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預防維修策略和延遲付款下生產過程不完美之經濟生產批量模式(第2年)

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中 文 摘 要 ： 生產過程是完美無缺，在傳統的生產模式中是基本的假設，然而，在現實的製造生產程序中，可能因操作人員的作業不當，及儀器設備的保養維修不確實等因素，致使生產的物件不符合產品規格。此外，傳統的生產模式中，製造商在收到原物料開始生產當下就須立即付清該原物料費用；不過，現今的交易市場中，供應商往往會提供允許延遲付款的優惠，以刺激買氣及獎勵購買者。因此，本研究計畫將針對生產過程不完美及供應商允許延遲付款這兩項議題，探討製造商的經濟生產批量模式。本研究計畫分為兩大部分。第一部份：針對不合格品，製造商會全部進行修復使其完全符合產品規格；探討在此生產策略及供應商提供允許延遲付款的優惠下，製造商該如何擬定其生產計劃，才能使得每年的總成本最小。就上述情況，本研究計畫將構建一數學模式，藉由模式的求解，決定製造商的最佳生產批量、生產週期及總成本；接著，引用數值範例，驗證模式的可行性，並且透過敏感度分析說明最佳生產策略在管理上的運用及意涵。第二部份：實務上，針對不完美的生產過程，製造商往往會對機器設備採行預防保養維修策略，及對顧客提供保證期間的產品免費修復服務。考慮以上情形及供應商提供允許延遲付款的優惠下，本研究計畫構建一數學模式，藉由模式的求解，提出使製造商每年總成本達到最小的最佳生產策略。接著，運用數值範例和敏感度分析，探討延遲付款、預防維修策略與保證期間的產品免費修復服務對最佳生產批量、生產週期及總成本的影響，同時說明此生產計劃在管理上的運用及意涵。

中文關鍵詞 ： 批量大小；經濟生產批量；預防維修；保證期間；信用交易

英 文 摘 要 ： This project will study economic production quantity (EPQ) models for the manufacturer under imperfect production process and permissible delay in payments. There are two scenarios in this project. The first scenario: We assume that all non-conforming items can be reworked and become to conforming items, as well as, shortage are allowed and completely backlogged. Based on the previous assumptions, we will develop an EPQ model with imperfect production process when the supplier provides a permissible delay in payments. The purpose of this project is to find the optimal production lot size and backorder level for minimizing the manufacturer’ s total inventory cost. The proposed model is illustrated through numerical examples and sensitivity analysis is reported. The second scenario: Because machine breakdown is inevitable in the imperfect production process, manufacturers frequently adopt the preventive maintenance to guarantee that the machine does not break down and runs up to a predetermined production up time in real manufacturing system. Hence, we assume that a regular preventive maintenance and free repair warranty policies are adopted by the manufacturer. To reflect the above assumptions and the permissible delay in payments offered by supplier, we will develop an EPQ model with imperfect production process, preventive maintenance and free repair warranty policies when the supplier permits a permissible delay in payments. The purposes of this project are to find the optimal production policy for minimizing the manufacturer’s total inventory. Numerical examples are provided to illustrate the solution procedure. Sensitivity analysis is carried out to discuss the influences of preventive maintenance and free repair warranty policies on the optimal solution and to investigate critical parameters.

英文關鍵詞： Lot size; Economic production quantity; Preventive maintenance; Warranty period; Trade credits

預防維修策略和延遲付款下生產過程不完美之經濟生產批量模式

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Abstract

It is tacitly assumed that the products are all perfect, and the manufacturer must pay off as soon as the raw materials are received in the traditional production model. However, in the real manufacturing circumstance, the non-conforming items are produced due to imperfect production process, and a supplier frequently permits his/her manufacturers a delay of payment for attracting new purchasers and increasing sales in today's competitive business environment. This project will study economic production quantity (EPQ) models for the manufacturer under imperfect production process and permissible delay in payments. There are two scenarios in this project. The first scenario: We assume that all non-conforming items can be reworked and become to conforming items, as well as, shortage are allowed and completely backlogged. Based on the previous assumptions, we will develop an EPQ model with imperfect production process when the supplier provides a permissible delay in payments. The purpose of this project is to find the optimal production lot size and backorder level for minimizing the manufacturer's total inventory cost. The proposed model is illustrated through numerical examples.

1. Introduction

In the classical economic production quantity (EPQ) model, a common unrealistic assumption is used that the products are all perfect. However, in the real manufacturing circumstance, the defective items are produced due to imperfect production process. To reflect the real-life situation, several scholars have developed various analytical models to study the EPQ model with imperfect production process. Rosenblatt and Lee (1986) were one of the early researchers who studied the effects of an imperfect production process on the optimal production cycle time for the classical economic manufacturing quantity (EMQ) model. Porteus (1986) introduced a relationship between process quality control and lot sizing. Zhang and Gerchak (1990) presented joint lot sizing and inspection policy in an economic order quantity (EOQ) model with random yield. Cheng (1991) developed an EOQ model with demand-dependent unit production cost and imperfect production processes. Ben-Daya (2002) formulated an integrated model with joint determination of EPQ and preventive maintenance level under an imperfect process. Lin et al. (2003) examined an integrated production-inventory model for imperfect production processes under inspection schedules. Recently, Sana (2010) developed a production inventory model in an imperfect production process. Many related articles in EPQ models with imperfect quality items can be found in such as Salameh and Jaber (2000), Sana et al. (2007), Yoo et al. (2009), Sana (2011), Sarker et al. (2010), Sarker and Moon (2011), Sarker (2012) and their references. In addition, it is well-known that the total production-inventory costs can be reduced by reworking the imperfect quality items produced with a relatively smaller additional reworking and holding costs. Numerous studies on the problems of EPQ model with rework process have been discussed by Liu and Yang (1996), Hayek and Salameh (2001), Chiu (2003), Jamal et al. (2004), Chiu and Chiu (2006), Chiu (2008), Chiu et al. (2010), Taleizadeh et al. (2010) and their references.

In the imperfect production system, machine breakdown is inevitable. It is more economical for an enterprise to implement preventive maintenance. Many researchers have done considerable researches in this area. Meller and Kim (1996) studied the influence of preventive maintenance on system cost and buffer size. Abboud et al. (2000) developed an economic lot sizing model with consideration of random machine unavailability time. Sheu and Chen (2004) presented an EPQ model to discuss the optimal lot-sizing problem with imperfect maintenance and imperfect production. Zequeira et al. (2004) proposed a production-inventory model to determine the optimal buffer inventory and preventive maintenance for an imperfect production process. Gharbi et al. (2007) considered joint preventive maintenance and safety stocks in unreliable manufacturing systems. Recently, Widyadana and Wee (2012) developed an EPQ model for deteriorating items with preventive maintenance policy and random machine breakdown. Sana (2012) presented an production model with preventive maintenance and warranty period in an imperfect production system.

