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Determining a subsidy rate for Taiwan's recycling glass industry: an application of bi-level programming

H-S Shih[1], C-B Cheng[2*], U-P Wen[3], Y-C Huang[3] and M-Y Peng[1]

[1]Graduate Institute of Management Sciences, Tamkang University, Taipei, Taiwan; [2]Tamkang University, Taipei, Taiwan; and [3]National Tsing Hua University, Hsinchu, Taiwan

This study attempts to optimize the operations of the Recycling Fund Management Board (RFMB), founded by the Environmental Protection Administration of the R.O.C. Government (on Taiwan), through the decision of a subsidy rate for the domestic glass recycling industry. The hierarchical and interactive nature between the two parties is modelled by bi-level programming, where the RFMB plays the upper-level decision unit while the recycling industry is the lower-level counterpart. In order to solve the problem by optimization software, the bi-level formulation is transformed to a single-level problem via Karush-Kuhn-Tucker optimality conditions and is further transformed to a 0−1 mixed integer programming problem by variable substitution. The problem is solved with real-world data, and the obtained solutions are analysed and compared with the RFMB's current operations. The results suggest that the proposed approach can improve the operations of the RFMB.

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Keywords: bi-level programming problem; glass recycling industry; subsidy rate; recycling and treatment fee; 0−1 mixed integer programming; KKT conditions

1. Introduction

Lacking sufficient landfill sites and incinerator capacity due to limited land resources, the solid-waste problem has become severe in Taiwan (Bor *et al*, 2004). To prevent and solve the environmental pollution problems caused by waste materials, the Environmental Protection Administration (EPA) of Taiwan established the Recycling Fund Management Board (RFMB) in 1998 to monitor and manage the operations of the recycling and reuse of waste materials, and to enhance recycling efficiency.

The RFMB has worked to establish a recycling, clearance, and disposal system for containers, end-of-life vehicles, waste tires, lubricating oil, dry cell and lead batteries, pesticide containers, electric and electronic products, and IT products. The RFMB's operations are conducted through the joint participation of industry representatives, the fee rate inspection committee, an auditing and verifying organization, the recycling industry, the government, and the general public. Manufacturers of designated materials or containers pay recycling fees (ie product charge) based on fee rates derived by the fee rate inspection committee. The money is then channeled into the recycling management fund and used to increase recycling incentives, part of which is used as the subsidy given to recycling industries to enhance their recycling ratio. Confirmation of the amount of resources actually recycled is conducted by an auditing and verifying organization selected by the EPA. Fee rates are determined based on materials, volume, mass, recycling and reuse value, and recycling, clearance, and disposal rates of the prior year.

The determination of the recycling and treatment fee and the subsidy rate has a great impact on manufacturers, recycling industries, as well as the RFMB. In particular, the recycling and treatment fee is the major income of the RFMB and is the source of the subsidy given to recycling industries. A greater subsidy would increase the recycling incentive of the industries. However, a too high recycling and treatment fee would discourage the manufacturers from reporting their true production and decrease social welfare as well.

The RFMB has four times revised its computation formula for fee rate determination since its establishment in 1998. Nevertheless, its fairness is still questioned by members involved in this recycling system. One major concern is that the computation of the fee rate and the

*Correspondence: *C-B Cheng, Department of Information Management, Tamkang University, 151 Ying-chuan Road, Tamsui, Taipei 25137, Taiwan.*
E-mail: cbcheng@mail.tku.edu.tw

subsidy rate is purely from the perspective of the RFMB without considering the interests of other members in the system. In fact, problems have been found with the RFMB's current operations in the glass recycling industry. For example, the low recycling ratio in this industry has caused a low utilization of equipment, and hence it is unable to reduce the recycling cost. This high cost boosts the subsidy rate that the RFMB has to pay to the glass recycling industry, and hence gives the RFMB no choice but to increase the recycling and treatment fee charged to glass container manufacturers. Thus, the present study attempts to model the decision-making process of the subsidy rate from a system-wide perspective, in which not only the RFMB's objectives are considered, but also the interests of the recycling industry are taken into account. Among the 33 items of recyclable materials published by Taiwan's EPA, we select glass containers as our study subject for the following reason: recycled glass is a pure material without the need of disassembly operations, which simplifies the cost estimation process.

Research on the determination of product charge and subsidy rate in the recycling system is limited. Kohn (1995) presented an equilibrium model to find optimal combination of three instruments: the disposal tax on land filled waste, the excise tax on disposable goods, and the subsidy for recycled raw material. Although this model provides a formulation of the relationship between manufactured products and recycled raw material, its strong assumptions have some unrealistic features as commented by the author. Chang (2008) performed an economic analysis for determining product charge and subsidy to support the scrap tire recycling programme in Taiwan. This analysis includes an econometric estimation of the demand for vehicle tires together with their recycling and a subsequent cost–benefit analysis to compute the associated product charge given the designated levels of subsidy. The approach of Chang provides an economic assessment platform for calculating reasonable levels of subsidy and product charge and evaluating the balance between them. However, the interactions between different decision units in the recycling system are not fully considered in this approach. For instance, the determination of the subsidy in the aforementioned approach does not consider the subsequent reaction of the recycling industry, which may in turn affect the benefit of the entire system.

