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期末報告

二段信用交易下生產-銷售供應鏈之供貨及訂貨策略的研究

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中 文 摘 要 ： 在現實的交易市場中，銷售者為刺激買氣，鼓勵消費，往往會提供允許延遲付款的優惠給購買者。在生產-銷售的系統中，除製造商常常允許零售商延遲付款外，零售商亦會提供延遲付款的優惠給他的顧客。因此，本研究計畫將針對生產-銷售的供應鏈，在追求零售商利潤最大的前提下，探討當製造商對零售商給予延遲付款優惠，而零售商亦提供延遲付款給其顧客時，延遲付款對零售商決定訂貨策略的影響。本研究假設製造商生產多項商品提供零售商進行銷售，零售商在訂購多項商品的數量時，除須考慮市場的需求外，亦須注意商品儲存空間的限制。針對延遲付款優惠及多項產品的儲存空間限制的假設，本研究構建一數學模式，探討使得零售商利潤最大的訂貨策略，藉由模式的求解，決定使零售商總利潤達到最大的最佳訂購數量。接著，利用數值範例應證及說明此數學模式的應用。

中文關鍵詞： 存貨;二段信用交易; 退化性商品; 空間限制

英 文 摘 要 ： In today’s business transaction, most sellers usually offer their buyers a delay period in payment to stimulate sales and encourage consumption. In a manufacturer-retailer channel, the manufacturer provides an upstream credit period to his/her retailer, while the retailer offers his/her customers a downstream credit period. This study will discuss that the influences of trade credit on the ordering policy of retailer in manufacturer-retailer chain. In this project, we assume that the manufacturer products multiple items and delivers the products to the retailer, as well as, the total storage capacity for all items is fixed. Based on the previous assumptions, this project establishes new mathematical models to find the optimal ordering policy which maximizes the profit of retailer. The numerical examples are provided to illustrate the solution procedure.

英文關鍵詞： inventory; two-level trade credit; deteriorating items; restriction of capacity

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Abstract

In today's business transaction, most sellers usually offer their buyers a delay period in payment to stimulate sales and encourage consumption. In a manufacturer-retailer channel, the manufacturer provides an upstream credit period to his/her retailer, while the retailer offers his/her customers a downstream credit period. This study will discuss that the influences of trade credit on the ordering policy of retailer in manufacturer-retailer chain. In this project, we assume that the manufacturer products multiple items and delivers the products to the retailer, as well as, the total storage capacity for all items is fixed. Based on the previous assumptions, this project establishes new mathematical models to find the optimal ordering policy which maximizes the profit of retailer. The numerical examples are provided to illustrate the solution procedure.

1. Introduction

It is tacitly assumed that the buyer must pay the total purchasing cost as soon as he/she receives the ordered items in the classical inventory model. However, this assumption is unrealistic in the real world. In today's business transactions, most sellers usually offer their buyers a delay period in payment (i.e., a trade credit) to stimulate sales and reduce inventory. The trade credit may also be seen as an alternative to a price discount because it does not provoke competitors to reduce their prices, and thus introduces lasting price reductions. Hence, the trade credit is widespread and represents an important proportion of company finance. Businesses, especially small businesses, with limited financing opportunities, may be financed by their suppliers rather than by financial institutions (Petersen and Rajan, 1997). On the other hand, offering trade credit to retailers may encourage the supplier sales and reduce the on-hand stock level (Emery, 1987). Goyal (1985) was the first to establish an EOQ model with a constant demand rate under the condition of a permissible delay in payments. Teng (2002) modified Goyal's (1985) model by considering the difference between the selling price and purchase cost, and found that the economic replenishment interval and order quantity decrease under the permissible delay in payments in certain cases. Chang *et al.* (2003) developed an EOQ model with deteriorating items under supplier's credits linked to ordering quantity. Huang (2003) established an EOQ model under two levels of trade credit in which the supplier provided the retailer a trade credit period M, and the retailer also offered its customer a trade credit period N (with $N < M$). Teng and Goyal (2007) amended Huang's model and a relaxed dispensable assumption, $N < M$. Jaggi et al. (2008) proposed an EOQ model with the credit-linked demand under permissible delay in payments. Thangam and Uthayakumar (2009) extended the model of Jaggi et al. (2008) and developed an EOQ model with selling price and credit linked demand for deteriorating items. Teng (2009) established an EOQ model that a supplier offers a full trade credit to good customers and a partial trade credit to bad customers. Teng et al. (2011) proposed the optimal ordering policy for stock-dependent demand under progressive payment scheme. Further, Teng et al. (2012) extended the demand from constant to non-decreasing pattern. Ouyang and Chang (2013) developed an EPQ model with imperfect production process under permissible delay in payments and complete backlogging. Sarkar et al. (2014) built up an integrated inventory model with lead time, defective units, and delay in payments. Likewise, Liao et al. (2014) derived optimal strategy for deteriorating items with capacity constraints under two-level trade credit. There were several interesting and relevant studies related to trade credits such as Chang et al. (2010), Liang and Zhou (2011), Chern et al. (2013), Lou and Wang (2013), Yang and Chang (2013), Chen et al. (2014) and so on.

