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不同付款方式對經濟訂購量的影響

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中文摘要：傳統的經濟訂購量(EOQ)模式中，有個很重要的基本假設，消費者在收到貨品的同時必須馬上繳清貨款。然而，在現實的交易市場中，供應商為控制或降低購買者違約或拖欠貨款的機率，往往會要求購買者在下訂單時就須先預付全部或部分貨款，亦即，購買者在收到貨品前就須先支付所有或部分的貨款；此外，為刺激買氣，鼓勵消費者，供應商亦往往會提供允許延遲付款的優惠或給予價格上的現金折扣。因此，本研究計畫將針對退化性貨品，探討當供應商對零售商要求部分預先付款及部分現金付款，或提供延遲付款優惠等不同的付款方式，對零售商的定價及訂貨策略有何影響。根據上述供應商對零售商所採行的不同付款方式，本研究計畫分別構建一數學模式，藉由模式的求解，提出使零售商每年總利潤達到最大的最佳定價及訂貨策略。接著，利用數值範例求算零售商最佳定價、訂購週期及總利潤。

中文關鍵詞：經濟訂購量；預先付款；延遲付款；退化性產品

英文摘要：In the classical economic order quantity (EOQ) model, a common unrealistic assumption is used that the purchasing cost is paid at the time of delivery. However, in practices, a supplier frequently asks his/her buyers to pay all or a fraction of the purchasing cost in a fixed period before the date of delivery, in order to reduce or control default risks. In addition, a supplier may offer a permissible delay in payments to the buyers to stimulate more sales or provide a cash discount to the buyers to encourage the buyers pay cash on delivery and avoid the default risks. This project will study economic order quantity models with deteriorating items for retailers when the supplier adopts three different strategies (prepayment, trade credits and cash payment) for paying the purchasing cost. Based on the previous assumptions, this project will establish a new mathematical model to find the optimal pricing and ordering policies for the retailer to obtain its maximum profit, when the supplier asks partial prepayment and cash payment and provides partial delay payment for paying the purchasing cost. Next, numerical examples are provided to illustrate the solution procedure.

英文關鍵詞：Economic order quantity; Advance payment; Permissible delay in payments; Perishable products

不同付款方式對經濟訂購量的影響

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Abstract

For perishable products, the seller usually asks for the buyer to prepay a fraction of the acquisition cost as a good-faith deposit, to pay some cash upon the receipt of the order, and then a permissible delay is granted on the remaining of the acquisition cost. Hence, an advance-cash-credit payment scheme is commonly used in real world business transactions. In this paper, we develop a supplier-retailer-customer chain in which the retailer receives an upstream advance-cash-credit payment from the supplier while in return offers a down-stream cash-credit (some in cash and the rest in credit) payment to customers. Additionally, the demand for perishable products is influenced by the combined effect of selling price and product freshness linked to expiration date. Furthermore, we demonstrate that the present value of total annual profit is strictly concave in unit price and strictly pseudo-concave in replenishment time, which simplifies the search for the global solution to a local maximum. Finally, we conduct a sensitivity analysis and obtain several managerial insights.

1. Introduction

In real world, to reduce or control default risks, a supplier frequently asks his buyer to pay all or a fraction of the purchasing cost in a fixed period before the date of delivery. Hence, the advance payment scheme is widespread and useful to diminish the estimation error in demand. The scheme is a real life phenomenon. Zhang (1996) proposed an optimal advance payment scheme involving fixed per-payment cost. A simple model is presented to resolve the tradeoff between the lost interest of the cash deposit and the fixed per-payment costs. Maiti et al. (2009) discussed the inventory model with advance payment incorporating stochastic lead-time, and proposed an inventory model with stochastic lead-time and price dependent demand incorporating advance payment. Taleizadeh et al. (2011) proposed a constraint multiproduct joint-replenishment inventory control problem that considers importing raw material from another country where a fraction of the purchasing cost is paid in advance.

Thangam (2012) considered both the advance payment scheme and two-echelon trade credit option in a supply chain with perishable items. Taleizadeh et al. (2013) considered that the buyer must pay a fraction of the purchasing cost as prepayment during several payments, and developed an EOQ model with multiple partial prepayments and partial backordering. Taleizadeh (2014) developed two classic EOQ models for deteriorating items with and without shortage under multiple prepayments. The prepayment can be paid in consecutive equaled size payments. Zhang et al. (2014) studied the buyer's optimal ordering policies when the vendor gives different kinds of payment terms, including all the payment paid in advance, as well as the partial-advanced-partial delayed payment.