The subsidy rate decision made by the RFMB directly affects the incentive of the recycling industry, while on the other hand the recycling ratio of the industry in turn affects the decision of the RFMB if it attempts to improve the recycling ratio in the industry. Such a leader-follower interaction between the RFMB and the recycling industry makes the bi-level programming a suitable tool for modelling this problem. Bi-level programming is a special case of multi-level programming (MLP) which is categorized as non-convex programming and proven to be NP-hard by Ben-Ayed and Blair (1990). Multi-level or bi-level problems have the following common features (Wen and Hsu, 1991; Shih *et al*, 1996): (i) interactive decision-making units exist within a predominantly hierarchical structure; (ii) execution of decisions is sequential, from the top to a lower level; (iii) each decision unit independently optimizes its own benefits, but is affected by the actions of other units through externalities; and (iv) the external effect on a decision maker's problem can be reflected in both the objective function and the set of constraints.

Previous studies have also reported the advantages of applying bi-level programming in modelling the government–industry interaction for policy formulation. Amouzegar and Moshirvaziri (1999) presented a hazardous waste capacity planning and facility location problem where the government seeks to regulate the firms in order to maximize the social welfare and the firms respond to these regulations. They formulated a central planning model and a bi-level model, respectively, to solve the problem showing that the complex behaviour of private firms in the presence of central planning decisions can best be captured by the bi-level model. Bard *et al* (2000) also presented a bi-level programming formulation for the government to determine the level of tax credits for each final product or bio-fuel produced by the agricultural sector. The conflict inherent in this problem is that the government intends to minimize its costs subject to a given level of land usage for non-food crops while the agricultural sector expects to maximize its profits subject to the technological constraints. The authors claimed that the bi-level model can help decision makers arrive at a rational policy for encouraging bio-fuel production.

The remainder of this paper is organized as follows. Section 2 presents the bi-level formulation of the subsidy decision problem and the transformation of the original formulation to a single-level problem to make the problem solvable by optimization software. The third section offers the solutions obtained for determining the subsidy rate for the glass recycling industry. Interpretation of these solutions and their comparisons with the RFMB's current operations are provided as well. Section 4 performs a sensitivity analysis on the optimum solution of the recycling and treatment fee and discusses the assumptions of the proposed model. Finally, concluding remarks are given in Section 5.

2. Model formulation

This section presents a system optimization model by bi-level programming for the RFMB to determine the subsidy rate for Taiwan's glass recycling industry. The objective of this model is to set an appropriate subsidy rate applying to the glass container recycling industry, in which the decision-making process is considered as a leader-follower interaction between the RFMB and the industry. In order to solve the problem, the proposed model will be reformulated as a single-level problem by the Karush-Kuhn-Tucker (KKT) transformation and is further transformed to a 0−1 mixed integer programming problem by introducing variable substitution.

2.1. Bi-level programming

In this subsidy decision problem, the recycling and treatment fee and the subsidy rate are decisions of the RFMB, the reported amount of recyclable products is the decision of the designated manufacturers, and the recycled amount is determined by the glass recycling industry. As a result, there should be three decision units involved in the problem. However, due to the considerations of data availability and model solvability, the present study assumes that there are two levels of decision units in the model: the RFMB at the upper level and the recycling industry the lower level. The manufacturers' interest is implicitly considered in the constraints.

The notations used in the model are described as follows.

Decision variables:

Upper level

- C_a Subsidy rate to recycling industry for waste recycling and treatment (NT\$/kg)
- C_f Recycling and treatment fee charged to manufacturers of designated materials (NT\$/kg)
- γ The ratio of administration expense over the fund of RFMB (%)

Lower level

- α Recycling ratio of waste material (%)

Parameters:

- τ Resource recycling ratio (%), which is the ratio of recycled waste that is turned into a reusable resource
- ω Amount of declared obsolescence of the year (kg/year)
- $v(\alpha)$ Unit resource recycling value, which is defined as a function of the recycling ratio (NT\$/kg)
- C_E Unit cost of environmental effects (NT\$/kg)
- $C(\alpha)$ Total recycling cost for the year (NT\$/year) by recyclers
- C_a^* Published subsidy rate by the RFMB for waste recycling and treatment
- C_f^* Current recycling and treatment fee charged to manufacturers of designated materials by the RFMB
- L Auditing cost of RFMB allocated to glass container recycling per year (NT\$/year)
- S Predicted amount of recyclable product produced by designated manufacturers (kg/year) in the coming year
- γ^L The lower limit of γ
- γ^U The upper limit of γ

Figure 1 depicts the operations of the RFMB and its interactions with designated material manufacturers and recycling industries. The flows in the diagram indicate the amount of waste materials transition, or the monetary transition between entities. In particular, the glass container manufacturers or importers pay the recycling and treatment fee $C_f S$ to the RFMB, the produced glasses amount, ω, are consumed by customers, the amount of glasses to be recycled, $\alpha\omega$, is collected by waste disposal plants and then sent to the recyclers while the un-recycled amount, $(1-\alpha)\omega$, becomes pollution to the environment. The recyclers receive subsidy $C_a\alpha\omega$ in total from the RFMB, and sell the recycled material amounted to $\tau\alpha\omega$ to the manufacturers and obtain the payment $v(\alpha)\tau\alpha\omega$ from the manufacturers.