It is an observed phenomenon that an increase in shelf space for an item induces more consumers to buy it. This occurs because of its visibility, popularity or variety. Conversely, low stocks of certain baked goods (*e.g.*, donuts) might raise the perception that they are not fresh. Therefore, demand is often inventory-level dependent. In the last several years, a considerable body of literature has been written in the operational research area on how inventory-level dependent demand should affect inventory control policies. Levin *et al*. (1972) observed that "large piles of consumer goods displayed in a supermarket will lead customers to buy more. Yet, too many goods piled up in everyone's way leaves a negative impression on buyers and employees alike." Silver and Peterson (1985) also noted that sales at the retail level tend to be proportional to the amount of inventory displayed. To quantify this, Baker and Urban (1988) established an economic order quantity model (or EOQ) for a power-form inventory-level-dependent demand pattern. Mandal and Phaujdar (1989) considered an economic production quantity model (or EPQ) for deteriorating items with constant production rate and linearly stock-dependent demand. Bar-Lev *et al.* (1994) developed an extension of the inventory-level-dependent demand-type EOQ model with random yield. Urban and Baker (1997) further generalized the EOQ model in which the demand is a multivariate function of price, time, and level of inventory. Teng and Chang (2005) developed and EOQ models for deteriorating items with price- and stock-dependent demand. Chang et al. (2010) established an inventory model with non-instantaneous deteriorating items with stock-dependent demand. Sajadieh et al. (2010) proposed an integrated vendor-buyer model with stock-dependent demand. Min et al. (2010) developed an inventory model for deteriorating items under stock-dependent demand and two-level trade credit.

Based on the previous discussions, the trade credit is real life phenomenon and it has an important influence on the inventory policies, either the cost to the seller or the benefit to the buyer, it is not ignored in practical business environment. In addition, in reality, many suppliers product several goods as well as retailers sell various items and stock them in their warehouse. Hence, there is a restriction on maximum warehouse space available for storage. In order to reflect the practical inventory management problem and real market phenomena, the project discusses the impact of the trade credit and the restriction of warehouse space on the decision variables for multi-items with stock-dependent demand. The project develops appropriate inventory model with multi-items under stock-dependent demand, two-level trade credit and the restriction of capacity. The objective of this project is to find the optimal ordering policy with various items for a retailer.

2. Notations and assumptions

The following notation and assumptions are used in the entire study.