Most suppliers grant their buyers varied credit terms to stimulate sales and reduce inventory in today's competitive markets. Hence, the trade credit is widespread and represents an important proportion of company finance. Businesses, especially small businesses, with limited financing opportunities, may be financed by their suppliers rather than by financial institutions (Petersen and Rajan, 1997). On the other hand, offering trade credit to retailers may encourage the supplier sales and reduce the on-hand stock level (Emery, 1987). Goyal (1985) was the first to establish an EOQ model with a constant demand rate under the condition of a permissible delay in payments. Teng (2002) modified Goyal's (1985) model by considering the difference between the selling price and purchase cost, and found that the economic replenishment interval and order quantity decrease under the permissible delay in payments in certain cases. Chang *et al.* (2003) developed an EOQ model with deteriorating items under supplier's credits linked to ordering quantity. Jaggi et al. (2008) proposed an EOQ model with the credit-linked demand under permissible delay in payments. Thangam and Uthayakumar (2009) extended the model of Jaggi et al. (2008) and developed an EOQ model with selling price and credit linked demand for deteriorating items. Teng (2009) established an EOQ model that a supplier offers a full trade credit to good customers and a partial trade credit to bad customers. Teng et al. (2011) proposed the optimal ordering policy for stock-dependent demand under progressive payment scheme. Further, Teng et al. (2012) extended the demand from constant to non-decreasing pattern. Ouyang and Chang (2013) developed an EPQ model with imperfect production process under permissible delay in payments and complete backlogging. Sarkar et al. (2014) built up an integrated inventory model with lead time, defective units, and delay in payments. Likewise, Liao et al. (2014) derived optimal strategy for deteriorating items with capacity constraints under two-level trade credit. There were several interesting and relevant studies related to trade credits such as Chang et al. (2010), Liang and Zhou (2011), Chern et al. (2013), Lou and Wang (2013), Yang and Chang (2013), Chen et al. (2014) and so on.

Based on the previous discussions, we adopt a generalized advance-cash-credit payment scheme and a discounted cash-flow analysis to set up the objective that maximizes the present value of total annual profit. Furthermore, we demonstrate that the present value of total annual profit is strictly pseudo-concave in both the selling price and the replenishment time. Finally, several numerical examples are solved by a discounted cash-flow analysis to gain managerial insights.

2. Notations and assumptions

The following notation and assumptions are introduced to model the EOQ model for perishable products with an advance-cash-credit payment scheme.

2.1. Notation

α	The fraction of procurement cost to be paid in advance, $0 \le \alpha \le 1$.
β	The fraction of procurement cost to be paid at the time of delivery, $0 \le \beta \le 1$.
τ	The fraction of procurement cost granted a permissible delay from the supplier to the retailer, $0 \le \tau \le 1$ and $\alpha + \beta + \tau = 1$.
δ	The downstream credit period by the retailer to customers, $\delta \ge 0$.
μ	The upstream credit period by the supplier to the retailer, $\mu \ge 0$.
ρ	The fraction of the sales revenue offered a permissible delay by the retailer to customers, $0 \le \rho \le 1$.
r	The annual compound interest paid per dollar per year.
A	The procurement cost in dollars when placing an order at time $-l$.
c	The procurement cost per unit in dollars, $c > 0$.
CC	The present value of capital cost per cycle in dollars.
h	The holding cost excluding interest charge per unit per year in dollars, $h > 0$.
HC	The present value of holding cost excluding interest charge per cycle in dollars.
l	The length of time in years during which the prepayments are paid, $l > 0$.
I_c	The interest charged by the supplier per dollar per year.
I_e	The interest earned per dollar per year.
m	The time to the expiration date or the maximum lifetime in years, $m > 0$.
O	The ordering cost in dollars per order, $O > 0$.
OC	The present value of ordering cost per cycle in dollars.
p	The price per unit in dollars, $p > c > 0$ (a decision variable).
Q	The order quantity in units.
SR	The present value of sales revenue per cycle in dollars.
PC	The present value of procurement cost per cycle in dollars.

t	The time in years, $t \geq 0$.
T	The length of cycle time in years, $T \leq m$ (a decision variable).
$D(p)$	The annual demand rate, $D(p) = ae^{-\lambda p}$ with $a, \lambda > 0$.
$\theta(t)$	The degrading (or deteriorating) rate at time t, $0 \leq \theta(t) \leq 1$, $\theta'(t) \geq 0$, and $\theta(m) = 1$.
$I(t)$	The inventory level in units at time t.
PTP	The present value of total annual profit in dollars.

