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Airline Flight Frequency Determination in Response to Competitive Interactions Using Fuzzy Logic

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Abstract—This study determines the flight frequencies on an airline network in response to anticipated airline competitive interactions using fuzzy logic. It is expressed as an optimization problem to be solved in the medium-term airline network-planning phase, and it deals with tactical decisions made one season or one year in advance. The proposed model includes three submodels—an airline market share model, a flight-frequency programming model, and a fuzzy-logic-based competitive interaction model. Flight frequencies on an airline network with competitive interactions are analyzed by combining these three submodels. A case study, demonstrating the feasibility of applying the proposed model, confirms that the competitive interactions will converge. The results of this study confirm the accuracy of the proposed model and their flexibility of the decision-making involved in determining flight frequencies on an airline network in competitive and uncertain environments.

Keywords—Fuzzy logic, Airline network, Competitive interactions, Flight frequency determination, Tactical decisions.

1. INTRODUCTION

Airline flight frequency determination is an important task since it encompasses decisions on flight frequencies and aircraft types on individual routes for airline networks [1–3]. Determining flight frequencies on an airline network is more complex when more competing airlines are making network and flight frequency decisions in the same market. In a competitive environment, an airline must consider its competitors' likely decisions concerning flight frequencies and basic airfares when their networks overlap [4]. Moreover, flight frequency determination on an airline network is a fundamental aspect of an airline's short-run operational planning including flight scheduling, routing and pricing, since the proposed flight frequencies and basic airfares in the airline planning phase are usually used as bases in future operational planning. However, the

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result of operational planning constitutes one of the most important of an airline's products, and such planning is certainly a factor that greatly affects a passenger's choice of a particular airline [5]. A flexible flight frequency plan that can better respond to anticipated competitive frequency/airfare interactions facilitates good operational planning since flight frequencies and basic airfares proposed in the airline planning stage are bases for operational planning in later stages.

Elucidating the competitive interactions among competing airlines has become a major area of research concerning competition among airlines. Previous studies have addressed these topics from the perspective of economic competition, e.g., [6–8]. Early study [6] developed a theoretical microeconomic model of airfare and schedule competition in a single origin-destination (OD) pair market. The studies [7,8] analyzed airline hubbing phenomena in attempts to capture the implications for airport economics and economic driving forces for airline networking behavior. Many studies focused on game-theory-based approaches to airline competition, e.g., [4,9–12]. Studies [9,10] modeled airline frequency competition setting as a noncooperative Nash game; while study [11] examined the airport and airline competition involving multiple departure airports. Previous studies [9–11] considered the airline competition with fixed origin-destination (OD) demands and airfares, and constructed models apart from network modeling. Others have addressed competitive airline network modeling and scheduling [4,12]; where a three-level hierarchical process developed in [4] investigated the competitive choices of flight schedules and airfares by airlines in a pure hub-and-spoke (with a single hub) system, and a two-stage Nash game-theoretical model presented in [12] evaluated airline hub-and-spoke networks. Published literature on airline network modeling focused primarily on determining airline network shapes, flight frequencies and aircraft types (e.g., [1–3,13–22]); however, in previous models, passenger demand patterns are assumed to be exogenous, and travel demand is assumed to be inelastic, even though passenger demand may in fact be elastic to flight frequencies and airfares in a competitive environment. The authors' earlier work [23] determined flight frequencies on an airline network with demand-supply interaction between passenger demand and flight frequencies of the airline. The present study further combines a competitive interaction model with the demand-supply interaction process proposed in [23].

When one airline determines flight frequencies and basic airfares on its network in a competitive environment, it normally does not know precisely its competitors' decisions and strategies concerning network planning, since the flight frequencies and airfares offered by competing airlines in any OD-pair market follow decisions that realize strategies optimized not only for the OD-pair but also for these competitors' overall networks. Moreover, in the network planning stage, one airline can only guess at the dispositions and the approximate range of its competitors' likely changes to frequencies and airfares. Given the uncertainties and indeterminacy associated with competition among airlines, fuzzy logic tools may be applied in describing competitive interactions, and further examining the convergence of airline competition. This study is the first attempt to develop a fuzzy-logic-based competitive interaction model of flight frequency and basic airfare interactions between any two competing airlines for individual OD pairs in a competitive environment. The fuzzy rules of this airline competitive interaction model are based on the findings of relevant game-theoretical and empirical studies (e.g., [9-11]). While formulating similar objective functions and mathematical programming models for all competitors, Adler [12] addressed the problem using the best-response game-theoretical model to determine reaction functions and solve for optimal frequencies and outputs. In contrast to best-response approaches (e.g., [12]), the fuzzy-logic-based competitive interaction model incorporates a rule-based IF-THEN approach to solving a flight frequency programming problem, rather than attempting to formulate precisely network models for all competitors and solving them simultaneously.

This study develops a model to determine flight frequencies on an airline network, by considering competitors' decisions and competitive interactions in individual OD-pair markets of the airline network well in advance. Herein, it is expressed as an optimization problem in the medium-term airline network-planning phase and it deals with tactical decisions made one season or one year in advance. The model consists of three submodels, including an airline flight frequency programming mode, a market share model, and a fuzzy-logic-based competitive interaction model. The flight frequency programming model determines flight frequencies and basic airfares on individual routes of an airline network, by maximizing the airline's total profit. The airline market share model formulates airline market shares for all OD-pairs of a network as functions of flight frequency shares and relative airline airfares. The airline competitive interaction model, based on fuzzy-logic, describes the competitive frequency/airfare interactions between two competing airlines. Then, an algorithm consisting of an iterative scheme that integrates these three submodels is presented to solve this problem. The rest of this paper is organized as follows. Section 2 presents the proposed airline market share model and flight frequency programming model. Section 3 develops the fuzzy-logic-based competitive interaction model, and proposes an iterative algorithm to solve it. Section 4 discusses a case study that demonstrates the effectiveness of the proposed model. Concluding remarks are made in Section 5.

2. MODEL FORMULATION

The model developed herein is a profit-maximizing flight frequency programming for an airline network. It aims to determine the optimal flight frequencies and basic airfares on individual routes of an object airline's network in response to competitive interactions in individual OD-pair markets well in advance. Figure 1 shows the inputs and outputs of each submodel and the connections between them. Airline market shares are expressed as functions of flight frequency shares and airline airfares in OD markets. The OD market demand and airline market shares are input parameters to the airline flight frequency programming model. Changes made by the object airline to route flight frequencies and/or airfares will trigger competitive interaction among all competing airlines in individual OD markets, affecting the airlines' market shares.

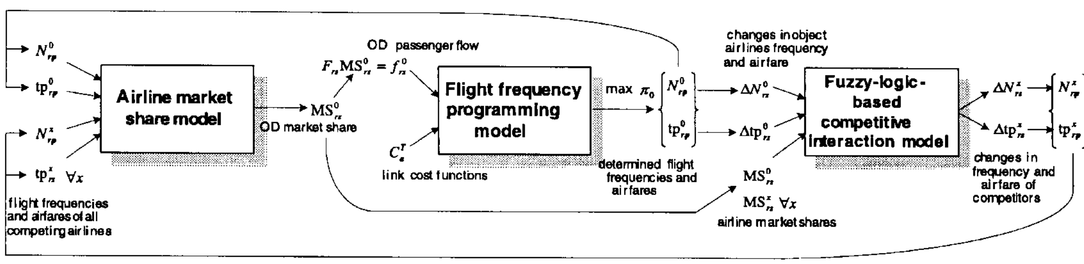

Figure 1. Model connection and input/output diagram.

