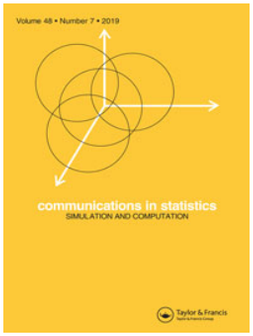

Communications in Statistics - Simulation and Computation

ISSN: 0361-0918 (Print) 1532-4141 (Online) Journal homepage: https://www.tandfonline.com/loi/lssp20

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To cite this article: Shu-Fei Wu (2019) A modified multiple comparisons with a control for exponential location parameters based on doubly censored sample under heteroscedasticity, Communications in Statistics - Simulation and Computation, 48:7, 2056-2064, DOI: 10.1080/03610918.2018.1433839

To link to this article: https://doi.org/10.1080/03610918.2018.1433839

Published online: 07 Mar 2018.

COMMUNICATIONS IN STATISTICS - SIMULATION AND COMPUTATION®
2019, VOL. 48, NO. 7, 2056–2064
https://doi.org/10.1080/03610918.2018.1433839

A modified multiple comparisons with a control for exponential location parameters based on doubly censored sample under heteroscedasticity

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ABSTRACT

In this paper, a modified one-stage multiple comparison procedures with a control for exponential location parameters based on the doubly censored sample under heteroscedasticity is proposed. A simulation study is done and the results show that the proposed procedures have shorter confidence length with coverage probabilities closer to the nominal ones compared with the one proposed in Wu (2017). At last, an example of comparing the duration of remission for four drugs as the treatment of leukemia is given to demonstrate the proposed procedures.

ARTICLE HISTORY
Received 9 July 2017
Accepted 23 January 2018

KEYWORDS
Doubly censored sample; Exponential distribution; Multiple comparisons with a control; One-stage procedure

1. Introduction

The field of ranking and selection for normal distributions has been extensively studies by Bechhofer (1954), Gupta (1956) and their followers. In this paper, let $\pi_1, \ldots, \pi_k$ denote k (≥ 2) independent two-parameter exponential populations, where π_k is a control population. The random sample from population π_i follows an exponential distribution denoted as $E(\theta_i, \sigma_i)$, $i = 1, \ldots, k$, where $\theta_1, \ldots, \theta_k$ are unknown location parameters and so-called the threshold values or "guaranteed time" parameters in reliability and engineering and $\sigma_1, \ldots, \sigma_k$ are unknown and possibly unequal scale parameters and are referred as the mean effective duration in addition to θ_i, $i = 1, \ldots, k$. There are many applications of exponential distribution in the analysis of reliability and the life test experiments. See for example, Johnson, Kotz, and Balakrishnan (1994), Bain and Engelhardt (1991), Lawless and Singhal (1980) and Zelen (1966). When k scale parameters are equal, i.e. $\sigma_1 = \cdots = \sigma_k = \sigma$, Ng, Lam, and Chen (1993) proposed multiple comparison procedures with the control. When k scale parameters are unknown and possibly unequal, Lam and Ng (1990) proposed the design-oriented two-stage multiple comparison procedures with the control under heteroscedasticity. When the additional sample at the second stage may not be available due to the lack of experimental budget or other uncontrollable factors in some experiments, one-stage multiple comparison procedures with a control are considered instead. Wu, Lin, and Yu (2010) proposed the one-sample multiple comparison procedures with a control using Lam's (1987, 1988) technique for complete sample. When the first r lifetimes and the last s lifetimes out of n inspected items are missing, the doubly Type II censoring is arisen. In this paper, we proposed a modified one-stage multiple comparison procedures based on

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doubly censored sample in Section Section 2. In Section 3, a simulation comparison is done between the proposed procedures and the ones in Wu (2017). In Section 4, an example of duration of remission by four drugs used in the treatment of leukemia is used to illustrate the proposed procedures. Finally, our conclusions are summarized in Section 5.