Actually, today trade credit is widespread and represents an important proportion of company finance. Businesses, especially small businesses, with limited financing opportunities, may be financed by their suppliers rather than by financial institutions (Petersen and Rajan, 1997). On the other hand, offering trade credit to retailers may encourage the supplier sales and reduce the on-hand stock level (Emery, 1987). Goyal (1985) was the first to establish an EOQ model with a constant demand rate under the condition of a permissible delay in payments. Teng (2002) modified Goyal's (1985) model by considering the difference between the selling price and purchase cost, and found that the economic replenishment interval and order quantity decrease under the permissible delay in payments in certain cases. Chang *et al.* (2003) developed an EOQ model with deteriorating items under supplier's credits linked to ordering quantity. Numerous interesting and relevant paper related to trade credits such as Aggarwal and Jaggi (1995), Jamal et al. (1997), Chang (2004), Ouyang et al. (2005), Teng et al. (2005), Goyal et al. (2007), Liao (2008), Teng and Chang (2009), Chang et al. (2010) and so on.

Based on the previous discussions, the imperfect production process is inevitable in most practical production environment, as well as, the preventive maintenance is adopted by the manufacturer in order to avoid breakdown of the system. In addition, the trade credit is a widespread and popular payment method in real business transaction. In order to reflect the practical production environment and real market phenomena, the project will develop appropriate EPQ models with imperfect production process to find the optimal production policy for the manufacturer when the supplier offers a permissible delay in payments.

Scenario I

Notation and Assumptions

The following notation and assumptions will be adopted in this project.

Notation:

P	production rate
λ	demand rate
K	setup cost for each production run
v	purchasing cost of raw material per unit
c	production cost per item including purchasing cost and inspecting cost, $c > v$
s	selling price per unit, $s > c$

h	holding cost per item per unit time, excluding the interest charge
b	shortage cost per item per unit time
x	the proportion of imperfect quality items produced, where $0 < x < 1$
d	the production rate of imperfect quality regular production process per unit time, where $d = Px$
P_1	the rate of reworking of imperfect quality items
c_R	reworking cost for each imperfect quality item
Q	production lot size for each cycle
B	allowable backorder level
T	production cycle length
H_1	maximum level of on-hand inventory when regular production process stops
H	maximum level of on-hand inventory in units, when the reworking ends
M	permissible delay period offered by the supplier
I_c	the interest charged per dollar per unit time in stocks by the supplier
I_e	the interest earned per dollar per unit time
$TC_i(Q, B)$	inventory total cost per cycle for case i, i = 1, 2, 3
$TCU_i(Q,B)$	inventory total cost per unit time for case i, i.e., $TCU_i(Q,B)=TC_i(Q,B)/T$, i = 1, 2, 3.

Assumptions:

(1) Each product is made by a raw material.

(2) Production rate for perfect items is larger than demand rate, i.e., $(1-x)P > \lambda$.

(3) All imperfect quality items can be reworked and become to perfect items.

(4) Shortages are allowed and completely backlogged.

(5) The supplier provides the manufacturer a permissible delay in payments. During the trade credit period the account is not settled, generated sales revenue is deposited in an interest bearing account with interest rate I_e. At the end of the permissible delay, the manufacturer pays off all units ordered, and starts paying for the interest charges on the raw material in stocks with interest rate I_c.

Mathematical formulation

The manufacturer buys all raw materials Q units per order from the supplier to product and the unit purchasing price of raw material is v. The supplier offers the manufacturer a permissible delay period M. A constant product rate P is considered during the regular production uptime. The process may generate x percent of imperfect quality items at a production rate $d = Px$. Thus, the produced items fall into two groups, the perfect and the imperfect. The production rate for perfect item $(1-x)P$ is larger than the demand rate λ. All imperfect quality items are assumed to be reworkable at a rate of P_1, and rework process starts when regular production process ends. In this situation, the production-inventory system follows the pattern depicted in Figure 1. From Figure 1, the expressions of production uptime t_1 and t_2, reworking time t_3, production downtime t_4, shortage permitted time t_5, the maximum levels of on-hand inventory H_1 and H, and the cycle length T are as follows:

$$t_1 = \frac{B}{P-d-\lambda}, \tag{1}$$

$$t_2 = \frac{H_1}{P-d-\lambda}, \tag{2}$$

$$t_3 = \frac{xQ}{P_1} = \frac{dQ}{P_1 P}, \tag{3}$$

$$t_4 = \frac{H}{\lambda}, \tag{4}$$

$$t_5 = \frac{B}{\lambda}, \tag{5}$$

$$H_1 = (P-d-\lambda)\frac{Q}{P} - B, \tag{6}$$

$$H = H_1 + (P_1 - \lambda)\, t_3 = Q\left(1 - \frac{\lambda}{P} - \frac{d\lambda}{P_1 P}\right) - B, \tag{7}$$

and

$$T = t_1 + t_2 + t_3 + t_4 + t_5 = \frac{Q}{\lambda}. \tag{8}$$

In addition, for convenience, we let $t_a \equiv t_1$, $t_b \equiv t_1 + t_2 = \dfrac{Q}{P}$, $t_c \equiv t_1 + t_2 + t_3 = \dfrac{Q}{P} + \dfrac{xQ}{P_1}$, and $t_d \equiv t_1 + t_2 + t_3 + t_4 = \dfrac{Q-B}{\lambda}$.

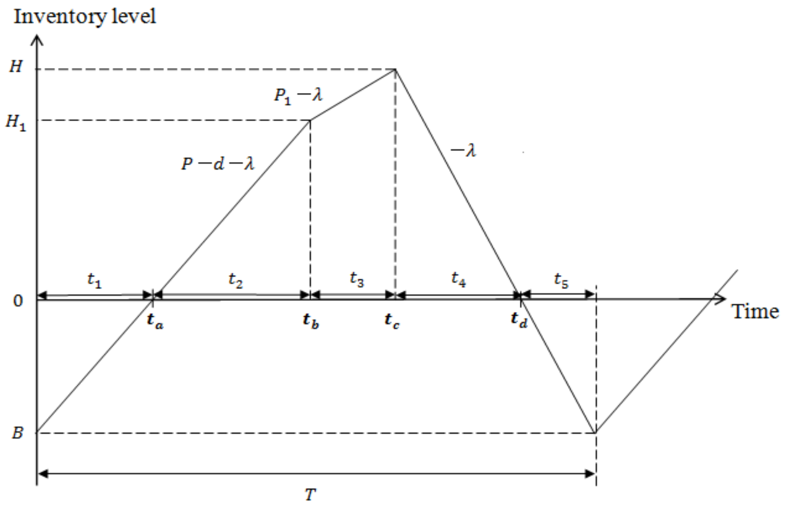

Figure 1: Graphical representation of the inventory system

The inventory total cost per cycle consists of the following components:

(a) The production cost is cQ.