2.2. Assumptions

When the procurement cost is high, the supplier usually demands that the retailer (*i*) prepay α fraction of procurement cost A (i.e., αA) in l years prior to the time of delivery, (*ii*) pay another β percentage of procurement cost (i.e., βA) at the time of delivery (i.e., $t = 0$) , and (*iii*) offer an upstream credit period of μ years on the remaining τ portion of procurement cost (i.e., τA , with $0 \leq \alpha, \beta, \tau \leq 1$, and $\alpha + \beta + \tau = 1$).

For simplicity, we assume that the retailer prepays α fraction of procurement cost at time $-l$ years when placing an order, pays another β percentage of procurement cost at time 0 upon the receipt of all items, and receives an upstream credit period of μ years on the remaining τ portion of procurement cost. Likewise, the retailer also offers customers a partial downstream trade credit, in which a customer is granted a credit period of δ years on ρ fraction of sales and pays the remainder (i.e., $1-\rho$ fraction of sales) in cash.

Since the product cannot be sold after the expiration date, we may assume WLOG that $\delta \leq m$, $\mu \leq m$, and $T \leq m$.

3. Mathematical formulations

The retailer pays the supplier α fractions of the procurement cost A in l years prior to the time of delivery. The order quantity (i.e., Q units) arrives at time 0. The quantity received is gradually depleted to zero at time T due to the combination of demand and deterioration. Then the replenishment cycle repeats. Consequently, the inventory level at time t is governed by the following differential equation:

$$\frac{dI(t)}{dt} = -D(p) - \theta(t)I(t), 0 \leq t \leq T, \tag{1}$$

with boundary condition $I(T) = 0$. The solution of the above differential equation is:

$$I(t) = e^{-\phi(t)} \int_t^T D(p) e^{\phi(v)} dv, \; 0 \leq t \leq T \;, \tag{2}$$

where

$\phi(t) = \int_0^t \theta(v)dv$ is non-decreasing, $0 \le t \le T$. (3)

The retailer's ordering time is l years prior to the time of delivery 0. So, the present value of ordering cost at time $-l$ is:

$$OC = O\,e^{rl}. \tag{4}$$

Since the retailer grants customers a partial downstream credit period δ (i.e., a customer receives items at time t, and must pay the credit payment at time $t+\delta$) on ρ fraction of sales. Hence, the retailer's present value of sales revenue per cycle time T is as follows:

$$SR = p\rho\int_{\delta}^{T+\delta} D(p)e^{-rt}dt + p(1-\rho)\int_0^T D(p)e^{-rt}dt . \tag{5}$$

From (2), we know the order quantity delivered at time 0 as:

$$Q = I(0) = \int_0^T D(p)e^{\phi(t)}dt . \tag{6}$$

The procurement cost without considering time value of money is given by:

$$A = cQ = cI(0) = c\int_0^T D(p)e^{\phi(t)}dt . \tag{7}$$

The payments for the procurement cost consists of three parts: (1) the advance payment at l years before time 0, (2) the cash payment at time 0, and (3) the credit payment at time μ. Therefore, the present value of procurement cost is give as:

$$PC = \alpha\,Ae^{rl} + \beta\,A + \tau Ae^{-r\mu} = A(\alpha e^{rl} + \beta + \tau e^{-r\mu}). \tag{8}$$

The present value of the holding cost excluding interest charged per cycle time T is as follows:

$$HC = h\int_0^T I(t)\,e^{-rt}dt = hD(p)\int_0^T\int_t^T e^{\phi(v)-\phi(t)-rt}dv\,dt \tag{9}$$

The present value of interest charged including advance and cash payments per cycle is given below:

$$IC_a = cD(p)TI_c\left[\int_{-l}^{\delta}\alpha e^{-rt}dt+\int_0^{\delta}\beta e^{-rt}dt\right]+(\alpha+\beta)cD(p)I_c\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt\,. \quad (10)$$

As for the credit payment, from the values of upstream and dowstream credit periods μ and δ, we have two potential cases: (1) $\delta \le \mu$, and (2) $\delta \ge \mu$. Let us discuss them separately.

3.1. Case 1 of $\delta \le \mu$

Based on the values of downstream credit period μ, and timing $T+\delta$ at which the retailer receives the customer's last payment, there are two sub-cases.

