next, in Section 4, we determine flight frequencies and assign the aircraft on routes by

minimizing the total airline costs and the total passenger travel costs based on multiobjective programming. Section 5 presents a case study which demonstrates the proposed model's effectiveness. Concluding remarks are finally made in Section 6.

2. Designing the shape of an airline network

Trips between two cities can be made by non-stop ones or by flights with one or more intermediate stops. With the latter, passengers usually incur extra travel time and inconvenience, thereby preferring the former. On the other hand, airlines tend to consolidate passenger flows from several city-pair routes and combine these individual routes into a hub-and-spoke network and, in doing so, realize economies of flow concentration and achieve the lowest possible operating cost. Both passengers and carriers are interested in the shortest possible routes which reduce operating costs for carriers and enhance the service level for passengers. Based on these considerations, assume herein that the route length index, the total number of intermediate stops and the concentration index of traffic flow are the basic parameters for designing the shape of an airline network. The first two indices are considered as indices to evaluate the structural characteristics of all potential routes (Teodorovic et al., 1994). Moreover, the concentration index of traffic flow is an important measure widely applied to evaluate the economic efficiency of air service for an airline network (Abrahams, 1983; Chou, 1993; Bania et al., 1998). We believe that these three indices should cover overall consideration for selecting the optimal network shape.

Swan (1979) stated that “both stops and extra miles have significant costs, so practical options for a route neither stop too often nor go too far around”. However, when designing the airline network, the future routing plan and schedule are unknown. Restated, in this initial phase, we do not proceed with planning the airline schedule and routing. Therefore, the exact number of passengers who will transfer to another plane in an intermediate stop and the total travel time of

individual routes are difficult to estimate. Thus, the vague and uncertain characteristics of the first two indices persist when evaluating route candidates. In addition, the concentration index of traffic flows is also not precise enough, as attributed to the impossibility of accurately estimating these traffic flows with enough precision. To resolve the above difficulties, in this study, we apply grey clustering to evaluate routes with uncertain and vague parameters. Moreover, although these three indices are independently defined and measured, grey clustering can provide a mechanism to transfer calculated index values into the strength of decision preferences and combine all indices for an overall evaluation (Deng et al., 1988).

The scale of designing the shape of an airline network connecting a large number of cities may become quite large. Teodorovic et al. (1994) made a two-step choice by applying the generalized Floyd algorithm and fuzzy logic to decrease the task complexity. Herein, we adhere to the initial step of that selection process using the Floyd algorithm to choose route candidates among a very large number of possible routes (Teodorovic et al., 1994). However, instead of fuzzy logic, we apply grey clustering in the second step for the final choice. Grey clustering closely resembles fuzzy logic in that it is also a highly effective means of determining route candidates contain uncertainty, ambiguity, vagueness and indetermination indices. However, grey clustering does not require mandate that the approximate reasoning algorithm contain the number of rules (fuzzy phrase) which increases with the number of the basic parameters that influence the route choice preference by an increasing rate. Thus, the steps of the evaluation procedure for grey clustering are substantially less than those for fuzzy logic in cases involving a larger number of basic evaluated parameters. This study adopts the two-step choice procedure to design an airline network shape. The first step applies the Floyd algorithm to construct a feasible network set with k shortest paths for any pair of cities as route candidates. Next, the route index and the total number of intermediate stops is used to evaluate the structural characteristics of all potential routes. Moreover, the concentration index

of traffic flow is used to measure the efficiency of service for all routes. The second step, based on grey clustering, determines the optimal route candidates.

Herein, an attempt is made to decrease the complexity of airline network shape design. Initially, we apply the Floyd algorithm to determine the route candidates between any two cities. The Floyd algorithm labels the route candidates by determining the k shortest paths for any pair of cities with transport demand between them. Let n^* denote the total number of route candidates. Then, the route candidates, n^* , include direct nonstop flights and flights with one or more intermediate stops.

Next, those route candidates obtained from the Floyd algorithm are ranked on the basis of three parameters: total length, total number of intermediate stops and concentration index of traffic flow. Teodorovic et al. (1994) defined the first two indices. Herein, the definition of route length index, Γ_{rsc} , made by Teodorovic et al., is adopted, i.e.

$$\Gamma_{rsc} = \frac{d_{rs}}{D_{rsc}}, \quad (1)$$

where d_{rs} denotes the length from city r to city s for a direct nonstop flight, and D_{rsc} represents the actual length of the route candidate c from city r to city s , for all route candidates $c = 1, 2, 3, \dots, n^*$ $\forall(r, s)$.

In Eq. (1), the actual length D_{rsc} is obviously longer than d_{rs} , i.e. $D_{rsc} \geq d_{rs}$, and then $\Gamma_{rsc} \leq 1$. In addition, shorter route candidates correspond to larger values of route length index, and vice versa. Denote the total number of intermediate stops along route candidate c from city r to city s by m_{rsc} , $c = 1, 2, 3, \dots, n^*$, $\forall(r, s)$. If $m_{rsc} = 0$, then route candidate c is a direct nonstop route; otherwise, if $m_{rsc} \geq 1$. Under this circumstance, candidate c is a route with one or more intermediate stops.

In addition to these two indices, the travel demand distribution on route candidates is also of relevant concern. Airlines are interested in achieving economies of flow concentrations by combining passengers from several city-pair

markets. From this perspective, we formulate the concentration index of traffic flow. Chou (1993) defined an index for measuring the spatial concentration of airline travel demand. Chou initially defined W_{ij} as a relative measure of the ratio of the one direction trip volume between i and j to the system's total traffic. That is,

$$W_{ij} = \frac{f_{ij}}{\sum_i \sum_j f_{ij}}, \quad (2)$$

where f_{ij} denotes the forecasted passenger traffic volume between city i and city j . In the next section, we present the forecasting method for city-pair traffic volumes.

For each intermediate airport node ℓ along route c from city r to city s , Chou also defined ϖ_ℓ as the proportion of total traffic directly linked to this node. In addition, ϖ_ℓ is defined as

$$\varpi_\ell = \sum_i W_{i\ell} + \sum_j W_{\ell j}. \quad (3)$$

By applying the same definition, we denote ϖ_r and ϖ_s , respectively, as the proportions of total traffic for original city r and destination city s .

Herein, we define the concentration index of traffic flow for route candidate c from city r to city s , θ_{rsc} , as the sum of the concentration proportion of the original, intermediate and destination nodes along the route candidate c . For those with two or more intermediate stops, we select the one with the maximum proportional value among those of all intermediate points to represent the intermediate node for these route candidates. In sum, the concentration index of traffic flow for route candidate c from city r to city s , θ_{rsc} , can be defined as

$$\theta_{rsc} = \begin{cases} \varpi_r + \varpi_s, & \text{if } m_{rsc} = 0, \\ \varpi_r + \varpi_s + \max_\ell \varpi_\ell, & \text{if } m_{rsc} \geq 1. \end{cases} \quad (4)$$

Eq. (4) indicates that when route candidate c is direct nonstop route, then $m_{rsc} = 0$, and the traffic concentration index θ_{rsc} is the sum of the proportional concentration of the original and destination nodes along the route. Route candidates with a higher concentration index value imply that a tremendous amount of traffic flow is concentrated

in one of the intermediate nodes along these routes. Restated, these routes achieve higher economies of flow concentration and reduce operating costs for airlines.

The objectives of designing an airline network shape are assumed herein to decrease airline operating costs and passenger travel costs. Therefore, the decision preferences for selecting route candidates should be a high route length index Γ_{rsc} , large traffic concentration index θ_{rsc} and small number of intermediate stops m_{rsc} . Herein, we denote \mathbf{B} as a set of decision categories,

$$\mathbf{B} = \{low (\kappa = 1), medium (\kappa = 2), high (\kappa = 3)\}.$$

As mentioned earlier, these decision categories are referred to as the decision preference for selecting route candidates. However, the statement for decision preferences is vague while grey clustering could be applied to deal with this vagueness. Doing so allows us to evaluate and select routes based on the values of route length index, the number of intermediate stops and traffic concentration index.