The model is then formulated as

$$\underset{c_f,\, c_a, \gamma}{\text{Minimize}}\ C_f S - (C_a\omega\alpha + \gamma C_f S) \tag{1}$$

where α solves:

$$\underset{\alpha}{\text{Maximize}}\ v(\alpha)\tau\omega\alpha + C_a\omega\alpha - C(\alpha) \tag{2}$$

Subject to

$$C_a\ \omega\alpha \geqslant [C(\alpha) - v(\alpha)\tau\omega\alpha] \tag{3}$$

$$C_a \leqslant C_a^* \tag{4}$$

$$C_f S \geqslant C_a\omega\alpha + \gamma C_f S \tag{5}$$

$$C_f \geqslant [C(\alpha) + \gamma C_f S - v(\alpha)\tau\omega\alpha]/S \tag{6}$$

$$C_f \leqslant C_f^* \tag{7}$$

$$0 \leqslant \alpha \leqslant 100\% \tag{8}$$

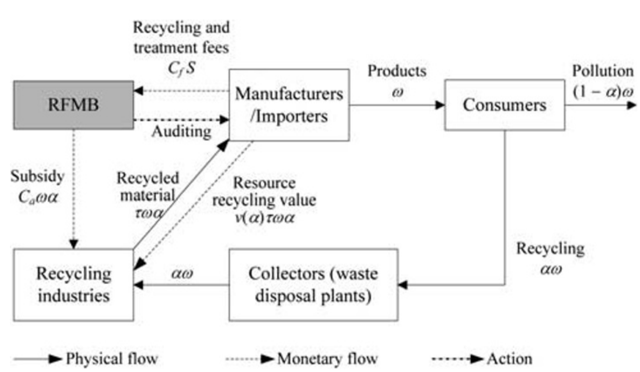

Figure 1 Operations of the RFMB.

$$\gamma^L \leqslant \gamma \leqslant \gamma^U \tag{9}$$

$$\gamma C_f S \geqslant L + C_E(1-\alpha)\omega \tag{10}$$

The objective of the upper-level decision maker, that is the RFMB, is to balance the income and expense of the RFMB fund and thus the objective function is defined as the minimization of the difference of these two terms, through setting an appropriate recycling and treatment fee and subsidy rate and by determining the administration expense budget. On the other hand, the objective of the lower-level decision maker, that is the glass recycling industry, is to maximize its profit.

Constraint (3) specifies that the subsidy rate must be able to cover (greater than or equal to) the net unit cost of waste recycling and treatment. Constraint (4) expresses the decision maker's expectation that an appropriate subsidy rate should be less than the current rate published by the RFMB, that is a cost reduction to the RFMB. The current equipment utilization of the glass recycling industry is poor due to low recycling volume. We expect that the recycling volume will increase by using the proposed model and hence reduce the recycling cost.

The income of the RFMB is the total recycling and treatment fee collected from manufacturers, and it must be abundant enough to cover its expense, that is the subsidy and the administration cost, as well as the environmental cost which occurs due to failing to fully recycle all recyclable materials, as shown in constraint (5). It is noted that though the produced amount of obsolescence is ω, the recycling and treatment fee is charged to the manufacturers based on the predicted production (S) in the coming year. This is based on the rate formulation by RFMB, where the total recycling and treatment fee caused by the produced obsolescence is shared by the future production of manufacturers.

From the system perspective, the recycling and treatment fee also has to cover the net cost of the recycling system as described by constraint (6). It is also noted that the current auditing cost of the RFMB is too high. This cost is expected to be reduced in the near future, and thus constraint (7) shows the expectation that the new recycling and treatment fee will be less than the current one. Constraint (8) expresses that the recycling ratio cannot exceed 100%. The reasonable administration expense ratio is bound by an interval $[\gamma^L, \gamma^U]$ as stated by constraint (9). By referring to the operations in European countries, this interval is set as $\gamma^L = 0.1$ and $\gamma^U = 0.3$. Constraint (10) describes that the administration expense has to support the auditing cost on glass containers, as well as the environmental cost that occurs when the recyclable material is not recycled. The environment cost occurs when glass containers are not recycled, and it is computed based on the un-recycled amount, $(1-\alpha)\omega$. The unit environmental effects cost was computed from estimated environmental costs by the RFMB based on the budget subsidized to local governments for garbage reduction, waste disposal, and resource recycling.

2.2. Parameter estimation

The resource recycling value $v(\alpha)$ in the above model is defined as the market value of recycled materials. We assume the market is perfectly competitive, and hence the price is inversely proportional to the supply. A demand curve (see Figure 2) suggested by Truett and Truett (2003) is used:

$$v(\alpha) = (d - \tau\omega\alpha)/e, \tag{11}$$

where d and e are coefficients to be estimated, and in which d/e is the interception and $-\tau/e$ is the slope of the curve, respectively. The only information available to estimate d and e is the resource recycling values obtained for years 2004 and 2005 in an investigation of the recycling industry conducted by (Wen, 2005). In which, the resource recycling values were estimated as NT$2.01/kg for 2004 and NT$3.25/kg for 2005. With these two values and the recycled amount in years 2004 and 2005, we are able to estimate d and e. However, considering the huge variation between the 2 years, the estimation of the parameters would be unreliable if these two extreme points are directly used. Thus, the values in between these two extreme points are taken and each pair of resource recycling values in 2004 and 2005 are used to estimate the parameters in the demand curve, respectively. Eleven pairs of resource recycling values in 2004 and 2005 are determined as shown in Table 1, and are used to estimate the values of d and e, respectively. The estimation results are provided in Table 1 as well.

