Rev Quant Finan Acc (2007) 28:203–225
DOI 10.1007/s11156-006-0008-5

Oil convenience yields estimated under demand/supply shock

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Published online: 29 December 2006

Abstract This paper extends the call option model of Milonas and Thomadakis (1997) to estimate oil convenience yields with futures prices. We define the business cycle of a seasonal commodity with demand/supply shocks and find that the convenience yield for crude oil exhibits seasonal behavior. The convenience yield for West Texas Intermediate (WTI) crude oil is the highest in the summer, while that for Brent crude oil is the highest in the winter. This implies that WTI crude oil is more sensitive to high summer demand and that Brent crude oil is more sensitive to shortages in winter supply. Convenience yields are negatively related to the inventory level of the underlying crude oil and positively related to interest rates due to the business cycle. We also show that convenience yields may explain price spread between WTI crude oil and Brent crude oil. Our computed convenience yields are consistent with Fama and French (1988) in that oil prices are more volatile than futures prices at low inventory level, verifying the Samuelson (1965) hypothesis that future prices are less variables than spot prices at lower inventory levels.

Keyword Business cycle · Convenience yield · Demand/supply shock · Theory of storage · Two-period model

JEL Classification E12 · E32 · G13 · Q00 · Q18 · Q40

An early version of this paper was presented at the 11th Annual Conference on Pacific Basin Finance, Economics and Accounting in Nov. 2003, the Conference on Financial Engineering Association of Taiwan in May 2004, and the Global Business and Economics Research Conference, Turkey, August 2004.

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1 Introduction

Convenience yield is important in the theory of storage and serves as an incentive to hold spot commodities. In this paper, we extend the call option model of Milonas and Thomadakis (1997) to estimate West Texas Intermediate (WTI) and Brent crude oil convenience yields. Our estimated convenience yields extend the Fisher (1978) model, which utilizes a non-traded asset as strike price. We instead set the price of a traded asset to be the strike price of the option in our model. In addition, our analyses differ from the Milonas and Thomadakis model in that we not only fit the price of a traded asset, such as crude oil futures prices, as the strike price but also add in storage cost when estimating the convenience yield. Our objective is to examine convenience yields while taking into account inventory, and interest rates, evaluating the consistency between our empirical study and the theory of storage.

Keynes (1930) points out that in the theory of liquid stocks the risk associated with holding spot goods is much higher than holding forward contracts. When inventory level is lower than planned due to unexpected demand/supply shock, the spot price will be higher than the forward price. This price difference is regarded as a risk premium, and such price behavior is referred to as "backwardation." In the theory of storage, Kaldor (1939), Working (1948, 1949), Brennan (1958), and Telser (1958) use convenience yields to explain this inverted market phenomenon, where spot prices are higher than futures price, and convenience yields can be viewed as a benefit of holding storable consumption goods. According to the theory of storage, a high inventory level implies a lower probability of a stock-out in the future. Open futures contracts help lower excess demand in the spot market, hence reducing the benefit of holding the commodity. Consequently, convenience yields are negatively related to inventory levels. Such a relationship is stronger for a commodity that is more sensitive to seasonality or supply/demand effect. Therefore, convenience yield plays a central role in explaining the benefits of holding inventory during periods of unexpected demand/supply shock.

Consistent with the theory of storage, Samuelson (1965) predicts that spot and futures price variations will be similar when a supply/demand shock occurs during higher inventory levels, but spot prices will be more variable than the futures prices at lower inventory levels. Fama and French (1987, 1988) verify the proposition of Samuelson (1965) and show that a negative relationship does exist between convenience yields and inventory levels of seasonal commodities. Heinkel et al. (1990) derive a model in which convenience yields behave like options. Their two-period call option model also supports an inverse relationship between inventory levels and convenience yields. Particularly, convenience yields arise from unexpected supply/demand in the cycle of spot markets, which needs to be considered when estimating convenience yield. In their empirical study of metal commodity, Fama and French (1988) suggest that metal inventory and prices are not affected by seasonality but by general business conditions. Their evidence indicates that metal production does not adjust quickly to positive demand shocks during business-cycle peaks.