(b) The repair cost is $c_R x Q$.

(c) The setup cost is K.

(d) The holding cost is

$$h\left(\frac{H_1}{2}t_2 + \frac{H_1 + H}{2}t_3 + \frac{H}{2}t_4\right) + h\left[\frac{d(t_1+t_2)}{2}(t_1+t_2) + \frac{d(t_1+t_2)\,t_3}{2}\right]$$

$$= \frac{h}{2\lambda}\left(\frac{1-x}{1-x-\lambda/P}\right)B^2 - \frac{h}{\lambda}QB + \frac{h}{2\lambda}\left(1-\frac{\lambda}{P}\right)Q^2.$$

(e) The shortage cost is $\dfrac{b}{2}B(t_1+t_5) = \dfrac{bB^2}{2}\left(\dfrac{1}{P(1-x)-\lambda} + \dfrac{1}{\lambda}\right)$.

(f) Interest earned and interest charged

Based on the values of M, t_a, and t_d, we have the following three possible cases: (1) $M < t_a$, (2) $t_a \le M < t_d$, and (3) $M \ge t_d$.

Case 1: $M < t_a$

In this case, the manufacturer starts production and replenishing shortage at time 0. As a result, the manufacturer accumulates revenue in an account that earns I_e per dollar per year starting from 0 to M. The interest earned per cycle is $sI_e(P-d)M^2/2$.

On the other hand, the manufacturer pays off all units sold by M at time M, keeps the profits, and starts paying for the interest charges on the items sold after M. The interest charged per cycle is

$$vI_c\left[\frac{(P-d)B^2}{2\lambda(P-d-\lambda)}+\ \frac{Q^2}{2\lambda}-\frac{QB}{\lambda}-QM+\frac{(P-d)M^2}{2}\right].$$

Case 2: $t_a \le M < t_d$

Since $t_a \le M < t_d$, the interest earned per cycle is

$$sI_e\left[\frac{\lambda M^2}{2}+MB-\frac{B^2}{2(P-d-\lambda)}\right].$$

On the other hand, the interest charged per cycle is

$$vI_c\left[\frac{Q^2}{2\lambda}+\frac{B^2}{2\lambda}+\frac{\lambda M^2}{2}-\frac{QB}{\lambda}-MQ+MB\right].$$

Case 3: $M \ge t_d$

In this case, the manufacturer receives the total revenue at time t_d, and is able to pay the supplier the total purchase cost at time *M*. Since t_d is shorter than or equal to the credit period *M*, the manufacturer faces no interest charged. On the other hand, the interest earned per cycle is

$$sI_e\left[-\frac{(P-d)B^2}{2\lambda(P-d-\lambda)}-\frac{Q^2}{2\lambda}+\frac{QB}{\lambda}+MQ\right].$$

According to the above arguments, we can obtain the inventory total cost per cycle as follows:

$TC_i(Q, B)$ = production cost + repair cost + setup cost + holding cost + shortage cost + interest charged – interest earned, $i = 1, 2, 3$.

Case 1: $M < t_a$

$TC_1(Q, B)$

$$= \ cQ+c_R xQ-vI_c MQ+K+(vI_c-sI_e)\,\frac{(P-d)M^2}{2}$$

$$+\ \frac{1}{2\lambda}\left(\frac{1-x}{1-x-\lambda/P}\right)(b+h+vI_c)B^2-\left(\frac{h}{\lambda}+\frac{vI_c}{\lambda}\right)QB+\left[\frac{h}{2\lambda}\left(1-\frac{\lambda}{P}\right)+\frac{vI_c}{2\lambda}\right]Q^2. \quad (9)$$

Hence, the inventory total cost per unit time is

$TCU_1(Q, B) = TC_1(Q, B)/T$

$$=\frac{1}{2Q}\Big\{2\lambda Q\,(c+c_R x-vI_c M\,)+2\lambda K+\lambda\,(vI_c-sI_e)\,P(1-x)M^2-2(h+vI_c)QB$$

$$+\left(b+h+vI_c\right)\left(\frac{1-x}{1-x-\lambda/P}\right)B^2+\left[h\left(1-\frac{\lambda}{P}\right)+vI_c\right]Q^2\Bigg\}. \tag{10}$$

For convenience, we let

$G_1 \equiv c+c_R x-vI_cM$,

$U_1 \equiv \left(b+h+vI_c\right)\left(\frac{1-x}{1-x-\lambda/P}\right)=\left(b+h+vI_c\right)\left(\frac{(1-x)P}{(1-x)P-\lambda}\right)>0,$

$V_1 \equiv h\left(1-\frac{\lambda}{P}\right)+vI_c>0$ and $W_1 \equiv h+vI_c>0.$

Then, Equation (10) can be rewritten as

$$TCU_1(Q, B) = \frac{1}{2Q}\left\{U_1\left(B-\frac{W_1}{U_1}Q\right)^2+2\lambda G_1Q+2K\lambda+\lambda\ (vI_c-sI_e)\ P(1-x)M^2\right.$$

$$\left.+\left(V_1-\frac{W_1^2}{U_1}\right)Q^2\right\}. \tag{11}$$

Case 2: $t_a \le M < t_d$

$TC_2(Q, B)$

$$= cQ+c_RxQ-vI_cMQ+K+(vI_c-sI_e)\lambda M^2/2$$

$$+\ \frac{1}{2\lambda}\left(\frac{1-x}{1-x-\lambda/P}\right)(b+h)B^2+\frac{1}{2\lambda}\left[vI_c+\frac{\lambda\, sI_e}{P(1-x-\lambda/P)}\right]B^2$$

$$-\left(\frac{h}{\lambda}+\frac{vI_c}{\lambda}\right)QB+(vI_c-sI_e)M\,B+\left[\frac{h}{2\lambda}\left(1-\frac{\lambda}{P}\right)+\frac{vI_c}{2\lambda}\right]Q^2. \tag{12}$$