2.1. Airline Market Share Model

Many studies have demonstrated that the relationship between airline market share and flight frequency share is nonlinear, and is normally described by an S-shaped curve [16,24,25]. The airline market share depends not only on the flight frequency share but also on the airfares. Moreover, the attractiveness of a given airline in an OD-pair market will depend on both its airfare and the average airfare offered by competitors.

The object airline's flight frequency share, FS^0_{rs}, between OD pair $r - s$, is,

$$\mathrm{FS}^0_{rs} = \frac{\sum\limits_p N^0_{rsp}}{\sum\limits_p N^0_{rsp} + \sum\limits_{x, \forall x \neq 0} \sum\limits_p N^x_{rsp}}, \tag{1}$$

where N^0_{rsp} and N^x_{rsp} represent the flight frequencies offered by airline '0' (the object airline) and by the competing airline x ($x \neq 0$), respectively, on their routes p, where $p \in P_{rs}$, P_{rs} is a set of routes that includes direct nonstop flights and flights with one or more intermediate stops between OD pair $r-s$, where the superscript x is the index of airline. Furthermore, the object airline's average basic airfare, tp_{rs}^0, weighted by route traffic between OD pair $r-s$can be expressed as,

$$\mathrm{tp}_{rs}^0 = \frac{\sum_p \mathrm{tp}_{rsp}^0 f_{rsp}^0}{\sum_p f_{rsp}^0}, \tag{2}$$

where tp_{rsp}^0 is the basic airfare of the object airline on its route p between OD pair $r-s$, and f_{rsp}^0 is the passenger traffic carried by the object airline on its route p between OD pair $r-s$. Similarly, tp_{rs}^x is the average basic airfare on OD-pair $r-s$ for competing airline x ($x \neq 0$); then, the average value of tp_{rs}^x, for all airlines x ($\forall x \neq 0$), $\overline{\mathrm{tp}}_{rs}^x$, can be calculated. For simplicity, the market share formulation is regarded as an aggregate model associated with OD markets, rather than disaggregate discrete choice model based on passengers' airline and flight choice behavior.

A multiplicative model of the object airline's market share for passenger demand in an OD market can be expressed as,

$$\mathrm{MS}_{rs}^0 = \gamma_0 \left(\mathrm{FS}_{rs}^0\right)^{a_1} \left(\mathrm{tp}_{rs}^0\right)^{a_2} \left(\overline{\mathrm{tp}}_{rs}^x\right)^{a_3}, \tag{3}$$

where MS_{rs}^0 is the object airline's market share of passengers who travel between OD pair $r-s$, and the superscript '0' refers to the object airline; FS_{rs}^0 represents the object airline's flight frequency share between OD pair $r-s$; tp_{rs}^0 represents the object airline's average basic airfare, weighted by route traffic, between OD pair $r-s$, and $\overline{\mathrm{tp}}_{rs}^x$ is the average airfare for all competing airlines x, $\forall x \neq 0$, parameters γ_0, a_1, a_2, and a_3 are estimated by regression analysis. A similar multiplicative form of the market share model was considered in [25]. Notably, given a multiplicative market share model, a_1, a_2, and a_3 are the elasticity of the airline market share with respect to frequency share, average airfare and the average airfare for all competitors, respectively. Intuitively, a_1 and a_3 would be expected to be positive and a_2 to be negative.

2.2. Flight Frequency Programming Model

This section outlines the flight frequency programming model proposed in [3,23]. Consider the object airline network, $G(\mathbf{N}, \mathbf{A})$, where $\mathbf{N}$ and $\mathbf{A}$ represent, respectively, the set of nodes and set of links in graph G. Let $\mathbf{R}$ ($\mathbf{R} \subseteq \mathbf{N}$) represent the set of origin cities, and $\mathbf{S}$ ($\mathbf{S} \subseteq \mathbf{N}$) represent the set of destination cities in graph G, where $\mathbf{R} \cap \mathbf{S} \neq \emptyset$. Any given OD pair $r-s$ is connected by a set of routes P_{rs} ($r \in \mathbf{R}$, $s \in \mathbf{S}$) through the network. An airline fleet that serves international routes typically includes several aircraft of various sizes. Consequently, the main decision variables in modeling the airline network are the flight frequencies on individual routes served by type of aircraft in the object airline network [2,3]. Let N_{rspq}^0 represent the flight frequency of the object airline's (airline '0') type q aircraft between OD pair $r-s$ along route p ($p \in P_{rs}$). Restated, if N_{rsp}^0 represents the total flight frequencies of all aircraft used by the object airline on its route p between OD pair $r-s$, then $N_{rsp}^0 = \sum_q N_{rspq}^0$, and the total flight frequency served by the object airline between OD pair $r-s$, N_{rs}^0, is $N_{rs}^0 = \sum_p N_{rsp}^0$.

Let N_{aq}^0 represent the flight frequency served by the object airline's type q aircraft on link $a (a \in A)$. It is the sum of the flight frequencies of all object airline's routes that contain link a, served by aircraft q. That is,

$$N_{aq}^0 = \sum_{r,s} \sum_p \delta_{a,p,q}^{r,s} N_{rspq}^0, \tag{4}$$

where $\delta_{a,p,q}^{r,s}$ is an indicator variable, given by

$$\delta_{a,p,q}^{r,s} = \begin{cases} 1, & \text{if link } arm \text{ is part of route } p \text{ served by type } q \text{ aircraft from city } r \text{ to city } s; \\ 0, & \text{otherwise.} \end{cases}$$

The total flight frequency on link a of the object airline, N_a^0, can now be expressed as $N_a^0 = \sum_q N_{aq}^0 = \sum_q \sum_{r,s} \sum_p \delta_{a,p,q}^{r,s} N_{rspq}^0$. Let f_a^0 represent the link flow of link a; f_a^0 is the sum of the flows on all routes of the object airline's network that pass through that link:

$$f_a^0 = \sum_{r,s} \sum_p \delta_{a,p}^{\prime r,s} f_{rsp}^0, \tag{5}$$

where $\delta_{a,p}^{\prime r,s}$ is the indicator variable, and,

$$\delta_{a,p}^{\prime r,s} = \begin{cases} 1, & \text{if link } a \text{ is part of route } p \text{ from city } r \text{ to city } s, \\ 0, & \text{otherwise.} \end{cases}$$

In airline network modeling, two-way OD passenger flows are assumed to be symmetric, an assumption made in practice by most airlines when designing their networks. A similar assumption was also made in [1–3]. Let F_{rs} represent the total expected OD demand (market size) between OD pair $r - s$ during a particular study period; f_{rs}^0 represents the total number of passengers carried by the object airline between OD pair $r - s$. It can be estimated as $f_{rs}^0 = F_{rs}\mathrm{MS}_{rs}^0$, where MS_{rs}^0 is the object airline's market share of passengers who travel between OD pair $r - s$, and can be obtained using equation (3). Moreover, the following condition must be satisfied such that the total number of passengers carried by the object airline on individual routes between OD pair $r - s$ equals the total number of passengers who travel between OD pair $r - s$, carried by the object airline: $\sum_p f_{rsp}^0 = f_{rs}^0 = F_{rs}\mathrm{MS}_{rs}^0$.