Previous studies estimate convenience yields using the cost-of-carry model, where the convenience yield is treated as an exogenous variable. Brennan (1986) finds that convenience yields follow a mean-reverting process. Gibson and Schwartz (1990) use a cost-of-carry model with stochastic mean-reverting convenience yields, which is connected to the time to maturity of a futures contract, but they assume an exogenously specified convenience yields measure. Casassus and Collin-Dufresne (2005) use a three-factor Gaussian model to capture commodity futures prices and integrates all three variables analyzed by Schwartz (1997). Their model allows convenience yields to depend on spot prices and interest rate, which leads to mean-reverting convenience yields as seen in Gibson and Schwartz (1990). Milonas and Thomadakis (1997) find that mean-reverting behavior does not necessarily occur for commodities with seasonal business cycles, so it is inappropriate to assume convenience yields as an exogenous stochastic mean-reverting variable. This is because the business cycle affects supply, inventory and demand in a systematic manner, and this is not necessarily consistent with mean reversion. Most theoretical models of convenience yields assume that storage costs are zero, such as Fama and French (1988). They use interest-adjusted basis as a proxy, which avoids the difficulty of estimating storage cost, and develop a relationship between convenience yield and inventory level without directly estimating the storage costs.

To the extent that studies above are subject to empirical verification, they do not offer explicit measures of the determining variables in the context of an option model. Milonas and Thomadakis (1997) extend the option approach of Heinkel et al. (1990) with the Black–Scholes model to estimate convenience yields. Although they model convenience yields as call options, the approach is problematic in that convenience yields reenter the call option equations when spot prices are used as underlying variables. That requires estimating an unknown variable. In addition, storage cost is also ignored, which could result in a negative convenience yield in their estimation with a cost of carry model.

When estimating convenience yields in this study, we consider unexpected demand/supply shock and business cycle, and we specify the beginning month, the intermediate month, and the final month of a crude oil business cycle. To deal with the two issues, we first choose, under an option pricing framework, the price of a futures contract maturing in the intermediate month as the underlying variable, hereafter termed the *nearby* contract. Second, unlike Fisher (1978) which utilizes a non-traded asset as the strike price, we instead set the price of a traded asset to be the strike price of the option in our model. For the traded asset, we use a futures contracts maturing in the final month, which will be termed hereafter as the *distant* contract. This resolves the problem of unknown variable encountered in Milonas and Thomadakis (1997). Convenience yields of WTI and Brent crude oil are then estimated assuming that the price of the underlying asset and the strike price are both stochastic variables. Storage costs are also incorporated to avoid potential problems in their study.

Fama and French (1988) use a simple proxy for the level of inventory to test of the theory of storage and is consistent with the Samuelson (1965) proposition that future prices are less variables than spot prices at lower inventory levels. Therefore, we adopt real inventory of crude oil to examine the theory of storage in our estimation and use the approach of Fama and French to test its implications about the variation of spot and futures prices.

Few studies examine how interest rates affect convenience yields. The theory of storage suggests that holding inventory becomes more costly in periods of high interest rates, so we may expect a negative correlation between interest rates and inventory. However, to the extent that inventory and interest rates are both relative measures, it seems consistent with the theory to find a relationship between interest rates and convenience yields. Casassus and Collin-Dufresne (2005) show that the sensitivity of convenience yields to interest rate is positive for crude oil, copper, gold and silver, which is consistent with the theory of storage. Furthermore, as interest rates are related to economic activity, interest rates in turn affect convenience yields of various commodities. So, we also examine the relationship between convenience yields and interest rate with a cross-sectional regression model.

In the sections that follow, Section 2 discusses the status of the crude oil markets, Section 3 presents the model. Section 4 explains the empirical analysis. Finally, Section 5 provides the concluding remarks.