2.1. Notation

c_i	unit purchase cost for Product *i*, *i* = 1, 2, …, *n*, in dollars
h_i	holding cost per unit per year for Product *i*, *i* = 1, 2, …, *n*, in dollars
I_c	interest charged per dollar per year
I_e	interest earned per dollar per year
K	ordering cost per order for multiple items in dollars

M	retailer's upstream credit period offered by the supplier in years
N	retailer's downstream credit period to the buyers in years
θ_i	perishable rate for Product *i*, *i* = 1, 2, …, *n* ($0 \le \theta \le 1$)
p_i	unit selling price for Product *i*, *i* = 1, 2, …, *n*, in dollars ($p_i \ge c_i$)
w_i	unit storage size of Product *i*, *i* = 1, 2, …, *n*,
W	total storage capacity,
E_i	ending inventory level for Product *i*, *i* = 1, 2, …, *n*, in units ($E_i \ge 0$)
T	replenishment cycle time in years ($T \ge 0$)
Q_i	order quantity for Product *i*, *i* = 1, 2, …, *n*, in units
$I_i(t)$	stock level at time *t* for Product *i*, *i* = 1, 2, …, *n*, in units
$TP_j(T)$	Total annual profit in dollars for Case *j*

Next, we propose some necessary assumptions in order to build up the mathematical model.

2.2. Assumptions

Levin et al. (1972) and Silver and Peterson (1985) observed that a large pile of fresh products displayed in a supermarket often induces more sales because of its visibility, variety, and freshness. Hence, we assume the same as in Mandel and Phaujdar (1989) that the demand rate $R_i(t)$ at time *t* for Product *i*, *i* = 1, 2,…, *n*, is as follows:

$$R_i(t) = D_i + \alpha_i I_i(t), \tag{1}$$

where α_i, and D_i are positive constants, *i* = 1, 2, …, *n*.

It is a well-known fact that it may be profitable to end the inventory cycle with non-zero ending inventory if the demand depends on the displayed stock level on-hand. The higher the inventory level, the higher the demand, which may result in the larger the profit for the retailer. Hence, for generality and profitability, we extend the traditional assumption of zero ending inventory to non-zero ending inventory.

To make the replenishment cycle repeatable, we assume that the initial and the ending inventory levels are the same. At time 0, the retailer has the initial inventory of E_i units for Product i, i = 1, 2, …, n, and receives the order quantity of Q_i units, for i = 1, 2, …, n. Hence, the retailer's inventory level at time 0 is increased to $E_i + Q_i$ units at time 0 for Product i, i = 1, 2, …, n. Due to demand consumption and deterioration, the inventory level is gradually depleted to E_i units at the end of the replenishment cycle time T, and the new replenishment cycle is repeating again.

The retailer receives an upstream credit period M from the supplier, hence pays no interest charges until time M, and earns interest on accumulative revenue during the upstream credit period [0, M]. In addition, the retailer grants a downstream credit period N to the customers. Hence, a customer buys the product at time 0 and pays at time N. Similarly, a customer buys the product at time T and pays at time $T + N$. So, the retailer receives the revenue from N to $T + N$.

For generality, we assume that shortages are prohibited, replenishment rate is instant and infinite, and the residual value of a perishable item is zero.

3. Mathematical model

In this note we establish the inventory level for Product i, i = 1, 2, …, n, at the end of the replenishment cycle time T to be not zero; that is $I_i(T) = E_i$, so:

$$\frac{dI_i(t)}{dt} = -D_i - \alpha_i I_i(t) - \theta_i I_i(t), \quad 0 \le t \le T, \quad I_i(T) = E_i, \quad i = 1, 2, \ldots, n. \tag{2}$$

Solving the differential equation in (2), we obtain:

$$I_i(t) = \left(\frac{D_i}{\alpha_i + \theta_i} + E_i\right)\left[e^{(\alpha_i+\theta_i)(T-t)} - 1\right] + E_i, \quad 0 \le t \le T, \quad i = 1, 2, \ldots, n. \tag{3}$$

Therefore, the demand rate $R_i(t)$ at time t for Product i, i = 1, 2, …, n, becomes:

$$R_i(t) = D_i + \alpha_i I_i(t) = \frac{1}{\alpha_i + \theta_i}\left\{D_i\theta_i + [\alpha_i(D_i + E_i(\alpha_i + \theta_i))]e^{(\alpha_i+\theta_i)(T-t)}\right\},$$

$$0 \le t \le T, \quad i = 1, 2, \ldots, n. \tag{4}$$

The order quantity for Product i, i = 1, 2, …, n, is:

$$Q_i = I_i(0) - E_i = \left(\frac{D_i}{\alpha_i + \theta_i} + E_i\right)\left[e^{(\alpha_i+\theta_i)T} - 1\right], \quad i = 1, 2, \ldots, n. \tag{5}$$