2. Modified one stage multiple comparison procedures based on doubly censored sample

In life testing experiments, the lifetimes of all products on the life test may not be completely observed due to the limitation of time, budgets or material resources. If the first r lifetimes and the last s lifetimes out of n inspected products are missing, then we can only obtain the doubly Type II censored sample denoted as $X_{i(r+1)}, \ldots, X_{i(n-s)}$ from the i*th* population $\pi_i, i = 1, \ldots, k$. Let the $(r+1)$*th* ordered observation from the i*th* population denoted as $Y_i = X_{i(r+1)}$. Consider the pivotal quantity $W_i = T_i/U_i$, where $T_i = \frac{X_{i(r+1)}-\theta_i}{\sigma_i}$ is the (r+1)*th* order statistic of a standard exponential distribution and $U_i = \frac{2\nu S_i}{\sigma_i}$ follows a chi-squared distribution with 2ν degrees of freedom and $\nu = n - r - s - 1$, where $S_i = \frac{2(s+1)(X_{i(n-s)}-Y_i)+\sum_{j=r+2}^{n-s-1}(X_{i(j)}-Y_i)}{\nu}$, $i = 1, \ldots, k$. From Wu (2017), the p.d.f. and c.d.f. of W_i are obtained as

$$f(w_i) = \frac{2\,\nu\,n!}{r!(n-r-1)!}\sum_{j=0}^{r}\binom{r}{j}(-1)^j(1+(2n-2r+2j)w_i)^{-(\nu+1)} \quad \text{and}$$

$$F(w_i) = 1 - \frac{n!}{r!(n-r-1)!}\sum_{j=0}^{r}\binom{r}{j}(-1)^j\frac{(1+(2n-2r+2j)w_i)^{-\nu}}{n-r+j}, \quad w_i > 0,\ i = 1, \ldots, k.$$

Let $F^*_{n,r,\nu}(P^*)$ be the $100P^{th}$ percentile of distribution of $W_i, i = 1, \ldots, k$ and then $F^*_{n,r,\nu}(P^*)$ is the solution of the following equation

$$F(w_i) = 1 - \frac{n!}{r!(n-r-1)!}\sum_{j=0}^{r}\binom{r}{j}(-1)^j\frac{(1+(2n-2r+2j)w_i)^{-\nu}}{n-r+j} = P^*. \tag{1}$$

The $100P^{th}$ percentile of distribution of W_i can be solved numerically. Let $c^* = \max_{i=1,\ldots,k} 2\nu S_i$. Using the inequality given in Lam (1987, 1988), Wu (2017) proposed the one-sided and two-sided confidence intervals summarized in the following Theorem.

Theorem 1. *For a given* $0 < P^* < 1$, *we have*

(a) $P(\theta_i - \theta_k \le Y_i - Y_k + c^*s^*_U,\ i = 1, \ldots, k-1) \ge P^*$ *if* $s^*_U = F^*_{n,r,\nu}(P^*)$, *where* $F^*_{n,r,\nu}(P^*)$ *is the solution of equation* (1).
Thus, $(-\infty, Y_i - Y_k + c^*s^*_U)$ *is a set of upper confidence intervals for* $\theta_i - \theta_k$ *with confidence coefficient* P^*, $i = 1, \ldots, k-1$.

(b) $P(\theta_i - \theta_k \ge Y_i - Y_k - c^*s^*_L,\ i = 1, \ldots, k-1) \ge P^*$ *if* $s^*_L = F^*_{n,r,\nu}(P^{*\frac{1}{k-1}})$, *where* $F^*_{n,r,\nu}(P^{*\frac{1}{k-1}})$ *is the solution of equation* (1)*by replacing* P^* *by* $P^{*\frac{1}{k-1}}$.
Thus, $(Y_i - Y_k - c^*s^*_L, \infty)$ *is a set of lower confidence intervals for* $\theta_i - \theta_k$ *with confidence coefficient* P^*, $i = 1, \ldots, k-1$.

(c) $P(Y_i - Y_k - c^*s^*_t \le \theta_i - \theta_k \le Y_i - Y_k + c^*s^*_t,\ i = 1, \ldots, k-1) \ge P^*$ *if* $s^*_t = F^*_{n,r,\nu}(P^{*\frac{1}{k}})$.
Thus, $(Y_i - Y_k \pm c^*s^*_t)$ *is a set of simultaneous two-sided confidence intervals for* $\theta_i - \theta_k$ *with confidence coefficient* P^*, $i = 1, \ldots, k-1$.