Hence, the inventory total cost per unit time is

$TCU_2(Q, B) = TC_2(Q, B)/T$

$$= \frac{1}{2Q}\Bigg\{2\lambda Q\,(c+c_Rx-vI_cM)+2\lambda K+(vI_c-sI_e)\lambda^2M^2-2(h+vI_c)QB$$

$$+\,2\,(vI_c-sI_e)\lambda M\,B+\left[(b+h)\left(\frac{1-x}{1-x-\lambda/P}\right)+vI_c+\frac{\lambda\, sI_e}{P}\left(\frac{1}{1-x-\lambda/P}\right)\right]B^2$$

$$+\left[h\left(1-\frac{\lambda}{P}\right)+vI_c\right]Q^2\Bigg\}. \tag{13}$$

For convenience, we let

$G_2 = G_1 \equiv c+c_R x-vI_cM$

$U_2 \equiv (b+h)\left(\frac{1-x}{1-x-\lambda/P}\right)+vI_c+\frac{\lambda\, sI_e}{P}\left(\frac{1}{1-x-\lambda/P}\right)>0,$

$V_2 = V_1 \equiv h\left(1-\dfrac{\lambda}{P}\right)+vI_c > 0$ and $W_2 = W_1 \equiv h+vI_c > 0$.

Then, Equation (13) can be rewritten as

$$TCU_2(Q, B)$$

$$= \frac{1}{2Q}\left\{U_2\left(B-\frac{W_2}{U_2}Q+\frac{(vI_c-sI_e)\lambda M}{U_2}\right)^2 + 2\left(\lambda G_2+W_2\frac{(vI_c-sI_e)\lambda M}{U_2}\right)Q\right.$$

$$\left.+(vI_c-sI_e)\lambda^2M^2\left(1-\frac{vI_c-sI_e}{U_2}\right)+2K\lambda+\left(V_2-\frac{W_2^2}{U_2}\right)Q^2\right\}. \quad (14)$$

Case 3: $M \ge t_d$

$TC_3(Q, B)$

$$= cQ+c_RxQ-sI_eMQ+K$$

$$+\frac{1}{2\lambda}\left(\frac{1-x}{1-x-\lambda/P}\right)(b+h+sI_e)B^2-\left(\frac{h}{\lambda}+\frac{sI_e}{\lambda}\right)QB+\left[\frac{h}{2\lambda}\left(1-\frac{\lambda}{P}\right)+\frac{sI_e}{2\lambda}\right]Q^2. \quad (15)$$

Hence, the inventory total cost per unit time is

$TCU_3(Q, B) = TC_3(Q, B)/T$

$$= \frac{1}{2Q}\left\{2\lambda Q(c-sI_eM+c_Rx)+2\lambda K-2(h+sI_e)QB\right.$$

$$\left.+(b+h+sI_e)\left(\frac{1-x}{1-x-\lambda/P}\right)B^2+\left[h\left(1-\frac{\lambda}{P}\right)+sI_e\right]Q^2\right\}. \quad (16)$$

For convenience, we let

$G_3 \equiv c-sI_eM+c_Rx$

$U_3 \equiv (b+h+sI_e)\left(\dfrac{1-x}{1-x-\lambda/P}\right) = (b+h+sI_e)\left(\dfrac{(1-x)P}{(1-x)P-\lambda}\right) > 0,$

$V_3 \equiv h\left(1-\dfrac{\lambda}{P}\right)+sI_e > 0$ and $W_3 \equiv h+sI_e > 0$.

Then, Equation (16) can be rewritten as

$$TCU_3(Q, B) = \frac{1}{2Q}\left\{U_3\left(B-\frac{W_3}{U_3}Q\right)^2+2\lambda G_3Q+2K\lambda+\left(V_3-\frac{W_3^2}{U_3}\right)Q^2\right\}. \quad (17)$$

Theoretical results

In this section, simple algebraic manipulations and an arithmetic-geometric mean inequality approach are used to find the optimal production lot size and backorder level. The arithmetic-geometric mean inequality is: if $a > 0$ and $b > 0$, then $(a+b)/2 \ge \sqrt{ab}$, and the inequality holds when $a = b$.

Case 1: $M < t_a$

For minimizing $TCU_1(Q, B)$ in Equation (11), we let $\left(B - \frac{W_1}{U_1}Q\right)^2 = 0$, then

$$B = \frac{W_1}{U_1}Q. \tag{18}$$

Substituting Equation (18) into Equation (11), it gets

$$\begin{aligned} TCU_1(Q) &\equiv TCU_1(Q, B) \\ &= \lambda G_1 + \frac{2K\lambda + \lambda(vI_c - sI_e)P(1-x)M^2}{2Q} + \frac{(U_1V_1 - W_1^2)Q}{2U_1}. \end{aligned} \tag{19}$$

The arithmetic-geometric mean inequality is used as optimization method to minimize the inventory total cost per unit time. Therefore, we obtain the optimal production lot size (say Q_1) is given by

$$Q_1 = \sqrt{\frac{U_1[2K\lambda + \lambda(vI_c - sI_e)P(1-x)M^2]}{U_1V_1 - W_1^2}}, \tag{20}$$

and the optimal backorder level (say B_1) can be obtained as

$$B_1 = \frac{W_1}{U_1}Q_1 = \frac{W_1}{U_1}\sqrt{\frac{U_1[2K\lambda + \lambda(vI_c - sI_e)P(1-x)M^2]}{U_1V_1 - W_1^2}}. \tag{21}$$

Case 2: $t_a \le M < t_d$

Similarly, for minimizing $TCU_2(Q, B)$ in Equation (14), we let

$$\left(B - \frac{W_2}{U_2}Q + \frac{(vI_c - sI_e)\lambda M}{U_2}\right)^2 = 0,$$

then $$B = \frac{W_2}{U_2}Q - \frac{(vI_c - sI_e)\lambda M}{U_2}. \tag{22}$$

Substituting Equation (22) into Equation (14), it gets

$$\begin{aligned} TCU_2(Q) &\equiv TCU_2(Q, B) \\ &= \frac{1}{2Q}\left\{2\left(\lambda G_2 + W_2\frac{(vI_c - sI_e)\lambda M}{U_2}\right)Q + (vI_c - sI_e)\lambda^2 M^2\left(1 - \frac{vI_c - sI_e}{U_2}\right)\right. \\ &\qquad \left. + 2K\lambda + \left(V_2 - \frac{W_2^2}{U_2}\right)Q^2\right\}, \\ &= \lambda G_2 + W_2(vI_c - sI_e)\lambda M / U_2 + \end{aligned}$$

$$+\frac{2K\lambda+(vI_c-sI_e)\lambda^2M^2(U_2-vI_c+sI_e)/U_2}{2Q}+\frac{(U_2V_2-W_2^2)Q}{2U_2}. \tag{23}$$