From (3) – (5), we know that the total profit per replenishment cycle time consists of the following elements:

1. The sales revenue for Product i, i = 1, 2, …,n, is

$$SR_i = p_i \int_0^T R_i(t)\,dt = \frac{p_i}{\alpha_i + \theta_i}\left[\theta_i D_i T + \alpha_i \left(\frac{D_i}{\alpha_i + \theta_i} + E_i\right)(e^{(\alpha_i+\theta_i)T} - 1)\right]. \tag{6}$$

2. The ordering cost is

$$OC = K. \tag{7}$$

3. The purchasing cost for Product i, i = 1, 2, …, n, is

$$PC_i = c_i Q_i = c_i \left(\frac{D_i}{\alpha_i + \theta_i} + E_i\right)\left[e^{(\alpha_i+\theta_i)T} - 1\right]. \tag{8}$$

4. The holding cost for Product i, i = 1, 2, …, n, is

$$HC_i = h_i \int_0^T I_i(t)\,dt = \frac{h_i}{\alpha_i + \theta_i}\left\{\left(\frac{D_i}{\alpha_i + \theta_i} + E_i\right)\left[e^{(\alpha_i+\theta_i)T} - 1\right] - D_i T\right\}. \tag{9}$$

5. For the interest earned IE and the interest payable IP, based on the values of N and M, there are two cases: $N \le M$ and $N \ge M$. Let us discuss the case in which $N \le M$ first, and then the other case.

Case 1: $N \le M$

The retailer receives all items at time zero and must pay the purchasing cost at time M. Based on the values of M (*i.e.*, the time at which the retailer must pay the supplier to avoid interest charge) and $T + N$ (*i.e.*, the time at which the retailer receives the payment from the last customer), we have two possible sub-cases: $T + N \ge M$ and $T + N < M$. Now, let us discuss the detailed formulation in each sub-case.

Sub-case 1-1: $M \le T + N$

In this sub-case, the retailer starts selling product at time 0, but receiving the money from the customer at time N. Consequently, the retailer accumulates revenue in an account that earns I_e per dollar per year starting from N through M. Hence, the interest earned per cycle is $p_i I_e$ multiplied by the area of *NAM* as shown in Fig. 1. On the other hand, the retailer pays off all units sold by $M-N$ at time M, keeps the profits and must finance all items sold after time $M-N$ at interest charged I_c per dollar per year. Therefore, the interest payable per cycle is $c_i I_c$ times the area of $MB(T+N)$ shown in Fig. 1. As a result, the interest earned IE_i and the interest payable IP_i are represented in Eqs. (6) and (7), respectively.

$$IE_{1i} = p_i I_e \left\{ \frac{D_i \theta_i (M-N)^2}{2(\alpha_i+\theta_i)} + \frac{\alpha_i [D_i + E_i(\alpha_i+\theta_i)]\,(M-N)\,e^{(\alpha_i+\theta_i)T}}{(\alpha_i+\theta_i)^2} + \frac{\alpha_i [D_i + E_i(\alpha_i+\theta_i)]\,(e^{(\alpha_i+\theta_i)(T+N-M)} - e^{(\alpha_i+\theta_i)T})}{(\alpha_i+\theta_i)^3} \right\}, \tag{10}$$

and

$$IP_{1i} = c_i I_c \left\{ \frac{D_i + E_i(\alpha_i+\theta_i)}{(\alpha_i+\theta_i)^2} \left[e^{(\alpha_i+\theta_i)(T+N-M)} - 1 \right] - \frac{D_i (T+N-M)}{\alpha_i+\theta_i} \right\}. \tag{11}$$

Notice that the retailer offers the customers a downstream credit period N, and hence receives the sales revenue from time N to time $T+N$.

Next, we discuss the sub-case of $M > T+N$.