The critical values s_U^*, s_L^* and s_t^* can be obtained by solving equation (1) by replacing P^* by P^*, $P^{*\frac{1}{k-1}}$ and $P^{*\frac{1}{k}}$ respectively and the results are listed in Table 1 in Wu (2017) for $k = 3,4,6$, $n = 20,30,60$, $r = 1(1)3$, s = 0(1)2 and $P^* = 0.90, 0.95$. The confidence interval length is $L_1 = 2c^*s_t$, where $c^* = \max_{i=1,\ldots,k} 2\nu S_i$, $s_t^* = F^*_{n,r,\nu}(P^{*\frac{1}{k}})$ and $s_t^* = F^*_{n,r,\nu}(P^{*\frac{1}{k}})$ is the $100(P^{*\frac{1}{k}})^{th}$ percentile of the distribution of W_i, $i = 1, \ldots, k-1$. It can be seen that the value c^* is very crucial and is proportional to confidence interval length L_1. To obtain shorter confidence interval lengths and smaller coverage probabilities, the value of c^* is changed appropriately to $c_i^* = \max_{l\neq i, l=1,\ldots,k} 2\nu S_l$, $c_i = 2\nu S_i$ for the simultaneous one-sided and two-sided confidence intervals (SCI) for $\theta_i - \theta_k$, $i = 1, \ldots, k-1$ in the following theorem:

Theorem 2. *For a given* $0 < P^* < 1$, *let* $c_i^* = \max_{l\neq i, l=1,\ldots,k} 2\nu S_l$, $c_i = 2\nu S_i$, *and* s_U^*, s_L^* and s_t^* are defined in Theorem 1, *we have*

(a) $P(\theta_i - \theta_k \le Y_i - Y_k + c_i^* s_U^*,\ i = 1, \ldots, k-1) \ge P^*$. *Thus,* $(-\infty, Y_i - Y_k + c_i^* s_U^*)$ *is a set of upper confidence intervals for* $\theta_i - \theta_k$ *with confidence coefficient* P^*, $i = 1, \ldots, k-1$.

(b) $P(\theta_i - \theta_k \ge Y_i - Y_k - c_i s_L^*,\ i = 1, \ldots, k-1) \ge P^*$. *Thus,* $(Y_i - Y_k - c_i s_L^*, \infty)$ *is a set of lower confidence intervals for* $\theta_i - \theta_k$ *with confidence coefficient* P^*, $i = 1, \ldots, k-1$.

(c) $P(Y_i - Y_k - c_i s_t^* \le \theta_i - \theta_k \le Y_i - Y_k + c_i^* s_t^*,\ i = 1, \ldots, k-1) \ge P^*$.

Thus, $(Y_i - Y_k - c_i, Y_i - Y_k + c_i^* s_t^*)$ *is a set of simultaneous two-sided confidence intervals for* $\theta_i - \theta_k$ *with confidence coefficient* P^*, $i = 1, \ldots, k-1$.

Proof of Theorem 2.

For (a), we have

$$\begin{aligned} &P\big(\theta_i - \theta_k \le Y_i - Y_k + c_i^* s_U^*,\ i = 1, \ldots, k-1\big) \\ &\quad = P\big(2\nu S_i W_l \ge 2\nu S_k W_k - c_i^* s_U^*,\ i = 1, \ldots, k-1\big) \\ &\quad \ge P\big(2\nu S_i W_i \ge c_i^* W_k - c_i^* s_U^*,\ i = 1, \ldots, k-1\big) \end{aligned}$$

Table 1. Coverage probabilities based on the doubly censored sample under $(\sigma_1, \sigma_2, \sigma_3, \sigma_4) =$ (1.0,1.0,1.0,1.0).

k = 4, $P^* = 0.90$			Lower		upper		two				
m	*r*	*s*	old	new	old	new	old	new	L_1	L_3	L_3/L_1
20	1	1	0.995	0.976	0.985	0.980	0.995	0.980	0.859	0.568	0.660
	2	1	0.998	0.986	0.993	0.990	0.998	0.988	1.190	0.783	0.658
	1	2	0.997	0.982	0.989	0.985	0.997	0.985	0.921	0.608	0.660
30	1	1	0.992	0.974	0.979	0.975	0.991	0.976	0.507	0.343	0.676
	2	1	0.996	0.984	0.989	0.986	0.996	0.987	0.684	0.461	0.674
	1	2	0.994	0.979	0.982	0.978	0.993	0.981	0.529	0.357	0.676
k = 4, $P^* = 0.95$			Lower		upper		two				
m	*r*	*s*	old	new	old	new	old	new	L_1	L_3	L_3/L_1
20	1	1	0.998	0.989	0.995	0.993	0.998	0.991	1.015	0.670	0.660
	2	1	1.000	0.994	0.998	0.996	1.000	0.995	1.385	0.911	0.658
	1	2	0.999	0.992	0.997	0.995	0.999	0.994	1.089	0.719	0.660
30	1	1	0.997	0.988	0.992	0.990	0.997	0.990	0.593	0.401	0.676
	2	1	0.999	0.993	0.996	0.995	0.999	0.994	0.786	0.530	0.674
	1	2	0.998	0.990	0.994	0.992	0.998	0.992	0.619	0.418	0.676