2 Characteristics of crude oil markets

The crude oil market is subject to a seasonal cycle of supply and demand, with prices price adjusting to supply/demand disequilibrium. This process of market adjustment from disequilibrium to equilibrium may be regarded as a business cycle, and, crude oil convenience yields should exist during periods of unexpected demand/supply shocks within a business cycle. Crude oil is traditionally traded by means of futures contracts. Most of the crude oil futures trades take place in the New York Mercantile Exchange (NYMEX) and London's International Petroleum Exchange[1] (IPE). NYMEX trades crude oil futures are based on West Texas Intermediate (WTI) crude oil, while IPE contracts are based on North Sea Brent Blend crude oil. Since contracts traded in these two markets are based on crude oil from different production areas, observing their convenience yields involves issues different from dealing with different commodities within a single market. The asked spot prices of crude oils from the North American and West African oil fields are quoted based on the value of WTI crude oil. On the other hand, the asked prices in the European market and from oil fields in the North Sea, Russia, northern African and the Middle East generally use Brent crude oil as their benchmark. Most of the crude oil spot markets around the world give quotes based either on WTI or Brent crude oil due to their stable supplies.

Table 1 illustrates the annual average spot prices, rates of return and volatilities of WTI and Brent crude oils. We find that the annual average spot price of WTI is always higher than that of Brent. The price advantage of WTI crude oil could be tested on the basis of demand and supply, as its futures price is influenced by economy, weather and consumer behavior. The oil fields and trading markets for WTI and Brent are located in the same climate zone. WTI crude oil supplies the vast North American and global consumer markets, while Brent supplies the relatively smaller European consumer market. Moreover, the delivery point for the WTI spot is closer to the refineries, and there is a standard settlement contract for WTI, while Brent's delivery point is located far away from the refineries, and there is no standard settlement contract. Tanker shipments of Brent oil to the refineries at times face the port freezing problem. Crude oil is traded mostly in the form of forward or futures contracts. In March 24, 2006,

[1] IPE has changed its name to IntercontinentalExchange (ICE) Futures. In June of 2001, ICE expanded its business into futures trading by acquiring the IPE.

Table 1 Summary statistics for WTI and brent crude oils

Year	WTI Spot Price ($/bbl)				WTI minus Brent	Brent Spot Price ($/bbl)			
	N	Mean	σ	Return	(spot price)	N	Mean	σ	Return
1989	251	19.60	1.17	0.09	1.39 (13.26)***	249	18.21	1.18	0.11
1990	252	24.50	6.64	0.11	0.69 (1.08)	250	23.81	7.61	0.12
1991	253	21.52	1.92	– 0.16	1.44 (8.17)***	251	20.08	2.04	– 0.19
1992	259	20.57	1.28	0.01	1.25 (11.88)***	258	19.32	1.12	0.00
1993	260	18.43	1.75	– 0.12	1.46 (10.02)***	259	16.97	1.58	– 0.12
1994	257	17.16	1.69	0.09	1.36 (10.02)***	259	15.80	1.39	0.08
1995	258	18.37	0.87	0.04	1.38 (17.25)***	256	16.99	0.94	0.06
1996	255	21.95	2.24	0.11	1.39 (7.03)***	257	20.56	2.23	0.10
1997	257	20.61	1.82	– 0.15	1.45 (8.91)***	254	19.16	1.86	– 0.16
1998	259	14.44	1.57	– 0.16	1.65 (11.93)***	257	12.79	1.56	– 0.17
1999	256	19.33	4.56	0.33	2.09 (5.22)***	254	17.24	4.49	0.32
2000	260	30.16	2.98	– 0.02	3.86 (14.58)***	253	26.30	3.03	– 0.07
2001	254	25.90	3.56	– 0.10	2.69 (9.61)***	254	23.21	2.68	0.00
2002	252	26.15	3.22	0.18	1.15 (4.21)***	254	25.00	2.93	0.20
2003	251	31.10	2.62	0.02	2.32 (10.15)***	254	28.78	2.51	– 0.03
2004	249	41.49	5.75	0.12	3.18 (6.16)***	256	38.31	5.82	0.11
2005	251	56.61	6.27	0.14	1.51 (2.70)***	254	55.10	6.24	0.15
1989–2005	4334	25.09	10.57	0.03	1.72 (7.63)***	4329	23.37	10.41	0.03
Skewness			1.8686					1.9690	
Kurtosis			3.5094					3.9391	