Airline operating costs are typically divided into direct and indirect operating costs. Direct operating costs are those expenses associated with a type of operated aircraft, including all flying costs, all maintenance, and all aircraft depreciation expenses. Indirect operating costs are those expenses related to passengers rather than aircraft. Let C_a^T represent the total airline operating costs for link a, such as,

$$C_a^T(N_{aq}^0) = \sum_q C_{aq}^D\left(N_{aq}^0\right) + C_a^I\left(N_{aq}^0\right), \tag{6}$$

where C_{aq}^D is the direct operating cost of type q aircraft for flights over link a with stage length d_a, in U.S. dollars, and C_a^I is the total indirect operating cost associated with link a, in U.S. dollars. The cost functions $C_{aq}^D(N_{aq}^0)$ and $C_a^I(N_{aq}^0)$ are, respectively,

$$C_{aq}^D\left(N_{aq}^0\right) = \left(\alpha_q + \beta_q d_a\right) N_{aq}^0, \tag{7}$$

$$C_a^I\left(N_{aq}^0\right) = c_h \sum_q n_q l_a N_{aq}^0, \tag{8}$$

where d_a is the stage length of link a in miles; α_q and β_q are parameters specific to type q aircraft; c_h is the unit handling cost per passenger in U.S. dollars; n_q is the number of available seats on type q aircraft, and l_a is the specified load factor associated with link a. If $N_a^0 = \sum_q N_{aq}^0 = 0$, then $C_a^T = 0$.

The flight frequency programming problem is typically considered apart from short-run yield management issues during the global airline network planning phase. For simplicity, yield management issues are beyond the scope of this study, and airfare setting is assumed to involve only determining basic airfares. The basic airfare is the backbone of the airfare structure, in that it applies to all passengers at all times and is the basis for all other airfare levels [5]. This study assumes that the object airline selects basic route airfares at or above average operating costs on every route. Lederer [26] made a similar assumption, and stated that it reflects expected behavior on routes served by an airline. The basic airfare, tp_{rsp}^0, per passenger on route p can then be determined as,

$$\mathrm{tp}_{rsp}^0 = \left(1 + \bar{r}_{rsp}^0\right) \sum_a \delta_{a,p}^{\prime r,s} \frac{C_a^T}{\sum_q n_q l_a N_{aq}^0}, \tag{9}$$

where $\bar{r}^0_{rsp}$ is the profit margin specified by the object airline on route p between OD pair $r-s$. The total revenue of the object airline can then be expressed as $\sum_{r,s}\sum_p \mathrm{tp}^0_{rsp} f^0_{rsp}$.

The flight frequency programming for the object airline network determined by maximizing its total profit π_0 can then be modeled as,

$$\max_{N^0_{aq}, N^0_{rspq}} \quad \pi_0 = \sum_{r,s}\sum_p \mathrm{tp}^0_{rsp} f^0_{rsp} - \sum_{a\in A} C^T_a(N^0_{aq}), \tag{10a}$$

$$\text{s.t.} \quad \sum_q n_q l_a N^0_{aq} - \sum_{r,s}\sum_p \delta'^{r,s}_{a,p} f^0_{rsp} \geqslant 0, \qquad \forall\, a \in A \tag{10b}$$

$$\sum_p f^0_{rsp} = F_{rs}\mathrm{MS}^0_{rs} \qquad p \in P_{rs}, \quad \forall\,(r,s), \tag{10c}$$

$$N^0_{aq} = \sum_{r,s}\sum_p \delta^{r,s}_{a,p,q} N^0_{rspq}, \qquad \forall\, a,q, \tag{10d}$$

$$\sum_a t^0_{aq} N^0_{aq} \leqslant u^0_q U^0_q, \qquad \forall\, q, \tag{10e}$$

$$N^0_{rspq}, N^0_{aq}, f^0_{rsp} \geqslant 0. \tag{10f}$$

Equation (10a) is the objective function that maximizes the total profit π_0 of the object airline network. Equation (10b) states that the numbers of seats available for each link must be equal to or greater than the total number of passengers on all routes that include that link. Equation (10c) indicates that the total number of passengers carried by the object airline on any route p between OD pair $r-s$ must equal the total number of passengers who travel between the OD pair and are carried by the object airline. Equation (10d) defines the relationship between link frequency and route frequency. Equation (10e) states that total aircraft utilization must be equal to or less than the maximum possible utilization, where t^0_{aq} is the block time for the object airline's type q aircraft on link a; u^0_q is the maximum possible utilization, and U^0_q is the total number of type q aircraft in the object airline's fleet. Finally, equation (10f) constrains all variables as nonnegative.

As stated above, the decision-makers may specify a profitable load factor l_a, for all links when determining the flight frequencies on an airline network. A minimum load factor, $\underline{l}_a$, must be met on flights for link a. At this load factor, the revenue is at the minimum tolerable. A maximum acceptable load factor, $\overline{l_a}$, at which a minimally acceptable level of service can be maintained for passengers is also assumed. Then, the load factor l_a must be specified within the interval $[\underline{l}_a, \overline{l_a}]$, $\forall\, a \in A$. In competitive interactions, airlines are assumed to be able to adjust their proposed flight frequencies and basic airfares by specifying different load factors l_a ($l^{\in}_a [\underline{l}_a, \overline{l_a}]$, $\forall\, a$), while maintaining demand constraints and overall objectives, in response to competitors' changing frequencies or airfares.

3. AIRLINE COMPETITIVE INTERACTION MODEL

In this study, the interaction model is assumed to describe simplistic competitive interactions, in which the airline competition focuses on the changes in the flight frequencies among competing airlines rather than on price competition. Since decisions regarding pricing strategies can be made in a short time-frame, whereas those regarding changes in flight frequency may involve a longer time horizon. Accordingly, this study assumes that the setting of airfares involves only determining basic airfares, rather than decisions concerning pricing strategies during the determination of flight-frequency. Knowledge of competitors' strategic choices is usually uncertain because of the incomplete information about the competitive interactions among the flight frequency and airfare decisions of competing airlines [9]. When an airline attempts to estimate its competitors' reactions to its strategies in the network planning stage, it can only guess at possible dispositions and the approximate range of its competitors' likely changes to frequencies and airfares. Fuzziness is introduced to accommodate incomplete knowledge about competitive interactions among airlines. Herein, fuzzy rules are applied to develop a competitive interaction model that describes competitive flight frequency and airfare interactions between competing airlines in individual OD markets. The market shares of an object airline and its competitors, and changes to the object airline's flight frequencies and airfares are the inputs of the fuzzy logic system, which approximates the reasoning that underlies competitors' reactions. The system outputs are changes in the flight frequencies and airfares of the competitors.

An object airline can characterize the reactions of its competitors, in terms of market shares and changes in flight frequencies and airfares, as fuzzy sets. Airline market shares, MS_{rs}^{0} and MS_{rs}^{x}, $\forall x$, $\forall r, s$, can be grouped in three fuzzy sets, $\{$*Small*, *Medium*, and *Large*$\}$. Changes in airline flight frequency and basic airfare are assumed to vary in seven terms, {NB, NM, NS, ZO, PS, PM, PB}, where NB represents the fuzzy set "negative big"; NM denotes "negative medium"; NS denotes "negative small"; ZO denotes "near zero"; PS denotes "positive small"; PM denotes "positive medium", and PB denotes "positive big". Figure 2 presents the membership functions of the above fuzzy sets for airline market shares and changes in flight frequencies and airfares. Figure 2a shows the membership functions of the fuzzy sets $\{$*Small*, *Medium*, *Large*$\}$ for airline market shares. The centroids of the fuzzy sets $\{$*Small*, *Medium*, *Large*$\}$ can be set for airline market shares according to the actual market shares all competing airlines in individual OD markets considered in the case study. Three or more airlines provide flight services to each OD market in the case study; therefore, fuzzy set $\{$*Large*$\}$ is defined for airline market shares "*larger*