Let $x_{i\zeta}$ denote certain calculated values of indices ζ , e.g., m_{rsc} ($\zeta = 1$), Γ_{rsc} ($\zeta = 2$), and θ_{rsc} ($\zeta = 3$), for route candidate i ($i = 1, 2, \dots, n^*$). In addition, each route candidate has certain calculated values, respectively, for indices ζ . Grey clustering is an approach given to assess what the i th route candidate's decision category is. Let $f_\zeta^\kappa(x)$ denote the whitening function associated with the decision category κ and index ζ . The whitening function is the weight function which can be used to describe the decision level of category, and serves as a criterion for assessing the category level. The whitening function in grey clustering is a mechanism to transfer calculated index values into the strength of decision preferences. The shape of the whitening function, when constructed of linear lines, is generally assumed to be a typical function. Herein, we adopt the typical function as whitening function to simplify our analysis. The whitening functions generally represent three types of meaning, e.g., "smaller than a certain number", "approximate a certain number" and "larger than a certain number". Fig. 1 plots the

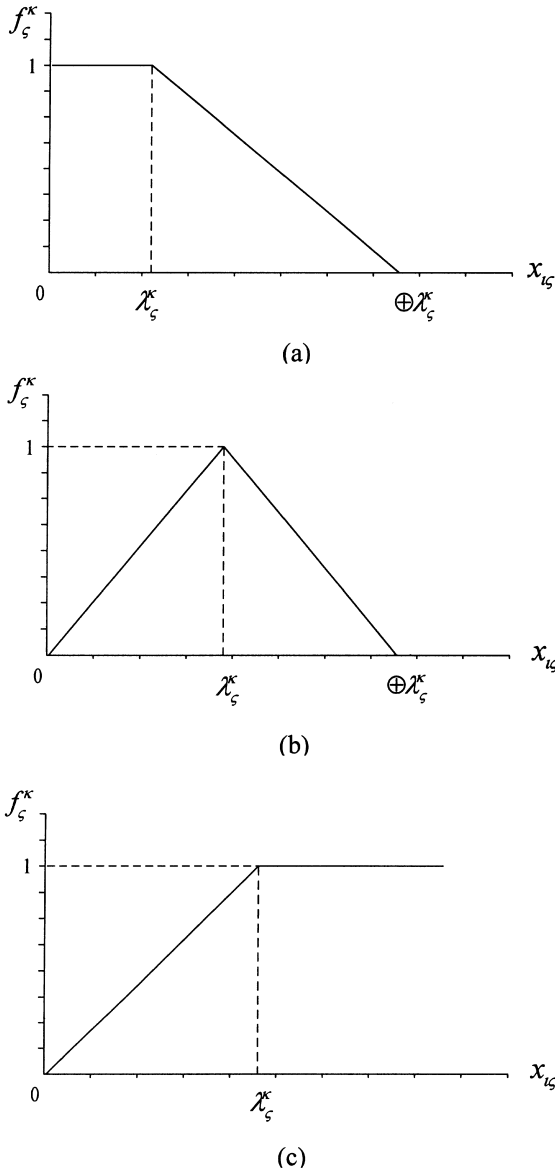


Fig. 1. Three types of whitening functions of the typical shape, (a) “smaller than a certain number λ_{ζ}^{κ} ”, (b) “approximate a certain number λ_{ζ}^{κ} ”, and (c) “larger than a certain number λ_{ζ}^{κ} ”.

typical shape for these types of whitening functions, and is given by

1. “Smaller than a certain number λ_{ζ}^{κ} ”:

$$f_{\zeta}^{\kappa}(x) = \begin{cases} 1, & 0 \leq x \leq \lambda_{\zeta}^{\kappa}, \\ \frac{1}{(\oplus \lambda_{\zeta}^{\kappa} - \lambda_{\zeta}^{\kappa})} (\oplus \lambda_{\zeta}^{\kappa} - x), & \lambda_{\zeta}^{\kappa} \leq x < \oplus \lambda_{\zeta}^{\kappa}, \\ 0, & x \geq \oplus \lambda_{\zeta}^{\kappa}. \end{cases} \quad (5)$$

2. “Approximate a certain number λ_{ζ}^{κ} ”:

$$f_{\zeta}^{\kappa}(x) = \begin{cases} \frac{1}{\lambda_{\zeta}^{\kappa}} x, & 0 \leq x \leq \lambda_{\zeta}^{\kappa}, \\ \frac{1}{(\oplus \lambda_{\zeta}^{\kappa} - \lambda_{\zeta}^{\kappa})} (\oplus \lambda_{\zeta}^{\kappa} - x), & \lambda_{\zeta}^{\kappa} \leq x \leq \oplus \lambda_{\zeta}^{\kappa}, \\ 0, & x > \oplus \lambda_{\zeta}^{\kappa}. \end{cases} \quad (6)$$

3. “Larger than a certain number λ_{ζ}^{κ} ”:

$$f_{\zeta}^{\kappa}(x) = \begin{cases} \frac{1}{\lambda_{\zeta}^{\kappa}} x, & 0 \leq x \leq \lambda_{\zeta}^{\kappa}, \\ 1, & x \geq \lambda_{\zeta}^{\kappa}, \end{cases} \quad (7)$$

where λ_{ζ}^{κ} and $\oplus \lambda_{\zeta}^{\kappa}$ denote the critical values and the upper limits of the values $x_{i_{\zeta}}$, respectively. The upper limits of the whitening functions, $\oplus \lambda_{\zeta}^{\kappa}$, are the maximum values of $x_{i_{\zeta}}$, such that $\oplus \lambda_{\zeta}^{\kappa} = \max x_{i_{\zeta}} \forall \zeta$. The critical values λ_{ζ}^{κ} of whitening functions could be arbitrarily set by decision makers either for fulfilling some objectives of network design, or from statistical distribution of empirical, investigated data. The values of whitening functions are maintained within the close interval $[0, 1]$ for all $f_{\zeta}^{\kappa}(x)$. That is, $0 \leq f_{\zeta}^{\kappa}(x_{i_{\zeta}}) \leq 1 \forall x_{i_{\zeta}}, \zeta, \kappa$.

The weights of the index, ω_{ζ} , are set to establish the relative importance of the indices (i.e., m_{rsc}, Γ_{rsc} and θ_{rsc}), and the sum of all weights is equal to 1, i.e., $\sum_{\zeta=1}^3 \omega_{\zeta} = 1$. Two fundamentally different ways of eliciting weights of index importance are direct elicitation and indirect elicitation. Direct elicitation can be achieved through interviews, questionnaires, preference or trade-off interrogation. Indirect elicitation, in which the decision maker performs a series of overall evaluations of indices, is achieved through the multiple regression approach to obtain the weights. Many functionally methods are available to elicit weights of index importance, such as rating method, entropy method, analytic hierarchy process (AHP), least-square method, logarithmic least-square method, etc. (e.g., Zeleny, 1974; Nijkamp, 1977; Saaty, 1980; Krovak, 1987; Cook and Kress, 1980).

Let σ_i^{κ} denote the decision weight parameter of grey clustering associated to category κ and route candidate i , then

$$\sigma_i^{\kappa} = \sum_{\zeta=1}^3 f_{\zeta}^{\kappa}(x_{i_{\zeta}}) \omega_{\zeta}. \quad (8)$$

For $\kappa = 1, 2, 3$ (e.g., *low* ($\kappa = 1$), *medium* ($\kappa = 2$), *high* ($\kappa = 3$)), we have

$$\sigma_i = (\sigma_i^1, \sigma_i^2, \sigma_i^3) \quad \forall i. \quad (9)$$

The decision weight parameter of grey clustering suggests the level to be subordinate to the category κ for candidate i in accordance with the parameter. In addition, a route candidate i can be classified into the category κ by ranking the weight parameters and selecting the highest. That is,

$$\begin{aligned} \text{if } \sigma_i^{\kappa^*} &= \max_{\kappa} \{ \sigma_i^{\kappa} \} \\ &= \max (\sigma_i^1, \sigma_i^2, \sigma_i^3), \text{ classify } i \text{ into } \kappa^*. \end{aligned} \quad (10)$$

The route candidates determined as “*high* decision category ($\kappa = 3$)” are chosen and to be included in the airline network to form the shape.

3. Grey time-series model for traffic forecasting of city-pairs

The input data of airline network design are the forecasted passenger traffic flows. These forecasted flows for all origin–destination (O–D) pairs are input parameters for evaluating route candidates (i.e., calculating index θ_{rsc}) and determining the flight frequencies on individual routes. However, the number of available observed traffic flows is generally not large. Therefore, in this study, we apply grey models, which require a small amount of data for forecasting city-pair traffic.

The grey models (GM) encompass a group of differential equations adapted for parameter variance. In grey models, there is a group of difference equations with variations in the structure along with time rather than being general difference equations. A GM series is defined as a time series in which the number of data points of the series must be more than or equal to four (Deng et al., 1988). Although applying the data from the original series to construct GM models would be unnecessary, the data must be taken at equal intervals and in consecutive order without bypassing any data (Deng et al., 1988). A condition which should be satisfied to establish a GM is that

the potency of the series must be more than four. Furthermore, assumptions regarding statistical distribution of data are unnecessary when applying the Grey theory. Accumulated generating operation (AGO), an important feature of grey models, focuses largely on reducing the randomness of data. In fact, functions derived from AGO formulations of original series are always well-fitted to exponential functions (Deng, 1985). As mentioned in Section 1.2, Deng emphasized that nonnegative smooth discrete functions can be transformed into a sequence with the approximate exponential law. A detailed derivation of the grey exponential law can be found in Deng et al. (1988). Grey forecasting models have been recently used in many applications (e.g., Deng et al., 1988; Sun, 1991; Chen and Tien, 1996; 1997; Tien and Chen, 1997; 1998; Hsu and Wen, 1997; 1998).

