Available online at www.sciencedirect.com
ScienceDirect
Mathematical and Computer Modelling 45 (2007) 801–813
MATHEMATICAL AND COMPUTER MODELLING
www.elsevier.com/locate/mcm

An extension of TOPSIS for group decision making

Hsu-Shih Shih[a,1], Huan-Jyh Shyur[b,2], E. Stanley Lee[c,*]

[a] *Graduate Institute of Management Sciences, Tamkang University, Tamsui, Taipei, 25137, Taiwan, ROC*
[b] *Department of Information Management, Tamkang University, Tamsui, Taipei, 25137, Taiwan, ROC*
[c] *Department of Industrial and Manufacturing Systems Engineering, Kansas State University, Manhattan, KA 66506, USA*

Received 14 March 2006; accepted 14 March 2006

Abstract

An extension of TOPSIS (technique for order performance by similarity to ideal solution), a multi-attribute decision making (MADM) technique, to a group decision environment is investigated. TOPSIS is a practical and useful technique for ranking and selection of a number of externally determined alternatives through distance measures. To get a broad view of the techniques used, we provide a few options for the operations, such as normalization, distance measures and mean operators, at each of the corresponding steps of TOPSIS. In addition, the preferences of more than one decision maker are internally aggregated into the TOPSIS procedure. Unlike in previous developments, our group preferences are aggregated within the procedure. The proposed model is indeed a unified process and it will be readily applicable to many real-world decision making situations without increasing the computational burden. In the final part, the effects of external aggregation and internal aggregation of group preferences for TOPSIS with different computational combinations are compared using examples. The results have demonstrated our model to be both robust and efficient.

Keywords: TOPSIS; Multi-attribute decision making; Group decision support; Preference aggregation; Distance function; Normalization

1. Introduction

TOPSIS (technique for order performance by similarity to ideal solution) is a useful technique in dealing with multi-attribute or multi-criteria decision making (MADM/MCDM) problems in the real world [1]. It helps decision maker(s) (DMs) organize the problems to be solved, and carry out analysis, comparisons and rankings of the alternatives. Accordingly, the selection of a suitable alternative(s) will be made. However, many decision making problems within organizations will be a collaborative effort. Hence, this study will extend TOPSIS to a group decision environment to fit real work. A complete and efficient procedure for decision making will then be provided.

The basic idea of TOPSIS is rather straightforward. It originates from the concept of a displaced ideal point from which the compromise solution has the shortest distance [2,3]. Hwang and Yoon [1] further propose that the

* Corresponding author. Tel.: +1 785 532 3730; fax: +1 785 532 3738.
E-mail addresses: hshih@mail.tku.edu.tw (H.-S. Shih), shyur@mail.im.tku.edu.tw (H.-J. Shyur), eslee@ksu.edu (E.S. Lee).
[1] Tel.: +886 2 8631 3221; fax: +886 2 8631 3214.
[2] Tel.: +886 2 2621 5656x2881; fax: +886 2 2620 9737.

0895-7177/$ - see front matter
doi:10.1016/j.mcm.2006.03.023

ranking of alternatives will be based on the shortest distance from the (positive) ideal solution (PIS) and the farthest from the negative ideal solution (NIS) or nadir. TOPSIS simultaneously considers the distances to both PIS and NIS, and a preference order is ranked according to their relative closeness, and a combination of these two distance measures. According to Kim et al. [4] and our observations, four TOPSIS advantages are addressed: (i) a sound logic that represents the rationale of human choice; (ii) a scalar value that accounts for both the best and worst alternatives simultaneously; (iii) a simple computation process that can be easily programmed into a spreadsheet; and (iv) the performance measures of all alternatives on attributes can be visualized on a polyhedron, at least for any two dimensions. These advantages make TOPSIS a major MADM technique as compared with other related techniques such as analytical hierarchical process (AHP) and ELECTRE (refer to [1]). In fact, TOPSIS is a utility-based method that compares each alternative directly depending on data in the evaluation matrices and weights [5]. Besides, according to the simulation comparison from Zanakis et al. [6], TOPSIS has the fewest rank reversals among the eight methods in the category. Thus, TOPSIS is chosen as the main body of development.

Because MADM is a practical tool for selection and ranking of a number of alternatives, its applications are numerous. TOPSIS has been deemed one of the major decision making techniques within the Asian Pacific area. In recent years, TOPSIS has been successfully applied to the areas of human resources management [7], transportation [8], product design [9], manufacturing [10], water management [11], quality control [12], and location analysis [13]. In addition, the concept of TOPSIS has also been connected to multi-objective decision making [14] and group decision making [15]. The high flexibility of this concept is able to accommodate further extension to make better choices in various situations. This is the motivation of our study.

It is not uncommon for certain groups to constantly make complex decisions within organizations. However, for using any MADM technique, e.g., TOPSIS, it is usually assumed that the decision information is provided in advance by a team or a task group. Thus, Shih et al. [16] propose post-work to enhance TOPSIS as a problem-solving tool. However, this compensation needs a group decision support system to fulfill its objectives. To simplify the decision making activities, we will suggest an integrated group TOPSIS procedure for solving real-world problems, with the goal of making effective decisions.

The paper is organized as follows. In the next section, the literature survey for the applications and group works for TOPSIS is given. Section 3 will focus on the proposed group TOPSIS model, with various combinations, in a step-by-step fashion. Afterwards, external and internal aggregation experiments, under varying circumstances, are tested. In the final section, conclusions are drawn and remarks made as regards future study.

2. Literature survey

To arrange the survey in various aspects, we will divide it into two parts: TOPSIS and group decision making, and the operations within TOPSIS.