It is similar to the arguments as in Case 1, the arithmetic-geometric mean inequality can be used as optimization method to minimize the inventory total cost per unit time. That is, we obtain the optimal production lot size (say Q_2) is given by

$$Q_2 = \sqrt{\frac{2K\lambda U_2+(vI_c-sI_e)\lambda^2M^2(U_2-vI_c+sI_e)}{U_2V_2-W_2^2}}, \tag{24}$$

and the optimal backorder level (say B_2) can be obtained as

$$B_2 = \frac{W_2}{U_2}Q_2-\frac{(vI_c-sI_e)\lambda M}{U_2}$$

$$=\frac{W_2}{U_2}\sqrt{\frac{2K\lambda U_2+(vI_c-sI_e)\lambda^2M^2(U_2-vI_c+sI_e)}{U_2V_2-W_2^2}}-\frac{(vI_c-sI_e)\lambda M}{U_2}. \tag{25}$$

Case 3: $M \geq t_d$

Likewise, for minimizing $TCU_3(Q, B)$ in Equation (17), we let $\left(B-\frac{W_3}{U_3}Q\right)^2 = 0$, then

$$B=\frac{W_3}{U_3}Q. \tag{26}$$

Substituting Equation (26) into Equation (17), it gets

$$TCU_3(Q) \equiv TCU_3(Q, B) = \lambda G_3+\frac{K\lambda}{Q}+\frac{(U_3V_3-W_3^2)Q}{2U_3}. \tag{27}$$

Using the arithmetic-geometric mean inequality, we obtain the optimal production lot size (say Q_3) is given by

$$Q_3 = \sqrt{\frac{2K\lambda U_3}{U_3V_3-W_3^2}}, \tag{28}$$

and the optimal backorder level (say B_3) can be obtained as

$$B_3=\frac{W_3}{U_3}Q_3 = \frac{W_3}{U_3}\sqrt{\frac{2K\lambda U_3}{U_3V_3-W_3^2}}. \tag{29}$$

Scenario II

Notation:

P	production rate
D	demand rate, where $P > D$
A	setup cost for each production run
C	production cost per unit item including purchasing cost and inspecting cost
C_0	cost per unit time for preventive maintenance
S	selling price per unit
C_h	holding cost per unit item per unit time, excluding the interest charge
C_S	shortage cost per unit item per unit time
C_W	repair cost for warranty per unit item
C_R	reworking cost for each non-conforming item
τ	time elapsed after which the production process shifts to the out-of-control state, which is a random variable
$f(\tau)$	probability density function of τ
N	number of non-conforming items
θ_1	probability of non-conforming items when the production process is in the in-control state
θ_2	probability of non-conforming items when the production process is in the out-of-control state, where $0 < \theta_1 < \theta_2 < 1$
T	total replenishment time
g	fraction of total products that are non-conforming items
w	warranty period
$h_1(x)$	failure rate function of conforming items
$h_2(x)$	failure rate function of non-conforming items
M	permissible delay period offered by the supplier
I_c	interest charged per dollar per unit time in stocks by the supplier
I_e	interest earned per dollar per unit time
TC	total inventory cost per cycle

Assumptions:

(1) Demand rate and production rate are constant over time.

(2) Products are sold with a warranty policy.

(3) A free repair warranty policy is adopted.

(4) All non-conforming items can be reworked immediately in a parallel

manufacturing system.

(5) The regular preventive maintenance guarantees that the probability of a breakdown of the manufacturing system during the production run time is zero.

(6) The supplier provides the manufacturer a permissible delay in payments. During the trade credit period the account is not settled, generated sales revenue is deposited in an interest bearing account with interest rate I_e. At the end of the permissible delay, the manufacturer pays off all units ordered, and starts paying for the interest charges on the raw material in stocks with interest rate I_c.

Mathematical formulation

The manufacturer buys all raw materials *Q* units per order from the supplier to product and the unit purchasing price of raw material is *C*. The supplier offers the manufacturer a permissible delay period *M*. A constant product rate *P* is considered during the regular production uptime. The production-inventory system follows the pattern depicted in Figure 2. From Figure 2, the expressions of production uptime t_1 and t_2, production downtime t_3, shortage permitted time t_4, and the cycle length *T* are as follows:

$$t_1 = \frac{B}{P-D}, \tag{30}$$

$$t_2 = \frac{Q}{P} - \frac{B}{P-D}, \tag{31}$$

$$t_3 = \frac{(P-D)Q}{PD} - \frac{B}{D}, \tag{32}$$

$$t_4 = \frac{B}{D}, \tag{33}$$

and

$$T = t_1 + t_2 + t_3 + t_4 = \frac{Q}{D}. \tag{34}$$

In addition, for convenience, we let $t_A \equiv t_1$, $t_B \equiv t_1 + t_2 = \frac{Q}{P}$ and $t_C \equiv t_1 + t_2 + t_3 = \frac{Q-B}{D}$.

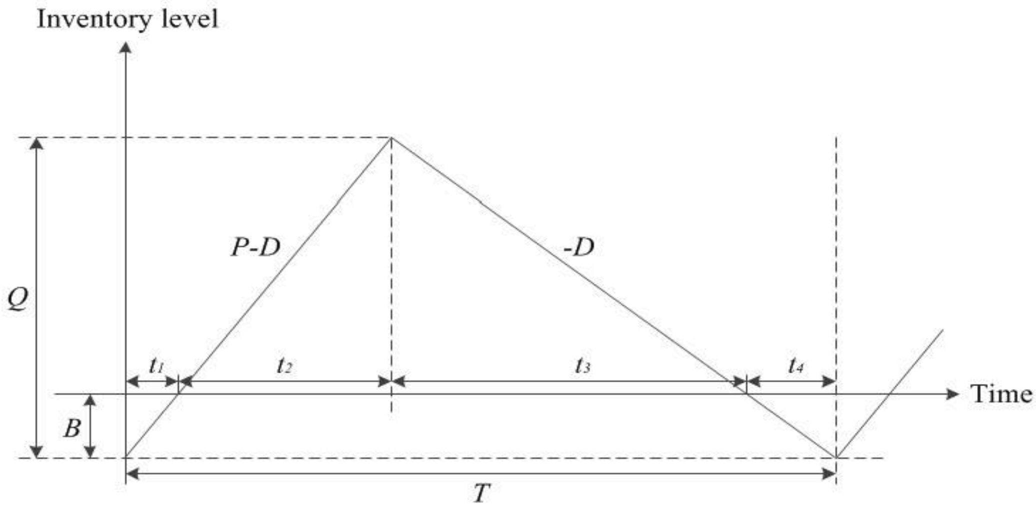

Figure 2: Graphical representation of the inventory system

The number of non-conforming item N can be obtained as

$$N = \begin{cases} \theta_1 P(t_1 + t_2) & ,\text{if } \tau \geq t_1 + t_2 \\ \theta_2 P\tau + \theta_2 P(t_1 + t_2 - 2) & ,\text{if } \tau < t_1 + t_2 \end{cases} . \tag{35}$$

