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INSTITUTE OF MATHEMATICS
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International Conference on Designs, Matrices and Enumerative Combinatorics

19 May – 22 May 2011

Auditorium, Institute of Mathematics, Academia Sinica

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Matrix Classes: Old and New

Richard A. Brualdi

May 19 - 22, 2011

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Abstract

Classes of $(0,1)$ and, more generally, nonnegative integral matrices are basic to combinatorics and graph theory and have had a long history. Early investigations were largely concerned with the structure of such classes and the extreme values of various combinatorial parameters. Some new classes (generalizing the Gale-Ryser classes in an unexpected way) and a new parameter (motivated by the study of combinatorial batch codes) have been recently introduced. We shall give an exposition of these old and recent developments.

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Context-Free Grammars for Combinatorial Polynomials

William Y. C. Chen [1] and Amy M. Fu [2]

May 19 - 22, 2011

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Abstract

A context-free grammar G over an alphabet A is defined as a set of substitution rules replacing a letter in A by a formal function over A. The formal derivative D_G is an operator defined with respect to a context-free grammar G. Given a monomial $g(x)$ and a formal derivative D_G, $D_G^n(g(x))$ might be a surprising polynomial with combinatorial meanings. This paper explains how to use this grammatical method to obtain some classic identities concerning Stirling polynomials, the generalized Stirling polynomials, Eulerian polynomials and so on. In particular, the relationship between Eulerian polynomials and the generalized Eulerian polynomials whose coefficient of x^i is the number of k-Stirling permutations with i ascents, and the relationship between the Euler numbers and the number of permutations with k exterior peaks, can be obtained through different Jabotinsky's matrices. Several combinatorial objects are studied in this paper, such as the Stirling permutations, the increasing trees and the set partitions.

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Graph Search

Hung-Lin Fu

May 19 - 22, 2011

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Abstract

In a *graph learning problem*, a hidden graph G is known to belong to a given family $\mathbf{H}$ of labeled graphs on vertex set [n] := {1, 2, 3,...,n}. Referring to the information of "belonging to $\mathbf{H}$", we wish to identify G by *edge-detecting queries*, each of which tells whether a subset of [n] induces an edge of G. Such a problem is motivated by applications in DNA physical mapping. Learning a hidden graph can be viewed as a variant of group testing namely "competitive group testing on complex model with each complex of size two". In this talk, I will present an algorithm to learn a general graph G on [n] in (m(log n) + 9m + 3n) edge-detecting queries where m is the number of edges of G. This algorithm performs as well as the designed algorithms known for some families of graphs of known topology.

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The scientific works of Philippe Flajolet

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May 19 - 22, 2011

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Abstract

Philippe Flajolet was born in 1948 in Lyon and passed away on March 22, 2011. He was one of the most influential figures in several scientific fields, notably in analysis of algorithms and in analytic combinatorics for each of which he published a book jointly with Robert Sedgewick. He was elected member of the French Academy of Science (l'Académie des Sciences) in 2003. The major events of his life is briefly summarized as follows.

1948 **Born in Lyon**

1966 **Baccalauréat**
1968
Ecole polytechnique
1970
1971 **DEA; Research Assistant at INRIA**
1973 **Thèse de 3e cycle, Université Paris 7**
Docteur ès Science, Université Paris 11
1979 **Research Director**

1981 **Head of Projet ALGO**
1986 **Prix Scientifique (Union des Assurances de Paris)**
Prix Michel Montpetit
1994 **corresponding member (Académie des Sciences)**
Doctorate Honoris Causa (U. Libre Bruxelles)
1996 **Analysis of Algorithms book published**
Fellow, Academia Europaea
2003 **Full member Académie des Sciences**
2004 **Médaille d'argent CNRS**
2007 **Simulation Prize of INFORMS**
2009 **Analytic Combinatorics book published**
2011 **Rest in peace**

We first give a more outsider's view of his major works through several figures and tables, and then provide a more thorough "guided tour" for almost all of his publications, indicating important ideas, original developments, philosophical thoughts, interdisciplinary connections, and a specific linguistic-complexity synthesis.