The recycling cost, $C(\alpha)$, is defined as the cost of waste treatment by the recycling industry to the status for landfill, incineration, or selling (in the case of recycled material). Thus, this cost is in fact the production cost of the recycling industry, and the marginal cost decreases when the recycled amount increases as shown in Figure 3. For simplicity,

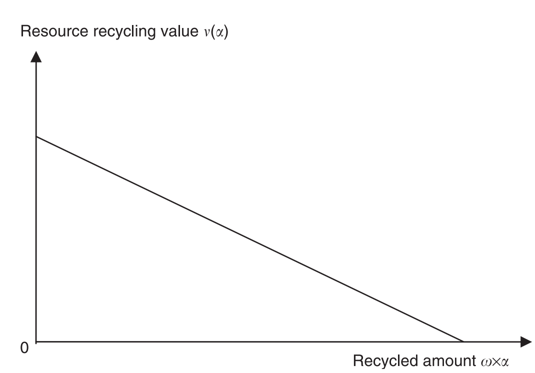

Figure 2 Demand curve of recycled materials.

the recycling cost is defined as a quadric function of the recycled amount:

$$C(\alpha) = a(\omega\alpha)^2 + b(\omega\alpha) + c, \tag{12}$$

where a, b, and c are coefficients to be estimated. The estimates of a, b, and c are obtained based on the data of production costs (consisting of fixed and variable costs) of firms in the glass recycling industry provided by Wen (2007). Table 2 presents the resultant estimates.

Table 1 Parameter estimation of resource recycling value function

Setting no.	*Pair**	*d*	*e*
1	(2.01, 3.25)	168 300 197	6 139 508
2	(2.01, 2.1)	325 983 231	84 588 779
3	(2.01, 2.2)	236 497 207	40 068 369
4	(2.01, 2.3)	208 725 683	26 251 690
5	(2.01, 2.4)	195 195 965	19 520 487
6	(2.01, 2.5)	187 188 582	15 536 715
7	(2.8, 3.25)	203 329 502	16 917 756
8	(2.9, 3.25)	219 038 847	21 751 400
9	(3.0, 3.25)	247 315 667	30 451 960
10	(3.1, 3.25)	313 294 915	50 753 267
11	(3.2, 3.25)	643 191 153	152 259 802.2

*Each pair denotes the resource recycling values assumed for 2004 and 2005.

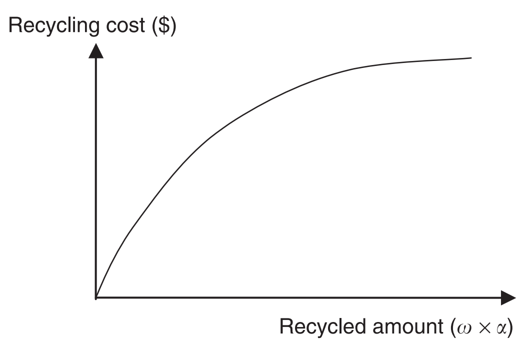

Figure 3 Cost function.

Table 2 provides a summary of the values of all parameters used in the bi-level programming model. The resource recycling ratio τ is the ratio of recycled waste that is turned into a reusable resource. Among the recycled glass containers, coloured glass cannot be reused. By a sampling, we found the ratio of coloured glass containers is about 10%, thus we estimated the value of τ as 90%. The unit environmental effect cost C_E is computed by proportioning the budget of the RFMB subsidized to local governments for garbage reduction, waste disposal, and resource recycling to the total amount of un-recycled obsolescence. The value of C_E used in our model is an average of such costs in the past 3 years.

2.3. Problem reformulation by KKT conditions

There are many methods for solving the bi-level programming problem. Among them, the KKT transformation is one of the most popular methods, for example Bard and Falk (1982), Bard (1984), Bialas and Karwan (1984), Fortuny-Amat and McCarl (1981), Ben-Ayed and Blair (1990), Onal (1993), Chen and Florian (1995), and Sinha and Sinha (2002).

The original bi-level model is thus transformed with KKT optimality conditions as:

Minimize Equation (1)

subject to:

Equations (3)–(10)

$$\begin{aligned} & v'(\alpha)\tau\omega + v(\alpha)\tau\omega + C_a\omega - C'(\alpha) - \lambda_1[C'(\alpha) \\ & \quad - v'(\alpha)\tau\omega\alpha - v(\alpha)\tau\omega - C_a\omega] \\ & \quad - \lambda_2 C_a\omega - \lambda_3[C'(\alpha) - v'(\alpha)\tau\omega\alpha \\ & \quad - v(\alpha)\tau\omega - C_a\omega] - \lambda_4 + \lambda_5 + \lambda_6 C_E\omega = 0 \end{aligned} \tag{13}$$

$$\lambda_1[C(\alpha) - v(\alpha)\tau\omega\alpha - C_a\omega\alpha] = 0 \tag{14}$$

$$\lambda_2[C_a\omega\alpha + \gamma C_f S - C_f S] = 0 \tag{15}$$

$$\lambda_3[C(\alpha) + \gamma C_f S - v(\alpha)\tau\omega\alpha - C_f S] = 0 \tag{16}$$