Note: *** Significant at the 0.01 level. T-statistic is in parentheses.

the turnover of NYMEX WTI Crude futures was 992,616 lots, while that of IPE Brent Crude futures reached only 128,934 lots.

WTI crude oil is produced in North America and shipped by pipelines to the delivery point in Cushing, Oklahoma, and then by trucks to the refineries of US oil companies. Due to the close proximity of its delivery locations to the refineries, the delivery cost of WTI crude oil is relatively low. Brent crude oil is settled and delivered to Sullom Voe in the Shetland Island, and then shipped to refineries by tankers, which faces port freezing problem in the wintertime. Relative to WTI crude oil, the production of Brent crude oil is more susceptible to the influence of climate. To the extent that there are price spreads among crude oils produced in different regions, WTI crude oil apparently possesses certain price advantages. Whether such advantages increase the convenience yield of holding crude oil is a topic that we examine in this study.

3 The model

The model we adopt to analyze oil convenience yields is a call option pricing model, with a particular focus on the demand/supply shocks of crude oils. Keynes (1930) assumes that an unexpected demand shock would cause the spot prices of commodities to exceed their futures prices and that a convenience yield from holding inventories would arise during a stock-out. The relationship between the convenience yield and the business cycle of a commodity is very close. Figure 1 presents the monthly average spot prices of WTI and Brent crude oils from 1989 to 2005, adjusted based on the producer price index for January 1990.

We find that WTI and Brent crude oil prices start to rise before the summer season and start to fall after peaking in September, with the lowest prices occurring in the winter, such as in February. Therefore, the behavior of the crude oil price is affected by seasonality and business cycles. The source of shocks result primarily from demand and supply rather than weather or technology, and both supply and

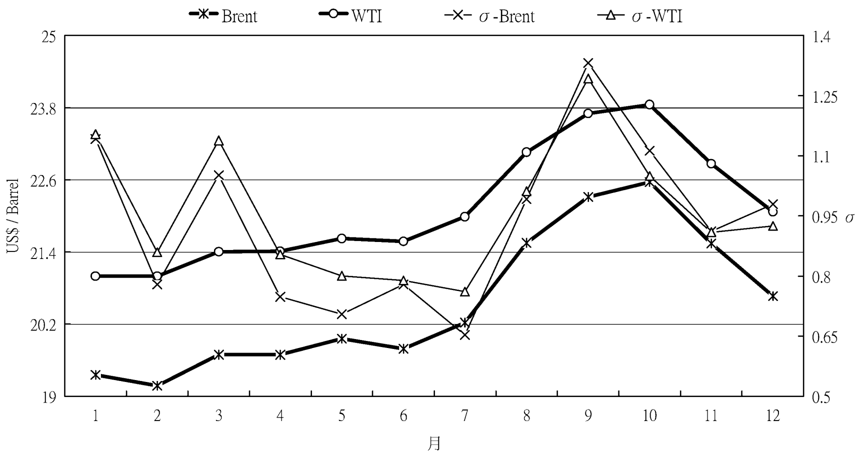

Fig. 1 Plot of means and volatilities of WTI and brent crude spot prices

demand effects influence convenience yields. As spot prices change to restore the equilibrium of the oil market, an oil business cycle is then defined as the start of the disequilibrium to the restoration of equilibrium. The crude oil busyness cycle is then from January to December, with the beginning as date 0, set in January, and the end as date T in December. In particular, January, with the lowest average spot price, is also the observation month since it is when the market equilibrium is restored. The month with the highest average spot price is defined as the intermediate month, which is also the event or shock month in which the convenience yield arises from holding the commodity. The intermediate month divides the business cycle into two periods. A nearby futures contract written in March is considered the starting contract for estimating the convenience yield. The convenience yield is computed in the observation month–September, when spot prices peak–and is set as the shock month to estimate convenience yields.