The expected value of N is given by

$$E(N) = \theta_1 P(t_1 + t_2)\int_{t_1+t_2}^{\infty} f(\tau)\, d\tau) + \int_0^{t_1+t_2} [\theta_1 P\tau + \theta_2 P(t_1 + t_2 - \tau)] f(\tau)\, d\tau$$

$$= \theta_1 P(t_1 + t_2) + (\theta_2 - \theta_1) P\int_0^{t_1+t_2} (t_1 + t_2 - \tau) f(\tau)\, d\tau . \tag{36}$$

The fraction of non-conforming item of the total produced is

$$g = \frac{E(N)}{P(t_1 + t_2)}$$

$$= \theta_1 + \frac{\theta_2 - \theta_1}{t_1 + t_2}\int_0^{t_1+t_2} (t_1 + t_2 - \tau) f(\tau)\, d\tau$$

$$= \theta_1 + \frac{\theta_2 - \theta_1}{Q/P}\int_0^{Q/P} (\frac{Q}{P} - \tau) f(\tau)\, d\tau = g(Q) . \tag{37}$$

The probability of a product failing within the warranty period [0, w] is

$$P_W = (1-g)\int_0^w h_1(x)\,dx + g\int_0^w h_2(x)\,dx$$

$$= (1-g)W_1 + gW_2$$

$$= W_1 + (W_2 - W_1)g\,,$$

where $W_1 = \int_0^w h_1(x)\,dx$ and $W_2 = \int_0^w h_2(x)\,dx$. (38)

The inventory total cost per cycle consists of the following components:

(1) The setup cost = A.

(2) The production cost $= CP(t_1 + t_2) = CQ$.

(3) The cost of maintenance = $C_0(t_3 + t_4) = C_0 \dfrac{(P-D)Q}{DP}$.

(4) The expected free repair cost for warranty =

$$C_W P(t_1 + t_2)P_W = C_W Q[W_1 + (W_2 - W_1)\, g(Q)].$$

(5) The expected rework cost = $C_R\, g\, P(t_1 + t_2) = C_R\, g(Q)Q$.

(6) The holding cost =

$$C_h[\frac{1}{2}(P-D)t_2^2 + \frac{1}{2}\frac{(P-D)^2}{D}t_2^2]$$

$$= C_h \frac{(P-D)P}{2D} t_2^2$$

$$= C_h \frac{[Q(P-D)-BP]^2}{2DP(P-D)} = C_h[\frac{(P-D)}{2DP}Q^2 + \frac{P}{2D(P-D)}B^2 - \frac{1}{D}QB].$$

(7) The shortage cost is $C_S \dfrac{B}{2}(t_1 + t_4) == C_s \dfrac{B}{2}\dfrac{PB}{(P-D)D} = C_S \dfrac{PB^2}{2(P-D)D}$.

(8) Interest earned and interest charged

Based on the values of M, t_A, and t_C, we have the following three possible cases:

(1) $M < t_A$, (2) $t_A \le M < t_C$, and (3) $M \ge t_C$.

Case 1: $M < t_A$

In this case, the interest earned per cycle IE_1 is

$$IE_1 = SI_e[\frac{DM^2}{2} + \frac{(P-D)M^2}{2}] = SI_e \frac{(P-D)M^2}{2}.$$

The interest charged per cycle IC_1 is

$$IC_1 = CI_c[\frac{(P-D)(t_A-M^2)}{2}+\frac{D(t_C-M)^2}{2}]$$

$$= CI_c[\frac{Q^2}{2D}+\frac{PB^2}{2D(P-D)}-\frac{QB}{D}-MQ+\frac{P}{2}M^2].$$

Case 2: $t_A \le M < t_C$

In this case, the interest earned per cycle IE_2 is

$$IE_2 = SI_e\{\frac{DM^2}{2}+\frac{[(M-t_A)+M)]B}{2}\}$$

$$= SI_e[\frac{DM^2}{2}+MB-\frac{B^2}{2(P-D)}].$$

The interest charged per cycle IC_2 is

$$IC_2 = CI_c[\frac{D}{2}(t_C-M)^2] = cI_c[\frac{Q^2}{2D}+\frac{B^2}{2D}-\frac{QB}{D}+\frac{DM^2}{2}-MQ+MB].$$

Case 3: $M \ge t_C$

In this case, the interest earned per cycle IE_3 is

$$IE_3 = SI_e\left\{\frac{[(M-t_A)+M]B}{2}+\frac{[(M-t_C)+M]Dt_C}{2}\right\}$$

$$= SI_e\left[MQ+\frac{QB}{D}-\frac{Q^2}{2D}-\frac{PB^2}{2(P-D)D}\right].$$

The interest charged per cycle IC_3 is zero, that is $IC_3 = 0$.

According to the above arguments, we can obtain the inventory total cost per cycle as follows:

$TC_i(Q, B)$ = setup cost + production cost + maintenance cost + warranty cost + rework cost + holding cost + shortage cost + interest charged – interest earned, i = 1, 2, 3.