Table 2 Parameter value

Parameter	*Value*	*Description*
τ	90%	Estimated by this study
$[\gamma^L, \gamma^U]$	[0.1, 0.3]	Assumed by this study (based on operations in Europe)
ω	245 719 248 kg/year	Reported by the industry in 2005
a	−0.00000000087689	Coefficient of $C(\alpha)$, estimated by this study
b	2.859379698	Coefficient of $C(\alpha)$, estimated by this study
c	3 575 305.705	Coefficient of $C(\alpha)$, estimated by this study
d	Referred to Table 1	Coefficient of $v(\alpha)$, estimated by this study
e	Referred to Table 1	Coefficient of $v(\alpha)$, estimated by this study
C_E	NT$0.7229/kg	Estimated by this study
C_a*	NT$2.50/kg	Published by EPA, Taiwan, in 2002
C_f*	NT$1.55/kg	Published by EPA, Taiwan, in 2005
S	233 004 918 kg/year	Published by EPA, Taiwan, in 2006
L	NT$58 236 500/year	Estimated by this study

$$\lambda_4(\alpha - 1) = 0 \quad (17)$$

$$\lambda_5(-\alpha) = 0 \quad (18)$$

$$\lambda_6[L + C_E(1-\alpha)\omega - \gamma C_f S] = 0$$
$$\lambda_i \geqslant 0, i = 1, \ldots, 6$$
$$C_f, C_a, \gamma, \alpha \geqslant 0, \quad (19)$$

where $-\lambda_i$ are Lagrange multipliers.

Equations (14)–(19) are called complementary slackness conditions (CSC), and they can be further simplified by variable substitution (Fortuny-Amat and McCarl, 1981). The problem then becomes a 0–1 mixed-integer programming problem as shown below:

Minimize Equation (1)
subject to:
Equations (3)–(10), (13)

$$\lambda_i \leqslant M(1-\eta_i),\ i = 1, \ldots, 6 \quad (20)$$

$$C(\alpha) - v(\alpha)\tau\omega\alpha - C_a\omega\alpha \leqslant M\eta_1 \quad (21)$$

$$C_a\omega\alpha + \gamma C_f S - C_f S \leqslant M\eta_2 \quad (22)$$

$$C(\alpha) + \gamma C_f S - v(\alpha)\tau\omega\alpha - C_f S \leqslant M\eta_3 \quad (23)$$

$$\alpha - 1 \leqslant M\eta_4 \quad (24)$$

$$-\alpha \leqslant M\eta_5 \quad (25)$$

$$L + C_E(1-\alpha)\omega - \gamma C_f S \leqslant M\eta_6$$
$$\eta_i \in \{0, 1\}, i = 1, \ldots, 6$$
$$\lambda_i \geqslant 0, i = 1, \ldots, 6$$
$$C_f, C_a, \gamma, \alpha \geqslant 0, \quad (26)$$

where M is a very large number.

3. Computational results analysis

The 0–1 non-linear problem formulated earlier is solved by employing the LINGO software. Relevant data of year 2006 are used. The problem is solved with different settings of the parameters in the resource recycling value function as provided in Table 1, respectively. Solutions are obtained as shown in Table 3.

3.1. Solution analysis

The obtained solutions are analysed as follows:

3.1.1. Recycling and treatment fee. The solution of this variable for all cases is 1.55, which is in fact the upper bound of this variable. This result indicates that the upper bound of the recycling and treatment fee is a tight constraint. The recycling ratio may be improved if this constraint is released. To justify our guess, the upper bound of the recycling and treatment fee and that of the subsidy rate are removed and the problem is solved again. Table 4 presents the newly obtained solutions. Table 4 shows that the recycling and treatment fee, and the subsidy rate, are both increased (also illustrated in Figures 4 and 5, respectively). Although the recycling ratio increases on average after removing the upper bounds of the recycling and treatment fee and the subsidy rate, a paired *t*-test shows that there is no significant evidence that the recycling ratio is improved (with *p*-value 0.125). However, the lower-level objectives are improved 8.8% on average as the subsidy rates increase.

Since the established model did not consider the minimization of the recycling and treatment fee from the manufacturer's perspective, the imposition of an upper bound on this fee contributes to the balance of all parties' interests. Without such a constraint, the RFMB would impose a much greater recycling and treatment fee on manufacturers, as demonstrated by our solutions presented in Table 4. Although an increase of this fee also increases the recycler's profit, manufacturers are likely to transfer their costs to consumers. Besides, as discussed earlier, the increment of the recycling and treatment fee did not improve the recycling ratio. Thus, this study argues that an

Table 3 Problem solutions with different settings of resource recycling value function

Setting no.	*Upper-level objective*	*Lower-level objective*	C_f	C_a	α (%)	γ	$v(\alpha)$	*Unit cost*
1	0.0000	1.7726×10^7	1.55	1.43	72.10	0.30	1.44	2.72
2	0.0000	8.1692×10^7	1.55	1.30	88.50	0.30	1.54	2.69
3	0.0000	7.7063×10^7	1.55	1.43	71.79	0.30	1.94	2.72
4	0.0000	5.9168×10^7	1.55	1.43	71.79	0.30	1.90	2.72
5	0.0000	5.7440×10^7	1.55	1.37	75.06	0.30	1.50	2.72
6	0.0000	5.4281×10^7	1.55	1.39	74.24	0.30	1.48	2.72
7	0.0000	1.6473×10^8	1.55	1.38	74.43	0.30	2.29	2.72
8	0.0000	1.9298×10^8	1.55	1.43	71.79	0.30	2.77	2.72
9	0.0000	1.9137×10^8	1.55	1.37	75.25	0.30	2.66	2.72
10	0.0000	1.8420×10^8	1.55	1.03	100.00	0.30	1.82	2.66
11	0.0000	2.4752×10^8	1.55	1.23	99.71	0.16	2.78	2.66