Sub-case 1-2: $M > T+N$

In this sub-case, the retailer receives the total revenue at time $T+N$, and is able to pay the supplier the total purchase cost at time M. Since the supplier credit period M is longer than the customer last payment time $T+N$, the retailer faces no interest charged. Hence, the interest payable $IP_{2i} = 0$. In addition, the interest earned per cycle is $p_i I_e$ multiplied by the area of *NABM* as shown in Fig. 2. Consequently, the interest earned IE_{1i} is given by

$$IE_{2i} = p_i I_e \left\{ \frac{-D_i\theta_i T^2}{2(\alpha_i+\theta_i)} + \frac{\alpha_i[D_i+E_i(\alpha_i+\theta_i)](1-e^{(\alpha_i+\theta_i)T})}{(\alpha_i+\theta_i)^3} + \frac{\alpha_i[D_i+E_i(\alpha_i+\theta_i)]T}{(\alpha_i+\theta_i)^2} \right.$$

$$\left. \left[\frac{D_i\theta_i T}{(\alpha_i+\theta_i)} + \frac{\alpha_i[D_i+E_i(\alpha_i+\theta_i)](e^{(\alpha_i+\theta_i)T}-1)}{(\alpha_i+\theta_i)^2} \right](M-N) \right\}. \tag{12}$$

Now, we are ready to discuss Case 2 of $N \geq M$.

Case 2: $N \geq M$

In this case, the downstream trade credit period N is equal to or larger than the upstream credit period M. Consequently, there is no interest earned for the retailer. That is, $IE_{3i} = 0$. In addition, the retailer must finance all items ordered at time M at an interest payable I_c per dollar per year, and start to pay off the loan after time N. Hence the interest payable per cycle is $c_i I_c$ times the area of $MAB(T+N)$ shown in Fig. 3. Hence, the interest payable per cycle is given by

$$IP_{3i} = c_i I_c \left\{ (N-M) \left[\frac{D_i\theta_i T}{\alpha_i+\theta_i} + \frac{\alpha_i[D_i+E_i(\alpha_i+\theta_i)](e^{(\alpha_i+\theta_i)T}-1)}{(\alpha_i+\theta_i)^2} \right] \right.$$

$$\left. + \frac{[D_i+E_i(\alpha_i+\theta_i)][e^{(\alpha_i+\theta_i)T}-1]}{(\alpha_i+\theta_i)^2} - \frac{D_i T}{\alpha_i+\theta_i} \right\}. \tag{13}$$

Based on the above argument, we obtain the retailer's total annual profit as

$$TP(T,E_1,E_2,\ldots,E_n) = \begin{cases} TP_1(T,E_1,E_2,\ldots,E_n), & N \leq M \text{ and } M \leq T+N; \\ TP_2(T,E_1,E_2,\ldots,E_n), & N \leq M \text{ and } M > T+N; \\ TP_3(T,E_1,E_2,\ldots,E_n), & N \geq M, \end{cases} \tag{14}$$

where

$$TP_j(T,E_1,E_2,\ldots,E_n) = \frac{1}{T}\left[\sum_{i=1}^{n} (SR_i + IE_{ji} - PC_i - HC_i - IP_{ji}) - OC \right], \; j = 1, 2, 3. \tag{15}$$

According to Eqs, (1) - (9), the total annual profit for each j is given as follows:

$$TP_1(T,E_1,E_2,\ldots,E_n) = \frac{-K}{T} + \frac{1}{T}\sum_{i=1}^{n}\left[F_i A_i e^{(\alpha_i+\theta_i)T} + B_i T + F_i O_i + U_i \right], \tag{16}$$

$$TP_2(T,E_1,E_2,\ldots,E_n) = \frac{-K}{T} + \frac{1}{T}\sum_{i=1}^{n}\left[F_i G_i[e^{(\alpha_i+\theta_i)T}-1] + J_i T - \frac{p_i I_e D_i\theta_i}{2(\alpha_i+\theta_i)} T^2 \right], \tag{17}$$