On Framed Vertex Operator Algebras

Ching Hung Lam

May 19 - 22, 2011

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Abstract

The theory of framed vertex operator algebras has been motivated by the study of the famous moonshine vertex operator algebra, whose full automorphism group is the Monster simple group. In this talk, we will explain some basic features of framed vertex operator algebras and discuss the applications to the study of some sporadic finite simple groups. We will also talk about our recent work toward the classification of holomorphic framed vertex operator algebras of central charge 24.

Resolvable Designs and (n,c)-Colorings of Complete Graphs

Hao Shen

May 19 - 22, 2011

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Abstract

Let v ,k and m be positive integers, agroup divisible design, denoted $GD(k, m; v)$, is a triple $(X, \mathbf{G}, \mathbf{A})$ where X is a set containing v elements,$\mathbf{G}$ is a set of m-subsets of $\mathbf{X}$ called groups, and $\mathbf{A}$ is a set of k-subsets of X called blocks such that $\mathbf{G}$ forms a partition of X, and each 2-subset of X is contained either in a unique group or in a unique block, but not both..

For given positive integers n and c,an (n, c)-coloring of a complete graph is a coloring of the graph with c colors such that all monochromatic connected subgraphs have at most n vertices. We are interested in the maximum number of vertices of complete graphs with (n, c)- colorings. Let f(n,c) be the smallest integer $v = v(n, c)$ with the following property: if the edges of the complete graph of v vertices are colored with c colors, then there exists a monochromatic connected subgraph of more than n vertices. From the point of view of Ramsey theory, $f(n, c) - 1$ is a lower bound for the Ramsey number $R(T_n, c)$, where T_n is any tree of n edges.

In this talk, we study applications of resolvable group divisible designs in determining the function $f(n, c)$.

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Generalized Ovals in PG$(3n-1, q)$, with q Odd

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May 19 - 22, 2011

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Abstract

In 1954 Segre proved that every oval of PG$(2, q)$, with q odd, is a nonsingular conic. The proof relies on the "Lemma of Tangents". A generalized oval of PG$(3n-1, q)$ is a set of $q^n + 1$ $(n-1)$-dimensional subspaces of PG$(3n-1, q)$, every three of them generate PG$(3n-1, q)$; a generalized oval with $n = 1$ is an oval. The only known generalized ovals are essentially ovals of PG$(2, q^n)$ interpreted over GF(q). If the oval of PG$(2, q^n)$ is a conic, then we call the corresponding generalized oval classical. Now assume q odd. We prove several properties of classical generalized ovals. Further we obtain a strong characterization of classical generalized ovals in PG$(3n-1, q)$ and an interesting theorem on generalized ovals in PG$(5, q)$, developing a theory in the spirit of Segre's approach. So for example a "Lemma of Tangents" for generalized ovals is obtained. We hope such an approach will lead to a classification of all generalized ovals in PG$(3n-1, q)$, with q odd.

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A Zero-Sum Ramsey-Type Problem and the Smith Form of Certain Incidence Matrices

Richard M. Wilson

May 19 - 22, 2011

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Abstract

This work is motivated in part by a problem introduced by Y. Caro and N. Alon.

Given a graph G with the number of edges divisible by an integer m, what is the least integer $R(G, m)$ so that if $n \geq R(G, m)$ and the edges of the complete graph K_n are colored with integers modulo m, there exists a subgraph G' of K_n that is isomorphic to G and so that the sum of the colors on the edges of G' is 0 modulo m? Caro answered this question completely when $m = 2$ and G has an even number of edges. Perhaps surprisingly, $R(G, 2)$ is either k, $k+1$, or $k+2$, where k is the number of vertices of G, and "almost always" equal to k.