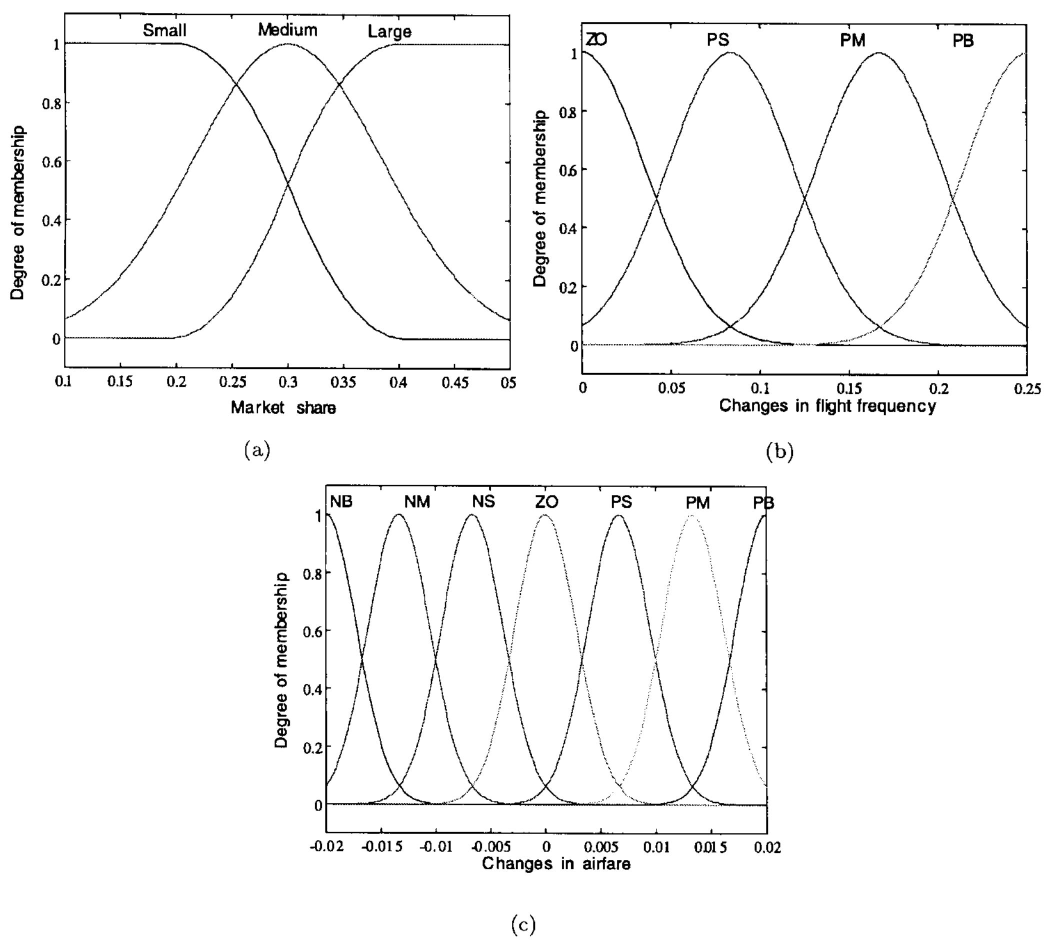

Figure 2. Membership functions of fuzzy sets for (a) airline market share, (b) changes in flight frequency, and (c) changes in airfare.

than 0.4", {*Small*} is defined for market shares "*smaller than* 0.2", and {*Medium*} is defined for market shares of "approximately 0.3". The average load factors vary between about 0.68 and 0.88, according to the present load factors of international flights flown by all competing airlines in year 2001. A value of 0.65 is adopted as the minimum load factor for all test routes. Moreover, the magnitude of changes in route flight frequencies and airfares of all competing airlines are investigated by testing different load factors within acceptable ranges, and are then used to specify the centroids of the fuzzy sets {NB, NM, NS, ZO, PS, PM, PB} of changes to flight frequencies and basic airfares. Figure 2b presents the membership functions of fuzzy sets {ZO, PS, PM, PB} of changes to flight frequencies. Figure 2c presents membership functions of fuzzy sets {NB, NM, NS, ZO, PS, PM, PB} of changes to basic airfares.

A priori analysis of airline competition is applied to develop a fuzzy logic system, to select the fuzzy rules. This *a priori* analysis may include studying relevant theory (including oligopolistic competition issues and game theory), actual or simulated practice, case studies or other sorts of examples, experiments on competitive behaviors of airlines, and other specific issues involving airline competition. In developing the fuzzy rules, this study applies the concepts of airline competitive interactions developed by [9] using reaction diagrams, and follows the findings of numerical results in [9,11].

Consider that in a competitive flight frequency interaction between any two airlines, an increase in the object airline's frequency initially stimulates the competitor to increase its flight frequencies to maintain market share. In contrast, when the object airline's flight frequency decreases, the competitor will not lose market share without decreasing its flight frequencies. If the object airline does not change its flight frequencies while its market share also remains unchanged, then the competitor also will not change its flight frequencies. Similar statements on competitive interactions between determinations of flight frequencies proposed by any two airlines were made in [9]. However, the competitive interactions between the basic airfare settings of any two airlines are more complex than frequency competition, since basic airfare setting of a profit-maximizing airline depends upon its determination of flight frequencies. Pels *et al.* [11] provided evidence that an airline can increase its airfares without immediately losing market share when competitors' frequencies decrease. From the statements in [11], if the object airline's frequency increases and its airfares decrease, the competitor will not only increases flight frequencies but will also reduce its airfares. As mentioned previously, the fuzzy-logic model determines changes in the flight frequencies and airfares of all competing airlines within acceptable ranges, in which profits are maintained. Accordingly, when airlines increase their frequencies, such that their costs also increase, they can decrease their airfares only when the increased demand allows the increase in revenues to offset the increase in costs. When the object airline increases its flight frequencies and airfares, the competitor may not change its airfares. However, if the object airline does not increase its frequency, or even reduces it, then when it increases airfares, its competitors can increase their airfares without losing market share, and will thereby increase profits. When the object airline reduces its airfares, while reducing frequency, competitors can reduce their airfares only when the object airline's airfare decreases markedly, such that the decline affects their market shares.

Furthermore, consider the competitive interaction between any two airlines, if an airline has stronger feed traffic availability than its competitor, then the changes made by the weaker competitor in its flight frequencies and airfare will be larger than those of the stronger airline, to prevent it from being forced out of the market. The asymmetries in competitive interaction due to differences in the amounts of feed traffic available to airlines were addressed in [9]. Specifically, a "stronger" airline is one with a greater market share. The above assumption was deduced from statements by [9]. It is justified since, as the weaker competitor increases frequencies and/or decreases airfares to enhance its market share, the stronger airline can dominate the market by making lesser increases in frequencies and/or smaller decreases in airfares. In contrast, when stronger airline increases its frequencies slightly and/or decreases its airfares slightly, the weaker competitor must greatly increase its frequencies and/or greatly reduce its airfares to maintain market share.

Based on the above descriptions concerning the general behaviors of competitive interactions between airlines, an approximate reasoning algorithm, consisting of fuzzy IF-THEN rules, was established to estimate competitors' reactions in terms of changes in flight frequencies and airfares. Defuzzified outputs are competitors' changed flight frequencies, ΔN_{rs}^{x}, and airfares, $\Delta \mathrm{tp}_{rs}^{x}$. In this study, defuzzification is the centroid of the area determined by the joint membership function of the fuzzy action. This center-of-gravity (COG) method is often employed in the literature [17,27]. In the i-th round of interaction, the changes to competitor xs flight frequencies, ΔN_{rs}^{xi}, and to its basic airfares, $\Delta \mathrm{tp}_{rs}^{xi}$, can be determined by applying the approximate reasoning algorithm and defuzzification method. Then, competitor xs flight frequencies and basic airfares between OD pair $r-s$ after i rounds of interaction, N_{rs}^{xi} and tp_{rs}^{xi}, can be calculated using $N_{rs}^{xi}=(1+\Delta N_{rs}^{xi})N_{rs}^{xi-1}$ and $\mathrm{tp}_{rs}^{xi}=(1+\Delta \mathrm{tp}_{rs}^{xi})\mathrm{tp}_{rs}^{xi-1}$, respectively.