Case 1: $M < t_A$

$$TC_1(Q, B) = A + CQ + C_0\frac{(P-D)Q}{DP} + C_W Q[W_1+(W_2-W_1)\,g(Q)] + C_R\,g(Q)Q +$$

$$C_h[\frac{(P-D)}{2DP}Q^2+\frac{P}{2D(P-D)}B^2-\frac{1}{D}QB]+C_S\frac{PB^2}{2(P-D)D}$$

$$+\ CI_c[\frac{Q^2}{2D}+\frac{PB^2}{2D(P-D)}-\frac{QB}{D}-MQ+\frac{P}{2}M^2]-SI_e\frac{(P-D)M^2}{2}.$$

Hence, the inventory total cost per unit time is

$TCU_1(Q, B) = TC_1(Q, B)/T$

$$=AD+CD+C_0\frac{(P-D)}{P}+C_W D[W_1+(W_2-W_1)\ g(Q)]+C_R\,g(Q)D+$$

$$C_h[\frac{(P-D)}{2P}Q+\frac{P}{2Q(P-D)}B^2-B]+C_S\frac{PB^2}{2(P-D)Q}$$

$$+\ CI_c[\frac{Q}{2}+\frac{PB^2}{2Q(P-D)}-B-MD+\frac{PD}{2Q}M^2]-SI_e\frac{(P-D)DM^2}{2Q}. \quad (39)$$

Case 2: $t_A \le M < t_C$

$$TC_2(Q, B) = A+CQ+C_0\frac{(P-D)Q}{DP}+C_W Q[W_1+(W_2-W_1)\ g(Q)]+C_R\,g(Q)Q+$$

$$C_h[\frac{(P-D)}{2DP}Q^2+\frac{P}{2D(P-D)}B^2-\frac{1}{D}QB]+C_S\frac{PB^2}{2(P-D)D}$$

$$+\ cI_c[\frac{Q^2}{2D}+\frac{B^2}{2D}-\frac{QB}{D}+\frac{DM^2}{2}-MQ+MB]$$

$$-SI_e[\frac{DM^2}{2}+MB-\frac{B^2}{2(P-D)}].$$

Hence, the inventory total cost per unit time is

$TCU_2(Q, B) = TC_2(Q, B)/T$

$$=AD+CD+C_0\frac{(P-D)}{P}+C_W D[W_1+(W_2-W_1)\ g(Q)]+C_R\,g(Q)D+$$

$$C_h[\frac{(P-D)}{2P}Q+\frac{P}{2Q(P-D)}B^2-B]+C_S\frac{PB^2}{2(P-D)Q}$$

$$+\ CI_C[\frac{Q}{2}+\frac{B^2}{2Q}-B+\frac{D^2M^2}{2Q}-MD+\frac{DMB}{Q}].$$

$$-SI_e[\frac{DM^2}{2}+MB-\frac{B^2}{2(P-D)}]\frac{D}{Q}. \quad (40)$$

Case 3: $M \ge t_C$

$$TC_3(Q, B) = A+CQ+C_0\frac{(P-D)Q}{DP}+C_W Q[W_1+(W_2-W_1)\ g(Q)]+C_R\,g(Q)Q+$$

$$C_h[\frac{(P-D)}{2DP}Q^2+\frac{P}{2D(P-D)}B^2-\frac{1}{D}QB]+C_S\frac{PB^2}{2(P-D)D}$$

$$-SI_e\left[MQ+\frac{QB}{D}-\frac{Q^2}{2D}-\frac{PB^2}{2(P-D)D}\right].$$

Hence, the inventory total cost per unit time is

$TCU_3(Q, B) = TC_3(Q, B)/T$

$$= AD+CD+C_0\frac{(P-D)}{P}+C_W D[W_1+(W_2-W_1)\,g(Q)]+C_R\,g(Q)D+$$

$$C_h[\frac{(P-D)}{2P}Q+\frac{P}{2Q(P-D)}B^2-B]+C_S\frac{PB^2}{2(P-D)Q}$$

$$-SI_e\left[MD+B-\frac{Q}{2}-\frac{PB^2}{2(P-D)Q}\right]. \quad (41)$$

Optimal solutions

Our objective is to minimize the inventory total cost per unit time $TCU_j(Q, B)$, $j=$ 1, 2, 3, which is a function of Q and B. The necessary conditions to make total cost per unit time minimum are

$$\frac{\partial TCU_j(Q,B)}{\partial Q}=0,\text{ and }\frac{\partial TCU_j(Q,B)}{\partial B}=0,\ j=1,2,3. \quad (42)$$

Solve equations for Q and B in (42).

$$\frac{\partial TCU_1(Q,B)}{\partial Q}= C_W D(W_2-W_1)\,g'(Q)+C_R\,g'(Q)D+C_h\frac{(P-D)}{2P}$$

$$-(C_h+C_S)\frac{PB^2}{2(P-D)\,Q^2}+CI_C[\frac{1}{2}-\frac{PB^2}{2(P-D)\,Q^2}]$$

$$+\ SI_e\frac{(P-D)DM^2}{2\,Q^2}=0. \quad (43)$$

$$\frac{\partial TCU_1(Q,B)}{\partial B}= (C_h+CI_C)[\frac{PB}{(P-D)\,Q}-1]+C_S\frac{PB}{(P-D)D}=0. \quad (44)$$

$$\frac{\partial TCU_2(Q,B)}{\partial Q}= C_W D(W_2-W_1)\,g'(Q)+C_R\,g'(Q)D+C_h\frac{(P-D)}{2P}$$

$$-(C_h+C_S)\frac{PB^2}{2(P-D)\;Q^2}+CI_C[\frac{Q^2-B^2}{2\,Q^2}]$$

$$+(SI_e-CI_C)[\frac{D^2M^2+2DMB}{2\,Q^2}]-SI_e\,\frac{DB^2}{2\,(P-D)Q^2}=0. \qquad (45)$$

$$\frac{\partial TCU_2(Q,B)}{\partial B}=\;C_h[\frac{PB}{(P-D)\;Q}-1]+C_S\,\frac{PB}{(P-D)D}+CI_C[\frac{B+DM}{Q}-1]$$

$$-SI_e[M-\frac{B}{(P-D)}]\frac{D}{Q}=0. \qquad (46)$$

$$\frac{\partial TCU_3(Q,B)}{\partial Q}=\;C_W D(W_2-W_1)\;g'(Q)+C_R\,g'(Q)D+C_h\,\frac{(P-D)}{2P}$$

$$-(C_h+C_S)\frac{PB^2}{2(P-D)\;Q^2}\;-SI_e[-\frac{1}{2}+\frac{PB^2}{2\,(P-D)Q^2}]=0. \qquad (47)$$

$$\frac{\partial TCU_3(Q,B)}{\partial B}=\;C_h[\frac{PB}{(P-D)\;Q}-1]+C_S\,\frac{PB}{(P-D)D}\;-SI_e[1-\frac{PB}{(P-D)Q}]=0. \qquad (48)$$

Using Equations (44)-(48), the optimal solution (Q^*, B^*) can be obtained. Substituting (Q^*, B^*) into (39), (40) and (41), the optimal total cost per unit time $TCU_1(Q^*, B^*)$, $TCU_2(Q^*, B^*)$ and $TCU_3(Q^*, B^*)$ can be determined, respectively.