and

$$TP_3(T, E_1, E_2, \ldots, E_n) = \frac{-K}{T} + \frac{1}{T}\sum_{i=1}^{n}\left[F_i L_i (e^{(\alpha_i+\theta_i)T} - 1) + S_i T\right], \tag{18}$$

where

$$F_i = \frac{D_i + E_i(\alpha_i + \theta_i)}{(\alpha_i + \theta_i)^2} > 0, \tag{19}$$

$$\begin{aligned} A_i &= p_i\alpha_i - h_i - c_i(\alpha_i + \theta_i) - c_i I_c e^{(\alpha_i+\theta_i)(N-M)} + p_i I_e \alpha_i \left((M-N) + \frac{e^{(\alpha_i+\theta_i)(N-M)} - 1}{(\alpha_i+\theta_i)}\right) \\ &= p_i\alpha_i - h_i - c_i(\alpha_i + \theta_i) + p_i I_e \alpha_i (M-N) \\ &\quad - c_i I_c e^{-(\alpha_i+\theta_i)(M-N)} - p_i I_e \alpha_i \frac{1 - e^{-(\alpha_i+\theta_i)(M-N)}}{(\alpha_i+\theta_i)}, \end{aligned} \tag{20}$$

$$B_i = \frac{D_i}{(\alpha_i + \theta_i)}[p_i\theta_i + h_i + c_i I_c] > 0, \tag{21}$$

$$O_i = -p_i\alpha_i + h_i + c_i(\alpha_i + \theta_i) + c_i I_c, \tag{22}$$

$$U_i = \frac{D_i}{(\alpha_i + \theta_i)}\left[\frac{p_i I_e \theta_i (M-N)^2}{2} - c_i I_c (M-N)\right], \tag{23}$$

$$G_i = p_i\alpha_i - h_i - c_i(\alpha_i + \theta_i) + p_i I_e \alpha_i \left((M-N) - \frac{1}{(\alpha_i+\theta_i)}\right), \tag{24}$$

$$J_i = \frac{1}{(\alpha_i + \theta_i)}\left[p_i\theta_i D_i + h_i D_i + p_i I_e \left(\theta_i D_i (M-N) + \frac{\alpha_i[D_i + E_i(\alpha_i+\theta_i)]}{\alpha_i + \theta_i}\right)\right], \tag{25}$$

$$L_i = p_i\alpha_i - h_i - c_i(\alpha_i + \theta_i) - c_i I_c[1 + \alpha_i(N-M)], \tag{26}$$

and

$$S_i = \frac{D_i}{(\alpha_i + \theta_i)}\left[p_i\theta_i + h_i + c_i I_c(1 - \theta_i(N-M))\right]. \tag{27}$$

The unit size of Product i, i = 1, 2, …, n, is w_i. Hence, the total storage capacity occupied by all products is

$$\sum_{i=1}^{n} w_i Q_i = \sum_{i=1}^{n}\frac{w_i[D_i + E_i(\alpha_i+\theta_i)]}{\alpha_i + \theta_i}\left(e^{(\alpha_i+\theta_i)T} - 1\right) \le W. \tag{28}$$

Consequently, the objective of this paper is to find the replenishment cycle time T and the ending inventory level for Product i, E_i i = 1, 2, …, n, such that the retailer's total annual profit $TP(T, E_1, E_2, \ldots, E_n)$ is maximized. Thus, the mathematical model of the problem here is simplified as

$$\text{Maximizing}_{\ T, E_1, E_2, \ldots, E_n} \qquad TP(T, E_1, E_2, \ldots, E_n)$$

$$\text{suject to}: \qquad \sum_{i=1}^{n} \frac{w_i\,[D_i + E_i(\alpha_i + \theta_i)]}{\alpha_i + \theta_i}\left[e^{(\alpha_i+\theta_i)T} - 1\right] \le W, \tag{29}$$

$$\text{and} \qquad T \ge 0.$$

4. Numerical examples

Now, we use a couple of examples to compare the total annual profit between two optimal solutions with and without zero-ending inventory.