3.1.1. Sub-case 1 of $\delta \le \mu$ and $\mu \le T+\delta$

In this sub-case, the present value of interest charged for credit payment per cycle time *T* is given by:

$$IC_1 = \tau cD(p)I_c\left[\rho\int_{\mu}^{T+\delta}(T+\delta-t)e^{-rt}dt+(1-\rho)\int_{\mu}^{T}(T-t)e^{-rt}dt\right].$$

The present value of interest earned for credit payment per cycle time *T* is as follows:

$$IE_1 = \tau pD(p)I_e\left[\rho\int_{\delta}^{\mu}(t-\delta)e^{-rt}dt+(1-\rho)\int_0^{\mu}te^{-rt}dt\right].$$

Consequently, the present value of capital cost per cycle time *T* is as follows:

$$\begin{aligned}CC &= IC_a + IC_1 - IE_1\\ &= cD(p)TI_c\left[\int_{-l}^{\delta}\alpha e^{-rt}dt+\int_0^{\delta}\beta e^{-rt}dt\right]+(\alpha+\beta)cD(p)I_c\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt\\ &\quad+\tau cD(p)I_c\left[\rho\int_{\mu}^{T+\delta}(T+\delta-t)e^{-rt}dt+(1-\rho)\int_{\mu}^{T}(T-t)e^{-rt}dt\right]\\ &\quad-\tau pD(p)I_e\left[\rho\int_{\delta}^{\mu}(t-\delta)e^{-rt}dt+(1-\rho)\int_0^{\mu}te^{-rt}dt\right]. \qquad (11)\end{aligned}$$

Combining (4) – (11), we have the present value of total annual profit given by:

$$\begin{aligned}PTP_1(p,T) &= \frac{1}{T}(SR-OC-PC-HC-CC)\\ &= \frac{1}{T}\Big\{p\rho D(p)\int_{\delta}^{T+\delta}e^{-rt}dt+p(1-\rho)D(p)\int_0^{T}e^{-rt}dt\\ &\quad -Oe^{rl}-(\alpha e^{rl}+\beta+\tau e^{-r\mu})cD(p)\int_0^{T}e^{\phi(t)}dt - hD(p)\int_0^{T}\int_t^{T}e^{\phi(v)-\phi(t)-rt}dv\,dt\end{aligned}$$

$$-cD(p)TI_c\left[\int_{-l}^{\delta}\alpha e^{-rt}dt+\int_0^{\delta}\beta e^{-rt}dt\right]-(\alpha+\beta)cD(p)I_c\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt$$

$$-\tau cD(p)I_c\left[\rho\int_{\mu}^{T+\delta}(T+\delta-t)e^{-rt}dt+(1-\rho)\int_{\mu}^{T}(T-t)e^{-rt}dt\right] \qquad (12)$$

3.1.2. Sub-case 2 of $\delta \le \mu$ and $\mu \ge T+\delta$

In this sub-case, the retailer receives all revenue at time $T+\delta$, and is able to pay the supplier the total procurement cost at time μ. Hence, there is no interest charge for credit payment. However, the present value of interest earned for credit payment per cycle time T is given as:

$$IE_2 = \rho\tau\, pD(p)I_e\left[\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt+\int_{T+\delta}^{\mu}Te^{-rt}dt\right]$$

$$+\ (1-\rho)\tau\, pD(p)I_e\left[\int_0^{T}(T-t)e^{-rt}dt+\int_T^{\mu}Te^{-rt}dt\right].$$

Hence, the present value of capital cost per cycle time T is as follows:

$$CC = IC_a - IE_2$$

$$=cD(p)TI_c\left[\int_{-l}^{\delta}\alpha e^{-rt}dt+\int_0^{\delta}\beta e^{-rt}dt\right]+(\alpha+\beta)cD(p)I_c\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt$$

$$-\rho\tau\, pD(p)I_e\left[\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt+\int_{T+\delta}^{\mu}Te^{-rt}dt\right]$$

$$-(1-\rho)\tau\, pD(p)I_e\left[\int_0^{T}(T-t)e^{-rt}dt+\int_T^{\mu}Te^{-rt}dt\right]. \qquad (13)$$