All other variables, such as yield, sales promotions, and other quality variables, concerning the object airline and any competitors in its market, are assumed to be fixed in the frequency and airfare competitive interaction model. With reference to other issues concerning competition among airlines, for example involving the domestic market, competitive responses to newly entering carriers, and other types of strategy, the fuzzy logic system of the presented model can be reconstructed by applying a specific theoretical work or examining case studies, examples, and the results of experiments. In future applications, actual airline practices and policies can also be integrated into the fuzzy logic system.

In this study, once the object airline's market shares are assessed to have converged for all OD pairs, the demand-supply interaction has converged. Changes of airline market shares for individual OD pairs are determined after each round to assess the convergence conditions of the competitive interaction problem. Based on the definition of [9], the relative change in airline market share of MS_{rs}^{0}, is,

$$\mathrm{RC}(\mathrm{MS}_{rs}^{0})=\frac{|\mathrm{MS}_{rs}^{0i}-\mathrm{MS}_{rs}^{0i-1}|}{0.5\left(\mathrm{MS}_{rs}^{0i}+\mathrm{MS}_{rs}^{0i-1}\right)}, \tag{11}$$

where $\mathrm{RC}(\mathrm{MS}_{rs}^{0})$ represents the relative change in MS_{rs}^{0}, and MS_{rs}^{0i} is the value of MS_{rs}^{0} after i rounds. Finally, the following heuristic algorithm, consisting of an iterative scheme, is proposed to solve airline flight frequency programming problems with competitive interactions among competing airlines.

- Step 1. Input the initial values N_{rsp}^{0} and tp_{rsp}^{0}, $\forall\, r,s,p$, and N_{rsp}^{x} and tp_{rsp}^{x}, $\forall\, r,s,p$, $\forall\, x\neq 0$, and other exogenous parameters; where N_{rsp}^{x} and tp_{rsp}^{x}, $\forall\, r,s,p$, $\forall\, x\neq 0$, are initially determined by all competing airlines ($\forall\, x\neq 0$) simultaneously, to maximize their profits for the initial flight frequencies, N_{rsp}^{0}, and airfares, tp_{rsp}^{0}, $\forall\, r,s,p$, offered by the object airline.
- Step 2. In the i^{th} round, input N_{rsp}^{0i-1} and tp_{rsp}^{0i-1}, $\forall\, r,s,p$, N_{rs}^{xi-1}, and tp_{rs}^{xi-1}, $\forall\, x\neq 0$, $\forall\, r,s$, to estimate the object airline's market shares, MS_{rs}^{0i}, $\forall\, r,s$, using the market share model (equation (3)), and estimate f_{rs}^{0i}, $\forall\, r,s$, using $f_{rs}^{0i}=F_{rs}\mathrm{MS}_{rs}^{0i}$. For the object airline network, use the flight frequency programming model (equations (10a)–(10f)) to obtain route flight frequencies, N_{rspq}^{0i}, $\forall\, r,s,p,q$; route basic airfares, tp_{rsp}^{0i}, $\forall\, r,s,p$, and the objective function value π_0^i.
- Step 3. Calculate ΔN_{rs}^{0i} and $\Delta \mathrm{tp}_{rs}^{0i}$, where $\Delta N_{rs}^{0i}=(N_{rs}^{0i}-N_{rs}^{0i-1})/N_{rs}^{0i-1}$ and $\Delta \mathrm{tp}_{rs}^{0i}=(\mathrm{tp}_{rs}^{0i}-\mathrm{tp}_{rs}^{0i-1})/\mathrm{tp}_{rs}^{0i-1}$. Run the fuzzy-logic-based competitive interaction model to estimate ΔN_{rs}^{xi} and $\Delta \mathrm{tp}_{rs}^{xi}$, $\forall\, r,s$, $\forall\, x\neq 0$, by inputting MS_{rs}^{0i}, ΔN_{rs}^{0i}, $\Delta \mathrm{tp}_{rs}^{0i}$, $\forall\, r,s$, and MS_{rs}^{xi}, $\forall\, x\neq 0$, $\forall\, r,s$; then calculate N_{rs}^{xi} and tp_{rs}^{xi}, $\forall\, x\neq 0$, $\forall\, r,s$, using $N_{rs}^{xi}=(1+\Delta N_{rs}^{xi})N_{rs}^{xi-1}$ and $\mathrm{tp}_{rs}^{xi}=(1+\Delta \mathrm{tp}_{rs}^{xi})\mathrm{tp}_{rs}^{xi-1}$, respectively.
- Step 4. If $\mathrm{RC}(\mathrm{MS}_{rs}^{0})<\varepsilon$ (a small number), then STOP. Otherwise, $i:=i+1$, and return to Step 2.

4. CASE STUDY

This section presents a case study that demonstrates the application of the proposed models. In this case study, the object airline is China Airlines (CI) of Taiwan, and the proposed models were applied to a simplified version of CI's international network. For simplicity, only ten cities (in eight countries) of 32 cities currently served by CI were selected, and 14 wide-body aircraft, including eight Boeing 747-400s (394 seats) and six Airbus 300s (268 seats) were assumed to serve these ten cities. The selected nine OD-pairs were Taipei (TPE)-Hong Kong (HKG), Tokyo (TYO), Bangkok (BKK), Singapore (SIN), Kuala Lumpur (KUL), Los Angeles (LAX), San Francisco (SFO), New York (NYC), and Amsterdam (AMS). Traffic between these selected OD-pairs represents about 70% of the total CI traffic. In this case study, the route flight frequencies and basic airfares of CI's network in year 2000 were used as the initial flight frequencies and airfares in the model. The study aims to determine CI's route flight frequencies under competitive interactions in the planning year 2001, not only to demonstrate the application of the proposed model, but also to compare the results of the model with the true situation. Base values for the cost-function-related parameters are given to solve the flight frequency programming problem for CI's network. However, some of CI's operating cost data were unavailable, so operating cost data reported in [28] was used to estimate these costs. Characteristics of the aircraft were taken from CI's fleet fact sheet and those reported by [29], and were used to estimate block times.

Before other parts of the models, the statistical estimates pertaining to the market share model are first discussed. Monthly data from 1999 to 2000 [30,31], including OD passenger demands, passenger traffic by airline, airline flight frequencies and average airfares for all OD pairs, were used. For each OD market, data concerning CI's market share, total number of passengers, CI's frequency share, and average airfares of competing airlines were available for each of the 24 months

Table 1. Results estimated using the market share models on individual OD-pairs of CI's network.