Numerical Example

Example 1: Consider a production system with the following data: P = 10,000, λ = 2,000, P_1 = 600, K = 750, v = 1.5, c = 2, c_R = 0.5, s = 4, b = 0.25, h = 0.2, x = 0.05, M = 0.3, I_c = 0.09 and I_e = 0.05 in appropriate units. Using the proposed results above, the optimal solutions are obtained as follows: optimal production cycle length T^* = 2.249, optimal production lot size Q^* = 4498, optimal backorder level B^* = 2039, and the optimal manufacturer's inventory total cost $TCU(Q^*, B^*)$ = 4613.06.

Example 2: Consider a production system with the following data: P = 10,000, D = 2,000, A = 750, C = 2, C_0 = 0.5, S = 4, C_R = 1.5, s = 4, C_S = 0.25, C_h = 0.2,

$C_W = 0.15$, $w = 1$, $\theta_1 = 0.10$, $\theta_2 = 0.20$, $f(\tau) - 0.5\,e^{-0.5\tau}$, $h_1(x) = 0.15x^2$, $h_2(x) = 0.45x^2$, $M = 0.3$, $I_c = 0.13$ and $I_e = 0.07$ in appropriate units. Using the proposed results above, the optimal solutions are obtained as follows: optimal production cycle length $T^* = 2.522$, optimal production lot size $Q^* = 5044$, optimal backorder level $B^* = 1988$, and the optimal manufacturer's inventory total cost $TCU(Q^*, B^*) = 4569.98$.

Conclusions

In order to reflect the real manufacturing circumstance and the practical business behavior, firstly, we establish two mathematical models to study the optimal production policy for an EPQ inventory system with imperfect production processes under permissible delay in payments, complete backlogging and preventive maintenance. Next, a simple algebraic manipulation and an arithmetic-geometric mean inequality method are employed to determine the optimal production lot size and backorder level. Finally, numerical examples are given to illustrate the theoretical results.

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科技部補助計畫衍生研發成果推廣資料表

日期:2016/01/04

<table>
<tr><td rowspan="3">科技部補助計畫</td><td>計畫名稱: 預防維修策略和延遲付款下生產過程不完美之經濟生產批量模式</td></tr>
<tr><td>計畫主持人: 張春桃</td></tr>
<tr><td>計畫編號: 102-2410-H-032-067-MY2　　　學門領域: 作業研究／數量方法</td></tr>
<tr><td colspan="2">無研發成果推廣資料</td></tr>
</table>

102年度專題研究計畫研究成果彙整表

<table>
<tr><td colspan="3">計畫主持人：張春桃</td><td colspan="5">計畫編號：102-2410-H-032-067-MY2</td></tr>
<tr><td colspan="8">計畫名稱：預防維修策略和延遲付款下生產過程不完美之經濟生產批量模式</td></tr>
<tr><td colspan="3" rowspan="2">成果項目</td><td colspan="3">量化</td><td rowspan="2">單位</td><td rowspan="2">備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）</td></tr>
<tr><td>實際已達成數（被接受或已發表）</td><td>預期總達成數（含實際已達成數）</td><td>本計畫實際貢獻百分比</td></tr>
<tr><td rowspan="12">國內</td><td rowspan="4">論文著作</td><td>期刊論文</td><td>0</td><td>0</td><td>100%</td><td rowspan="3">篇</td><td></td></tr>
<tr><td>研究報告/技術報告</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>研討會論文</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專書</td><td>0</td><td>0</td><td>100%</td><td>章/本</td><td></td></tr>
<tr><td rowspan="2">專利</td><td>申請中件數</td><td>0</td><td>0</td><td>100%</td><td rowspan="2">件</td><td></td></tr>
<tr><td>已獲得件數</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="2">技術移轉</td><td>件數</td><td>0</td><td>0</td><td>100%</td><td>件</td><td></td></tr>
<tr><td>權利金</td><td>0</td><td>0</td><td>100%</td><td>千元</td><td></td></tr>
<tr><td rowspan="4">參與計畫人力（本國籍）</td><td>碩士生</td><td>5</td><td>5</td><td>100%</td><td rowspan="4">人次</td><td></td></tr>
<tr><td>博士生</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>博士後研究員</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專任助理</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td rowspan="12">國外</td><td rowspan="4">論文著作</td><td>期刊論文</td><td>0</td><td>2</td><td>100%</td><td rowspan="3">篇</td><td></td></tr>
<tr><td>研究報告/技術報告</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>研討會論文</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
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<tr><td rowspan="2">專利</td><td>申請中件數</td><td>0</td><td>0</td><td>100%</td><td rowspan="2">件</td><td></td></tr>
<tr><td>已獲得件數</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
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<tr><td>權利金</td><td>0</td><td>0</td><td>100%</td><td>千元</td><td></td></tr>
<tr><td rowspan="4">參與計畫人力（外國籍）</td><td>碩士生</td><td>0</td><td>0</td><td>100%</td><td rowspan="4">人次</td><td></td></tr>
<tr><td>博士生</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>博士後研究員</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td>專任助理</td><td>0</td><td>0</td><td>100%</td><td></td></tr>
<tr><td colspan="2">其他成果
（無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。）</td><td colspan="6">無</td></tr>
</table>

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

科技部補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估 ■達成目標 □未達成目標（請說明，以100字為限） 　　□實驗失敗 　　□因故實驗中斷 　　□其他原因 說明：
2. 研究成果在學術期刊發表或申請專利等情形： 論文：□已發表 □未發表之文稿 ■撰寫中 □無 專利：□已獲得 □申請中 ■無 技轉：□已技轉 □洽談中 ■無 其他：（以100字為限）
3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以500字為限） 本研究計畫將針對生產過程不完美及供應商允許延遲付款這兩項議題，探討製造商的經濟生產批量模式。本研究計畫分為兩大部分。第一部份：針對不合格品，製造商會全部進行修復使其完全符合產品規格；探討在此生產策略及供應商提供允許延遲付款的優惠下，製造商該如何擬定其生產計劃，才能使得每年的總成本最小。第二部份：針對不完美的生產過程，製造商往往會對機器設備採行預防保養維修策略，及對顧客提供保證期間的產品免費修復服務。考慮以上情形及供應商提供允許延遲付款的優惠下，本研究構建數學模式，藉由模式的求解，提出使製造商每年總成本達到最小的最佳生產策略。接著，引用數值範例，驗證模式的可行性，並且透過敏感度分析說明最佳生產策略在管理上的運用及意涵。研究成果有部分已撰寫成學術論文，目前正在投稿中；另一部分本人也已開始著手撰寫成學術論文形式，準備投稿至國際期刊。