2.1. TOPSIS and group decision making

Functionally associated with problems of discrete alternatives, MADM techniques are practical tools for solving real-world problems. The DM is to select, prioritize, and rank a finite number of courses of action [1]. Since there are too many techniques involved, Hwang and Yoon also provide a taxonomy for classifying the techniques as: the types of information from DMs, salient features of information, and a major class of methods. The classification indeed gives us a clear direction for learning MADM techniques.

Among these techniques, the category of information on attributes from DMs with cardinal information is convenient for making decisions owing to an explicitly represented procedure. In this category, TOPSIS, the concept of distance measures, of the alternatives from the PIS and the NIS, proposed by Hwang and Yoon [1] is the most straightforward technique in MADM. TOPSIS has been an important branch of decision making since then. To clarify its features, the characteristics of TOPSIS and AHP [17] are compared in Table 1. We can see that the major weaknesses of TOPSIS are in not providing for weight elicitation, and consistency checking for judgments. However, AHP's employment has been significantly restrained by the human capacity for information processing, and thus the number seven plus or minus two would be the ceiling in comparison [18]. From this viewpoint, TOPSIS alleviates the requirement of paired comparisons and the capacity limitation might not significantly dominate the process. Hence,

Table 1
Comparison of characteristics between AHP and TOPSIS

	Characteristics	AHP	TOPSIS
1	Category	Cardinal information, information on attribute, MADM	Cardinal information, information on attribute, MADM
2	Core process	Pairwise comparison (cardinal ratio measurement)	The distances from PIS and NIS (cardinal absolute measurement)
3	Attribute	Given	Given
4	Weight elicitation	Pairwise comparison	Given
5	Consistency check	Provided	None
6	No. of attributes accommodated	7 ± 2 or hierarchical decomposition	Many more
7	No. of alternatives accommodated	7 ± 2	Many more
8	Others	Compensatory operation	Compensatory operation

Note: Please refer to Hwang and Yoon [1], Saaty [17], and Saaty and Ozdemir [18].

Table 2
Some applications of TOPSIS

	Application areas	No. of attributes	No. of alternatives	Proposed by
1	Company financial ratios comparison	Four attributes	7 alternatives	Deng et al. [19]
2	Expatriate host country selection	Six major attributes (25 sub-attributes)	10 alternatives	Chen and Tzeng [7]
3	Facility location selection	Five attributes	4 alternatives	Chu [20]
4	Gear material selection	Five attributes	9 alternatives	Milani et al. [10]
5	High-speed transport system selection	Fifteen attributes	3 alternatives	Janic [8]
6	Manufacturing plant location analysis	Five major attributes (16 sub-attributes)	5 alternatives	Yoon and Hwang [13]
7	Multiple response selection	Two attributes (or responses)	18 alternatives (or scenarios)	Yang and Chou [12]
8	Rapid prototyping-process selection	Six attributes	6 alternatives	Byun and Lee [21]
9	Robot selection	Four attributes	27 alternatives	Parkan and Wu [22]
10	Solid waste management	Twelve attributes	11 alternatives	Cheng et al. [5]
11	Water management	Six attributes (with 3 demand points)	12 alternatives (or scenarios)	Srdjevic et al. [11]

it would be suitable for cases with a large number of attributes and alternatives, and especially handy for objective or quantitative data given.

Due to its logical reasoning, TOPSIS has solved many real-world problems, especially in recent years in the Asian Pacific region. Its applications are various, and Table 2 illustrates eleven typical applicable areas. In addition, the attributes and alternatives involved are also listed in the corresponding cases. This advantage will accommodate many applications in the near future.

Now that certain groups are constantly making complex decisions within organizations, group decision making is drawing a lot more attention. In extending TOPSIS to a group decision environment, a couple of works have involved the preference aggregation among the group. We can classify these works as external aggregation and internal aggregation as shown in Table 3. The former utilizes some operations to manipulate the alternative ratings and weight ratings, or uses a social welfare function to obtain a final ranking from individual DMs of the group (i.e., outside the traditional TOPSIS procedure; see Chen [23], Chu [20], Parkan and Wu [24], Shih et al. [15,16]). The latter tries to aggregate the preference of individuals within the TOPSIS procedure as in our study (i.e., an integrated procedure). Moreover, in the external aggregation class, we can further distinguish the methods as pre-operation (i.e., mathematical operators for cardinal information) and post-operation (i.e., Borda's count or function for ordinal information), which depend on whether the aggregation is done before or after the TOPSIS procedure. However, in the external aggregation class, there is only cardinal information. It seems that external aggregation aims to provide more information to support a complex decision, and the internal aggregation focus is on an integrated decision making procedure. From a practical viewpoint, MADM is known for its ease of use, and the integrated procedure will keep this strength and obtain multiple sources of knowledge and experience. Thus, this study will concentrate on the integrated TOPSIS for achieving an efficient decision.

Table 3
The preference aggregation for TOPSIS in the group decision environment

Aggregation method	Target	Proposed by
I External aggregation		
I.1 Pre-operation (cardinal information)		
1 Weighted sum	Alternative rating on subjective attributes (1–9 scale)	Parkan and Wu [24]
2 Arithmetical mean	Alternative rating on attributes (fuzzy) weights rating of criteria (fuzzy)	Chen [23]
3 Arithmetical mean	Alternative rating on subjective attributes (fuzzy) weights rating of criteria (fuzzy)	Chu [20]
I.2 Post-operation (ordinal information)		
4 Borda's count	TOPSIS ranking	Shih et al. [15,16]
II Internal aggregation (cardinal information)		
1 Arithmetic mean	Separation measure	This study
2 Geometric mean	Separation measure	This study

Note: Beside Borda's count (or function), there are many social choice functions for group syntheses. Please check Hwang and Lin [25].

2.2. The operations within TOPSIS

In addition to making group environments more manageable, many operations in each step of TOPSIS are scrutinized so that a broad view of TOPSIS can be established. The operations within the TOPSIS process include: decision matrix normalization, distance measures, and aggregation operators, which will be described in the background information to be put forward in the following section.