since $c_i^* \geq 2\nu S_k$,

$$\geq P\big(0 \geq c_i^* W_k - c_i^* s_U^*,\ i = 1, \ldots, k-1\big)$$
$$= P\big(W_k \leq s_U^*\big)\big(\text{Cancelling } c_i^*\big) = P^*.$$

Then we have s_U^* is the $100P^{th}$ percentile of distribution of W_i and the proof is thus obtained. For (b), we have

$$P\big(\theta_i - \theta_k \geq Y_i - Y_k - c_i s_L^*,\ i = 1, \ldots, k-1\big)$$
$$= P\big(2\nu S_k W_k \geq 2\nu S_i W_i - 2\nu S_i s_L^*,\ i = 1, \ldots, k-1\big)$$
$$\geq E_{S_1,\ldots,S_k} P\big(0 \geq 2\nu S_i W_i - 2\nu S_i s_L^*,\ i = 1, \ldots, k-1\big) \text{ (since } 2\nu S_i W_i > 0)$$
$$= P\big(W_i \leq s_L^*,\ i = 1, \ldots, k-1\big) \text{ (Cancelling } 2\nu S_i) = P\big(W_i \leq s_L^*\big)^{k-1} = P^*.$$

Solving the above equation, then we have $s_L^* = F^*_{n,r,\nu}(P^{*\frac{1}{k-1}})$ and the proof is thus obtained. For (c), combining (a) and (b), we have

$$P\left(Y_i - Y_k - c_i s_t^* \leq \theta_i - \theta_k \leq Y_i - Y_k + c_i^* s_t^*,\ i = 1, \ldots, k-1\right)$$
$$= E_{S_1,\ldots,S_k} P\left(2\nu S_k W_k \geq 2\nu S_i W_i - 2\nu S_i s_t^*\right.$$
$$\left. \cap\ 2\ \nu S_i W_i \geq c_i^* W_k - c_i^*\ s_t^*,\ i = 1, \ldots, k-1\right)$$
$$\geq P\left(W_i \leq s_t^*,\ W_k \leq s_t^*,\ i = 1, \ldots, k-1\right) = P(W_i \leq s_t^*)^k = P^*.$$

Solving the above equation, then we have $s_t^* = F^*_{n,r,\nu}(P^{*\frac{1}{k}})$ and the proof is thus obtained. □

The modified procedures proposed in Theorem 2 is called the new method. Let L_3 be the average length of k-1 two-sided confidence intervals for $\theta_i - \theta_k$ based on the new method, then we have $L_3 = (\ddot{c} + \bar{c})s_t^*$, where $\ddot{c} = \sum_{i=1}^{k-1} c_i^*/(k-1)$ and $\bar{c} = \sum_{i=1}^{k-1} c_i/(k-1)$. The expected length of $L_1 = 2c^* s_t^*$ and L_3 are given by $EL_1 = 2E(c^*)s_t^* = 2E(\max_{i=1,\ldots,k}\ 2\nu S_i)s_t^*$ and