The problem can be stated for t-uniform hypergraphs as well as graphs (the case $t = 2$). This author evaluated $R(H, 2)$ when H is the complete t-uniform hypergraph on k vertices, and proved that $R(H, 2) \leq k + t$ for any t-uniform hypergraph on k vertices with an even number of edges.

Given a t-uniform hypergraph H and an integer n, we consider the incidence matrix N or $N(H, n)$ whose rows are indexed by the t-subsets of an n-set X and whose columns correspond to all isomorphic copies of H in the complete t-uniform hypergraph on vertex set X. The zero-sum Ramsey-type problem asks how large must n be to ensure that every vector in the Z-module generated by the rows of N has a coordinate which is zero modulo m. When $m = 2$, we are asking how large must n be so that the vector of all 1's is *not* in the binary code generated by N.

Given H and n, we investigate the Smith form (or diagonal forms) of N. This is joint work with Tony Wong. (A diagonal form is known when $n \geq k + t$ where k is the number of vertices of H.) We can determine a diagonal form for N when H is any simple graph. This lets us reprove Caro's result and also answer the question of when the vector of all 1's is in the p-ary code generated by N. Our results for t-uniform hypergraphs, though not complete, lend evidence to the conjecture that $R(H, 2)$ is "almost always" the number k of vertices of H.

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The Symmetry between Crossings and Nestings in Combinatorial Structures

Catherine Yan

May 19 - 22, 2011

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Abstract

This talk is a survey of recent progresses on the enumeration of crossings and nestings in combinatorial structures. We describe a new combinatorial model– the fillings of moon polyominoes, which provides a unified approach to classical combinatorial analysis on permutations, words, matchings, set partitions, multigraphs, and Young tableaux. In the talk we will concentrate on three pairs of combinatorial statistics over the fillings

(1)the longest northeast (NE) and southeast (SE) chains,

(2)the number of NE and SE chains of length 2, and

(3)four families of mixed statistics (to be defined in the talk).

We present enumerative results and show that there is an elegant symmetry between each pair of statistics. These results are connected to many other areas, for example, free probability theory, random matrix theory, representation theory, and mathematical biology.

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Construction of Cyclic Sieving Phenomenona

Sen-Peng Eu

May 19 - 22, 2011

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Abstract

The cyclic sieving phenomenon is introduced by Reiner, Stanton and White in 2004. Since then this new topic attracts more and more attentions of researchers in recent years. In this talk we will introduce the cyclic sieving phenomenon and introduce some interesting old and new results. Finally we present a new development on constructing new cyclic sieving phenomena from new ones via elementary representation theory.

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The Existence of 3-sun Systems

Chin-Mei Fu

May 19 - 22, 2011

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Abstract

Let K_n be the complete graph of order n. A k-cycle is a cycle of length k, denoted by C_k. A matching of size k or a k-matching in G is a set of k mutually non-adjacent edges, denoted by M_k. If M_k covers all vertices of G, then M_k is called a perfect matching of G. A k-sun graph $S(C_k)$ is obtained from C_k by adding a pendent edge to each vertex of C_k. Thus each k-sun graph $S(C_k)$ contains exactly one C_k and one matching M_k. Let G be a simple graph. A decomposition D of G is a collection of edge-disjoint subgraphs $G_1, G_2, \ldots, G_t$ of G such that every edge of G belongs to exactly one G_j for $j = 1, 2, \ldots, t$. D is called an H-decomposition of G if each member of D is isomorphic to H. An H-decomposition of G is also called a (G, H)-design. In particular, if G is K_n and H is C_k with $k \geq 3$ then a (K_n, C_k)-design is known as a k-cycle system of order n. If H is an $S(C_3)$, then a $(K_n, S(C_3))$-design is called a 3-sun system of order n.