Based on the nature of the production, we assume that the demand (D) for crude oil in different periods of time is given. The supply at date 0 is a function of the production (Q_0) determined in the previous cycle, while storage (S_0) is determined at date 0 in the current cycle. The supply at any date t during the cycle is determined by the variation in the stock level, and the supply available at the end of the cycle is determined by the variation in the stock level. Production (Q_T) in the current cycle is determined at the beginning of the cycle, and then the spot prices (P) on the following days depend on the demand and supply available. Available supplies are defined in each period after subtracting inventory:

$$
\begin{aligned}
P_0 &= f(D_0; Q_0 - S_0),\\
P_t &= f(D_t; S_0 - S_t),\\
P_T &= f(D_T; S_t - S_T + Q_T).
\end{aligned}
$$

In a perfect market, the expectations model shows that the futures price (F) today equals the spot price that traders expect to prevail for the underlying asset on the delivery date of the futures contract:

$$F_{0,t} \equiv E(P_t).$$

The storage rises when the demand is low or the supply is high. The futures price that expires at date T observed at date t is a function of three variables (i.e., storage as decided at date t; production at date T as decided at the beginning of the cycle; and demand at date T, as expressed below):

$$F_{t,T} \equiv E\{f(D_T; S_t + Q_T)\}. \tag{1}$$

Equation (1) shows that the current futures price at date t is determined by the current storage and demand and production levels at date T. When the market faces higher demand or lower supply, storage gradually falls to zero. If the production at date T is known, there is a negative relationship between the futures price and storage, and the futures price will reach the upper bound:

$$F_{t,T}^{U} \equiv E\{f(D_T; Q_T)\},$$
$$F_{t,T} < F_{t,T}^{U}. \tag{2}$$

According to the theory of storage, the net cost of holding a futures contract under an arbitrage-free framework is the spot price plus the storage cost (*SC*):

$$F_{t,T} = P_t + SC_{t,T}.$$

Thus when supply/demand is in equilibrium, we obtain $S_t > 0$, and then

$$f(D_t; S_0) + SC_{t,T} < F_{t,T}^{U}. \tag{3}$$

When there is excess demand, $S_t = 0$, $F_{t,T} = F_{t,T}^{U}$, then

$$f(D_t; S_0) + SC_{t,T} > F_{t,T}. \tag{4}$$

Given that the futures price cannot completely explain the cost-of-carry model, the spot price will be higher than the futures price when the market faces excess demand, and the convenience yield from holding the commodity over the period from t to T may be expressed as:

$$CY_{t,T} = P_t + SC_{t,T} - F_{t,T}. \tag{5}$$

When Eq. (3) holds, we have $CY_{t,T} = 0$. When Eq. (5) holds, we obtain $CY_{t,T} > 0$. Therefore, a temporary shock in demand/supply during the cycle will cause the storage to drop and the spot price to rise, which gives rise to a risk premium from holding the commodity and results in a positive convenience yield.

Given that the storage cost in Eq. (5) is difficult to estimate, observing the convenience yield becomes infeasible. Fama and French (1988) considers the behavior of the convenience yield on an interest-adjusted basis. Such an approach avoids directly estimating the convenience yield but is unable to give the whole picture of the convenience yield from holding the commodity. Milonas and Thomadakis (1997) treats the convenience yield as a call option. With the assumption of zero storage cost and the spot price as the underlying variable, the payoff function from the convenience yield (*OCY*) at maturity, or the boundary condition of the option pricing formula, can be written as