$EL_3 = (E\ddot{c} + E\bar{c})s_t^* = (\sum_{i=1}^{k-1} E(\max_{l\neq i, l=1,\ldots,k}\ 2\nu S_l)/(k-1) + \sum_{i=1}^{k-1} E(2\nu S_i)/(k-1))$ $s_t^* = \sum_{i=1}^{k-1} E(\max_{l\neq i, l=1,\ldots,k}\ 2\nu S_l)/(k-1)\, s_t^* + +\bar{\sigma} s_t^*$, where $\bar{\sigma} = \sum_{i=1}^{k-1} \sigma_i/(k-1)$. The distribution of $\max_{l\neq i, l=1,\ldots,k}\ 2\nu S_l/\sigma_i$ is given by $(\int_0^x \frac{t^{\nu-1}e^{-\frac{t}{2}}}{\Gamma(\nu)2^\nu} dt)^{k-1}$. Then the expected value of $E(\max_{l\neq i, l=1,\ldots,k}\ 2\nu S_l)$ is given by $\eta_i = (k-1)\sigma_i \int_0^\infty (\int_0^x \frac{t^{\nu-1}e^{-\frac{t}{2}}}{\Gamma(\nu)2^\nu} dt)^{k-2} \frac{x^\nu e^{-\frac{x}{2}}}{\Gamma(\nu)2^\nu} dx$. Then the expected length for L_3 is derived as $EL_3 = (\bar{\eta} + \bar{\sigma})s_t^*$, where $\bar{\eta} = \sum_{i=1}^{k-1} \eta_i/(k-1)$.

Since $\max(2\nu S_k, 2\nu S_i) < \max_{i=1,\ldots,k}\ 2\nu S_i$, the ratio of expected length of L_3 over L_1 must be less than one and results in a more conservative coverage probability for procedures in Wu (2017) (called the old procedures) than the proposed procedures (called the new procedures).

When the scale parameters are unequal and known, the unbiased estimator S_i of σ_i is replaced by σ_i throughout Theorem 2 and the statistic W_i is replaced by the statistic $T_i/(2\nu)$ which is r+1 order statistic of n i.i.d. standard exponential distribution having p.d.f given in (1) divided by 2ν. Let $\tilde{c} = 2\nu\sigma_k$, $\tilde{c}_i = 2\nu\sigma_i$ and then Theorem 2 is modified as the following Theorem.

Theorem 3. *Let* $\tilde{c}_i^* = \max_{l\neq i, l=1,\ldots,k}\ 2\nu\sigma_l$, $\tilde{c}_i = 2\nu\sigma_i$, $\nu = n - r - s - 1$ *and* $F_{2(r+1),2(n-r)}(P^*)$ *be the right-tailed* P^* *percentile of F distribution with parameters* $2(r+1)$ *and* $2(n-r)$. *For a given* $0 < P^* < 1$, *when the scale parameters are unequal and known, we have*

(a) $P(\theta_i - \theta_k \leq Y_i - Y_k + \tilde{c}_i^* s_U,\ i = 1, \ldots, k-1) \geq P^*$
if $s_U = \ln(\frac{r+1}{n-r} F_{2(r+1),2(n-r)}(P^*) + 1)/(2\nu)$.

Thus, $(-\infty, Y_i - Y_k + \tilde{c}_i^* s_U)$ *is a set of upper confidence intervals for* $\theta_i - \theta_k$ *with confidence coefficient* P^*, $i = 1, \ldots, k-1$.

(b) $P(\theta_i - \theta_k \geq Y_i - Y_k - \tilde{c}_i s_L,\ i = 1, \ldots, k-1) \geq P^*$

if $s_L = \ln(\frac{r+1}{n-r} F_{2(r+1),2(n-r)}(P^{*\frac{1}{k-1}}) + 1)/(2\nu)$.

Thus, $(Y_i - Y_k - \tilde{c}_i s_L, \infty)$ *is a set of lower confidence intervals for* $\theta_i - \theta_k$ *with confidence coefficient* P^*, $i = 1, \ldots, k-1$.

(c) $P(Y_i - Y_k - \tilde{c}_i s_t \leq \theta_i - \theta_k \leq Y_i - Y_k + \tilde{c}_i^* s_t,\ i = 1, \ldots, k-1) \geq P^*$

if $s_t = \ln(\frac{r+1}{n-r} F_{2(r+1),2(n-r)}(P^{*\frac{1}{k}}) + 1)/(2\nu)$.

Thus, $(Y_i - Y_k - \tilde{c}_i s_t, Y_i - Y_k + \tilde{c}_i^* s_t)$ *is a set of simultaneous two-sided confidence intervals for* $\theta_i - \theta_k$ *with confidence coefficient* P^*, $i = 1, \ldots, k-1$.