For generalization, we firstly introduce the general form of the grey model, GM(h, N), where h stands for the h th order derivative of AGO-series of dependent variables, and N stands for N variables (i.e., one dependent variable and $N-1$ independent variables) in the differential equation in the model. GM(h, N) is defined as a linear differential equation (Deng, 1986; Chen and Tien, 1996; Tien and Chen, 1998):

$$\begin{aligned} \frac{d^h Y^{(1)}(k)}{dk^h} + a_1 \frac{d^{h-1} Y^{(1)}(k)}{dk^{h-1}} + \dots + a_h Y^{(1)}(k) \\ = b_1 X_1^{(1)}(k) + b_2 X_2^{(1)}(k) + \dots + b_{N-1} X_{N-1}^{(1)}(k), \end{aligned} \quad (11)$$

where Y is the dependent variable; X_1, X_2, \dots, X_{N-1} are independent variables; $Y^{(1)}(k), X_1^{(1)}(k), \dots, X_{N-1}^{(1)}(k)$ are their AGO-series, respectively (we will show AGO formulation later, see Eq. (14)), and a_1, a_2, \dots, a_h and b_1, b_2, \dots, b_{N-1} are parameters. Grey models are commonly represented in the form of first-order derivatives or as polynomial expressions (i.e., $h=0$), e.g., GM(1, 1), GM(1, N) or GM(0, N); where GM(1, 1) is a time-series forecasting model, GM(1, N) is a polyfactor system forecasting model and GM(0, N) is a smoothed polynomial interpolation (Deng, 1986). Furthermore, the h -order differential equation can be used to represent continuous dynamic systems

(Chen and Tien, 1996). Chen and Tien (1996) and Tien and Chen (1998) extended GM(2, 2) to be a deterministic grey dynamic model DGDM(2, 2, 1) for forecasting series in dynamic and changeable systems.

Herein, we develop time series GM(1, 1) models to predict all city-pairs' traffic of the airline network. Hsu and Wen (1997, 1998) presented time-series GM(1, 1) models for forecasting total passenger traffic and country-pair passenger traffic in the Trans-Pacific market. Herein, routes flow forecasts are predicted by using the models proposed by Hsu and Wen (1998). The formulation of the time series GM(1, 1) model is briefly described as follows.

Assume an original series of a given city-pair annual traffic $f_{rs}^{(0)}$ to be

$$f_{rs}^{(0)} = [f_{rs}^{(0)}(1), f_{rs}^{(0)}(2), \dots, f_{rs}^{(0)}(n)], \tag{12}$$

where n denotes the number of years observed. The AGO formation of $f_{rs}^{(0)}$ is

$$f_{rs}^{(1)} = [f_{rs}^{(1)}(1), f_{rs}^{(1)}(2), \dots, f_{rs}^{(1)}(n)], \tag{13}$$

where $f_{rs}^{(1)}(1) = f_{rs}^{(0)}(1)$,

$$f_{rs}^{(1)}(k) = \sum_{t=1}^k f_{rs}^{(0)}(t), \quad k = 1, 2, \dots, n. \tag{14}$$

The GM(1, 1) model can be constructed by establishing a differential equation for $f_{rs}^{(1)}$. That is,

$$\frac{df_{rs}^{(1)}}{dk} + u_1 f_{rs}^{(1)} = u_2. \tag{15}$$

The first-order differential equation, $(df_{rs}^{(1)}/dk)$, is represented as

$$(df_{rs}^{(1)}/dk) = \lim_{\Delta k \rightarrow 0} (f_{rs}^{(1)}(k + \Delta k) - f_{rs}^{(1)}(k))/(\Delta k).$$

Since the traffic is time-series data (i.e., $\Delta k = 1$), the derivative can be transformed into a forward difference equation, such as

$$(\Delta f_{rs}^{(1)}/\Delta k) = f_{rs}^{(1)}(k + 1) - f_{rs}^{(1)}(k).$$

As mentioned above, GM(1, 1) is a group of difference equations with variations in the structure along with time. The differential equation (Eq.

(15)) can be seen as a transformation (i.e., whitening process) which approximates to the grey difference equations.

The solution of Eq. (15) can be obtained using the least-square method to estimate the parameters u_1, u_2 . That is

$$\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} = (B^T B)^{-1} B^T y_N \tag{16}$$

and

$$B = \begin{bmatrix} -\frac{1}{2}(f_{rs}^{(1)}(1) + f_{rs}^{(1)}(2)), & 1 \\ -\frac{1}{2}(f_{rs}^{(1)}(2) + f_{rs}^{(1)}(3)), & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(f_{rs}^{(1)}(n-1) + f_{rs}^{(1)}(n)), & 1 \end{bmatrix}, \tag{17}$$

$$y_N = [f_{rs}^{(0)}(2), f_{rs}^{(0)}(3), \dots, f_{rs}^{(0)}(n)]^T. \tag{18}$$

For the feasibility to solve Eq. (15) by the least-square method to obtain the parameters \hat{u}_1, \hat{u}_2 , e.g., $[\hat{u}_1, \hat{u}_2]^T = (B^T B)^{-1} B^T y_N$, the potency, n , should be defined to be more than or equal to four.

Applying the inverse accumulated generating function (IAGO) to reduce generating equations leads to

$$\hat{f}_{rs}^{(1)}(k) = \left(f_{rs}^{(0)}(1) - \frac{\hat{u}_2}{\hat{u}_1} \right) (1 - e^{\hat{u}_1}) e^{-\hat{u}_1(k-1)}, \tag{19}$$

$$k = 2, 3, \dots$$

when $k = 1$,

$$\hat{f}_{rs}^{(0)}(1) = f_{rs}^{(0)}(1);$$

$$\hat{f}_{rs}^{(0)} = [\hat{f}_{rs}^{(0)}(1), \hat{f}_{rs}^{(0)}(2), \dots, \hat{f}_{rs}^{(0)}(n), \hat{f}_{rs}^{(0)}(n + 1), \dots]$$

is the fitted and forecast series. GM differs from conventional statistical models in not demanding a large amount of data with a good statistical distribution. In other words, GM(1, 1) is useful for modeling when there is only a small amount of data available, with poor statistical distribution. Moreover, GM(1, 1) may be a good model when the time series does exhibit exponential growth. However, GM(1, 1) is inappropriate for forecast-

ing future traffic over an extended period of time owing to its exponential feature. Forecasters should regularly use new data to update the model, to enhance forecasting accuracy. In GM(1, 1) modeling, the least-square solution for solving parameters \hat{u}_1, \hat{u}_2 may be asymptotically biased in the presence of intensive noises in the system, and may affect the accuracy of GM (Deng, 1996; Tien and Chen, 1998).

The upper and lower limits of original and forecast series capture the extents of variations in annual air passenger traffic evolution trends. The upper and lower series limits can be separately estimated by applying GM(1, 1) modeling; i.e. Eqs. (12)–(19) using, respectively, the upper and lower points and other appropriately selected points along the boundaries of the original series.

Moreover, the grey number is defined as follows to describe the variations of forecasted traffic between the upper and lower limits. That is,

$$\tilde{\otimes}f_{rs} = \underline{\otimes}f_{rs} + \alpha (\bar{\otimes}f_{rs} - \underline{\otimes}f_{rs}), \quad (20)$$

where $\tilde{\otimes}f_{rs}$ denotes the grey number relationship, and $\alpha \in [0, 1]$, $\bar{\otimes}f_{rs}$ and $\underline{\otimes}f_{rs}$ represent the upper and lower limits, respectively. These forecasted grey numbers of all city-pair traffic are used as input data to design network shape and determine flight frequencies on individual routes. In doing so, a group of designed networks coordinating is obtained in correspondence with these different forecasted values. According to the results of Hsu and Wen (1998), the accuracy of forecasted results by applying grey models yields more precise forecasts than conventional statistical models such as the ARIMA and multiple regression models.

4. Determining flight frequencies on airline network routes

Teodorovic et al. (1994) determined flight frequencies on the routes by considering the interests of airlines and passengers. Their approach is adopted herein to define cost functions related to airlines and passengers. The set of chosen route candidates described in Section 2 is used to comprise and form the shape of the airline

network. Consider the chosen airline network $G(\mathbf{N}, \mathbf{A})$, where \mathbf{N} and \mathbf{A} represent the set of nodes and set of links of graph G , respectively. Let \mathbf{R} ($\mathbf{R} \subseteq \mathbf{N}$) denote the set of origin cities, and \mathbf{S} represent the set of destination cities ($\mathbf{S} \subseteq \mathbf{N}$, where $\mathbf{R} \cap \mathbf{S} \neq \emptyset$). Next, any given O–D city-pair $r-s$ is connected by a set of routes (the chosen route candidates) P_{rs} ($r \in \mathbf{R}, s \in \mathbf{S}$) through the network. A carrier’s fleet serving its international routes normally contain many aircraft with various sizes. The decision makers should decide how to allocate all of their aircrafts on individual routes profitably. Airlines may either select a strategy of using larger aircraft and fewer flights or one of using smaller aircraft and more flights to fulfill the given demand. The former strategy, although lowering the airline’s unit operating cost and average fare, raises the passenger schedule delay costs; the latter would raise the fare, but reduce the delay. Notably, both schedule delay and fare prices influence the airline’s ability to attract passengers. Therefore, a tradeoff arises between these two strategies. Such a tradeoff should be considered when determining flight frequencies on routes.