For MADM, a decision matrix is usually required prior to the beginning of the process. The decision matrix contains competitive alternatives row-wise, with their attributes' ratings or scores column-wise. Normalization is an operation to make these scores conform to or reduced to a norm or standard. To compare the alternatives on each attribute, the normalized process is usually made column-wise, and the normalized value will be a positive value between 0 and 1. In this way, computational problems, resulting from different measurements in the decision matrix, are eliminated [26]. At the same time, Yoon and Hwang partition attributes into three groups: benefit attributes, cost attributes, and non-monotonic attributes. In addition, on the basis of the works of Hwang and Yoon [1], Milani et al. [10] and Yoon and Hwang [26], a few common normalization methods are organized in Table 4. These are classified as vector normalization, linear normalization, and non-monotonic normalization to fit real-world situations under different circumstances. Additionally, three forms for linear normalization are listed here.

Noted that we presume the available data being completed in the given decision matrix, including quantitative and qualitative information. The normalization of qualitative data or linguistic data could be first transformed to a linear scale, e.g., 1–10; then the above mentioned methods will be applicable (ref. to [1]).

Distance is the degree or amount of separation between two points, lines, surfaces, or objectives. Originally TOPSIS utilized Euclidean distances to measure the alternatives with their PIS and NIS so that the chosen alternative should have the shortest distance from the PIS and the farthest distance from the NIS. In fact, there are a couple of common distance measures, i.e., Minkowski's L_p metric in an n-dimensional space, where $p \geq 1$ (see Berberian [27]). Steuer [28] introduces weighted L_p metric, and the measures can be considered as the judgment in making a choice. The popular formulae of distance measures are depicted in Table 5.

On the basis of the proposed foundation, we will establish a generalized TOPSIS model in a group decision environment.

3. The proposed model

To include the multiple preferences of more than one DM, we will consider the separation measures by taking the geometric mean or arithmetic mean of the individuals for TOPSIS. The normalization methods and distance measures are also taken into consideration as well. Compared to the original TOPSIS procedure, the proposed model offers a general view of TOPSIS with group preference aggregation. The detailed procedure, with a few options within each step, is illustrated in the following.

Step 1. Construct decision matrix $\boldsymbol{D}^k$, $k = 1, \ldots, K$, for each DM.

Table 4
Some normalization methods for TOPSIS

1	Vector normalization $r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1, \ldots, m;\ j = 1, \ldots, n.$
2	Linear normalization (1) $r_{ij} = \frac{x_{ij}}{x_j^*}, i = 1, \ldots, m;\ j = 1, \ldots, n;\ x_j^* = \max_i\{x_{ij}\}$ for benefit attributes $r_{ij} = \frac{x_j^{\sim}}{x_{ij}}, i = 1, \ldots, m;\ j = 1, \ldots, n;\ x_j^{\sim} = \min_i\{x_{ij}\}$ or $r_{ij} = 1 - \frac{x_{ij}}{x_j^*}, i = 1, \ldots, m;\ j = 1, \ldots, n;\ x_j^* = \max_i\{x_{ij}\}$ for cost attributes
3	Linear normalization (2) $r_{ij} = \frac{x_{ij} - x_j^{\sim}}{x_j^* - x_j^{\sim}}$ for benefit attributes $r_{ij} = \frac{x_j^* - x_{ij}}{x_j^* - x_j^{\sim}}$ for cost attributes
4	Linear normalization (3) $r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}, i = 1, \ldots, m;\ j = 1, \ldots, n.$
5	Non-monotonic normalization $e^{-\frac{z^2}{2}}, z = \frac{(x_{ij} - x_j^0)}{\sigma_j}$; x_j^0 is the most favorable value and σ_j is the standard deviation of alternative ratings with respective to the jth attribute.

Note: (1) Please refer to Hwang and Yoon [1], Milani et al. [10], and Yoon and Hwang [26]. (2) Non-monotonic normalization is less used in the literature.

Table 5
Distance measures (functions) for TOPSIS

I	Minkowski's L_p metrics $L_p(x, y) = \{\sum_{j=1}^{n} \lvert x_j - y_j \rvert^p\}^{1/p}$, where $p \geq 1$ and with n dimensions (i) Manhattan (city block) distance $p = 1$ (ii) Euclidean distance $p = 2$ (iii) Tchebycheff distance $p = \propto$
II	Weighted L_p metrics $L_p(x, y) = \{(w_j \sum_{j=1}^{n} \lvert x_j - y_j \rvert)^p\}^{1/p}$, where $p \in \{1, 2, 3, \ldots\} \cup \{\propto\}$ w_j is the weight on the jth dimension or direction. The distance names are defined in the same way as above (i)–(iii).

Note: (1) Please refer to Berberian [27], Jones and Mardle [29], and Steuer [28] for details. (2) In general, p is a real number ≥ 1, but three positive integer values are common for Minkowski's p metric with $p = 1, 2$, and $\propto$. In addition, for $0 < p < 1$, the distance is a hyper-rectilinear distance, but it no longer has the properties of a norm.

The structure of the matrix can be expressed as follows:

$$\boldsymbol{D}^k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccccc} X_1 & X_2 & \cdots & X_j & \cdots & X_n \end{array} \\ \begin{bmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1j}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2j}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{i1}^k & x_{i2}^k & \cdots & x_{ij}^k & \cdots & x_{in}^k \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mj}^k & \cdots & x_{mn}^k \end{bmatrix} \end{array} \quad (1)$$

where A_i denotes the alternative i, $i = 1, \ldots, m$; X_j represents the attribute or criterion j, $j = 1, \ldots, n$; with quantitative and qualitative data. x_{ij}^k indicates the performance rating of alternative A_i with respect to attribute X_j by decision maker k, $k = 1, \ldots, K$, and x_{ij}^k is the element of $\boldsymbol{D}^k$. It is noted that there should be K decision matrices for the K members of the group.