Proof. Similar to the proof of Theorem 2, the W_i in Theorem 2 is replaced by $T_i/(2\nu)$ for this proof. □

The critical values s_U, s_L and s_t are the P^*, $P^{*\frac{1}{k-1}}$ and $P^{*\frac{1}{k}}$ percentile of the distribution of $T_i/(2\nu)$ and are derived in Wu (2017) and the critical values s_U, s_L and s_t are listed in Table 2 of Wu (2017) for k = 3,4,6, n = 20,30,60, r = 1(1)3,s = 0(1)2 and P^* = 0.90, 0.95. Let L_4 be the confidence length of the two-sided confidence intervals for $\theta_i - \theta_k$ by the one-sample procedure when the scale parameters are unequal and known, then we have $L_4 = (\tilde{c} + \bar{c})s_t$, where $\tilde{c} = \sum_{i=1}^{k-1} \tilde{c}_i^*/(k-1)$ and $\bar{c} = \sum_{i=1}^{k-1} \tilde{c}_i/(k-1)$. Likewise, L_4 is less than the confidence length $L_4 = 4\nu s_t$ proposed in Theorem 4 of Wu (2017).

3. Simulation comparison

The difference between the proposed new procedures and the old procedures in Wu (2017) is that the proposed procedures have shorter confidence lengths and better confidence coverage probabilities which are closer to the nominal confidence coefficients compared to the old procedures. In order to demonstrate this difference, a simulation study is conducted to find the lower, upper and two-sided confidence intervals for $\theta_i - \theta_k$, $i = 1, \ldots, k-1$ using two procedures under P^* = 0.90, 0.95, m = 20,30, (r,s) = (1,1),(2,1),(1,2) with various structures

Table 2. Coverage probabilities based on the doubly censored sample under $(\sigma_1, \sigma_2, \sigma_3, \sigma_4)$ = (1.0,2.0,3.0,4.0).

k = 4, P^* = 0.90			Lower		upper		two				
m	r	s	old	new	old	new	old	new	L_1	L_3	L_3/L_1
20	1	1	0.999	0.989	0.946	0.943	0.989	0.980	2.834	1.568	0.553
	2	1	1.000	0.995	0.958	0.955	0.992	0.987	3.911	2.160	0.552
	1	2	0.999	0.992	0.957	0.955	0.992	0.985	3.036	1.680	0.553
30	1	1	0.999	0.988	0.938	0.936	0.985	0.976	1.719	0.958	0.557
	2	1	0.999	0.994	0.949	0.948	0.989	0.984	2.313	1.288	0.557
	1	2	0.999	0.990	0.946	0.944	0.988	0.981	1.791	0.998	0.557
k = 4, P^* = 0.95			Lower		upper		two				
m	r	s	old	new	old	new	old	new	L_1	L_3	L_3/L_1
20	1	1	1.000	0.995	0.977	0.975	0.995	0.991	3.344	1.850	0.553
	2	1	1.000	0.998	0.983	0.981	0.997	0.994	4.546	2.511	0.552
	1	2	1.000	0.996	0.982	0.981	0.997	0.994	3.591	1.986	0.553
30	1	1	1.000	0.994	0.971	0.970	0.993	0.989	2.010	1.121	0.557
	2	1	1.000	0.997	0.976	0.975	0.995	0.993	2.659	1.481	0.557
	1	2	1.000	0.996	0.976	0.975	0.995	0.991	2.096	1.168	0.557

of scale parameters $(\sigma_1, \sigma_2, \sigma_3, \sigma_4) = (1.0,1.0,1.0,1.0),(1.0,2.0,3.0,4.0)$ for $k = 4$ using 500,000 simulation runs in this section. The coverage probabilities of wo procedures and the confidence lengths L_1, L_3 and the length ratio L_3/L_1 are listed in Tables 1–2.

From the simulation results, it can be seen that both of old procedures and new ones have coverage probabilities higher than the nominal confidence coefficients. The confidence interval length of the new procedures is a increasing function of total censored number of sample $r+s$ for any given m and P^*, a decreasing function of m for any given r, s and P^*a nd a increasing function of P^* for any given n, r and s. The confidence interval length of the new one-stage multiple comparison procedures with a control over the old ones is reduced from 66% to 67.6% under $(\sigma_1, \sigma_2, \sigma_3, \sigma_4) = (1.0,1.0,1.0,1.0)$ and from 55.2% to 55.7% under $(\sigma_1, \sigma_2, \sigma_3, \sigma_4) = (1.0,2.0,3.0,4.0)$. The reduction increases when the scale parameters are more dispersed. From this table, the proposed new procedures have shorter confidence length with coverage probability closer to the nominal confidence coefficients. It's also found that the coverage probability is a increasing function of $r+s$ for any given m and P^* and a decreasing function of m for any given r, s and P^*.