Table 4 Solutions of problems that remove upper bounds of C_f and Ca

Setting no.	*Upper-level objective*	*Lower-level objective*	C_f	C_a	α (%)	γ	$v(\alpha)$	*Unit cost*
1	0.0000	3.9304×10^8	4.30	4.82	76.10	0.10	0.00*	2.71
2	0.0000	2.5461×10^8	2.95	2.52	100.00	0.10	1.24	2.66
3	0.0000	1.0184×10^9	9.70	6.44	100.00	0.30	0.38	2.66
4	0.0000	7.4974×10^8	5.14	5.21	65.45	0.30	2.44	2.74
5	0.0000	1.6207×10^9	12.50	1.37	82.09	0.30	0.70	2.70
6	0.0000	2.5887×10^5	2.18	1.39	80.00	0.18	0.66	2.71
7	0.0000	2.3962×10^9	14.31	1.38	91.94	0.10	0.00†	2.68
8	0.0000	1.0112×10^9	6.16	1.43	68.83	0.30	3.07	2.73
9	0.0000	6.0160×10^8	1.55	3.29	63.49	0.30	3.51	2.75
10	0.0000	7.6377×10^8	1.55	6.19	100.00	0.30	1.82	2.66
11	0.0000	2.3187×10^8	1.55	1.56	93.89	0.30	2.86	2.67

*$C_f \approx 0.001$.
†$C_f \approx 0.0003$.

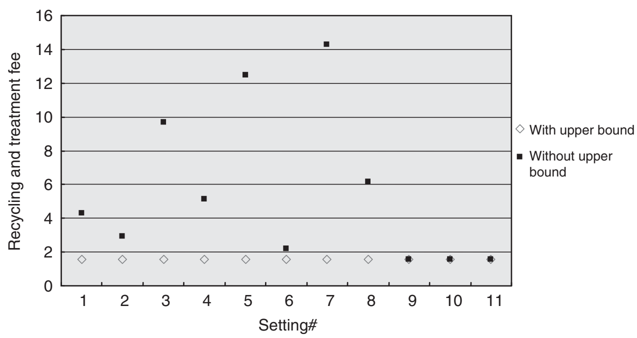

Figure 4 Values of recycling and treatment fee with/without upper bounds of C_f and Ca.

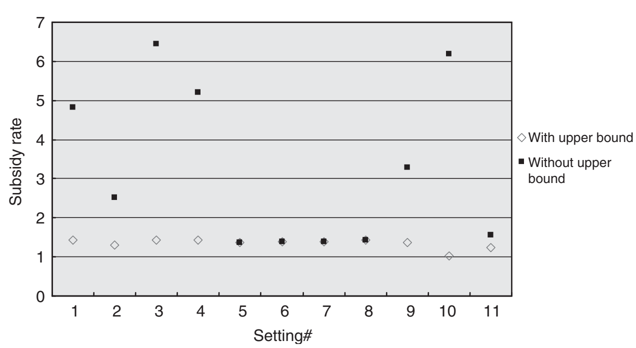

Figure 5 Values of subsidy rate with/without upper bounds of C_f and Ca.

upper bound for the recycling and treatment fee is necessary.

Since the objective of RFMB is to balance its income and expense, the increasing of recycling and treatment fee leads to the increasing of subsidy rate. However, a higher subsidy rate did not always translate to a greater recycling ratio as discussed above. Such an incompatible result is further discussed in the next subsection.

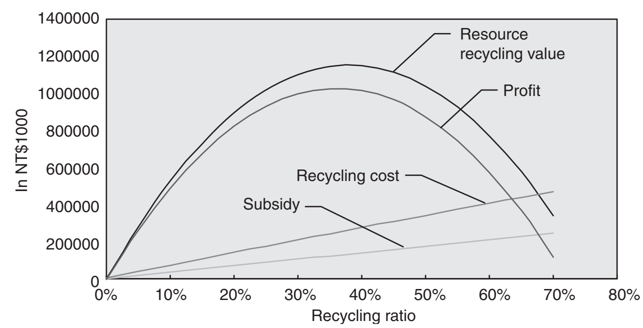

Figure 6 Resource recycling value, subsidy, and recycler's profit over recycling ratio.

3.1.2. Subsidy rate. All solutions of the subsidy rate presented in Table 3 are less than the current subsidy rate, that is NT$2.50/kg, published by the RFMB. This result also demonstrates that by employing the bi-level programming model the subsidy rate can be reduced. However, as shown in the previous subsection, the increasing of subsidy rate may not encourage the improvement of recycling ratio. This is due to the impediment by the resource recycling value function. As depicted in Figure 2, the resource recycling value decreases as the recycling ratio increases, and hence hinders the recycler from increasing the recycling ratio but holds it to maximize his profit instead. The change of the recycler's profit over the recycling ratio is presented in Figure 6, which demonstrates that the recycler would retain his recycling ratio at a certain level to maximize his own profit. If not considering the objective of RFMB, the recycler would hold his recycling ratio around 40%.