OD-Pairs	Estimated Model Coefficient				Adjusted R^2	F
	γ_0	a_1	a_2	a_3		
TPE-HKG	0.0113 (−1.77)	0.696 (2.01)	-5.229 (−1.93)	6.085 (1.82)	0.755	24.68
TPE-TYO	0.0002 (−1.83)	0.880 (9.15)	−1.784 (−1.86)	3.341 (2.55)	0.921	90.24
TPE-BKK	0.5762 (−1.73)	0.760 (3.29)	−2.387 (−1.83)	2.439 (2.73)	0.896	67.25
TPE-SIN	0.0002 (−1.87)	0.893 (12.34)	−2.344 (−3.41)	3.797 (3.17)	0.975	303.6
TPE-KUL	4.9×10^{-5} (−1.93)	0.953 (6.57)	−1.011 (−1.84)	2.813 (2.24)	0.945	133.8
TPE-LAX	0.6568 (−4.96)	0.933 (9.99)	−6.636 (−9.06)	6.749 (5.77)	0.957	171.4
TPE-SFO	0.0237 (−4.97)	0.894 (8.71)	−4.475 (−7.45)	5.068 (6.38)	0.957	172.1
TPE-NYC	1.3489 (3.92)	0.838 (9.94)	−6.663 (−5.07)	6.558 (8.19)	0.961	190.1
TPE-AMS	0.0122 (7.6)	0.884 (9.72)	−5.654 (−5.97)	6.216 (8.79)	0.981	401.8

Note: t-ratios are listed in parentheses.

Data source: Department of Statistics, M.O.T.C., R.O.C. [30], and Civil Aeronautics Administration, M.O.T.C., R.O.C. [31]. Detail data are publicly available upon request.

of the study period. Table 1 lists the estimated results obtained using the market share models of the nine OD-pairs of CI's network. Table 1 reveals that the signs of the estimated parameters, γ_0, a_1, a_2, and a_3, are as expected. The estimated models closely fitted the historical data, with strong statistical significance. The adjusted R^2 values range from 0.76 to 0.98. In seven out of nine OD markets, adjusted R^2 exceeds 0.92. F-statistics range from 24.6 to 401.8, indicating strongly significant estimated regressions. Moreover, the t-statistics associated with each of the

Table 2. Initial values of flight frequencies and basic airfares for all competing airlines, and market demands on individual OD pairs.

OD-Pairs	Market Demands* in 2001 (Annual Traffic)	Airlines	Flight Frequencies* (Flights/Week)	Route Airfares* (U.S.$)
TPE-HKG	2712706	CI	64	205.882
		BR	24	201.238
		CX	54	204.108
		EG	7	170.075
		SQ	3	182.147
		TG	14	173.375
TPE-TYO	953546	CI	21	208.978
		EG	22	211.789
		SQ	3	198.142
		CX	7	213.622
TPE-BKK	656578	CI	21	229.924
		BR	22	235.949
		TG	21	226.405
		KL	7	203.287
TPE-SIN	288531	CI	7	227.554
		BR	7	246.904
		SQ	8	247.678
TPE-KUL	254064	CI	8	206.333
		BR	7	206.778
		MH	9	225.000
TPE-LAX	588103	CI	13	446.717
		BR	14	455.050
		MH	4	397.475
		SQ	7	429.356
		UA	7	434.898
TPE-SFO	333160	CI	7	414.898
		BR	10	473.367
		UA	7	434.898
TPE-NYC	129089	CI	6	476.894
		BR	7	498.568
		UA	7	489.898
TPE-AMS	146867	CI	6	528.363
		BR	3	596.212
		KL	7	569.409

Note: *one direction.

Table 3. Round-by-round results.

(a). Route flight frequencies and objective function values.

Routes	Weekly Flight Frequencies (Flights/Week) (One Direction)							
	Initial	Aircraft	Round of Interaction					
			1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
TPE-HKG	64	B747-400	1	0	0	1	0	0
		A300	72	74	71	68	69	69
TPE-TYO	21	B747-400	21	22	22	22	22	22
		A300	5	4	4	4	4	4
TPE-BKK	21	B747-400	0	0	0	0	0	0
		A300	18	18	18	18	18	18
TPE-SIN	7	B747-400	5	5	5	5	5	5
		A300	0	0	0	0	0	0
TPE-KUL	8	B747-400	5	4	4	4	4	4
		A300	0	0	0	0	0	0
TPE-LAX	13	B747-400	13	13	13	13	13	13
TPE-SFO	7	B747-400	8	8	8	8	8	8
TPE-TYO-NYC	6	B747-400	6	6	6	6	6	6
TPE-BKK-AMS	6	B747-400	7	7	7	7	7	7
Objective function values (U.S.$):			2721745	2687077	2587108	2554429	2541703	2532241

(b). Route basic airfares.

Routes	Route Basic Airfares (U.S.$) (One Direction)						
	Initial	Round of Interaction					
		1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
TPE-HKG	205.882	205.385	205.564	205.564	205.374	205.564	205.564
TPE-TYO	208.978	209.830	209.414	209.414	209.414	209.414	209.414
TPE-BKK	229.924	212.454	215.904	215.904	215.904	215.904	215.904
TPE-SIN	227.554	209.729	209.729	209.729	209.729	209.729	209.729
TPE-KUL	206.333	205.206	203.529	203.529	203.529	203.529	203.529
TPE-LAX	446.717	445.155	445.155	445.155	445.155	445.155	445.155
TPE-SFO	414.898	410.908	410.908	410.908	410.908	410.908	410.908
TPE-TYO-NYC	476.894	490.858	490.858	490.858	490.858	490.858	490.858
TPE-BKK-AMS	528.363	513.918	513.918	513.918	513.918	513.918	513.918

estimated γ_0, a_1, a_2, and a_3 are also significant. Table 2 shows that the estimated elasticity of the frequency share varies from about 0.7 to 0.95, and the estimates reflect a relatively inelastic response of market share to frequency share. Similar findings were reported in [25], and the same results were analytically obtained in [11]. According to the model estimation results, the direct airfare elasticity ranges from about -6.66 to -1.01, while the cross elasticity of airfare ranges from about 2.44 to 6.75. Therefore, the relationship between airline market share and airline airfare is elastic. Moreover, the direct and cross elasticities of airfare seem to be large, because about 65% to 78% of all passengers are nonbusiness travelers in these selected markets. Similar ranges were presented in [25].

In this case study, actual total passenger demands between each OD pair in 2001 were taken as the OD market sizes, F_{rs}, $\forall r, s$. The market sizes for OD pairs in future planning years

Table 3. (cont.)

(c). OD market shares.

OD Market Shares (%) (One Direction)								
			Round of Interaction					
OD-pairs		Initial	1st	2nd	3rd	4th	5th	6th
TPE-HKG		31.933%	32.022%	30.806%	30.241%	29.994%	29.837%	29.837%
	Relative changes:		0.00278	0.0387	0.01852	0.00821	0.00523	0
TPE-TYO		39.320%	39.824%	39.820%	39.820%	39.820%	39.820%	39.820%
	Relative changes:		0.01276	0.00011	0	0	0	0
TPE-BKK		28.309%	28.300%	27.854%	27.844%	27.844%	27.844%	27.844%
	Relative changes:		0.00031	0.01589	0.00038	0	0	0
TPE-SIN		24.764%	24.177%	24.075%	24.065%	24.065%	24.065%	24.065%
	Relative changes:		0.02398	0.00423	0.00042	0	0	0
TPE-KUL		29.425%	21.473%	18.3385	18.329%	18.329%	18.329%	18.329%
	Relative changes:		0.31249	0.1575	0.00048	0	0	0
TPE-LAX		32.326%	32.231%	32.097%	32.083%	32.083%	32.083%	32.083%
	Relative changes:		0.00294	0.00416	0.00044	0	0	0
TPE-SFO		35.127%	36.019%	36.019%	36.019%	36.019%	36.019%	36.019%
	Relative changes:		0.02508	0	0	0	0	0
TPE-NYC		32.732%	26.008%	25.912%	25.902%	25.902%	25.902%	25.902%
	Relative changes:		0.22896	0.00369	0.0004	0	0	0
TPE-AMS		29.924%	34.813%	34.813%	34.813%	34.813%	34.813%	34.813%
	Relative changes:		0.15104	0	0	0	0	0