As a result, the present value of total annual profit is obtained as:

$$PTP_2(p,T)=\frac{1}{T}(SR-OC-PC-HC-CC)$$

$$= \frac{1}{T}\left\{p\rho D(p)\int_{\delta}^{T+\delta}e^{-rt}dt + p(1-\rho)D(p)\int_0^{T}e^{-rt}dt\right.$$

$$-Oe^{rl}-(\alpha e^{rl}+\beta+\tau e^{-r\mu})cD(p)\int_0^{T}e^{\phi(t)}dt\ -hD(p)\int_0^{T}\int_t^{T}e^{\phi(v)\text{-}\phi(t)\text{-}rt}dv\,dt$$

$$-cD(p)TI_c\left[\int_{-l}^{\delta}\alpha e^{-rt}dt+\int_0^{\delta}\beta e^{-rt}dt\right]-(\alpha+\beta)cD(p)I_c\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt$$

$$+\rho\tau\, pD(p)I_e\left[\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt+\int_{T+\delta}^{\mu}Te^{-rt}dt\right]$$

$$\left.+(1-\rho)\tau\, pD(p)I_e\left[\int_0^{T}(T-t)e^{-rt}dt+\int_T^{\mu}Te^{-rt}dt\right]\right\}. \qquad (14)$$

3.2. Case 2 of $\delta \geq \mu$

In this case, there is no interest earned for credit payment. The present value of interest charged for credit payment per cycle time *T* is derived as:

$$IC_3 = \tau cD(p)I_c\left\{\rho\left[\int_{\mu}^{\delta} Te^{-rt}dt + \int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt\right] + (1-\rho)\int_{\mu}^{T}(T-t)e^{-rt}dt\right\}$$

Thus, the present value of capital cost per cycle time *T* is given by:

$$\begin{aligned}CC &= IC_a + IC_3\\ &= cD(p)TI_c\left[\int_{-l}^{\delta}\alpha e^{-rt}dt + \int_{0}^{\delta}\beta e^{-rt}dt\right] + (\alpha+\beta)cD(p)I_c\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt\\ &\quad +(\tau cD(p)I_c\left\{\rho\left[\int_{\mu}^{\delta}Te^{-rt}dt + \int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt\right] + (1-\rho)\int_{\mu}^{T}(T-t)e^{-rt}dt\right\}. \end{aligned} \tag{15}$$

Hence, the present value of total annual profit is as follows:

$$\begin{aligned}PTP_3(p,T) &= \frac{1}{T}(SR-OC-PC-HC-CC)\\ &= \frac{1}{T}\Big\{p\rho D(p)\int_{\delta}^{T+\delta}e^{-rt}dt + p(1-\rho)D(p)\int_{0}^{T}e^{-rt}dt\\ &\quad -Oe^{rl} - (\alpha e^{rl}+\beta+\tau e^{-r\mu})cD(p)\int_{0}^{T}e^{\phi(t)}dt - hD(p)\int_{0}^{T}\int_{t}^{T}e^{\phi(v)-\phi(t)-rt}dv\,dt\\ &\quad -cD(p)TI_c\left[\int_{-l}^{\delta}\alpha e^{-rt}dt + \int_{0}^{\delta}\beta e^{-rt}dt\right] - (\alpha+\beta)cD(p)I_c\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt\\ &\quad -\tau cD(p)I_c\left[\rho\left(\int_{\mu}^{\delta}Te^{-rt}dt + \int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt\right) + (1-\rho)\int_{\mu}^{T}(T-t)e^{-rt}dt\right]\Big\}.\end{aligned} \tag{16}$$

4. Optimal solutions

4.1. Case 1 of $\delta \leq \mu$

Applying fraction concave theory, we have the following results:

Theorem 1. For any given selling price *p*,

(1) $PTP_1(p,T)$ in (12) is a strictly pseudo-concave funtion of *T*, and hence there exists a unique maximum solution T_1.

(2) $PTP_2(p,T)$ in (14) is a strictly pseudo-concave funtion of *T*, and hence there exists a unique maximum solution T_2.

For any given price *p*, applying Theorem 1, taking the first-order derivative of $PTP_1(p,T)$ with respect to *T*, setting the result to zero, and re-arranging terms, we get the necessary and sufficient condition for the optimal replenishment cycle time T_1^* as follows:

$$
\begin{aligned}
&p\rho D(p)[\int_{\delta}^{T+\delta} e^{-rt}dt - Te^{-r(T+\delta)}] + p(1-\rho)D(p)[\int_0^T e^{-rt}dt - Te^{-rT}] \\
&\quad - Oe^{rl} - (\alpha e^{rl} + \beta + \tau e^{-r\mu})cD(p)[\int_0^T e^{\phi(t)}dt - Te^{\phi(T)}] \\
&\quad - hD(p)[\int_0^T \int_t^T e^{\phi(v)\text{-}\phi(\text{t})\text{-}rt}dv\,dt - T\int_0^T e^{\phi(T)\text{-}\phi(\text{t})\text{-}rt}dt] \\
&\quad - (\alpha+\beta)cD(p)I_c\int_{\delta}^{T+\delta}(\delta - t)e^{-rt}dt \\
&\quad - \tau cD(p)I_c\left[\rho\int_{\mu}^{T+\delta}(\delta - t)e^{-rt}dt - (1-\rho)\int_{\mu}^{T} te^{-rt}dt\right] \\
&\quad + \tau\, pD(p)I_e\left[\rho\int_{\delta}^{\mu}(t-\delta)e^{-rt}dt + (1-\rho)\int_0^{\mu} te^{-rt}dt\right] = 0.
\end{aligned}
\tag{17}
$$