3.1.3. Recycling rate and resource recycling value. The resource recycling value is an incentive to the recycling industry. In other words, when the resource recycling value increases, the industry has a stronger will to recycle waste, and vice versa. As discussed earlier, the resource recycling value is controlled by a market mechanism, and the supply of recycled materials directly affects this value. The recycled amount of waste glass slightly decreased from 157 535 137 kg in 2004 to 149 845 248 kg in 2005 (ie decreased by 5.13%), while the resource recycling value greatly increased from NT$2.01/kg to NT$3.25/kg (ie increased by 61.69%). This statistic reveals that the resource recycling value is highly sensitive to the recycled amount. The sensitivity degree of the resource recycling value can be measured by the slope of the value function, that is $-\tau\omega/e$. Slopes of the resource recycling value function are computed and presented in Table 5. When the resource recycling value is less sensitive to the recycled amount, that is with a slope close to 0, or in other words, the resource recycling value is not decreased dramatically when the recycled amount increases, the industry is more willing to recycle the waste, that is recycling ratio increases. The relationship between the slope of the recycling resource value function and the recycling ratio is depicted in Figure 7, where a trend between the two sets of data supports the above argument.

Table 5 Slope of resource recycling value function

Setting no.	*Slope of $v(\alpha)$*	*α (%)*	*$v(\alpha)$*
1	-1.63×10^{-7}	72.10	1.44
2	-1.18×10^{-8}	88.50	1.54
3	-2.50×10^{-8}	71.79	1.94
4	-3.81×10^{-8}	71.79	1.90
5	-5.12×10^{-8}	75.06	1.50
6	-6.44×10^{-8}	74.24	1.48
7	-5.91×10^{-8}	74.43	2.29
8	-4.60×10^{-8}	71.79	2.77
9	-3.28×10^{-8}	75.25	2.66
10	-1.97×10^{-8}	100.00	1.82
11	-6.57×10^{-9}	99.71	2.78

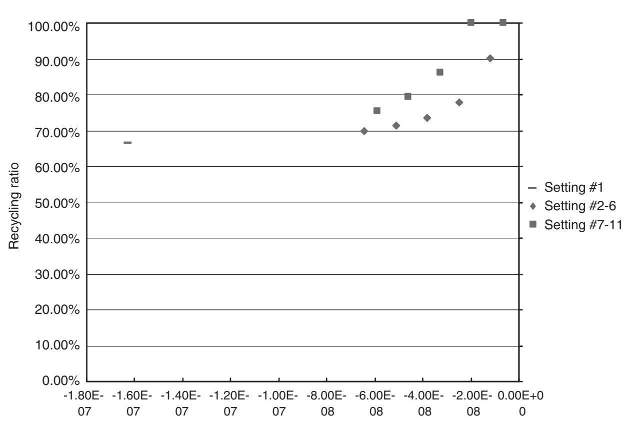

Figure 7 Relationship between slope of recycling resource value function and recycling ratio.

3.1.4. Recycling cost. The unit recycling cost is the total recycling cost divided by the recycled amount. When the recycling facilities are not fully utilized, the increase of the recycling ratio will decrease the unit recycling cost. Figure 8 depicts the relationship between the recycling ratio and the unit recycling cost, which shows an inverse proportion between these two values. In most cases, the recycling ratios are around 72%$\sim$75%, which indicate that the facility utilization in the recycling industry is still under its capacity and hence caused a higher unit recycling cost. An improvement in the recycling ratio reduces recycling firms' costs as indicated by those cases with greater recycling ratio. The unit recycling cost is an important factor in determining the subsidy rate by RFMB. Therefore, a decreasing recycling cost also benefits the RFMB for the reduction of the subsidy rate.

3.2. Comparison with current fund usage by RFMB

The solutions obtained in Table 3 are compared with the operations of the RFMB in 2006 as presented in Table 6. The expense of the RFMB exceeded its income in 2006, in which the operating expense (ie subsidy) was already greater than its income. The non-operating expense also took 68% of the fund, which is much higher than that in Europe and implies management inefficiency. By contrast, the subsidy rate solved by the proposed approach for all cases is less than the current rate (NT$2.50/kg) published by the RFMB, and the resulting operating expense takes only 70% for most cases. Furthermore, by setting the upper bound on the ratios of administration expense, we were still able to solve the problem with satisfactory solutions, where the overall expense (ie operating and non-operating expenses) can be covered by the RFMB's income for most cases. These results demonstrate that the proposed approach is able to improve the operations of the RFMB.

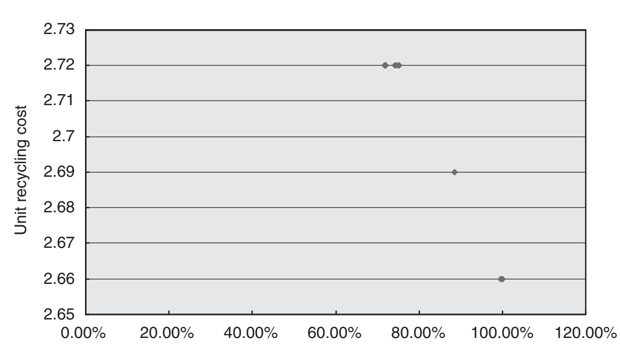

Figure 8 Relationship between the recycling ratio and unit recycling cost.