can be predicted by applying the forecasting model presented in the authors' earlier work [2,32]. The market shares of all competing airlines in year 2000 were used as initial market share levels in determining their initial flight frequencies, N^x_{rsp}, and basic airfares, tp^x_{rsp}, $\forall\, r, s, p$, $\forall\, x \neq 0$, corresponding to OD passenger demands in year 2001. Table 2 lists the initial values of N^0_{rspq} and tp^0_{rsp}, $\forall\, r, s, p, q$, and N^x_{rsp} and tp^x_{rsp}, $\forall\, x \neq 0$, $\forall\, r, s, p$. The value $\bar{r}^0_{rsp} = 0.15$ was also assumed, based upon slight adjustments in the average ratio of the actual route airfare to the average operating cost per available seat, for all routes in CI's network. In determining flight frequencies, the load factors were set to 75% for all routes on CI's network. LINGO was used to run the IP-based flight frequency programming, and the fuzzy-logic-toolbox of MATLAB was used to run the fuzzy-logic-based competition model. The competitive interaction problems were then solved using the proposed algorithm, in which stop criteria of $\mathrm{RC}(\mathrm{MS}^0_{rs}) < 0.001$, $\forall\, r, s$, were set.

The competitive interactions that determine the flight frequencies of the object airline network converged after six rounds. Table 3 lists the related determined route flight frequencies, airfares and market shares, objective function values, and estimated competitors' flight frequencies and airfare changes are listed round by round. Table 3c also lists the measured relative changes in airlines' market shares, RC (MS^0_{rs}), in each round. Table 3 shows that the competitive interactions on most routes converged soon after three or four rounds, except on route TPE-HKG. Moreover, model sensitivities were examined by setting the load factors to 70% and 80% without relaxing the initial assumptions. When determining flight frequencies on CI's network with 70% and 80% load factors, the competitive interactions converged after eight rounds, since convergence occurred on route TPE-TYO reached in the eighth round. The competitive interactions on routes TPE-HKG and -BKK converged after six rounds, while for other routes, the convergence of competitive interactions was also reached soon after three or four rounds.

Figure 3 depicts the competitive interactions of flight frequencies between the object airline (CI)

Table 3 (cont.)

(d) Changes in competitors' flight frequencies and airfares.

OD-pairs/ Competing Airlines	1st Weekly Freq. (Flights)	1st Airfare (U.S.$)	2nd Weekly Freq. (Flights)	2nd Airfare (U.S.$)	3rd Weekly Freq. (Flights)	3rd Airfare (U.S.$)	4th Weekly Freq. (Flights)	4th Airfare (U.S.$)	5th Weekly Freq. (Flights)	5th Airfare (U.S.$)
TPE-HKG										
BR	28	200.122	31	201.106	31	201.106	31	200.726	31	201.288
CX	63	202.976	69	203.974	69	203.974	69	203.589	69	204.368
EG	8	169.131	9	169.963	9	169.963	9	169.642	9	170.127
SQ	3	180.833	4	181.933	4	181.933	4	181.386	4	181.829
TG	16	172.412	18	173.261	18	173.261	18	172.933	18	173.595
TPE-TYO										
EG	26	211.789	27	211.281	27	211.281	27	211.281	27	211.281
SQ	4	198.142	4	197.029	4	197.029	4	197.029	4	197.029
CX	8	213.622	8	212.749	8	212.749	8	212.749	8	212.749
TPE-BKK										
BR	22	233.764	25	233.764	25	233.764	25	233.764	25	233.764
TG	21	224.472	24	224.472	24	224.472	24	224.472	24	224.472
KL	7	200.868	8	200.868	8	200.868	8	200.868	8	200.868
TPE-SIN										
BR	7	244.139	7	244.139	7	244.139	7	244.139	7	244.139
SQ	8	245.102	8	245.102	8	245.102	8	245.102	8	245.102
TPE-KUL										
BR	7	205.609	7	205.609	7	205.609	7	205.609	7	205.609
MH	9	223.763	9	223.763	9	223.763	9	223.763	9	223.763
TPE-LAX										
BR	14	452.197	14	452.197	14	452.197	14	452.197	14	452.197
MH	4	394.864	4	394.864	4	394.864	4	394.864	4	394.864
SQ	7	426.535	8	426.535	8	426.535	8	426.535	8	426.535
UA	7	432.041	8	432.041	8	432.041	8	432.041	8	432.041
TPE-SFO										
BR	12	469.173	12	469.173	12	469.173	12	469.173	12	469.173
UA	8	431.254	8	431.254	8	431.254	8	431.254	8	431.254
TPE-NYC										
BR	7	502.227	8	502.227	8	502.227	8	502.227	8	502.227
UA	7	494.15	8	494.15	8	494.15	8	494.15	8	494.15
TPE-AMS										
BR	4	589.058	4	589.058	4	589.058	4	589.058	4	589.058
KL	8	562.576	8	562.576	8	562.576	8	562.576	8	562.576

Note: One direction.

and its main competitor (CX) on route TPE-HKG. Table 3 and Fig. 3 show that the determined flight frequencies increased and the airfares decreased from the initial round to the first round on route TPE-HKG, such that the competing airlines were prompted to increase their flight frequencies and reduce their airfares as determined by the fuzzy-logic-based competitive model. In this situation, CI's market share on route TPE-HKG slightly increased in the subsequent round. When CI's flight frequencies continued to decline after two rounds, CI's market shares declined, since the competing airlines also decreased their airfares as CI decreased its airfares

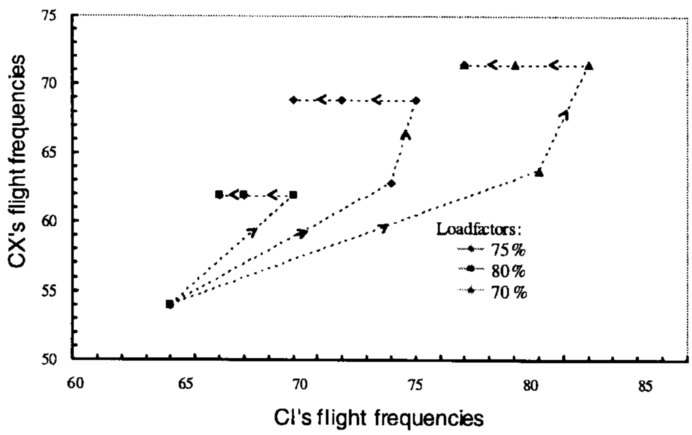

Figure 3. Competitive interactions between flight frequencies of CI and CX on route TPE-HKG.

Table 4. Comparisons among flight frequency programming results with different load factors in the competitive convergent state.