Example 1: We consider an inventory system with 5 items (n = 5), the associated parameters are {K, W, I_e, I_c, M, N} = {200, 1000, 0.10, 0.12, 0.3, 0.25}, and others are listed in Table 1. Since $M > N$, the Case 1 is considered. By using software MATHEMATICA 9, we obtain the optimal solutions with zero-ending inventory and nonzero-ending inventory as shown in Table 2. Table 2 reveals the optimal total annual profit with nonzero-ending inventory is 11% higher than that with zero-ending inventory because 5208.06 / 4681.34 = 1.1125.

Table 1: Parameters of inventory system for Example 1

i	D_i	α_i	θ_i	h_i	c_i	p_i	w_i
1	100	0.5	0.05	1.5	6	15	5
2	80	0.55	0.03	2.0	5	14	3
3	90	0.5	0.02	2.5	7	18	4
4	110	0.45	0.03	1.0	4	14	3
5	120	0.7	0.05	2.0	8	17	4

Table 2: Optimal Solution with and without zero-ending inventory

Optimal solution	Zero-ending inventory	Nonzero-ending inventory
TP^*	$TP^* = TP_1 = 4681.34$	$TP^* = TP_1 = 5208.06$
T^*	0.456457	0.368854
(Q_1^*, E_1^*)	(51.8866, 0)	(48.4262, 33.4927)
(Q_2^*, E_2^*)	(41.8067, 0)	(43.4063, 44.0334)
(Q_3^*, E_3^*)	(46.3664, 0)	(45.1647, 40.5335)
(Q_4^*, E_4^*)	(56.1355, 0)	(56.4660, 62.3600)
(Q_5^*, E_5^*)	(65.3189, 0)	(69.3964, 57.7536)

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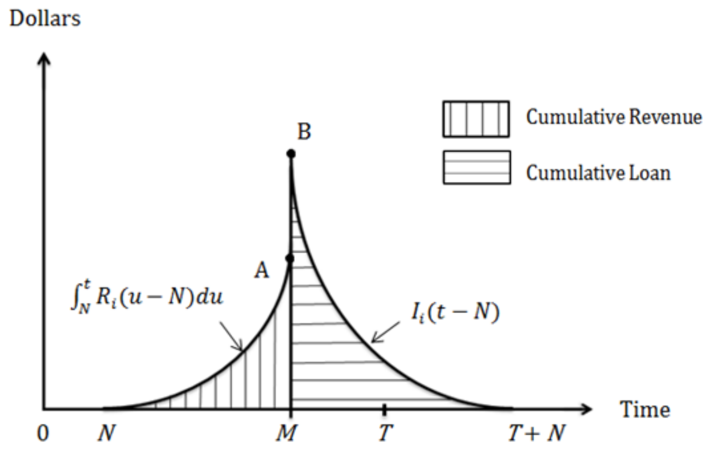