4. Example

The duration of remission for four drugs as the treatment of leukemia in Wu and Wu (2005) is used to illustrate our proposed new one-stage multiple comparison procedures with a control given in Theorem 2 based on doubly censored sample. Twenty patients are receiving one of the four drugs and the duration of remission are recorded (See the raw data in Wu and Wu (2005)). The longer guarantee duration of remission time (location parameter of two-parameter exponential distribution) is desired for this example. The likelihood ratio asymptotic χ^2 test (Lawless (2003)) had shown a significant difference among the scale parameters of four exponential distributions. Consider the doubly censoring schemes of $(r,s) = (1,1)$, $(1,2)$, $(2,1)$. Applying Theorem 2, the required statistics and critical values of s_U^*, s_L^* and s_t^* for $P^* = 0.90,\ 095$ are summarized in Table 3.

Using parts (a) and (b) of Theorem 2, we can obtain the one-sample one-sided confidence bounds with confidence coefficients 0.90 and 0.95 given in Table 4. From Table 4, drugs 1, 2

Table 3. The required statistics and critical values s_U^*, s_L^* and s_t^* for $P^* = 0.90,\ 095$.

(r,s)	Statistics	Drug1	Drug2	Drug3	Drug4	P^*	s_U^*	s_L^*	s_t^*
(1,1)	Y_i	1.0340	2.2140	3.1400	4.5130	0.90	0.006394	0.008870	0.009541
	S_i	1.3001	1.6696	3.5052	4.4348	0.95	0.008000	0.010568	0.011263
	c_i	44.2034	56.7664	119.1768					
	c_i^*	150.7832	150.7832	150.7832					
	Y_i-Y_4	− 3.4790	− 2.2990	− 1.3730					
(2,1)	Y_i	1.1090	2.2390	3.1470	4.5330	0.90	0.009725	0.013016	0.013903
	S_i	1.2970	1.7459	3.7164	4.6894	0.95	0.011865	0.015255	0.016170
	c_i	44.0980	59.3606	126.3576					
	c_i^*	159.4396	159.4396	159.4396					
	Y_i-Y_4	− 3.4240	− 2.2940	− 1.3860					
(1,2)	Y_i	1.0340	2.2140	3.1400	4.5130	0.90	0.006830	0.009498	0.010224
	S_i	1.2933	1.4486	3.5129	4.2798	0.95	0.008560	0.011335	0.012088
	c_i	43.9722	49.2524	119.4386					
	c_i^*	145.5132	145.5132	145.5132					
	Y_i-Y_4	− 3.4790	− 2.2990	− 1.3730					

Table 4. The 90% and 95% one-sided confidence intervals for three drugs compared with the control drug (drug 4).

		$(-\infty, Y_i - Y_k + c_i^* s_U^*), (Y_i - Y_k - c_i s_L^*, \infty)$		
P^*	(r,s)	$\theta_1 - \theta_4$	$\theta_2 - \theta_4$	$\theta_3 - \theta_4$
90%	(1,1)	(−∞,−2.5149), (−3.8711,∞)	(−∞,−1.3349),(−2.8025,∞)	(−∞,−0.4089), (−2.4301,∞)
	(2,1)	(−∞,−1.9284), (−3.9980,∞)	(−∞,−0.7484), (−3.0666,∞)	(−∞,0.1776), (−3.0307,∞)
	(1,2)	(−∞,−0.4089), (−3.9286,∞)	(−∞,−1.3636), (−2.8026,∞)	(−∞,−0.4376), (−2.5941,∞)
95%	(1,1)	(−∞,−2.2727), (−3.9461,∞)	(−∞,−2.8989), (−3.8925,∞)	(−∞,−0.1667), (−2.6324,∞)
	(2,1)	(−∞,−1.6435), (−4.0967,∞)	(−∞,−0.5135), (−3.1995,∞)	(−∞,0.3945), (−3.3136,∞)
	(1,2)	(−∞,−2.3067), (−3.9774,∞)	(−∞,−1.1267), (−2.8573,∞)	(−∞,−0.2007), (−2.7268,∞)

Table 5. The 90% and 95% two-sided confidence intervals for three drugs compared with the control drug (drug 4).