Observe that we can also set the outcomes of qualitative attributes from each alternative as discrete values, e.g., 1 to 10 or linguistics values, so that the quantitative values will be placed in the above decision matrix [1].

Step 2. Construct the normalized decision matrix $\boldsymbol{R}^k$, $k = 1, \ldots, K$, for each DM.

For DM k, the normalized value r_{ij}^k of the decision matrix $\boldsymbol{R}^k$ can be any linear-scale transformation to keep $0 \le r_{ij}^k \le 1$. For a general definition, we introduce three operators $\odot$, $\bigcirc$, and $\otimes$. Then the normalized value can be represented as

$$r_{ij}^k = x_{ij}^k \odot \{x_{i1}^k \bigcirc x_{i2}^k \bigcirc \cdots \bigcirc x_{in}^k\} \otimes x_j^{k*} \tag{2a}$$

or

$$r_{ij}^k = x_{ij}^k \odot \{x_{i1}^k \bigcirc x_{i2}^k \bigcirc \cdots \bigcirc x_{in}^k\} \otimes x_j^{k\sim} \tag{2b}$$

where $x_j^{k*} = \max_i\{x_{ij}^k\}$ and $x_j^{k\sim} = \min_i\{x_{ij}^k\}$ for $i = 1, \ldots, m$, $j = 1, \ldots, n$, and $k = 1, \ldots, K$.

For normalization, Eq. (2a) for benefit criterion j will be

$$r_{ij}^k = \frac{x_{ij}^k}{x_j^{k*}}; \tag{3a}$$

And Eq. (2b) for cost criterion j will be

$$r_{ij}^k = \frac{x_j^{k\sim}}{x_{ij}^k}. \tag{3b}$$

Moreover, if we simply consider that the normalized value of r_{ij}^k is the value of the corresponding element x_{ij}^k divided by the operation of its column elements, i.e., vector normalization, then

$$r_{ij}^k = \frac{x_{ij}^k}{\sqrt{\sum_{j=1}^{n} (x_{ij}^k)^2}}, \tag{4}$$

where $i = 1, \ldots, m$; $j = 1, \ldots, n$; and $k = 1, \ldots, K$.

Note that while utilizing Eq. (4) for normalization, we will make a distinction as to which one is a cost criterion for further manipulation.

In addition, we do not go directly to the process weighted normalized matrix as in the original TOPSIS. Please see Shipley et al. [30] for the reason.

Step 3. Determine the ideal and negative ideal solutions V^{k+} (PIS) and V^{k-} (NIS), respectively, for each DM $k = 1, \ldots, K$.

For DM k, his or her PIS and NIS are

$$\begin{aligned} V^{k+} &= \{r_1^{k+}, \ldots, r_n^{k+}\} = \{(\max_i r_{ij}^k \mid j \in J), (\min_i r_{ij}^k \mid j \in J')\}, \\ V^{k-} &= \{r_1^{k-}, \ldots, r_n^{k-}\} = \{(\min_i r_{ij}^k \mid j \in J), (\max_i r_{ij}^k \mid j \in J')\}, \end{aligned} \tag{5}$$

where J is associated with the benefit criteria and J' is associated with the cost criteria; $i = 1, \ldots, m$; $j = 1, \ldots, n$; and $k = 1, \ldots, K$.

Step 4. Assign a weight vector $\boldsymbol{W}$ to the attribute set for the group.

Each DM will elicit weights for attributes as w_j^k, where $j = 1, \ldots, n$, and $\sum_{j=1}^{n} w_j^k = 1$; and for each DM $k = 1, \ldots, K$. Each element of the weight vector $\boldsymbol{W}$ will be the operation of the corresponding elements of the attributes' weights per DM.

Step 5. Calculate the separation measure from the ideal and the negative ideal solutions, $\overline{S_i^+}$ and $\overline{S_i^-}$, respectively, for the group.

There are two sub-steps to be considered in Step 6. The first one concerns the distance measure for individuals; the second one aggregating the measures for the group.

Step 5a. Calculate the measures from PIS and NIS individually.

For DM k, his or her separation measures from PIS and NIS are computed through Minkowski's L_p metric. The individual separation measures of each alternative from the PIS and NIS are

$$S_i^{k+} = \left\{ \sum_{j=1}^{n} w_j^k (v_{ij}^k - v_j^{k+})^p \right\}^{1/p}, \quad \text{for alternative } i, i = 1, \dots, m \tag{6}$$

and

$$S_i^{k-} = \left\{ \sum_{j=1}^{n} w_j^k (v_{ij}^k - v_j^{k-})^p \right\}^{1/p}, \quad \text{for alternative } i, i = 1, \dots, m \tag{7}$$

where $p \geq 1$ and integer, w_j^k is the weight for the attribute j and DM k, and $\sum_{j=1}^{n} w_j^k = 1, k = 1, \dots, K$.

If we let $p = 2$, the metric is a Euclidean distance. Eqs. (6) and (7) will be

$$S_i^{k+} = \sqrt{\sum_{j=1}^{n} w_j^k (v_{ij}^k - v_j^{k+})^2}, \quad \text{for alternative } i, i = 1, \dots, m \tag{8}$$

and

$$S_i^{k-} = \sqrt{\sum_{j=1}^{n} w_j^k (v_{ij}^k - v_j^{k-})^2}, \quad \text{for alternative } i, i = 1, \dots, m. \tag{9}$$

Step 5b. Calculate the measures of PIS and NIS for the group.