In 2008, Anitha and Lekshmi decomposed K_{2k} into k-1 k-sun graphs and a perfect matching when k is odd and k-2 k-sun graphs, a perfect matching, and a Hamilton cycle when k is even. This motivates us to study the existence of k-sun systems. In 1988, Jian-Xing Yin and Bu-Sheng Gong proved the following result: *There exists a* 3*-sun system of order* n*, if and only if* $n \equiv 0, 1, 4, 9 \pmod{12}$*.*

In this talk, we will show that if $n \equiv 1 \pmod{12}$, then there exists a cyclic 3-sun system of order n. If $n \equiv 0 \pmod{12}$, then there exists a 1-rotational 3-sun system of order n. After this we want to show whether we can embed a Steiner triple system into a 3-sun system.

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The Moment - Transfer Approach and Its Applications in Computer Science

Michael Fuchs

May 19 - 22, 2011

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Abstract

Over the last decade, the moment-transfer approach has evolved into a standard tool in the analysis of algorithms. However, this approach was not applicable to certain "one-sided" recurrences. Such recurrences arise in the analysis of depths of random trees, cost of insertion in random trees, counting the number of cycles in random permutations, etc. In this talk, we discuss a simply modification of the moment-transfer approach which is applicable to a great number of "one-sided" recurrences (including the ones mentioned before).

This is a joint work with Che-Hao Chen.

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(c)-Riordan Arrays and Their Sequence Characterization

Tian-Xiao He

May 19 - 22, 2011

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Abstract

Here we present the definitions of (c)-Riordan arrays and (c)-Bell polynomials which are extensions of the classical Riordan arrays and Bell polynomials. Then, we consider the characterization of (c)-Riordan arrays by means of the A- and Z-sequences. It corresponds to a horizontal construction of a (c)-Riordan array, whereas the traditional approach to the classical Riordan arrays is through column generating functions. We shall show how the A- and Z-sequences of the product of two (c)-Riordan arrays are derived from those of the two factors in the same class; similar results are obtained for the inverse. We also show how the sequence characterization is applied to construct easily a (c)-Riordan array.

We define (c)-Riordan group and its several subgroups and present the characterizations relative to the subgroups. In addition, a relationship between a pair of Laurent series and (c)-Riordan arrays is formulated. A type of generalized Sheffer groups is defined using (c)-Riordan arrays with respect to power series with non-zero coefficients. The isomorphism between a generalized Sheffer group and the group of the (c)-Riordan arrays associated with Laurent series is established. Furthermore, the equivalence of the (c)-Riordan array pairs and generalized Stirling number pairs is given. A type of inverse relations of various series is constructed using pairs of (c)-Riordan arrays. Finally, we study (c)-Bell polynomials and its identities by means of convolution families. We also give the characterization of (c)-Riordan arrays in terms of the convolution families and (c)-Bell polynomials.

In the presentation, several applications involving various arrays, polynomial sequences, special formulas and identities are given as illustrative examples, which include an algorithm of the computation of the generalized Stirling numbers and classical Stirling numbers by using the characterizations of their (c)-Riordan arrays.

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Block Intersection Numbers of Certain Block Designs

Wen-Fong Ke

May 19 - 22, 2011

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Abstract

Let $(G,+)$ be a finite group of order v, and U a fixed-point free group of automorphisms of G with order $k \geq 2$. We refer to such a pair (G,U) a Ferrero pair. If one defines *blocks* as the subsets of G of the type $Ua + b = \{u(a) + b \mid u \in U\}$ for $a, b \in G$ with $a \neq 0$, then one obtains a simple 2-$(v, k, k-1)$ design with interesting combinatorial, geometrical, and statistical applications.

Important examples of finite Ferrero pairs are the *field generated* ones. Such one comes from a finite field $(F,+,\cdot)$ and a nontrivial subgroup U_k of order k of the multiplicative group of F, whose elements are viewed as automorphisms of the additive group $(F,+)$ via multiplication. In this case, we denote the 2-design obtained by $\mathcal{B}_{F,k}$, or $\mathcal{B}_{q,k}$, where $q = |F|$ is a power of some prime.

