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A MULTI-LOCATION NEWSBOY PROBLEM WITH LOST SALE DEPENDS ON HOLDING TIME

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Abstract

The classical newsboy problem is an important issue in inventory theory. The characteristic of newsboy problem is that the goods are perishable. In the selling period, the goods will decrease the degree of freshness as the expiration date of the goods gets closer. Then this will decrease the buying willingness of the customer. In this article, we assume that there are several selling locations and buying willingness will depend on the degree of freshness of the goods. Then the decision makers should determine the optimal selling period and distribute suitable quantity of goods to each location such that the expected profit per unit time is maximized.

1. Introduction

The classical newsboy problem is an important issue in inventory theory. It is a single-period single-product inventory problem which considers the inventory size to be ordered for the sake of meeting random demand so as to maximize expected profit while balancing holding and shortage costs.

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In the past works, many researchers discuss it by different methods, assumptions and constraints. For example, Koulamas [4] considered a standard newsvendor problem in a single manufacturer-retailer channel and compares the expected profits that each party receives in a traditional ordering environment with those that can be achieved under a revenue-sharing policy designed to completely eliminate double marginalization. Shao and Ji [6] considered the multi-product newsboy problem with fuzzy demands under budget constraint. They developed three types of models under different criteria: EVM model, DCP model and CCP model. In these models, the objective functions are to maximize the expected profit of newsboy, the chance of achieving a target profit and the profit which satisfies some chance constraints with at least some given confidence level, respectively. Ching et al. [3] proposed a higher-order Markov model whose number of states and parameters are linear with respect to the order of the model. They also developed efficient estimation methods for the model parameters, then apply the model and method to solve the generalized Newsboy's problem. Chen and Chen [1] presented a newsboy model with a simple reservation arrangement by introducing the willingness rate, represented as the function of the discount rate, into the models. Mathematical models are developed, and the solution procedure is derived for determining the optimal discount rate and the optimal order quantity. Khouja [5] formulated and solved a newsboy problem with multiple discounts. Also, he showed that multiple discounts, when possible, provide higher expected profit than using a single discount. Chen and Chen [2] considered a production planning to produce suitable quantity of goods to meet the random demand at the end of the time period.

But, they did not consider the degree of freshness of the food which would affect the buying willingness of the customer. Many products will decrease their degree of freshness when the holding time increases, such as milk, cereal food, etc. On the other hand, if the decision makers rented several locations (such as supermarkets) for selling their product. The demand of each location is a random variable with different mean. The decision maker faces the random demand and the risk of lost sale. Therefore, how frequent the suitable quantity of goods should distribute to each location? It is important to the decision maker.

Thus, in this article, assuming that the buying willingness of the customer will reduce as the expiration date of the product gets closer and existence of several selling locations each with Poisson demand rate in the selling period $[0, t]$, we construct a mathematical model to determine the optimal supply cycle and the suitable quantity of the goods distributed to each location such that the expected total profit per unit time is maximized.

2. Notation and Assumptions

To formulate the mathematical model, the following notation and assumptions are adopted throughout the article:

- $[0, t]$: The selling period.
- t : The length of selling period.
- v : The unit price of the goods.
- c : The unit cost of goods.
- k : The setup cost.
- c_p : The penalty cost per unit of goods.
- s_i : Slotting fee or rent fee per unit time of the *i*th location.
- r_i : The transportation cost per unit time of the *i*th location.
- δ : The expiration date of the goods.
- N_i : The total demand of the *i*th location in the time interval $[0, t]$.

Here, we assume that it is a random variable and has a Poisson distribution with parameter $\lambda_i t$, where λ_i is the mean arrival rate of the *i*th location per unit time, $i = 1, 2, ..., \ell$. Thus, the total demand is

$$N = N_1 + N_2 + \cdots + N_\ell$$

and thus *N* also has a Poisson distribution with parameter $\sum_{i=1}^{\ell} \lambda_i t$.

- X_{ij} : The time that the *j*th customer arrives to the *i*th location. We say that it is the arrival time of the customer and is also a random variable with p.d.f. $f_{X_{ij}}(x)$, and by the assumption of Poisson process, $f_{X_{ij}}(x) = \frac{1}{t}$, where $0 \leq x \leq t$, $j = 1, 2, \ldots, N_i$, $i = 1, 2, \ldots, \ell$.

However, when the customer comes to purchase the goods at time *x*, $x \in [0, t]$, then the length of time to expired date is $\delta - x$. Indeed, the lesser the length of $\delta - x$, the lesser the degrees of freshness of the goods. So, there are some customers will purchase and the other will leave. In other words, the buying willingness (probability) will decrease as the degrees of freshness decrease.