In addition, the group separation measure of each alternative will be combined through an operation $\otimes$ for all DMs, $k = 1, \dots, K$. Thus, the two group measures of the PIS and NIS are the following two equations:

$$\overline{S_i^+} = S_i^{1+} \otimes \cdots \otimes S_i^{K+}, \quad \text{for alternative } i, \tag{10}$$

$$\overline{S_i^-} = S_i^{1+} \otimes \cdots \otimes S_i^{K+}, \quad \text{for alternative } i. \tag{11}$$

The operation can offer many choices, geometric mean, arithmetic mean, or their modification. If we take the geometric mean of all individual measures, the group measures, Eqs. (8) and (9), from PIS and NIS will be

$$\overline{S_i^+} = \left(\prod_{k=1}^{K} S_i^{k+} \right)^{\frac{1}{K}}, \quad \text{for alternative } i, \tag{12}$$

and

$$\overline{S_i^-} = \left(\prod_{k=1}^{K} S_i^{k-} \right)^{\frac{1}{K}}, \quad \text{for alternative } i, \tag{13}$$

where $i = 1, \dots, m; k = 1, \dots, K$.

Step 6. Calculate the relative closeness $\overline{C_i^*}$ to the ideal solution for the group.

Calculate the relative closeness to the ideal solution and rank the alternatives in descending order. The relative closeness of the ith alternative A_i with respect to PIS can be expressed as

$$\overline{C_i^*} = \frac{\overline{S_i^-}}{\overline{S_i^+} + \overline{S_i^-}}, \quad i = 1, \dots, m \tag{14}$$

where $0 \leq \overline{C_i^*} \leq 1$. The larger the index value, the better the performance of the alternative.

Table 6a
Decision matrix of Example 1—objective attributes

No.	Candidates	Objective attributes				
		Knowledge tests			Skill tests	
		Language test	Professional test	Safety rule test	Professional skills	Computer skills
1	James B. Wang	80	70	87	77	76
2	Carol L. Lee	85	65	76	80	75
3	Kenney C. Wu	78	90	72	80	85
4	Robert M. Liang	75	84	69	85	65
5	Sophia M. Cheng	84	67	60	75	85
6	Lily M. Pai	85	78	82	81	79
7	Abon C. Hsieh	77	83	74	70	71
8	Frank K. Yang	78	82	72	80	78
9	Ted C. Yang	85	90	80	88	90
10	Sue B. Ho	89	75	79	67	77
11	Vincent C. Chen	65	55	68	62	70
12	Rosemary I. Lin	70	64	65	65	60
13	Ruby J. Huang	95	80	70	75	70
14	George K. Wu	70	80	79	80	85
15	Philip C. Tsai	60	78	87	70	66
16	Michael S. Liao	92	85	88	90	85
17	Michelle C. Lin	86	87	80	70	72

Note: (1) There is no difference in objective attributes among the group. (2) There are a total of 17 candidates for evaluation. (3) All listed attributes are benefit attributes.

Step 7. Rank the preference order.

A set of alternatives can now be preference ranked according to the descending order of the value of $\overline{C_i^*}$.

As the integrated model has outlined, we will now illustrate the process through an example.

Example 1. A human resources selection example.

A local chemical company tries to recruit an on-line manager. The company's human resources department provides some relevant selection tests, as the benefit attributes to be evaluated. These include knowledge tests (language test, professional test and safety rule test), skill tests (professional skills and computer skills), and interviews (panel interview and 1-on-1 interviews). There are 17 qualified candidates on the list, and four decision makers are responsible for the selection. The basic data, including objective and subjective attributes (only quantitative information here), for the decision are listed in Tables 6a and 6b. In addition, the weights of attributes, elicited by DMs, are shown in Table 7.

Following the suggested steps, each DM will construct a normalized decision matrix, determine PIS/NIS, and calculate the separation measures. Then, through aggregation by geometric mean, the relative closeness can be calculated as illustrated in Table 8. We can see that the 16th candidate is ranked first, and the 11th candidate is ranked last.

4. External aggregation versus internal aggregation under certain circumstances

For extending TOPSIS to a group decision environment, Table 3 has illustrated some typical past works. Most of the works are classified as external aggregations that use simple operators to aggregate the importance of the criteria and/or the rating of alternatives with respect to each criterion from individuals of the group [20,23,24]. Alternatively, we propose a group TOPSIS model for deriving group priorities provided by multiple DMs and synthesizing them into a single integrated stage. This can be categorized as internal aggregation because all individual preferences are grouped into the TOPSIS procedure and weighted distances are utilized instead of a weighted decision matrix. This development seems more meaningful since it is truly a group decision process for TOPSIS.

As pointed out by Shih et al. [15], the weights rating of attributes and the rating of alternatives on attributes are usually externally defined through teamwork or a task group; at the least, there exists some kind of consent in the use of common MADM techniques. From this viewpoint, the works of Chen [23], Chu [20], and Parkan and Wu [24], a

Table 6b
Decision matrix of Example 1—subjective attributes

No.	Subjective attributes							
	DM #1		DM #2		DM #3		DM #4	
	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview
1	80	75	85	80	75	70	90	85
2	65	75	60	70	70	77	60	70
3	90	85	80	85	80	90	90	95
4	65	70	55	60	68	72	62	72
5	75	80	75	80	50	55	70	75
6	80	80	75	85	77	82	75	75
7	65	70	70	60	65	72	67	75
8	70	60	75	65	75	67	82	85
9	80	85	95	85	90	85	90	92
10	70	75	75	80	68	78	65	70
11	50	60	62	65	60	65	65	70
12	60	65	65	75	50	60	45	50
13	75	75	80	80	65	75	70	75
14	80	70	75	72	80	70	75	75
15	70	65	75	70	65	70	60	65
16	90	95	92	90	85	80	88	90
17	80	85	70	75	75	80	70	75

Note: (1) There are four decision makers (DMs) selected for the evaluation. (2) There are a total of 17 candidates for evaluation. (3) All listed attributes are benefit attributes.