Since $\mu - \delta \le T_1 \le m$, we know from Theorem 1 that $T_1^* = T_1$, if $\mu - \delta \le T_1 \le m$. If $T_1 \ge m$, then $T_1^* = m$. If $T_1 \le \mu - \delta$, then $T_1^* = \mu - \delta$. Similarly, we get the necessary and sufficient condition for the optimal replenishment cycle time T_2^* as follows:

$$
\begin{aligned}
&p\rho D(p)[\int_{\delta}^{T+\delta} e^{-rt}dt - Te^{-r(T+\delta)}] + p(1-\rho)D(p)[\int_0^T e^{-rt}dt - Te^{-rT}] \\
&\quad - Oe^{rl} - (\alpha e^{rl} + \beta + \tau e^{-r\mu})cD(p)[\int_0^T e^{\phi(t)}dt - Te^{\phi(T)}] \\
&\quad - hD(p)[\int_0^T \int_t^T e^{\phi(v)\text{-}\phi(\text{t})\text{-}rt}dv\,dt - T\int_0^T e^{\phi(T)\text{-}\phi(\text{t})\text{-}rt}dt] \\
&- (\alpha+\beta)cD(p)I_c\int_{\delta}^{T+\delta}(\delta - t)e^{-rt}dt \\
&\quad + \rho\tau\, pD(p)I_e\left[\int_{\delta}^{T+\delta}(\delta - t)e^{-rt}dt + T^2e^{-r(T+\delta)} - T\int_{\delta}^{T+\delta} e^{-rt}dt\right] \\
&\quad + (1-\rho)\tau\, pD(p)I_e\left[\int_0^T - te^{-rt}dt + T^2e^{-rT}\right] = 0.
\end{aligned}
\tag{18}
$$

Likewise, it is clear from Theorem 1 and $T_2 \le \mu - \delta$ that $T_2^* = T_2$, if $T_2 \le \mu - \delta$. Otherwise, $T_2^* = \mu - \delta$.

Theorem 2. For any given cycle time *T*, if $(2-p\lambda)>0$, then

(1) $PTP_1(p,T)$ in (12) is a strictly concave funtion of *p*, and hence there exists a unique maximum solution p_1.

(2) $PTP_2(p,T)$ in (14) is a strictlyconcave funtion of *p*, and hence there exists a unique maximum solution p_2.

For any given cycle time *T*, applying Theorem 2, taking the first-order derivative of $PTP_1(p,T)$ with respect to *p*, setting the result to zero, and re-arranging terms, we have the optimal price as follows:

$$
\begin{aligned}
p_1^* = \frac{1}{\lambda} + \Big\{ &(\alpha e^{rl} + \beta + \tau e^{-r\mu}) c\int_0^T e^{\phi(t)} dt + h\int_0^T \int_t^T e^{\phi(v)-\phi(t)-rt} dv\, dt \\
&+ cTI_c\left[\int_{-l}^{\delta} \alpha e^{-rt} dt + \int_0^{\delta} \beta e^{-rt} dt\right] + (\alpha+\beta)cI_c\int_{\delta}^{T+\delta} (T+\delta-t)e^{-rt} dt \\
&+ \tau cI_c\left[\rho\int_{\mu}^{T+\delta} (T+\delta-t)e^{-rt} dt + (1-\rho)\int_{\mu}^{T} (T-t)e^{-rt} dt\right]\Big\} \\
&/ \Big\{\rho\int_{\delta}^{T+\delta} e^{-rt} dt + (1-\rho)\int_0^T e^{-rt} dt + \tau I_e\left[\rho\int_{\delta}^{\mu} (t-\delta)e^{-rt} dt + (1-\rho)\int_0^{\mu} t e^{-rt} dt\right]\Big\}. \quad (19)
\end{aligned}
$$