Let η_{rspq} denote the load factor of aircraft q flying from r to s along route p , and η_{rspq} is

$$\eta_{rspq} = \frac{f_{rspq}}{n_q N_{rspq}}, \quad (21)$$

where f_{rspq} represents the weekly number of passengers carried by aircraft q from r to s along route p , N_{rspq} is weekly flight frequency and n_q denotes the number of available seats of the aircraft type q . When determining the phase of flight frequency, the decision maker could specify a profitable load factor, η_{rspq}^* . Then, the average number of passengers served can be obtained as $\eta_{rspq}^* n_q N_{rspq}$.

Let f_a and N_a represent the weekly number of passengers and the flight frequency, respectively, on link a ($a \in \mathbf{A}$). Furthermore, the link flow is the sum of the flows on all routes going through that link and can be expressed as a function of the route flows. That is,

$$f_a = \sum_r \sum_s \sum_p \sum_q \delta_{a,p}^{r,s} f_{rspq}, \quad (22)$$

where $\delta_{a,p}^{r,s}$ is the indicator variable, and if $\delta_{a,p}^{r,s} = 1$, then link a is a part connecting O–D pair r – s ; otherwise, $\delta_{a,p}^{r,s} = 0$. By using the same indicator variable, the relationship between the link frequency and the route frequency is

$$N_a = \sum_r \sum_s \sum_p \sum_q \delta_{a,p}^{r,s} N_{rspq}. \quad (23)$$

4.1. Air carrier and passenger costs

Air carrier costs can be classified into operating costs and nonoperating costs. Nonoperating costs include those expenses not directly related to the operation of an air carrier. Therefore, while considering the air carrier costs, we simply take operating costs into account. Kanafani and Ghobrial (1982) systematized air carrier costs and passenger costs. Herein, we follow the formulation of these costs.

Air carrier operating costs are normally divided into direct operating cost and indirect operating costs. Direct operating costs are all those expenses associated with the type of operated aircraft, including all flying costs, all maintenance, and all aircraft depreciation expenses. According to Kanafani and Ghobrial (1982), the direct operating costs are considered to be a linear function of stage length. The direct operating cost of aircraft q for a flight over link a ($a \in \mathbf{A}$) with stage length d_a is denoted by doc'_{aq} . That is,

$$doc'_{aq} = \alpha_q + \beta_q d_a, \quad (24)$$

where α_q and β_q are constant parameters specific to aircraft type q . Then the total direct operating cost for flights on link a of the air carrier, $TDOC_a$, is

$$TDOC_a = \sum_q doc'_{aq} N_a, \quad (25)$$

where N_a denotes the weekly flight frequencies.

Indirect operating costs are those expenses related to passengers rather than related to aircrafts. Kanafani and Ghobrial (1982) noted that the unit indirect operating cost per passenger can be considered as a constant. Then, the total indirect operating cost on link a , $TIOC_a$, is

$$TIOC_a = c_h f_a, \quad (26)$$

where c_h represents the unit handling cost per passenger in dollars.

Airline network design problem is generally addressed from a long-run perspective. Therefore, assume that all carriers have approximately the same average tariff on same routes in the long run, and neglect whatever pricing strategies the carrier uses. Teodorovic et al. (1994) also assumed that the travel demand is inelastic and the average tariff between any link a is independent of the route gone through. This assumption is made herein. Moreover, the generic network design problem is usually stated as a design involving a single airline's network and a routing policy which satisfy the demand and minimize the total cost (Jaillet et al., 1996). Similar assumption on constant average fare of individual route was made by Teodorovic and Krmar-Nozic (1989). Numerous studies considered related airline network design problems as cost-minimization problems (e.g., O'Kelly, 1987; Aykin, 1995; Jaillet et al., 1996). Consequently, the total revenue of the air carrier on link a can be considered as a constant. In doing so, the problem of maximizing air carrier profit becomes the problem of minimizing costs.

In sum, the total operating costs of the air carrier on link a , TC_a^c , can be expressed as

$$TC_a^c = TDOC_a + TIOC_a. \quad (27)$$

Kanafani and Ghobrial (1982) also formulated the costs of passenger travel time, and divided those costs into two components. The first component is the total passenger line-haul travel cost on link a , TT_a , which could be expressed by

$$TT_a = \sum_r \sum_s \sum_p \sum_q \delta_{a,p}^{r,s} c_t (\gamma_q + \rho_q d_{rsp} + \Delta_{rsp}) f_{rspq}, \quad (28)$$

where γ_q , ρ_q are travel time function parameters and depend on the speed of aircraft type q ; d_{rsp} denotes the stage length of route p ; Δ_{rsp} represents the airport time including ground time in the nonstop route, stopover time, or transfer time in the multilink route; c_t is the average time value, a unit time-cost transformation reflecting the per-

ceived money cost of line-haul travel time. Denote Δ_g and Δ_l as the ground time for the origin airport and the destination airport, respectively. For a flight consisting of several intermediate stops, the airport time is represented by stopover or transfer time. However, in this phase of planning flight frequencies, the future routing plan and schedule are not yet known. In addition, whether or not passengers physically transfer planes or remain on the planes at some intermediate airport is unknown. Let Δ_m denote the average intermediate airport time, regardless of whether or not the passengers remained in the same plane or physically changed plane before flying that node. Thus, the component of travel time referring to airport time along route p from r to s , Δ_{rsp} , is

$$\Delta_{rsp} = \Delta_g + m_{rsp} \Delta_m + \Delta_l, \tag{29}$$

where m_{rsp} denotes the total number of intermediate stops along route p from r to s .

The second component is schedule delay cost and stochastic delay cost. The schedule delay is the time difference between the time that the passenger desires to travel and the time that it is actually possible due to the existing flight schedule. Following the formulations provided by Swan (1979), Kanafani and Ghobrial (1982), Teodorovic (1983) and Teodorovic and Krmar-Nowic (1989), we obtain total schedule delay cost on link a , TSC_a , as

$$TSC_a = c_{d1} \tau \frac{OT}{N_a} f_a, \tag{30}$$

where OT represents the average operating time of the airport over a specific period; OT/N_a is the average headway on link a ; c_{d1} denotes a unit time–cost transformation reflecting the perceived money cost of schedule delay time; τ represents the multiplier affected by flight scheduling, and τ is proved by Teodorovic (1983), Teodorovic and Krmar-Nowic (1989) to equal 1/4. Teodorovic (1988) also assumed that OT is 22.8 hours a day, and the weekly OT equals 7×22.8 hours.

Swan (1979) derived the stochastic delay cost per passenger. The stochastic delay is the time difference between the departure time of the flight chosen by the passenger and the departure time of

the flight on which the passenger received an available seat. The total stochastic delay cost on link a , TST_a , can be given by following the cost in Swan (1979) as

$$TST_a = c_{d2} v_1 \frac{OT}{N_a} \left(\frac{\mu}{\bar{n}} \right)^{v_2} f_a, \tag{31}$$

where c_{d2} denotes a unit time–cost transformation reflecting the perceived money cost of stochastic delay time, μ represents the average demand per flight, \bar{n} is the average seat capacity, and v_1, v_2 denote constant parameters. Moreover, v_1 and v_2 are assumed to equal 2.5 and 9, respectively, in Swan (1979), and μ/\bar{n} is the average load factor.

Finally, the total cost of passenger travel time on link a , TC_a^p , is

$$TC_a^p = TT_a + TSC_a + TST_a. \tag{32}$$

4.2. Flight frequency programming problem

The fact that not all aircrafts can be assigned to flights due to factors such as maintenance and turnover accounts for why the aircraft utilization should be considered when determining flight frequencies. Let A_q represent the total number of aircraft type q in fleet, and u_q denote the maximum possible utilization of aircraft type q . The maximum possible utilization implies the maximum possible daily use of aircraft for a period of time (Kane, 1990; Teodorovic, 1983). In our case, the weekly maximum possible total hours flown are $u_q \times A_q \times 7(\text{days})$. The u_q normally depends on the technical maintenance system and the network structure (Teodorovic, 1983). This is attributed to the fact that the total aircraft utilization must be less than or equal to the maximum possible utilization, we use the relation by Teodorovic (1983) as

$$\sum_r \sum_s \sum_p f_{rspq} N_{rspq} \leq u_q A_q \times 7 \quad \forall q, \tag{33}$$

where f_{rspq} denotes the block time of aircraft type q on route p between city r and city s .