Table 7
Weights on attributes of Example 1

No.	Attributes	The weights of the group			
		DM #1	DM #2	DM #3	DM #4
	Knowledge tests				
1	Language test	0.066	0.042	0.060	0.047
2	Professional test	0.196	0.112	0.134	0.109
3	Safety rule test	0.066	0.082	0.051	0.037
	Skill tests				
4	Professional skills	0.130	0.176	0.167	0.133
5	Computer skills	0.130	0.118	0.100	0.081
	Interviews				
6	Panel interview	0.216	0.215	0.203	0.267
7	1-on-1 interview	0.196	0.255	0.285	0.326
	Sum	1.000	1.000	1.000	1.000

Note: There are four DMs selected for the evaluation.

part of external aggregation, seem not to have much meaning if we get rid of the argument between fuzzy and crisp sets. In addition, some choice functions (e.g., Borda's function, Copeland's function) and statistical methods can be applied to aggregate the preference orders of MADM results [1]. It is rather natural and logical to make the extension of MADM techniques to a group decision environment as in the development by Shih et al. [15,16].

Moreover, for a further investigation, some operations in each step of TOPSIS might have an effect on the results of aggregation. The first operation is as regards the methods of normalization. These include vector normalization, several forms of linear normalization, and non-monotonic normalization (see Table 4). The first two methods are popular for decision making, and the third one is used only on some occasions. The second operation is related to distance measures (see Table 5). There are three common distances of Minkowski's L_p metrics to be recognized: Manhattan distance $p = 1$, Euclidean distance $p = 2$, and Tchebycheff distance $p = \propto$. The first two seem to most easily fit our purpose. In addition, there are also three common means to be considered: arithmetical mean, geometric

Table 8
The relative closeness and rank by group TOPSIS of Example 1

No.	Separation measure of the group		Relative closeness	Rank	Note
	$\overline{S_i^+}$	$\overline{S_i^-}$	$\overline{C_i^*}$		
1	0.0437	0.0735	0.6272	5	
2	0.0653	0.0514	0.4404	14	
3	0.0260	0.0956	0.7860	3	
4	0.0684	0.0566	0.4527	12	
5	0.0640	0.0558	0.4660	11	
6	0.0388	0.0757	0.6611	4	
7	0.0653	0.0539	0.4523	13	
8	0.0503	0.0667	0.5701	8	
9	0.0141	0.1031	0.8797	2	
10	0.0578	0.0596	0.5080	10	
11	0.0916	0.0243	0.2097	16	
12	0.0965	0.0194	0.1678	17	#
13	0.0523	0.0657	0.5568	9	
14	0.0483	0.0702	0.5924	6	
15	0.0717	0.0497	0.4091	15	
16	0.0120	0.1032	0.8960	1	*
17	0.0485	0.0704	0.5920	7	

Note: (1) The separation measure of the group is counted through the geometric mean of all DMs with Euclidean distance and vector normalization. (2) There are a total of 17 candidates for evaluation. (3) "*" and "#" mark the first and the last candidate, respectively.

mean, and harmonic mean. The first two are common for group aggregation (see Table 3). In general, these operations could be organized in different combinations for decision making. To understand how robust a decision will be, we will observe the effects of these combinations on external and internal aggregations through the following example.

Example 2. Aggregation comparison for Example 1.

On the basis of the same decision information as for Example 1 with 17 candidates or alternatives to be ranked, we will further investigate the effects of external and internal aggregations on the group decision under the circumstances of linear and vector normalization, Manhattan and Euclidean distances, and arithmetical and geometric means as well. Due to space limitations, just parts of the results from these combinations are illustrated, in Tables 9a, 9b and 9c, respectively.

In the portion of internal aggregation, Table 9a is the direct extension of Table 8. The ranking results from the Manhattan/Euclidean distance and arithmetical/geometric mean, with vector normalization, are rather consistent. The ranks of the first ten and the last three candidates do not change. Because the results of several forms of linear normalization are consistent, we will omit them to save space. Under external aggregation, Table 9b illustrates the ranking results from external operation aggregation with a number of combinations. The ranks of the first five and the last three candidates do not change. The ranking results are also consistent with vector normalization and linear normalization; however, only the cases using vector normalization are presented here. Table 9c illustrates the ranking results from post-operation in external aggregation with several combinations. The ranking results are rather inconsistent. Among the three sub-tables, the ranks in Table 9c are the most diverse.

After examining the three tables, we feel there is not much difference between external and internal aggregation with pre-operational methods based on the human resources selection example, though the ranks in the former seem more robust than the latter's. Since cardinal information is processed throughout all the steps of TOPSIS, the demonstrated solutions do not make much difference between these two. Moreover, the results from various combinations have only slight differences as regards rank. Thus, these two types of aggregations can be considered methods for acquiring strong decisions. On the other hand, dissimilar ranking results are generated by external aggregation using a post-operation method, e.g., Borda's function. The phenomenon shows us that ordinal information might not be suitable for aggregation with cardinal information due to heterogeneous data. Although social choice functions try to aggregate group preferences, further work is necessary to understand the contents of aggregation of preferences.