Likewise, we derive the optimal price p_2^* as follows:

$$
\begin{aligned}
p_2^* = \frac{1}{\lambda} + \Big\{ &(\alpha e^{rl} + \beta + \tau e^{-r\mu}) c\int_0^T e^{\phi(t)} dt + h\int_0^T \int_t^T e^{\phi(v)-\phi(t)-rt} dv\, dt \\
&+ cTI_c\left[\int_{-l}^{\delta} \alpha e^{-rt} dt + \int_0^{\delta} \beta e^{-rt} dt\right] + (\alpha+\beta)cI_c\int_{\delta}^{T+\delta} (T+\delta-t)e^{-rt} dt\Big\} \\
&/ \Big\{\rho\int_{\delta}^{T+\delta} e^{-rt} dt + (1-\rho)\int_0^T e^{-rt} dt + \rho\tau I_e\left[\int_{\delta}^{T+\delta} (T+\delta-t)e^{-rt} dt + \int_{T+\delta}^{\mu} T e^{-rt} dt\right] \\
&+ (1-\rho)\tau I_e\left[\int_0^T (T-t)e^{-rt} dt + \int_T^{\mu} T e^{-rt} dt\right]\Big\}. \quad (20)
\end{aligned}
$$

4.2. Case 2: $\delta \ge \mu$

Applying the same analogous argument as in Case 1, we get the following results.

Theorem 3. For any given selling price *p*, $PTP_3(p,T)$ in (16) is a strictly pseudo concave funtion of *T*, and hence there exists a unique maximum solution T_3.

For any given price *p*, applying Theorem 3, taking the first-order derivative of $PTP_3(p,T)$ with respect to *T*, setting the result to zero, and re-arranging terms, we obtain the necessary and sufficient condition for the optimal replenishment cycle time T_3^* as follows:

$$
\begin{aligned}
& p\rho D(p)[\int_{\delta}^{T+\delta} e^{-rt}dt - Te^{-r(T+\delta)}] + p(1-\rho)D(p)[\int_0^T e^{-rt}dt - Te^{-rT}] \\
&\quad - Oe^{rl} - (\alpha e^{rl} + \beta + \tau e^{-r\mu})cD(p)[\int_0^T e^{\phi(t)}dt - Te^{\phi(T)}] \\
&\quad - hD(p)[\int_0^T \int_t^T e^{\phi(v)\text{-}\phi(\text{t})\text{-}rt}dv\,dt - T\int_0^T e^{\phi(T)\text{-}\phi(\text{t})\text{-}rt}dt] \\
& -(\alpha+\beta)cD(p)I_c\int_{\delta}^{T+\delta}(\delta - t)e^{-rt}dt \\
&\quad - \tau cD(p)I_c\left[\rho\int_{\mu}^{T+\delta}(\delta - t)e^{-rt}dt - (1-\rho)\int_{\mu}^{T} te^{-rt}dt\right] = 0.
\end{aligned} \tag{21}
$$

Since $0 < T_3 \le m$, and from Theorem 3, we know that $T_3^* = T_3$, if $T_3 \le m$. Otherwise, $T_3^* = m$.

Theorem 4. For any given cycle time *T*, if $(2 - p\lambda) > 0$, then $PTP_3(p,T)$ in (16) is a strictly concave funtion of *p*, and hence there exists a unique maximum solution p_3.

For any given cycle time *T*, applying Theorem 4, taking the first-order derivative of $PTP_3(p,T)$ with respect to *p*, setting the result to zero, and re-arranging terms, we get the optimal price p_3^* for the case of $\delta \ge \mu$ as follows:

$$
\begin{aligned}
p_3^* = & \frac{1}{\lambda} + \Big\{(\alpha e^{rl} + \beta + \tau e^{-r\mu})c\int_0^T e^{\phi(t)}dt + h\int_0^T\int_t^T e^{\phi(v)\text{-}\phi(\text{t})\text{-}rt}dv\,dt \\
&\quad + cTI_c\left[\int_{-l}^{\delta}\alpha e^{-rt}dt + \int_0^{\delta}\beta e^{-rt}dt\right] + (\alpha+\beta)cI_c\int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt \\
&\quad + \tau cI_c\left[\rho\left(\int_{\mu}^{\delta}Te^{-rt}dt + \int_{\delta}^{T+\delta}(T+\delta-t)e^{-rt}dt\right) + (1-\rho)\int_{\mu}^{T}(T-t)e^{-rt}dt\right]\Big\} \\
&\quad / \left[\rho\int_{\delta}^{T+\delta}e^{-rt}dt + (1-\rho)\int_0^T e^{-rt}dt\right].
\end{aligned} \tag{22}
$$