Thus, we shall introduce another random variable *W* as follows:

- *W*: The withdrawal time of the customer, i.e., the time that he is not willing to buy and leave.

Consider the following conditional probability:

$$P(W > x \mid X_{ij} = x) = \int_x^{\delta} f_{W \mid X_{ij}}(w \mid X_{ij} = x) dw. \tag{1}$$

This conditional probability means that the customer is willing to buy given that he arrives at time *x*, and denoted it by $\theta(x)$. Clearly, $\theta(x)$ decreases as *x* increases. This means that the buying willingness decreases when *x* towards the expiration date δ. Conversely, $1 - \theta(x)$ is the probability that the customer is not willing to buy and leave when he arrives at time *x*.

Then we have

(1) Expected total revenue R:

$$R = (v - c)\sum_{i=1}^{\ell}\sum_{j=1}^{n_i}\int_0^t\int_x^{\delta} f_W(w \mid X_{ij} = x)\, f_{X_{ij}}(x)\, dwdxP(N_i = n_i)$$

$$= (v - c)\sum_{i=1}^{\ell}\sum_{n_i=0}^{\infty}\sum_{j=1}^{n_i}\int_0^t \theta(x)\, f_{X_{ij}}(x)\, dxP(N_i = n_i). \qquad (2)$$

(2) Expected total penalty cost P:

$$P = c_p\sum_{i=1}^{\ell}\sum_{n_i=0}^{\infty}\sum_{j=1}^{n_i}\int_0^t\left[1 - \int_x^{\delta} f_W(w \mid X_{ij} = x)\right] f_{X_{ij}}(x)\, dwdxP(N_i = n_i)$$

$$= c_p\sum_{i=1}^{\ell}\sum_{n_i=0}^{\infty}\sum_{j=1}^{n_i}\int_0^t (1 - \theta(x))\, f_{X_{ij}}(x)\, dxP(N_i = n_i). \qquad (3)$$

Here, we assume that N_i, X_{ij}, W are independent random variables, where $i = 1, 2, ..., \ell;\ j = 1, 2, ..., N_i$.

To discuss the distribution of W, we first define the withdrawal rate as follows:

$$r_w = \frac{f_W(w)}{1 - F_W(w)}, \qquad (4)$$

where $F_W(w) = \int_0^w f_W(x)dx$. In the lifetime study, it is the so-called instantaneous rate of death or failure rate. $1 - F_W(w) = \theta(w)$ is the probability of survive. In this case, $\theta(w)$ is the probability of the customer that is willing to buy given that he arrives at time w. In real world, r_w is an increasing function of w, i.e., the instantaneous rate of withdraw is an increasing function of w. For this reason, a simple and reasonable assumption of the distribution of W is uniformly distributed on $[0, \delta]$. Under this assumption, we have

$$r_w = \frac{\frac{1}{\delta}}{\frac{\delta - w}{\delta}} = \frac{1}{\delta - w}$$

which is an increasing function of w. This means that the instantaneous rate of withdraw (unwilling to buy and leave) increases as w increases. Also,

$$\theta(x) = \int_x^\delta f_{W \mid X_{ij}}(w \mid X_{ij} = x)dw = \frac{\delta - x}{\delta}.$$

So, R in (2) becomes

$$R = (v - c)\sum_{i=1}^{\ell} \lambda_i t \int_0^t \theta(x)\frac{1}{t}\,dx \tag{5}$$

and P in (3) becomes

$$P = c_p \sum_{i=1}^{\ell} \lambda_i t \int_0^t (1 - \theta(x))\frac{1}{t}\,dx. \tag{6}$$

From (5), (6), and together with transportation cost, setup cost and slotting fees, we have the expected profit per unit time as follows:

$$a(t) = \frac{v - c}{t}\sum_{i=1}^{\ell} \lambda_i \int_0^t \theta(x)dx - \frac{c_p}{t}\sum_{i=1}^{\ell} \lambda_i \int_0^t (1 - \theta(x))dx - \sum_{i=1}^{\ell} s_i - \frac{k}{t}. \tag{7}$$

3. Model and Optimal Solution

The purpose of this article is to find t so that average profit per unit time is maximized, i.e.,

$$\max_{t<\delta} a(t), \tag{8}$$

where

$$a(t) = \frac{v - c}{t}\sum_{i=1}^{\ell} \lambda_i \int_0^t \theta(x)dx - \frac{c_p}{t}\sum_{i=1}^{\ell} \lambda_i \int_0^t (1 - \theta(x))dx - \sum_{i=1}^{\ell} s_i - \frac{k}{t}.$$