For a block design, one talks about the block intersection numbers. A positive integer r is said to be a *block intersection number* of the design if there are two blocks intersecting exactly at r points.

In this talk, we will discuss the maximal block intersection numbers for field generated 2-design $\mathcal{B}_{q,k}$ as described above.

A joint work with Tim Boykett, Po-Yi Huang, and Günter F. Pilz

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New Results on the Hamilton-Waterloo Problem

Hongchuan Lei[1], Hung-Lin Fu[2] and Hao Shen[3]

May 19 - 22, 2011

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Abstract

For given positive integers n, m and t, and non-negative integers r and s with $r + s = \lfloor \frac{n-1}{2} \rfloor$, an $HW(n; r, s; m, t)$ is a decomposition of the complete graph K_n(when n is odd) or $K_n - I$(when n is even) into r 2-factors consisting of m-cycles and s 2-factors consisting of t-cycles. In this talk we determine necessary and sufficient conditions for the existence of an $HW(n; r, s; n, 4k)$ where k is any given positive integer. This gives a complete solution to the Hamilton-Waterloo problem for the case $m = n$ and $t \equiv 0 \pmod 4$. New progresses for the case $m = n$ and $t = 3$ are also obtained.

Studies on Coding Theory Based on Finite Geometries

Xiuli Li

May 19 - 22, 2011

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Abstract

Studying linear codes with projective geometries on finite fields is an effective method. We will introduce some applications of finite geometries on coding theory.

For given k, d, q, there is a famous upper bound-Greismer bound about the code length. We will study the minimum length bound of some kinds of linear codes with finite geometries. If the weight of the code has only two possible values, then call it a 2-weight code. The dual of the 2-weight code is an important kind of codes- uniformly packed codes. There are intimate relations between projective 2-weight codes and strongly regular graphs. We will study the constructions of projective 2-weight codes. Lastly, we will introduce the application of (α, β)-geometries on LDPC codes. The experiments show that the LDPC codes constructed from geometry structures have respectable express in minimum distance, girth and bit error rate.

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Catalan Numbers Modulo a Prime Power

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May 19 - 22, 2011

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Abstract

We develop a systematic tool to calculate the congruences of some combinatorial numbers involving $n!$. Using this tool, we re-prove Kummer's and Lucas' Theorems. And also classify the congruences of the Catalan numbers $c_n \pmod{64}$ by only considering the binary representation $[n]_2$ of n. Several general properties for calculating $c_n \pmod{2^k}$ are developed. For instance, a formula with powers of 2 and 5 can evaluate $c_n \pmod{2^k}$ for any k. An equivalence $c_n \equiv_{2^k} c_{\bar{n}}$ can show a rough classification, where $\bar{n}$ is the number obtained by shortening some runs of 0 and runs of 1 in the binary string $[n]_2$. By this equivalence relation, we observe that only few numbers in $[0, 2^k - 1]$ can be the congruences of $c_n \pmod{2^k}$ for $k \geq 2$. Besides, our newest study is on c_n modulo a odd prime power.

References

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Stirling Numbers and Ball Intersections in Cayley Graphs

Johannes Siemons

May 19 - 22, 2011

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Abstract

In a finite graph Γ with vertex set V let $\mathrm{d}(x, y)$ denote the length of a shortest path between the vertices $x \neq y$ in V. So $\mathrm{d}(x, y)$ is the usual graph metric on Γ and for $r \geq 0$ the set $B_r(x) := \{ y \in V : \mathrm{d}(x, y) \leq r \}$ can be considered as a *ball of radius* r around x. The size of such balls for $r \geq 0$ as x varies over V is one of the basic statistics of the graph. Connected to this is the generally difficult problem of finding the *ball intersection numbers* $|B_r(x) \cap B_r(y)|$ as $x \neq y$ range over V.