Fig.1. $N \leq M$ and $M \leq T+N$

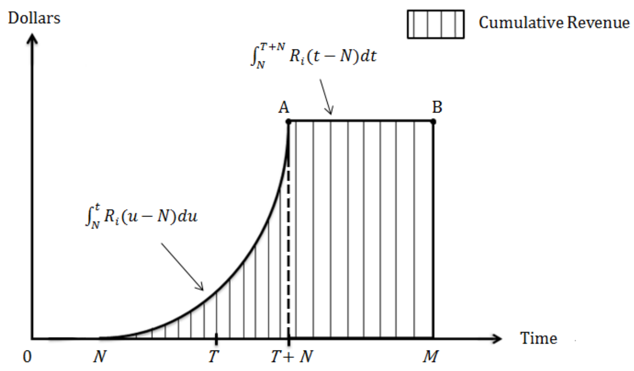

Fig.2. $N \leq M$ and $M > T+N$

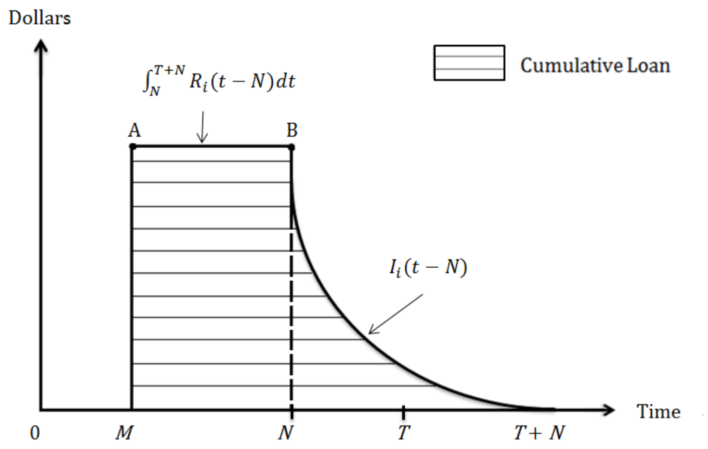

Fig.3. $N \geq M$

106年度專題研究計畫成果彙整表

計畫主持人：張春桃	計畫編號：106-2410-H-032-020-
計畫名稱：二段信用交易下生產-銷售供應鏈之供貨及訂貨策略的研究	

成果項目					量化	單位	質化（說明：各成果項目請附佐證資料或細項說明，如期刊名稱、年份、卷期、起訖頁數、證號...等）
國內	學術性論文	期刊論文			0	篇	
		研討會論文			0		
		專書			0	本	
		專書論文			0	章	
		技術報告			0	篇	
		其他			0	篇	
	智慧財產權及成果	專利權	發明專利	申請中	0	件	
				已獲得	0		
			新型/設計專利		0		
		商標權			0		
		營業秘密			0		
		積體電路電路布局權			0		
		著作權			0		
		品種權			0		
		其他			0		
	技術移轉	件數			0	件	
		收入			0	千元	
國外	學術性論文	期刊論文			0	篇	
		研討會論文			1		Manufacturer's production plan for products sold with warranty in an imperfect production system under preventive maintenance and trade credits, The 9th International Conference on Inverse Problems and Related Topics (ICIP), National University of Singapore in Singapore during 13-17 Aug. 2018
		專書			0	本	
		專書論文			0	章	
		技術報告			0	篇	
		其他			0	篇	
	智慧財產權及成果	專利權	發明專利	申請中	0	件	
				已獲得	0		

			新型/設計專利	0		
		商標權		0		
		營業秘密		0		
		積體電路電路布局權		0		
		著作權		0		
		品種權		0		
		其他		0		
	技術移轉	件數		0	件	
		收入		0	千元	
參與計畫人力	本國籍	大專生		0	人次	
		碩士生		3		陳伯杰、洪嘉敏與黃冠傑3位碩士班研究生擔任兼任助理。
		博士生		0		
		博士後研究員		0		
		專任助理		0		
	非本國籍	大專生		0		
		碩士生		0		
		博士生		0		
		博士後研究員		0		
		專任助理		0		
其他成果 （無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。）						

科技部補助專題研究計畫成果自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現（簡要敘述成果是否具有政策應用參考價值及具影響公共利益之重大發現）或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估 ■達成目標 □未達成目標（請說明，以100字為限） 　　□實驗失敗 　　□因故實驗中斷 　　□其他原因 說明：
2. 研究成果在學術期刊發表或申請專利等情形（請於其他欄註明專利及技轉之證號、合約、申請及洽談等詳細資訊） 論文：□已發表　□未發表之文稿　■撰寫中　□無 專利：□已獲得　□申請中　■無 技轉：□已技轉　□洽談中　■無 其他：（以200字為限）
3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性，以500字為限） 藉由本研究計畫中問題的討論闡述、數學模式的構建、推導及求解，除了解延遲付款方式 對零售商在擬定策略時的影響狀況，更可進一步探究對整體供應鏈的影響程度。就學術研究而言，明瞭數學模式在管理上的運用及貢獻；就企業而言，此研究結果可 提供零售商在擬定訂貨策略之參考。
4. 主要發現 本研究具有政策應用參考價值：■否　□是，建議提供機關 （勾選「是」者，請列舉建議可提供施政參考之業務主管機關） 本研究具影響公共利益之重大發現：□否　□是 說明：（以150字為限）