5. Numerical Examples

Example 1. For a perishable product, annual demand rate is $D(p) = 2000e^{-0.05p}$, and degrading rate is $\theta(t) = 1/(1+m-t)$. The parameter values are: $\alpha = 0.2$, $\beta = 0.2$, $\tau = 0.6$, $\rho = 0.4$, unit cost c = \$10, holding cost h = \$5 per unit per year, lead time l = 0.1 years, maximum lifetime m = 0.5 years, ordering cost O = \$20 per order, annual compounded interest rate r = 0.04 per dollar, downstream credit period $\delta = 0.25$ years, upstream credit period μ = 0.3 years, interest rate charged I_c = 0.05 per dollar per year, and interest rate earned I_e = 0.04 per dollar per year. Using Theorems 1 and 2, and applying Equations (17) – (20), we obtain optimal solutions to $PTP_1(p,T)$ in (12) and $PTP_2(p,T)$ in (14) respectively as follows:

p_1^* = \$46.655, T_1^* = 0.1084 years, and $PTP_1(p,T)$ = \$1,372.52.

p_2^* = \$45.755, T_2^* = 0.05 years, and $PTP_2(p,T)$ = \$1,222.06.

As a result, the optimal solution to the problem is

p^* = \$46.655, T^* = 0.1084 years, and PTP^* = \$1,372.52.

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105年度專題研究計畫成果彙整表

計畫主持人：張春桃				計畫編號：105-2410-H-032-040-		
計畫名稱：不同付款方式對經濟訂購量的影響						
成果項目				量化	單位	質化（說明：各成果項目請附佐證資料或細項說明，如期刊名稱、年份、卷期、起訖頁數、證號...等）
國內	學術性論文	期刊論文		0	篇	
		研討會論文		0		
		專書		0	本	
		專書論文		0	章	
		技術報告		0	篇	
		其他		0	篇	
	智慧財產權及成果	專利權	發明專利 申請中	0	件	
			發明專利 已獲得	0		
			新型/設計專利	0		
		商標權		0		
		營業秘密		0		
		積體電路電路布局權		0		
		著作權		0		
		品種權		0		
		其他		0		
	技術移轉	件數		0	件	
		收入		0	千元	
國外	學術性論文	期刊論文		1	篇	Pricing and lot-sizing policies for perishable products with advance-cash-credit payments by a discounted cash-flow analysis. International Journal of Production Economics, 2017, Vol.193, 578-589.
		研討會論文		0		
		專書		0	本	
		專書論文		0	章	
		技術報告		0	篇	
		其他		0	篇	
	智慧財產權及成果	專利權	發明專利 申請中	0	件	
			發明專利 已獲得	0		
			新型/設計專利	0		
		商標權		0		
		營業秘密		0		

		積體電路電路布局權	0		
		著作權	0		
		品種權	0		
		其他	0		
	技術移轉	件數	0	件	
		收入	0	千元	
參與計畫人力	本國籍	大專生	0	人次	
		碩士生	2		林宜潔與王斯霈兩位碩士班研究生擔任兼任助理。
		博士生	0		
		博士後研究員	0		
		專任助理	0		
	非本國籍	大專生	0		
		碩士生	0		
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		專任助理	0		
其他成果 （無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。）					

科技部補助專題研究計畫成果自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現（簡要敘述成果是否具有政策應用參考價值及具影響公共利益之重大發現）或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估 ■達成目標 □未達成目標（請說明，以100字為限） 　　□實驗失敗 　　□因故實驗中斷 　　□其他原因 說明：
2. 研究成果在學術期刊發表或申請專利等情形（請於其他欄註明專利及技轉之證號、合約、申請及洽談等詳細資訊） 論文：■已發表　□未發表之文稿　□撰寫中　□無 專利：□已獲得　□申請中　■無 技轉：□已技轉　□洽談中　■無 其他：（以200字為限）
3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性，以500字為限） 1. 本計畫的研究成果，可使零售商了解在面對供應商提出不同付款方式的要求時，該如何擬訂其定價及訂購數量，方能使其總利潤達到最大，提供廠商在決定定價及訂貨策略之參考。 2. 就學術發展而言，藉由本計畫數學模式的構建及求解，了解如何將數量方法應用於實際的管理問題，進而運用此研究成果明瞭數學模式在管理決策上的貢獻及影響。
4. 主要發現 本研究具有政策應用參考價值：■否　□是，建議提供機關 （勾選「是」者，請列舉建議可提供施政參考之業務主管機關） 本研究具影響公共利益之重大發現：□否　□是 說明：（以150字為限）