In this talk we will be concerned with the situation when $\Gamma = \Gamma_n$ is the Cayley graphs on the symmetric group Sym_n generated by a conjugacy class of elements of order 2. For instance, if the generating set are all transpositions on $\{1..n\}$ then Γ_n is known as the transposition Cayley graph on Sym_n.

For such Cayley graphs we have shown in [2, 3] that the ball intersection numbers are determined by the Stirling recursion for the unsigned Stirling numbers of the first kind, subject to various initial conditions. This problem arises in the combinatorics of genome rearrangements and reconstruction problems, see [1].

References

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On the Maximal Q-index Problem

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May 19 - 22, 2011

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Abstract

By the signless Laplacian of a (simple) graph G we mean the matrix $Q(G) = D(G) + A(G)$, where $A(G), D(G)$ denote respectively the adjacency matrix and the diagonal matrix of vertex degrees of G. This is a survey talk on our recent progress in the problem of determining connected graphs (or, graphs) that maximize the Q-index (i.e., the largest signless Laplacian eigenvalue) over all connected graphs (or graphs) with given numbers of vertices and edges, and related problems.

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Posets and Dependent Arcs

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May 19 - 22, 2011

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Abstract

Let x and y be two distinct elements in a poset S. If $x\tilde{y}$ and there doesn't exist an element $z \in S - \{x, y\}$ such that $x\tilde{z}\tilde{y}$, then we say that y covers x. Define D as a digraph with $V(D) = S$ and $A(D) = \{(x, y) : ycoversx\}$. D is called the Hasse diagram of a poset S. A graph G is called a cover graph if it is the underlying graph of the Hasse diagram of a poset S. An arc is dependent in an acyclic orientation if its reversal concretes a directed cycle. In the talk, we will introduce some studies between posets and dependent arcs. Pretzel proved that a graph G is a cover graph if and only if there exists an acyclic of G without dependent arcs. In the talk, we will introduce some studies between posets and dependent arcs.

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Reeder's Puzzle on a Tree

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Abstract

Let $F_2 = \{0, 1\}$ be a binary field and F_2^n the set of n-dimensional column vectors over F_2. Let T denote a tree with the vertex set $VT = \{1, 2, \ldots, n\}$, adjacency matrix A and the i-th standard basis e_i of the vector space F_2^n. For $1 \leq i \leq n$, the *i-th transvection* T_i of Γ is defined to be the $n \times n$ matrix $T_i = I + e_i e_i^T A$. Let G be the group generated by transvections $T_1, T_2, \ldots, T_n$. A vector $u \in F_2^n$ is *movable* if $Au \neq 0$. Let $q : F_2^n \to F_2$ be the function defined by $q(u) = 1$ if the number of connected components in the subgraphs induced on the vertices i with $u_i = 1$ is odd, and else $q(u) = 0$. A binary star is a tree obtained by attaching some leafs to the two ends of a path. We prove the following.

Theorem. Let T be a tree but not a binary star. Let O be an orbit under the action of G on F_2^n. Then exactly one of the following holds.

(i) $|O| = 1$.

(ii) $O = \{u \in F_2^n \text{ is movable} \mid q(u) = 1\}$.

(iii) $O = \{u \in F_2^n \text{ is movable} \mid u \neq 0, q(u) = 0\}$.

In particular there are $2^{\text{null } A} + 2$ orbits, where null A is the nullity of A over F_2.

The orbit description of binary star is also completed.

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On the Rank of a Cograph

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Abstract

The rank of a graph G is defined to be the number of nonzero eigenvalues (counted with multiplicities) of its adjacency matrix. Royle (The Electronic Journal of Combinatorics, 10 (2003) #N11) proved a somewhat surprising result that the rank of a cograph is equal to the number of distinct non-zero rows of its adjacency matrix. In this talk we answer a question posed by Royle by giving an elementary short proof for a more general setting of this rank property of cographs. This talk is based on a joint work with Gerard J. Chang and Liang-Hao Huang appeared in Linear Algebra and its Applications 429 (2008) 601-605.

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