On some nonstop routes, the carrier may be required to provide a flight frequency equal to or larger than a certain minimum frequency. Denote

L_{rs}^o as the minimum number of direct flights per week between city r and city s , and P_{rs}^o as the set of nonstop routes between r – s . Restated, the following inequality must hold for some nonstop routes in the network. That is,

$$\sum_q N_{rspq} \geq L_{rs}^o, \quad p \in P_{rs}^o. \quad (34)$$

This study not only determines the flight frequencies on individual routes, but also solves the routing problem of an airline network by minimizing the total air carrier costs and by minimizing the total passenger travel costs. Herein, a two-objective programming problem is formulated to determine flight frequencies. This problem is formulated as follows:

$$\text{Min } Z_1 = \sum_{a \in A} \text{TC}_a^c, \quad (35a)$$

$$\text{Min } Z_2 = \sum_{a \in A} \text{TC}_a^p, \quad (35b)$$

subject to:

$$\sum_r \sum_s \sum_p \sum_q \delta_{a,p}^{r,s} \eta_{rspq}^* n_q N_{rspq} - f_a \geq 0 \quad \forall a \in A, \quad (35c)$$

$$f_{rs} = \sum_p \sum_q f_{rspq}, \quad p \in P_{rs} \quad \forall (r, s), \quad (35d)$$

$$\sum_p \sum_q N_{rspq} = \sum_p \sum_q N_{srpq}, \quad p \in P_{rs} \quad \forall r, \quad (35e)$$

$$\sum_r \sum_s \sum_p f_{rspq} N_{rspq} \leq u_q A_q \times 7 \quad \forall q, \quad (35f)$$

$$\sum_q N_{rspq} \geq L_{rs}^o, \quad p \in P_{rs}^o, \quad (35g)$$

$$\text{all } N_{rspq}, f_{rspq} \geq 0. \quad (35h)$$

Eqs. (35a) and (35b) are two objective functions, and Eqs. (35c)–(35h) are constraints. Eq. (35c) represents that the transportation capacities

offered in terms of the number of seats on each link must be equal to or greater than the number of passengers on all routes that contain that link. Eq. (35d) defines that the sum of the passengers carried by any aircraft q along any route p from r to s equals the total number of passengers traveling between these two cities. Eq. (35e) determines that an equal number of take-off and landing operations occur at each airport in the network during a certain period of time. Eq. (35f) suggests that the total aircraft utilization must be equal to or less than the maximum possible utilization. Eq. (35g) indicates that the flight frequencies for some direct nonstop flights must be equal to or larger than a certain minimum frequency. Finally, Eq. (35h) confines all variables to be nonnegative.

In practice, the variables N_{rspq} and f_{rspq} should be integer and, therefore, the problem is a nonlinear integer programming one. However, nonlinear integer programming problems are extremely difficult to solve, particularly for problems with large dimensions. Owing to difficulties in obtaining optimal solutions to such large combinatorial problems, the nonlinear programming problem is rounding relaxation of the nonlinear integer programming problem. A similar relaxation approach can be found in several investigations, e.g., Teodorovic et al. (1994) and Teodorovic and Krmar-Nozic (1989) used linear programming to approximate the solution of integer programming by neglecting the variables' integer aspect. Furthermore, N_{rspq} is considered herein as the average weekly flight frequency of flights over one year covering season and off-season period and used merely as a basis for future operating planning.

4.3. Proposed solution to two-objective nonlinear programming problem

Determining flight frequency on routes, expressed by Eqs. (35a)–(35h) above, is a two-objective nonlinear programming problem of the general form:

$$\text{Min } \{Z_1(\mathbf{x}), Z_2(\mathbf{x})\}, \quad \mathbf{x} \in \mathbf{X}, \quad (36)$$

where \mathbf{x} is the set of decision variables, i.e. $\mathbf{x} = \{N_{rspq}, f_{rspq}, \forall r, s, p, q\}$; $\mathbf{X} \in \mathbb{R}^n$ is the set of feasible points defined by given constraints, i.e. Eqs. (35c)–(35h); $Z_1(\mathbf{x})$ and $Z_2(\mathbf{x})$ in Eqs. (35a) and (35b) are the two objective functions, respectively, to be minimized. Directly applying the notion of optimality for single-objective nonlinear programming to this two-objective nonlinear programming allows us to arrive a complete optimal solution that simultaneously minimizes these two objective functions. However, in general, such a complete optimal solution does not always exist when the objective functions conflict with each other (Sakawa, 1993). In our problem, these two objectives conflict with each other. Consequently, instead of a complete optimal solution, the Pareto optimality concept is introduced herein. The Pareto optimality is the solution where no objective can be reached without simultaneously worsening at least one of the remaining objectives (Cohon, 1978). The Pareto optimal solutions can be solved by the constraint method for our two-objective programming. The constraint method for characterizing Pareto optimal solutions attempts to solve the following constraint problem formulated by taking one objective function, $Z_1(\mathbf{x})$, as the objective function and allowing the other objective function, $Z_2(\mathbf{x})$, to be an inequality constraint for some selected values of e_2 (Haimes and Hall, 1974; Steuer, 1986; Sakawa, 1993):

$$\begin{aligned} \text{Min} \quad & Z_1(\mathbf{x}) \\ \text{s.t.} \quad & Z_2(\mathbf{x}) \leq e_2, \\ & \mathbf{x} \in \mathbf{X}. \end{aligned} \quad (37)$$

The relationships between the optimal solution \mathbf{x}^* to the constraint problem and the Pareto optimality of the two-objective programming problem have been proven to follow the theorem (Sakawa, 1993), such that: $\mathbf{x}^* \in \mathbf{X}$ is a Pareto optimal solution of the two-objective nonlinear programming problem, if and only if \mathbf{x}^* is an optimal solution of the constraint problem for some e_2 (Sakawa, 1993). Consider the Lagrange function, $L(\mathbf{x}, \lambda) = Z_1(\mathbf{x}) + \lambda_{12}(Z_2(\mathbf{x}) - e_2)$, for the constraint problem with respect to the e -constraints. If the Lagrange multiplier, λ_{12} , associated with the active con-

straint, i.e. $Z_2(\mathbf{x}) - e_2 = 0$, then the corresponding Lagrange multiplier can be proven to lead to the trade-off rates between $Z_1(\mathbf{x})$ and $Z_2(\mathbf{x})$ by $\lambda_{12} = -\partial Z_1(\mathbf{x})/\partial Z_2(\mathbf{x})$ (Sakawa, 1993). The trade-off rate means the marginal decrease of $Z_1(\mathbf{x})$ with one unit increase in $Z_2(\mathbf{x})$. Herein, we use $\lambda_{12} = -\Delta Z_1/\Delta Z_2$ to approximate $-\partial Z_1/\partial Z_2$. Then, both Pareto optimal solutions and trade-off rates can be obtained by altering the values of e_2 and solving the corresponding constraint problems. In this manner, a variety of frequency plans for routes can be generated from Pareto optimal solutions for decision makers. These Pareto optimal solutions can be plotted as a Pareto optimal boundary. Along this Pareto optimal boundary, this study attempts to obtain a solution nearest to the ideal point. The ideal point is defined as the point $Z^{\text{ideal}} = (Z_2^{\text{min}}, Z_1^{\text{min}})$, where Z_1^{min} and Z_2^{min} are the values of the objective function for single-objective programming that minimize Z_1 and Z_2 , respectively. Realizing the ideal point is generally infeasible, Yu (1973) and Zeleny (1974) introduced the concept of compromise programming. The compromise solution is a Pareto optimal solution which has the shortest geometrical distance from the ideal point. In the following case study, compromise programming is applied to determine and derive a compromise solution from these Pareto optimal solutions.

5. Case study

A case study is presented as follows to demonstrate the application of proposed models based on some available data from China Airlines (CAL). The objective of the case study attempts to design CAL's international network for the year 2000. CAL's international fleet comprises of fifty aircraft, including Boeing 747s and 737s, McDonnell Douglas 11s, and Airbus 300s. Its passenger routes include more than thirty destinations in Asia, Europe, North America, Africa and the Oceania. For simplicity, we simply select thirteen nodes in twelve countries among them. The selected thirteen nodes consist of the origin city Taipei (TPE) and twelve destinations, including

Hong Kong (HKG), Manila (MNL), Tokyo (TYO), Bangkok (BKK), Kuala Lumpur (KUL), Singapore (SIN), Jakarta (JKT), Los Angeles (LAX), San Francisco (SFO), Frankfurt (FRA), Rome (ROM) and Amsterdam (AMS). The traffic among these selected countries is the major part of the traffic carried by CAL. However, the historic data of the airline's city-pair traffic are unavailable. Herein, we use annual total country-pair passenger among these twelve countries for forecasting; these forecasted values are then translated to CAL's city-pair traffic by multiplying its relative average market share. These CAL's relative market shares are roughly estimated on the basis of its historic data and time table. The city-pair passenger traffic is forecasted by Eqs. (12)–(19) in Section 3. The data year, forecast values, and their upper and lower limits for each city-pair's passenger traffic in years 2000 are listed in Table 1. These annual values are divided by 52 (weeks) to obtain average weekly traffic. In the case study, we choose traffic forecasts, and their upper and lower limits for the year 2000, as three levels of input data.