		$(Y_i - Y_k - c_i s_t^*, Y_i - Y_k + c_i^* s_t^*)$					
P^*	(r,s)	$\theta_1 - \theta_4$	$\theta_2 - \theta_4$	$\theta_3 - \theta_4$	L_1	L_3	L_3/L_1
90%	(1,1)	(−3.9007,−2.0404)	(−2.8406,−0.8604)	(−2.5101,0.0656)	2.8772	2.1387	0.743
	(2,1)	(−4.0371, −1.3377)	(−3.1193, −0.2077)	(−3.1428, 0.7003)	4.1726	3.1514	0.755
	(1,2)	(−3.9286, −2.0788)	(−2.8026, −0.8988)	(−2.5941, 0.0272)	2.8004	2.1250	0.759
95%	(1,1)	(−3.9769, −1.7807)	(−2.9384, −0.6007)	(−2.7153, 0.3253)	3.3966	2.5248	0.743
	(2,1)	(−4.1371, −0.9975)	(−3.2539, 0.1325)	(−3.4292, 1.0405)	4.8530	3.6652	0.755
	(1,2)	(−4.0105, −1.8235)	(−2.8944, −0.6435)	(−2.8168, 0.2825)	3.3110	2.5124	0.759

are selected in a worse than the control (drug 4) subset with the probability of correct selection being at least 0.90 and 0.95 the corresponding upper confidence bounds are less than zero. For drug 3, it is selected in a worse than the control subset for $(r,s) = (1,1), (1,2)$ with $P^* = 0.90$ and for $(r,s) = (1,1),(1,2)$ with $P^* = 0.95$. We can conclude that drug 3 is is not significantly different from drug 4 for $(r,s) = (2,1)$ with $P^* = 0.90$ and for $(r,s) = (2,1)$ with $P^* = 0.95$.

Using parts (c) of Theorem 2, we can obtain the two-sided confidence bounds with confidence coefficients 0.90 and 0.95 given in Table 5. Since the upper limits of all simultaneous two-sided confidence intervals are less than zero, drugs 1, 2 are worse than drug 4 (the control population) for all censoring schemes except the case of $(r,s) = (2,1)$ with confidence coefficients 0.95. However, drugs 3 is not significantly different from drug 4 for all cases of censoring with confidence coefficients 0.90 and 0.95. Furthermore, $L_3 = (\ddot{\bar{c}} + \bar{c})s_t^*$ is increasing when r is increasing for given s and P^* or when s is decreasing for given r and P^* or when P^* is increasing for given r and s.

5. Conclusion

In practical world, the experimenters or researcher would like to conduct a multiple comparison procedure for comparing several treatments simultaneously with a control or standard treatment. For comparing the location parameters of exponential distributions when scale parameters are unequal and unknown, the two-stage procedures proposed by Lam and Ng (1990) can be employed to resolve this problem. However, the two-stage multiple comparison procedure with a control requires additional samples at the second stage, which can be large due to heterogeneous variances. The additional sample at the second stage may not

be collected because there may be an early termination due to time limitation and budgetary reasons in an experiment. In this situation, a one-sample multiple comparison procedure with a control is a good remedy for two-stage procedure when the second stage sample is not available. In this paper, we proposed the multiple comparison procedures with a control in Theorem 2 for doubly censored samples under heteroscedasticity. The simulation results show that the proposed methods have shorter confidence length with coverage probability closer to the nominal ones. Therefore, the proposed methods are recommended for the use for experimenters to make the multiple comparison procedures with the control for the location parameters of exponential distributions based on the doubly censored samples.

Acknowledgments

The author wish to thank an associate editor and referees for their careful reading and valuable suggestions so that the article is more readable and applicable. The author's research was supported by The National Science Council NSC101-2118-M-032-003-, NSC 102-2118-M-032-006- and Ministry of Science and Technology MOST 103-2118-M-032 -004 -MY2 and MOST 105-2118-M-032 -005 -MY2 in Taiwan, ROC.

Funding

Ministry of Science and Technology, Taiwan MOST 103-2118-M-032 -004 -MY2 and MOST 105-2118-M

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