Table 6 Comparison between the proposed approach and RFMB

		C_f	C_a	α (%)	*Total income of RFMB (in NT\$1000)*	*Operating expense*		*Non-operating expense*	
						In NT\$1000	*Percentage*	*In NT\$1000*	*Percentage*
Operations of RFMB in 2006		1.55	2.50	71	361 158	436 152	121	258 659	71
Setting no.	1	1.55	1.43	72.10	361 158	253 344	70	108 347	30
	2	1.55	1.30	88.50	361 158	282 700	78	108 347	30
	3	1.55	1.43	71.79	361 158	252 255	70	108 347	30
	4	1.55	1.43	71.79	361 158	252 255	70	108 347	30
	5	1.55	1.37	75.06	361 158	252 679	70	108 347	30
	6	1.55	1.39	74.24	361 158	253 567	70	108 347	30
	7	1.55	1.38	74.43	361 158	252 387	70	108 347	30
	8	1.55	1.43	71.79	361 158	252 255	70	108 347	30
	9	1.55	1.37	75.25	361 158	253 318	70	108 347	30
	10	1.55	1.03	100.00	361 158	253 091	70	108 347	30
	11	1.55	1.23	99.71	361 158	301 358	83	57 785	16

4. Discussion

The proposed bi-level programming model considers only the decision making of the RFMB and the recycling industry, while the manufacturers' objective is not taken into account explicitly in the model, but is implemented through an upper bound on the recycling and treatment fee. In order to examine the effect of omitting the manufacturers' objective, we conduct a sensitivity analysis on the optimum of the recycling and treatment fee obtained in Table 3. The value of the recycling and treatment fee varies above and below the optimum with an increment/decrement of 0.1, and the problem is solved again with these artificial values inserted to the model. Only the results for the first setting (ie $d = 168300197$ and $e = 6139508$) are presented here (Table 7), in which no feasible solution was found when this fee is less than 1.55, and the recycling ratio was not improved either when the fee is greater than 1.55. This result indicates that setting the recycling and treatment fee to 1.55 might be the optimum to this case.

Table 7 Sensitivity analysis on recycling and treatment fee

C_f	C_a	α (%)	γ (%)	$v(\alpha)$	$C(\alpha)$
1.15	No feasible solution				
1.25	No feasible solution				
1.35	No feasible solution				
1.45	No feasible solution				
1.55	1.43	72.07	30	1.45	2.7243
1.65	1.61	67.94	30	2.94	2.7344
1.75	1.78	65.30	30	3.89	2.7410
1.85	1.93	63.66	30	4.48	2.7451
1.95	1.77	73.07	30	1.09	2.7218
2.05	2.50	54.43	29	7.81	2.7688

Recycling cost is a critical factor in determining the subsidy rate in both the RFMB's rate formula and our proposed model. This cost in fact consists of two parts: the cost of *recycled and treated waste* and the cost paid to the *waste collectors*. The recycling cost is also a factor in the RFMB's fee formula. This cost is obtained through a survey every year, which is time-consuming and costly. The recycling cost is considered as a quadratic function and its parameters are estimated based on both surveyed and historical data. The use of this cost function not only saves the time and cost in estimating the cost of recycled and treated waste, but also enables the projection of future cost.

The present study assumed a linear resource recycling value function and estimated its parameters with the historical resource recycling values investigated by a project granted by the RFMB (Wen, 2005). However, the accuracy and the size of the sample data of the historical resource recycling values are limited. The available data of historical resource recycling values are the ones of years 2004 and 2005. This small amount of data created difficulty in accurately estimating the resource recycling value function. Furthermore, the assumption of linearity of the function may oversimplify the demand-supply relation in practice. A more elaborate approach to establish the resource recycling value model would be required in our future study.

5. Concluding remarks

The present study has proposed a new formulation for solving the subsidy rate decision problem for the RFMB of Taiwan to determine such a rate as applied to the domestic glass container recycling industry. The objectives of the RFMB are to balance its income and expense and to improve the recycling ratio, while the objective of the industry is to maximize its profit. Considering the hierarchical and interactive nature between the two parties, this study has employed bi-level programming to model the problem, where the RFMB plays the upper-level decision maker while the industry is the lower-level counterpart. In order to solve the problem, the original formulation was reformulated as a single-level problem by KKT transformation and is further transformed to a 0−1 mixed integer programming problem by introducing variable substitution.

The current computation of the subsidy rate by the RFMB is based on a formula that considers the occurring cost in the glass recycling industry. By contrast, the proposed model attempts to improve the operations of the RFMB and increase the profit of the industry through appropriate decisions at this moment in time. The solutions obtained from the model provide a satisfactory result. For example, the solved subsidy rate is less than the one published by the RFMB currently and thus reduces the operating expense of RFMB. Our model suggested the administration expense of the RFMB not to exceed 30% of its fund, while the actual expense is 68%, which caused the deficit of RFMB. Our simulated experiments also indicated that the current recycling and treatment fee (=NT$1.55/Kg) formulated by RFMB is appropriate. A value that was below or above the current one did not yield better solutions.

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