While considering CAL's international fleet expansion to satisfy future traffic growth and to simplify its fleet composition, we assume that fifty wide-body aircraft are available, including eighteen Boeing 747-400s (397 seats), twelve Boeing 747-200s (356 seats) and twenty Airbus 300s (263 seats) in CAL's fleet to serve the selected thirteen nodes. On the other hand, some of CAL's operating cost data are unavailable. Thus, the operating cost data reported in Kane (1990) are employed to estimate them. Aircraft characteristic data reported in Horonjeff and McKelvey (1994) are also used to estimate flight time and airport time.

This study uses the CAL's mileage data between any two cities for all these thirteen nodes to proceed with the initial step of shape designing. All-to-all route candidates are obtained by the Floyd algorithm, as proposed in Section 2. CAL is a major Taiwanese carrier with its base at TPE, and its international passenger routes are all connected routes serving TPE and all other destinations. Thus, we only select route candidates with its origin at TPE among those possible route

Table 1

The traffic forecasts, estimated upper and lower limits of city-pair passengers carried by CAL in 2000^a

City-pair	Lower limits	Forecasts	Upper limits
TPE↔HKG*	814684	910327	1198722
TPE↔MNL*	121678	158674	188116
TPE↔TYO*	297425	305060	330581
TPE↔BKK*	195242	267177	302733
TPE↔KUL*	114250	122937	139122
TPE↔SIN*	156040	192165	236499
TPE↔JKT*	123633	153167	182825
TPE↔LAX*	513593	642535	706384
TPE↔SFO*	179758	224887	247234
TPE↔FRA*	20249	22053	25664
TPE↔ROM*	7064	8448	9319
TPE↔AMS*	5819	6119	7447
TYO↔LAX [†]	227275	279020	338415
TYO↔SFO [†]	159092	195314	236890
BKK↔FRA [†]	201835	336457	347409
BKK↔ROM [†]	137916	229904	237387
BKK↔AMS [†]	142575	237670	245407
KUL↔FRA [†]	55752	93203	93604
KUL↔ROM [†]	15852	26500	26614
KUL↔AMS [†]	18760	31362	31497

^a Source: Historic data, *Department of Statistics, M.O.T.C., R.O.C. (1997), and [†]Boeing Commercial Airplane Group (1995). *9 historic data, years 1988 to 1996, and [†]10 historic data, years 1984 to 1993, respectively, for model forecasting.

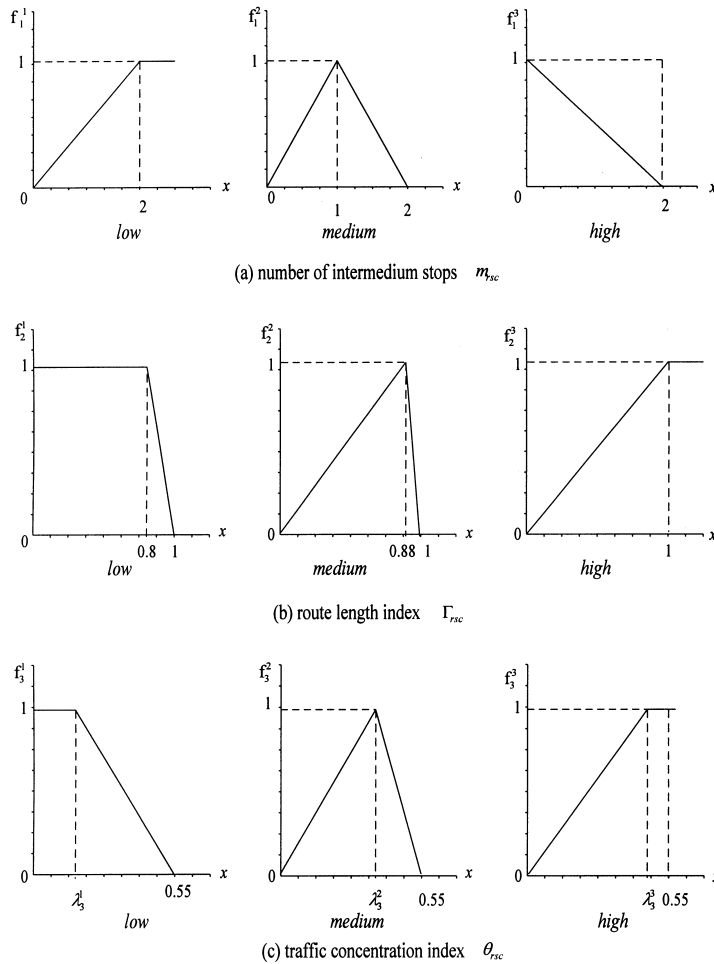


Fig. 2. The whitening functions associated with three decision categories (*low*, *medium* and *high*) and three indices: (a) number of intermediate stops, (b) route length index and (c) traffic concentration index.

candidates in this case study. The two-step network design approach proposed herein can also work for network design without any pre-positive constraint. For other cases, e.g., major domestic airlines in U.S., the all-to-all route candidates obtained by the Floyd algorithm should be directly ranked in the next step based on the grey clustering method in the future study. To simplify the calculation, the route flows on two-way directions are assumed to match a symmetric pattern.

The critical values for index m_{rsc} are obtained by a phone interview survey with CAL's planners

to investigate their decision preferences and, then, used to set its whitening functions of *high*, *medium* and *low* decision categories. These categories are set as “equal to 0”, “approximate to 1” and “larger than 2”, respectively, for index m_{rsc} . On the other hand, assume that the critical values for the other two indices, Γ_{rsc} and θ_{rsc} , are according to the statistical distribution of the calculated values $x_{i\zeta}$ for all route candidates. Then, set the whitening functions of *high*, *medium* and *low* decision categories for route length index Γ_{rsc} to be “equal to 1”, “approximate to 0.88,” and “smaller than 0.8,” respectively, where 0.88 and

0.8 are the median and the 20th percentile, respectively. The whitening functions of each decision category for traffic concentration index, θ_{rsc} , are set in a similar manner, and the critical values of *high*, *medium* and *low* decision categories for θ_{rsc} are given by the 80th percentile, the median and the 20th percentile of calculated values for all route candidates, respectively. Moreover, the upper limit for index m_{rsc} is set at 2 according to the assumption of Teodorovic et al. (1994). On the other hand, since $\Gamma_{rsc} \leq 1$ from the definition of Γ_{rsc} in Eq. (1), the maximum value of index Γ_{rsc} is 1; thus, the upper limit for index Γ_{rsc} is 1. In addition, the maximum value among all the calculated values of index θ_{rsc} is approximately equal to 0.55; thus, the upper limit for θ_{rsc} is set to 0.55. The above whitening functions for each index and decision category are constructed by Eqs. (5)–(7) in Section 2. Figs. 2(a)–(c) plot these whitening functions.

According to the results obtained from the interviews which elicit CAL's planners' weights on the importance of each index, these three indices do not markedly differ from each other. Therefore, we assume that these three indices are equally important and the weight ω_c of each index is assumed to be 1/3. Next, Eq. (8) is used to calculate the decision weight parameters and use Eq. (10) to assess these weight parameters and determine a decision category for each route candidate. Finally, the route candidate determined as "*high* decision category" is chosen to be the optimal candidate and to comprise and form the shape of the airline network. Moreover, three designed networks are obtained by using forecast values, as well as their upper and lower limits. The designed shape with traffic forecasts is the same as that with their upper limits. Thus, Figs. 3(a) and (b) plot two designed network shapes, one for forecast values and their upper limits and the other for their lower limits.