To find the optimal solution t^* of $a(t)$, substitute $\theta(x) = \frac{\delta - x}{\delta}$ into $a(t)$ and differentiate $a(t)$ with respect to *t*, we have

$$a'(t) = -\frac{v-c}{t^2}\sum_{i=1}^{\ell}\lambda_i\int_0^t \frac{\delta - x}{\delta}\,dx + \frac{v-c}{t}\sum_{i=1}^{\ell}\lambda_i\frac{\delta - t}{\delta}$$

$$+\frac{c_p}{t^2}\sum_{i=1}^{\ell}\lambda_i\int_0^t \frac{x}{\delta}\,dx - \frac{c_p}{t}\sum_{i=1}^{\ell}\lambda_i\frac{t}{\delta} + \frac{k}{t^2}$$

$$= -\frac{(v-c+c_p)}{2\delta}\sum_{i=1}^{\ell}\lambda_i + \frac{k}{t^2}. \tag{9}$$

Set $a'(t) = 0$, then we have

$$t^* = \sqrt{\frac{2\delta k}{(v-c+c_p)\sum_{i=1}^{\ell}\lambda_i}} \tag{10}$$

and the second order differentiation of $a(t)$ with respect to *t* is

$$a''(t) = -\frac{2k}{t^3} < 0. \tag{11}$$

Since $a(t)$ is strictly concave downward, thus, if t^* given in (10) is greater than δ, then $t^* = \delta$. Otherwise, t^* given in (10) is the optimal solution of (8), i.e.,

$$t^* = \begin{cases} \sqrt{\dfrac{2\delta k}{(v-c+c_p)\sum_{i=1}^{\ell}\lambda_i}}, & \text{if } \dfrac{2k}{(v-c+c_p)\sum_{i=1}^{\ell}\lambda_i} < \delta, \\ \delta, & \text{if } \dfrac{2k}{(v-c+c_p)\sum_{i=1}^{\ell}\lambda_i} \geq \delta. \end{cases} \tag{12}$$

4. Conclusion

First, consider the condition probability $\theta(x)$ given in (1), this probability means that the customer is willing to buy the goods given that he arrives at time *x*. Thus, the unconditional probability is

$$\int_0^t \int_x^\delta f_{W \mid X_{ij}}(w \mid x) f_{X_{ij}}(x) dw dx = \int_0^t \theta(x) \frac{1}{t} dx = 1 - \frac{t}{2\delta}, \qquad (13)$$

where $\theta(x) = \frac{\delta - x}{\delta}$. This is the buying probability of the customers who arrive in the time interval $[0, t]$.

As mentioned above, t^* given in (12), maximized the expected profit per unit time. As defined above, $N = N_1 + N_2 + \cdots + N_\ell$ is the total number of customers coming in the time interval $[0, t]$, which has Poisson distribution $\sum_{i=1}^{\ell} \lambda_i t$. If the buying probability is $1 - \frac{t}{2\delta}$ as given in (13), then the truly expected total demand becomes

$$q = \begin{cases} \sqrt{\frac{2\delta k \sum_{i=1}^{\ell} \lambda_i}{v - c + c_p}} \left[1 - \frac{1}{2\delta} \sqrt{\frac{2\delta k}{(v - c + c_p) \sum_{i=1}^{\ell} \lambda_i}} \right], & \text{if } \frac{2k}{(v - c + c_p) \sum_{i=1}^{\ell} \lambda_i} < \delta, \\ \frac{\sum_{i=1}^{\ell} \lambda_i \delta}{2}, & \text{if } \frac{2k}{(v - c + c_p) \sum_{i=1}^{\ell} \lambda_i} \geq \delta. \end{cases} \qquad (14)$$

However, the demand of the *i*th location is Poisson distribution with mean λ_i, so the quantity of goods distributed to the *i*th location is

$$q_i = \frac{\lambda_i}{\sum_{i=1}^{\ell} \lambda_i} q,$$

i.e.,

$$q_i = \begin{cases} \dfrac{\lambda_i}{\sqrt{\sum_{i=1}^{\ell} \lambda_i}} \sqrt{\dfrac{2\delta k}{v - c + c_p}} \left[1 - \dfrac{1}{2\delta} \sqrt{\dfrac{2\delta k}{(v - c + c_p) \sum_{i=1}^{\ell} \lambda_i}} \right], & \text{if } \dfrac{2k}{(v - c + c_p) \sum_{i=1}^{\ell} \lambda_i} < \delta, \\ \dfrac{\lambda_i \delta}{2}, & \text{if } \dfrac{2k}{(v - c + c_p) \sum_{i=1}^{\ell} \lambda_i} \geq \delta. \end{cases}$$

Clearly, q_i is the optimal inventory level of ith location, and t^* is the optimal length of supply cycle.

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