Routes	Market Shares (%) Model With Load Factors: 70%	75%	80%	Actual*	Determined Flight Frequencies (Flights/Week) Aircraft	Model With Load Factors: 70%	75%	80%	Actual Flight Frequencies (Flights/Week) Aircraft	Freq.
TPE-HKG	36.32%	29.84%	30.79%	31.35%	B747-400 A300	0 76	0 69	1 65	A340/A300/ B737-800/ B747-400	68
TPE-TYO	30.67%	39.82%	35.04%	36.26%	B747-400 A300	23 2	22 4	0 30	B747-400	22
TPE-BKK	26.55%	27.84%	30.21%	31.02%	B747-400 A300	0 18	0 18	0 18	MD11/A340/ B737-800/ B747-400	23
TPE-SIN	24.185%	24.07%	23.94%	23.53%	B747-400 A300	5 0	5 0	0 7	A300	6
TPE-KUL	18.41%	18.33%	14.75%	23.54%	B747-400 A300	4 0	4 0	3 0	B737-800/ A300	6
TPE-LAX	30.12%	32.08%	31.35%	38.49%	B747-400	13	13	12	B747-400	13
TPE-SFO	38.41%	36.02%	36.09%	32.45%	B747-400	9	8	8	B747-400	7
TPE-TYO-NYC	25.91%	25.90%	26.01%	29.57%	B747-400	6	6	6	A340	7
TPE-BKK-AMS	34.69%	34.81%	34.81%	31.84%	B747-400	7	7	7	B747-400	7
					Objective function values (U.S.$):	2353553	2532241	2635733		

Note: One direction.

* Source: Civil Aeronautics Administration, M.O.T.C., R.O.C. [31], in year 2001.

on route TPE-HKG. Only after six rounds did the market share on route TPE-HKG remain constant. Moreover, on route TPE-TYO, CI's determined flight frequencies first increased from the initial round to the first round, and then also caused competitors to increase their flight frequencies. However, the competing airlines did not change their airfares when CI increased its airfares. In this situation, CI's market shares slightly increased on route TPE-TYO from the first round to the second round. CI's market share on route TPE-TYO started decreasing from the third round because of reductions in competitors' airfares, and did not change until convergence.

Table 4 lists the outputs of the flight frequency programming model with different load factors (70%, 75% and 80%) in the competitive convergent state. The table also lists the corresponding route market shares. For comparison, Table 4 provides CI's actual flight frequencies and market shares by routes in 2001. Table 4 reveals that the route market shares for these three cases are similar to each other in the competitive convergent state. Perhaps, therefore, the competitive interaction did converge. From Table 4, higher load factors lead to the airlines' determining lower flight frequencies of larger aircraft, perhaps reducing operating costs and realizing higher profits. However, the determined flight frequencies and airfares for the routes in these three cases are also similar to those determined in the competitive convergent state. In the competitive convergent state for year 2001, the converged market share results were accurate and the flight frequencies and airfares for each route obtained from the proposed models were reasonable, as determined by comparing them with CI's actual market shares and route flight frequencies in 2001.

As stated, many studies (e.g., [12,23]) have modeled airline frequency competition using the best-response approach. Hence, the results of the presented fuzzy-logic-based model were compared with those of the best-response approach, tested on the same dataset. Table 5 lists the flight frequencies and market shares obtained from the proposed model and the best-response model, respectively, in the convergent state for the year 2001. The convergence condition was reached after eight rounds in solving the best-response model. However, solving the best-response game took much longer than solving the presented interaction model. In this case study, the proposed model took 5.7 minutes, whereas the best-response approach with the same dataset took 47.25 minutes to reach convergence (using the same Pentium-4 2.4 GHz PC). The best-response approach needs to solve various network programming models for all competitors simultaneously in each round. In contrast, the fuzzy-logic-based competitive interaction model incorporates a rule-based IF-THEN approach to solving a flight frequency programming problem, rather than attempting in each round to solve network programming models for all competitors. Therefore, the proposed model is not NP-hard, nor does its complexity increase exponentially with the number of competitors. Moreover, Table 5 shows that the flight frequency and the market share results of the proposed model are more accurate than those of the best-response model. However, both final solutions under the convergence condition seem to be similar to the initial values. These results imply that CI is in fact currently maximizing profitability in an approximately optimal manner under conditions of converging competitive interaction.

This case study demonstrates how anticipated competitive interactions can be considered well in advance of when solving airline flight frequency programming problems under uncertain competitive conditions. The fuzzy-logic-based competitive interaction model, incorporating theoretical airline competitive concepts, can be a useful tool with which to identify iteratively the competitive convergence. Consequently, taking into account competitive interactions makes the results of the presented flight frequency programming model of an airline network practically useful in decision-making in airline network-planning under uncertain competitive conditions.

5. CONCLUSIONS

This study developed a model for determining optimal flight frequencies and airfares on airline network routes, which took into account competitive interactions. The model is comprised of three submodels, including an airline market share model, an airline flight frequency programming model, and a fuzzy-logic-based competitive interaction model. The airline market share model is formulated as functions of flight frequency shares and relative airline airfares on OD-pairs. The passenger demands and the airline market shares of all OD pairs are used as input parameters in the airline flight frequency programming model. Airlines' competitive interactions concerning route flight frequencies and airfares are also considered. The competitive interaction model, using

Table 5. Comparisons between results of proposed model and those of the best-response approach, in the competitive convergent state.

Routes	Market Share (%) Proposed Model	Market Share (%) Best-Response Approach	Market Share (%) Actual*	Determined Flight Frequencies (Flights/Week) Proposed Model Aircraft	Proposed Model Freq.	Best-Response Approach	Actual Flight Frequencies* (Flights/Week)	
TPE-HKG	29.84%	26.81%	31.35%	B747-400 A300	0 69	1 62	A340/A300/ B737-800/ B747-400	68
TPE-TYO	39.82%	35.53%	36.26%	B747-400 A300	22 4	0 29	A340/A300/ B747-400/ B747-400	22
TPE-BKK	27.84%	23.57%	31.02%	B747-400 A300	0 18	2 14	MD11/A340/ B737-800/ B747-400	23
TPE-SIN	24.07%	24.55%	23.53%	B747-400 A300	5 0	6 0	A300	6
TPE-KUL	18.33%	30.43%	23.54%	B747-400 A300	4 0	7 0	B737-800 A300	6
TPE-LAX	32.08%	38.59%	38.49%	B747-400	13	17	B747-400	13
TPE-SFO	36.02%	22.90%	32.45%	B747-400	8	6	B747-400	7
TPE-TYO-NYC	25.90%	21.55%	29.57%	B747-400	6	6	A340	7
TPE-BKK-AMS	34.81%	31.24%	31.84%	B747-400	7	7	B747-400	7
				Objective function values (U.S.$):	2532241	2436761		
				Number of rounds to reach convergence:	6	8		

Note: One direction.

*Source: Civil Aeronautics Administration, M.O.T.C., R.O.C. [31], in year 2001.

fuzzy logic tools, is developed to estimate competitors' reactions. An algorithm that combines all the three submodels is presented to solve this problem.

The developed models were applied to the CI network that serves ten selected cities, as a case study. The proposed market share model yielded a very good fit between the estimated models and historical data. The case study addressed three cases with different load factors, and competitive interaction results always converged. Moreover, the results of the proposed model are more accurate than those of the best-response model with the same dataset. The fuzzy-logic-based model was solved in much less time than the best-response game. The results of this case study were shown to be reasonable by comparing the obtained solutions with CI's actual market shares and route flight frequencies.

This study demonstrates how anticipated airline competitive interactions may be considered well in advance of solving airline flight frequency programming problems. The proposed models represent an analytical tool with which airlines can evaluate the impact of their strategies for adjusting flight frequencies and basic airfares in competitive environments. They are also useful devices for iteratively identifying competitive convergence. The proposed competitive interaction model can be extended to cover other issues in airline competition by reconstructing the fuzzy logic system of the model by undertaking further specific theoretical work or considering case studies. Actual airline practices and policies also can be integrated into the fuzzy logic system in future applications.

Consequently, the results of this study not only verify that an airline flight frequency programming model with the anticipated competitive interactions using fuzzy logic is practical, but also that it provides flexibility in the decision-making involved in airline network-planning in competitive and uncertain environments.

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