Most routes of the designed network are non-stop flights, as shown in Fig. 3. Nevertheless, some routes from Taipei to US and Europe destinations are designed with one intermediate stop, e.g. Tokyo and Bangkok. According to Fig. 3(b), route TPE-BKK-AMS was removed from the designed network shape with lower limits of forecast values

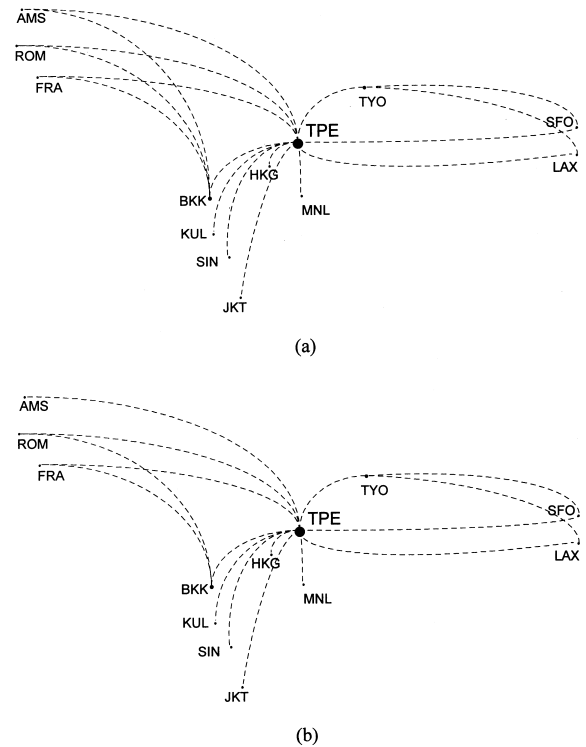


Fig. 3. The designed network shapes with different forecasted inputs: (a) traffic forecasts and upper limits; (b) lower limits.

because it was determined as the "*median* decision category".

The appropriate types of aircraft are selected for each flight on each link comprising these designed flight networks by their range. Tables 3(a)–(c) list these chosen aircraft on individual routes. Then, base values for the parameters of cost functions are given to resolve the problem to determine flight frequencies. Assume that the average load factor μ/\bar{n} is 70%, and specify profitable load factor η_{rspq}^* for each route according to the average value of load factor for those routes operated by CAL during 1997. Moreover, the average unit time-cost reflecting line-haul travel time and delay time are assumed to be \$23.15/hour and \$30.29/hour, respectively, according to slight adjustments on the values of time obtained by Furuichi and Koppelman (1994). In this case study, the maximum possible utilization of three types of aircrafts is given to 16.8 hours per day. The least numbers

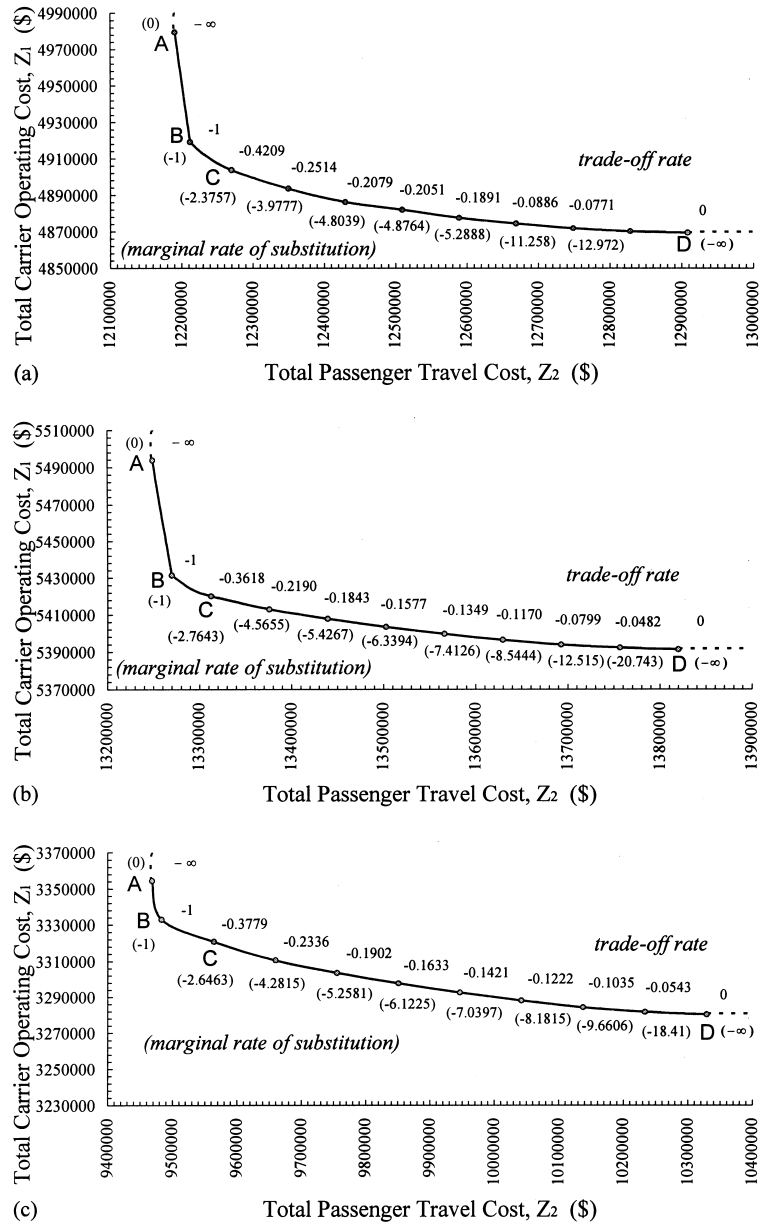


Fig. 4. The Pareto optimal boundaries with different forecasted inputs: (a) traffic forecasts; (b) upper limits; (c) lower limits.

for direct flights on routes TPE-FAR, TPE-ROM and TPE-AMS are set to 1. Consequently, the average weekly frequencies of routes can be obtained by solving formulated two-objective non-linear programming models.

In this study, these two-objective programming models are solved by using GINO, a computer-modeling program developed by Liebman et al. (1986) based on a generalized reduced gradient algorithm. The Pareto optimal solutions and

Table 2

The objective function values, the trade-off rates and the marginal rates of substitution of Pareto optimal solutions with different forecasted inputs: (a) traffic forecasts; (b) upper limits; (c) lower limits

Objective function value (\$)		Trade-off rate	Marginal rate of substitution
Z_1	Z_2		
<i>(a) Traffic forecasts</i>			
4979523	12190132	$-\infty$	0
4919223	12211502	-1	-1
4903710	12269929	-0.4209	-2.3757
4893571	12349725	-0.2514	-3.9777
4886225	12429522	-0.2079	-4.8093
4881995	12509318	-0.2051	-4.8764
4877513	12589115	-0.1891	-5.2888
4874368	12668912	-0.0886	-11.285
4871790	12748709	-0.0771	-12.972
4869419	12908302	0	$-\infty$
<i>(b) Upper limits of traffic forecasts</i>			
5493702	13248990	$-\infty$	0
5431301	13269696	-1	-1
5420194	13312453	-0.3618	-2.7643
5413048	13375916	-0.219	-4.5655
5407828	13439378	-0.1843	-5.4267
5403400	13502840	-0.1577	-6.3394
5399600	13566303	-0.1349	-7.4126
5396337	13629764	-0.117	-8.5444
5393778	13693226	-0.0799	-12.515
5392177	13756689	-0.0482	-20.743
5391188	13820151	0	$-\infty$
<i>(c) Lower limits of traffic forecasts</i>			
3354371	9468962	$-\infty$	0
3332974	9483452	-1	-1
3320723	9564615	-0.3779	-2.6463
3310591	9660269	-0.2336	-4.2815
3303537	9755922	-0.1902	-5.2581
3297614	9851575	-0.1633	-6.1225
3292494	9947228	-0.1421	-7.0397
3288037	10042882	-0.1222	-8.1815
3284253	10138534	-0.1035	-9.6606
3281621	10234188	-0.0543	-18.41
3280207	10329841	0	$-\infty$

trade-off rates are obtained by the constraint method described in the previous section. Moreover, the compromise solutions on Pareto optimal boundaries are obtained by obtaining the solutions with the minimum Geometrical distance from the ideal point. Figs. 4(a)–(c) plot the Pareto optimal boundaries for forecast values, as well as their upper and lower limits, respectively. Each point on these boundaries represents a Pareto optimal solution obtained by the constraint method. Table

2(a)–(c) list the objective function values of Pareto optimal solutions. The trade-off rates are also shown for each point in Fig. 4 and listed in Table 2.