Table 9a
Summary of the results of Example 2—internal aggregation by the proposed model

Candidate no.	Manhattan distance $p = 1$				Euclidean distance $p = 2$				Note
	Arithmetical mean		Geometric mean		Arithmetical mean		Geometric mean		
	Relative closeness $\overline{C_i^*}$	Rank	Relative closeness $\overline{C_i^*}$	Rank	Relative closeness $\overline{C_i^*}$	Rank	Relative closeness $\overline{C_i^*}$	Rank	
1	0.6359	5	0.6369	5	0.6295	5	0.6272	5	
2	0.4476	12	0.4463	11	0.4407	14	0.4404	14	
3	0.8285	3	0.8361	3	0.7583	3	0.7860	3	
4	0.4471	13	0.4394	13	0.4523	13	0.4527	12	
5	0.4549	11	0.4446	12	0.4651	11	0.4660	11	
6	0.6678	4	0.6698	4	0.6591	4	0.6611	4	
7	0.4440	14	0.4383	14	0.4551	12	0.4523	13	
8	0.5708	8	0.5683	8	0.5692	8	0.5701	8	
9	0.9071	2	0.9131	2	0.8749	2	0.8797	2	
10	0.5055	10	0.5076	10	0.5063	10	0.5080	10	
11	0.1930	16	0.1637	16	0.2302	16	0.2097	16	
12	0.1465	17	0.1322	17	0.1745	17	0.1678	17	#
13	0.5574	9	0.5579	9	0.5562	9	0.5568	9	
14	0.6044	6	0.6033	6	0.5936	6	0.5924	6	
15	0.3926	15	0.3942	15	0.4077	15	0.4091	15	
16	0.9126	1	0.9201	1	0.8905	1	0.8960	1	*
17	0.5958	7	0.5976	7	0.5908	7	0.5920	7	

Note: (1) There are a total of 17 candidates for evaluation. (2) There are two distance measures used, and each one has two types of aggregation for the group, arithmetic mean and geometric mean with vector normalization. (3) "*" and "#" mark the first and the last candidate, respectively.

Table 9b
Summary of the results of Example 2—internal aggregation/internal operation (1)

Candidate no.	Manhattan distance $p = 1$				Euclidean distance $p = 2$				Note
	Arithmetical mean		Geometric mean		Arithmetical mean		Geometric mean		
	Relative closeness $\overline{C_i^*}$	Rank	Relative closeness $\overline{C_i^*}$	Rank	Relative closeness $\overline{C_i^*}$	Rank	Relative closeness $\overline{C_i^*}$	Rank	
1	0.6247	5	0.6264	5	0.6306	5	0.6345	5	
2	0.4176	12	0.4221	11	0.3819	14	0.3871	14	
3	0.8451	3	0.8460	3	0.8401	3	0.8416	3	
4	0.4116	13	0.4139	13	0.3979	12	0.3973	12	
5	0.4286	11	0.4215	12	0.4141	11	0.4044	11	
6	0.6712	4	0.6741	4	0.6643	4	0.6685	4	
7	0.4065	14	0.4039	14	0.3937	13	0.3955	13	
8	0.5409	9	0.5399	9	0.5214	9	0.5197	9	
9	0.9404	2	0.9047	2	0.9308	2	0.9326	2	
10	0.4854	10	0.4887	10	0.4660	10	0.4705	10	
11	0.1135	16	0.1195	16	0.1266	16	0.1330	16	
12	0.0809	17	0.0793	17	0.1032	17	0.1006	17	#
13	0.5424	8	0.5455	8	0.5337	8	0.5372	8	
14	0.5941	6	0.5981	6	0.5729	7	0.5781	7	
15	0.3540	15	0.3572	15	0.3573	15	0.3595	15	
16	0.9554	1	0.9555	1	0.9352	1	0.9366	1	*
17	0.5915	7	0.5933	7	0.5807	6	0.5829	6	

Note: (1) There are a total of 17 candidates for evaluation. (2) There are two distance measures used, and each one has two types of aggregation for the group, arithmetic mean and geometric mean with vector normalization. (3) "*" and "#" marked the first and the last candidate, respectively.

The experiment results tell us that our model can provide a robust decision for TOPSIS in a group decision environment. Regardless of the sizes of examples being tested, we think the model would be meaningful and useful as an integrated procedure.

Table 9c
Summary of the results of Example 2—external aggregation/internal operation (2)

Candidate no.	Vector normalization				Linear normalization (1)				Note
	Manhattan distance		Euclidean distance		Manhattan distance		Euclidean distance		
	Borda's score	Rank	Borda's score	Rank	Borda's score	Rank	Borda's score	Rank	
1	61	1	42	5	61	1	41	6	
2	16	14	19	12	15	14	19	12	
3	48	3	57	3	48	4	57	3	
4	17	13	18	13	18	13	19	12	
5	22	11	25	11	21	11	25	11	
6	48	3	49	4	48	4	49	4	
7	20	12	17	14	21	11	17	14	
8	39	9	34	8	40	8	34	8	
9	50	2	62	1	50	2	62	1	
10	26	10	28	10	26	10	28	10	
11	2	17	2	16	2	16	2	16	
12	3	16	2	16	2	16	2	16	
13	43	7	33	9	43	7	33	9	
14	41	8	42	5	40	8	42	5	
15	12	15	12	15	12	15	28	15	
16	48	3	61	2	49	3	61	2	
17	48	3	41	7	48	4	41	6	

Note: (1) There are a total of 17 candidates for evaluation. (2) There are two distance measures used, and each one has two types of aggregation for the group, arithmetic mean and geometric mean. (3) Please check Table 4 for the formula for linear normalization used here, and only the results of the first form of linear normalization are listed.

5. Conclusions and remarks

We have proposed a group TOPSIS model for decision making. After checking the aggregations under various circumstances, we can see that the model is rather simple to use and meaningful for aggregation, and it will not cause more computational burden than the original TOPSIS. In addition, two examples have demonstrated the model is efficient and robust. It is quite good for real-world applications.

We did not involve the topic of weight elicitation in this study, as it is usually assumed that the weights of attributes are given as TOPSIS begins. Some authors suggest using AHP or other techniques to obtain the weights, as in the study of Shih et al. [15]. Moreover, interested readers can also refer to Olson [31] for different weighting schemes in TOPSIS models.

Although some observations are obtained from the given examples, we are confident the results for various examples would give us similar conclusions. However, we still think a large number of examples (as regards the aspects of weighting combinations, normalization methods and scaling techniques, distance measures, and group synthesis) should be recommended for test in future studies.

We have not discussed consensus and other group interactions in the study. Interested readers can refer to Shih et al. [16] as regards consensus on weights of attributes. Any topic related to group interactions would be an interesting one for group decision making, and will be left for future study.