The trade-off rate can be considered as the ratio of weights for objectives Z_2 and Z_1 in the sense of decision making. According to Fig. 4 and Table 2, the higher absolute value of trade-off rate implies a lower value of the objective Z_2 . This observation implies that if decision-makers

Table 3a
The Pareto optimal solutions with different forecasted inputs: Traffic forecasts

Marked points		A	B	C	D
Trade-off rate		$-\infty$	-1	-0.4209	0
Marginal rate of substitution		0	-1	-2.3757	$-\infty$
Objective function value	Z_1	4979523	4919223	4903710	4869419
(\$)	Z_2	12190132	12211502	12269929	12908302
Routes	Aircraft	Weekly flight frequencies (one direction)			
<i>Traffic forecasts</i>					
TPE-HKG	B747-400	0	0	0	0
	B747-200	0	0	0	35
	A300	48	48	48	0
TPE-MNL	B747-400	0	0	0	0
	B747-200	0	0	0	7
	A300	9	9	9	0
TPE-TYO	B747-400	0	0	5	0
	B747-200	0	0	0	10
	A300	14	14	7	0
TPE-BKK	B747-400	0	0	9	8
	B747-200	0	0	0	1
	A300	14	14	0	0
TPE-KUL	B747-400	0	0	0	5
	B747-200	0	0	0	0
	A300	7	7	7	0
TPE-SIN	B747-400	0	0	0	7
	B747-200	0	0	0	0
	A300	10	10	10	0
TPE-JKT	B747-400	0	0	0	6
	B747-200	0	0	0	0
	A300	8	8	8	0
TPE-LAX	B747-400	13	13	13	13
TPE-TYO-LAX	B747-400	15	15	15	15
TPE-SFO	B747-400	5	5	5	5
TPE-TYO-SFO	B747-400	9	9	9	9
TPE-FRA	B747-400	0	0	0	0
TPE-BKK-FRA	B747-200	1	1	1	1
	B747-400	0	13	13	13
TPE-ROM	B747-200	15	0	0	0
	B747-400	1	1	1	1
TPE-BKK-ROM	B747-400	9	9	9	9
TPE-AMS	B747-400	0	0	0	0
TPE-BKK-AMS	B747-200	1	1	1	1
	B747-400	0	9	9	9
TPE-BKK-AMS	B747-200	10	0	0	0

pay more attention to the service levels, they may use those optimal solutions with higher absolute trade-off rates. On the other hand, if decision-makers decide to aim at the operating economies, then they may obtain those optimal solutions with lower absolute trade-off rates. In addition,

according to our results, the trade-off rates of the compromise solutions equal -1 , implying that the compromise solution is equivalent to the solution of single-objective programming that minimize $Z_1 + Z_2$. Furthermore, the concept of marginal rate of substitution is introduced herein between

Table 3b
The Pareto optimal solutions with different forecasted inputs: Upper limits

Marked points		A	B	C	D
Trade-off rate		$-\infty$	-1	-0.3618	0
Marginal rate of substitution		0	-1	-2.7643	$-\infty$
Objective function value	Z_1	5493702	5431301	5420194	5391188
(\$)	Z_2	13248990	13269696	13312453	13820151
Routes	Aircraft	Weekly flight frequencies (one direction)			
<i>Upper limits of traffic forecasts</i>					
TPE-HKG	B747-400	0	0	0	0
	B747-200	0	0	0	46
	A300	63	63	63	0
TPE-MNL	B747-400	0	0	0	0
	B747-200	0	0	0	8
	A300	11	11	11	0
TPE-TYO	B747-400	0	0	0	0
	B747-200	0	0	6	11
	A300	15	15	7	0
TPE-BKK	B747-400	0	0	4	0
	B747-200	0	0	7	11
	A300	15	15	0	0
TPE-KUL	B747-400	0	0	0	0
	B747-200	0	0	0	7
	A300	8	8	8	0
TPE-SIN	B747-400	0	0	0	0
	B747-200	0	0	0	9
	A300	12	12	12	0
TPE-JKT	B747-400	0	0	0	3
	B747-200	0	0	0	4
	A300	10	10	10	0
TPE-LAX	B747-400	14	14	14	14
TPE-TYO-LAX	B747-400	17	17	17	17
TPE-SFO	B747-400	5	5	5	5
TPE-TYO-SFO	B747-400	10	10	10	10
TPE-FRA	B747-400	0	0	0	0
TPE-BKK-FRA	B747-200	1	1	1	1
	B747-400	0	14	14	14
TPE-ROM	B747-200	15	0	0	0
	B747-400	1	1	1	1
TPE-BKK-ROM	B747-400	9	9	9	9
TPE-AMS	B747-400	0	0	0	0
TPE-BKK-AMS	B747-200	1	1	1	1
	B747-400	0	10	10	10
	B747-200	11	0	0	0

these two objectives for airline's decision-makers. The marginal rate of substitution between objectives Z_2 and Z_1 is defined as the amount of passengers' service levels the decision-maker willing to sacrifice in exchange for lowering one unit of airline's operating costs. The marginal rate of

substitution represents the marginal increase of total passenger travel costs when the carrier's operating cost is decreased by one unit. The trade-off rate between objectives Z_1 and Z_2 , λ_{12} , is the inverse number of the marginal rate of substitution between objectives Z_2 and Z_1 ; and can be

Table 3c
The Pareto optimal solutions with different forecasted inputs: Lower limits

Marked points		A	B	C	D
Trade-off rate		$-\infty$	-1	-0.3779	0
Marginal rate of substitution		0	-1	-2.6463	$-\infty$
Objective function Value	Z_1	3354371	3332974	3320723	3280207
(\$)	Z_2	9468962	9483452	9564615	10329841
Routes	Aircraft	Weekly flight frequencies (one direction)			
<i>Lower limits of traffic forecasts</i>					
TPE-HKG	B747-400	0	0	0	28
	B747-200	0	0	0	0
	A300	43	43	43	0
TPE-MNL	B747-400	0	0	0	5
	B747-200	0	0	0	0
	A300	7	7	7	0
TPE-TYO	B747-400	0	0	7	9
	B747-200	0	0	0	0
	A300	14	14	4	0
TPE-BKK	B747-400	0	0	5	7
	B747-200	0	0	0	0
	A300	10	10	3	0
TPE-KUL	B747-400	0	0	0	4
	B747-200	0	0	0	0
	A300	6	6	6	0
TPE-SIN	B747-400	0	0	0	5
	B747-200	0	0	0	0
	A300	8	8	8	0
TPE-JKT	B747-400	0	0	0	4
	B747-200	0	0	0	0
	A300	7	7	7	0
TPE-LAX	B747-400	10	10	10	10
TPE-TYO-LAX	B747-400	12	12	12	12
TPE-SFO	B747-400	4	4	4	4
TPE-TYO-SFO	B747-400	7	7	7	7
TPE-FRA	B747-400	0	0	0	0
TPE-BKK-FRA	B747-200	1	1	1	1
	B747-400	0	8	8	8
TPE-ROM	B747-200	9	0	0	0
	B747-400	1	1	1	1
TPE-BKK-ROM	B747-400	5	5	5	5
TPE-AMS	B747-400	0	0	0	0
	B747-200	1	1	1	1

denoted as $1/\lambda_{12} = -\Delta Z_2/\Delta Z_1$. The marginal rates of substitution between objectives Z_2 and Z_1 are shown on each point on Pareto optimal boundaries in Fig. 4 and are also listed in Table 2. The solutions with lower objective Z_1 values have higher absolute values of marginal rates of substitution; meanwhile, those with higher objective Z_1 values have lower absolute values of

marginal rates of substitution. This finding implies that if the decision-makers decide to choose solutions with higher operating economies and want to lower one more unit of airline's operating cost, they must simultaneously worsen larger amount of the service levels. Furthermore, among three Pareto optimal boundaries in Figs. 4(a)–(c), the curve associated with upper limits of

estimated traffic has the flattest slope, while the curve associated with lower limits has the steepest slope. This observation also implies that the overall substitution rates between passengers' traveling costs and airline's operating costs with high traffic surpass those associated with lower traffic.

Tables 3a–c lists the solutions for four groups of points marked by A, B, C and D, on each Pareto optimal boundary in Figs. 4(a)–(c). The results of points B are compromise solutions, while those of points A are the solutions that minimize total passenger travel cost and those of points D are the solutions minimizing total carrier operating cost. Results obtained from points C are efficient solutions whose objective values lie between those of points B and those of points D. These four groups of points correspond to different objectives and fulfill various traffic levels; they are different groups of frequency plans. The solutions are reasonable, as evidenced by comparing the results of compromise solutions with the actual frequencies shown on CAL's existing time table with adjustment for future traffic growth. Moreover, results obtained from other Pareto optimal solutions (e.g., the results of points A, C and D) provide flexibility for planners to determine different routing and frequency plans corresponding to different decision objectives. Restated, these Pareto optimal solutions could provide airlines with higher flexibility of aiming and weighting on different objectives on decision making. Consequently, a group of frequency plans can be generated to satisfy different objectives and to fulfill various traffic levels on prerequisite planning for a carrier's route network design.

6. Conclusions

This study presents a series of models to forecast airline city-pair passenger traffic and to determine the shape of a carrier's airline network and its corresponding flight frequencies. The models proposed herein are formulated by applying Grey theory and multiobjective programming.

Also considered herein is the route length, the total number of intermediate stops and the concentration of traffic flow as indices and applying grey clustering to assess the route candidates so as to comprise and form the designed airline networks with different traffic levels. In addition, grey time-series models are developed to forecast the airline's city-pair traffic and to estimate their upper and lower limits to capture the extents of variations in future trends. Moreover, a group of optimal frequency plans on routes of these designed networks is determined by applying multiobjective programming. These groups of solutions not only provide flexibility in decision-making with two different objectives, but also show the trade-off rates and the marginal rates of substitution between two objectives that minimize the total airline costs and/or the passenger travel costs. The case study not only demonstrates that the shapes and optimal solutions of the designed networks yield promising results, but also verify that the models are practical in airline network design.

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