Acknowledgements

The authors would like to express their gratitude to Professor So-Chin Chen at the Department of Mathematics, National Tsing Hua University, and Professor Chi-chiang Jou at the Department of Information Management, Tamkang University, Taiwan, for their valuable help. Special thanks are given to National Science Council, Taiwan, ROC, for financial support under the grant numbers: 90-2218-E-214-010, NSC 91-2213-E-032-035, NSC 92-2213-E-032-032, and NSC 92-2213-E-032-020.

References

[1] C.L. Hwang, K. Yoon, Multiple Attribute Decision Making, Springer-Verlag, Berlin, 1981.

[2] S.M. Belenson, K.C. Kapur, An algorithm for solving multicriterion linear programming problems with examples, Operational Research Quarterly 24 (1) (1973) 65–77.
[3] M. Zeleny, A concept of compromise solutions and the method of the displaced ideal, Computers and Operations Research 1 (1974) 479–496.
[4] G. Kim, C.S. Park, K.P. Yoon, Identifying investment opportunities for advanced manufacturing systems with comparative-integrated performance measurement, International Journal of Production Economics 50 (1997) 23–33.
[5] S. Cheng, C.W. Chan, G.H. Huang, Using multiple criteria decision analysis for supporting decision of solid waste management, Journal of Environmental Science and Health, Part A 37 (6) (2002) 975–990.
[6] S.H. Zanakis, A. Solomon, N. Wishart, S. Dublish, Multi-attribute decision making: A simulation comparison of selection methods, European Journal of Operational Research 107 (1998) 507–529.
[7] M.F. Chen, G.H. Tzeng, Combining gray relation and TOPSIS concepts for selecting an expatriate host country, Mathematical and Computer Modelling 40 (2004) 1473–1490.
[8] M. Janic, Multicriteria evaluation of high-speed rail, transrapid maglev, and air passenger transport in Europe, Transportation Planning and Technology 26 (6) (2003) 491–512.
[9] C.K. Kwong, S.M. Tam, Case-based reasoning approach to concurrent design of low power transformers, Journal of Materials Processing Technology 128 (2002) 136–141.
[10] A.S. Milani, A. Shanian, R. Madoliat, The effect of normalization norms in multiple attribute decision making models: A case study in gear material selection, Structural Multidisciplinary Optimization 29 (4) (2005) 312–318.
[11] B. Srdjevic, Y.D.P. Medeiros, A.S. Faria, An objective multi-criteria evaluation of water management scenarios, Water Resources Management 18 (2004) 35–54.
[12] T. Yang, P. Chou, Solving a multiresponse simulation–optimization problem with discrete variables using a multi-attribute decision-making method, Mathematics and Computers in Simulation 68 (2005) 9–21.
[13] K. Yoon, C.L. Hwang, Manufacturing plant location analysis by multiple attribute decision making: Part I—single-plant strategy, International Journal of Production Research 23 (1985) 345–359.
[14] Y.J. Lai, TOPSIS for MODM, European Journal of Operational Research 76 (1994) 486–500.
[15] H.-S. Shih, W.Y. Lin, E.S. Lee, Group decision making for TOPSIS, in: Joint 9th IFSA World Congress and 20th NAFIPS International Conference, IFSA/NAFIPS 2001, 25–28 July, Vancouver, Canada, 2001, pp. 2712–2717.
[16] H.S. Shih, C.H. Wang, E.S. Lee, A multiattribute GDSS for aiding problem-solving, Mathematical and Computer Modelling 39 (11–12) (2004) 1397–1412.
[17] T.L. Saaty, The Analytic Hierarchy Process, 2nd ed., RWS Pub., Pittsburgh, PA, 1990.
[18] T.L. Saaty, M.S. Ozdemir, Why the magic number seven plus or minus two, Mathematical and Computer Modelling 38 (2003) 233–244.
[19] H. Deng, C.H. Yeh, R.J. Willis, Inter-company comparison using modified TOPSIS with objective weights, Computers and Operations Research 27 (2000) 963–973.
[20] T.C. Chu, Facility location selection using fuzzy TOPSIS under group decision, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 10 (6) (2002) 687–701.
[21] H.S. Byun, K.H. Lee, A decision support system for the selection of a rapid prototyping process using the modified TOPSIS method, International Journal of Advanced Manufacturing Technology 26 (11–12) (2005) 1338–1347.
[22] C. Parkan, M.L. Wu, Decision-making and performance measurement models with applications to robot selection, Computers and Industrial Engineering 36 (1999) 503–523.
[23] C.T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems 114 (1) (2000) 1–9.
[24] C. Parkan, M.L. Wu, Process selection with multiple objective and subjective attributes, Production Planning and Control 9 (2) (1998) 189–200.
[25] C.L. Hwang, M.J. Lin, Group Decision Making under Multiple Criteria, Springer-Verlag, Berlin, 1987.
[26] K.P. Yoon, C.L. Hwang, Multiple Attribute Decision Making: An Introduction, Sage Pub., Thousand Oaks, CA, 1995.
[27] S.K. Berberian, Fundamentals of Real Analysis, Springer, NY, 1999.
[28] R.E. Steuer, Multiple Criteria Optimization: Theory, Computation, and Application, John Wiley, NY, 1986.
[29] D.F. Jones, S.J. Mardle, A distance-metric methodology for the derivation of weights from a pairwise comparison matrix, Journal of the Operational Research Society 55 (2004) 869–875.
[30] M.F. Shipley, D.K. Korvin, R. Obit, A decision making model for multi-attribute problems incorporating uncertainty and bias measures, Computers and Operations Research 18 (1991) 335–342.
[31] D.L. Olson, Comparison of weights in TOPSIS models, Mathematical and Computer Modelling 40 (2004) 721–727.