$$OCY_{t,T} = Max(P_t - F_{t,T}, 0). \tag{6}$$

Instead of using spot price as an underlying variable, as in Eq. (6), we set the price of a nearby futures contract as the underlying variable, and the price of a distant contract as the exercise price. Under a cost-of-carry framework with zero storage cost, the convenience yield is the difference between the net cost of carrying a nearby and a distant futures contract observed at time 0,

$$OCY_{t,T}^{0} = Max(F_{0,t} - F_{t,T}, 0). \tag{7}$$

From the above equation the convenience yield from t to T is observed at date 0 in the beginning month of business cycle. However, it ignores storage cost, which might result in a negative convenience yield. The storage cost ($SC_{t,T}$) is incurred from date t, when the nearby contract matures, to date T, when the commodity is delivered and the distant contract matures. Equation (7) is then revised as:

$$OCY_{t,T}^{0} = Max(F_{0,t}^{*} - F_{t,T}, 0), \tag{8}$$

where

$$F_{0,t}^{*} = F_{0,t} + SC_{t,T}.$$

Both the underlying variable and the exercise price in Eq. (8) are uncertain variables, which follow standard diffusion processes. We assume that the nearby futures contract price ($F_{0,t}^{n}$) with a maturity date at time t and the distant futures contract price ($F_{t,T}^{d}$) with a maturity date at time T, and they both follow geometric Brownian motion (GBM):

$$dF_{0,t}^{n} = \mu_n F_{0,t}^{n} dt + \sigma_n F_{0,t}^{n} dz_n,$$
$$dF_{t,T}^{d} = \mu_d F_{t,T}^{d} dt + \sigma_d F_{t,T}^{d} dz_d.$$

Based on the deduction of Fisher (1978) that utilizes a non-traded asset as strike price, the expected rate of return on the hypothetical security may be solved by the capital asset pricing model (CAPM). Unlike Fisher, where the hedge against the uncertainty of strike price is through a non-traded asset, the strike price in our model is the price of a traded asset such as crude oil futures. The expected rate of return will be zero. Simplifying $F_{0,t}^{*}$ as $F_{0,t}^{n}$ and $F_{t,T}$ as $F_{t,T}^{d}$, referring to Eq. (8), the boundary condition can be rewritten as:

$$F^{n} Max(X_T - 1, 0), \tag{9}$$

where $X_T = F^{n}/F^{d}$. Applying Itô's lemma on the transformed variables, we obtain a closed-form solution to the call option (OCY):

$$OCY_{t,T}^{0} = F^{n} N(d_1) - F^{d} N(d_2), \tag{10}$$

where

$$d_1 = \frac{\ln(X_t) + (\sigma_p^2/2) \times \tau}{\sigma_p \sqrt{\tau}},$$

$$d_2 = \frac{\ln(X_t) - (\sigma_p^2/2) \times \tau}{\sigma_p \sqrt{\tau}},$$

and

$$\sigma_p^2 = \sigma_{F^n}^2 + 2\sigma_{F^n}\sigma_{F^d}\rho_{F^nF^d} + \sigma_{F^d}^2. \quad (11)$$

σ, ρ, r_f and τ are the volatility, correlation coefficient, risk-free rate, and the period between the nearby and distant contracts, respectively. We use the variance of daily logarithmic futures price changes from all trading days during the beginning of the first month of the business cycle as an estimate of volatility of the futures contract in the model. The correlation coefficient is estimated using the nearby and distant daily logarithmic futures price changes from all trading days during the beginning month.

4 Data and empirical analysis

We now turn to the estimation of convenience yield, as well as the design of the business cycle for commodities. For each commodity in our sample, we first need to estimate the convenience yield and then determine how these values relate to inventory and interest rate by cross-sectional regression. Our analyses test the consistency and validity of the theory of storage. Finally, we also test Samuelson (1965) hypothesis for our estimation of convenience yield with the same approach as that in Fama and French (1988).

The issue of whether crude oil prices and convenience yields have seasonal cycles is closely related to whether they are subject to supply/demand shocks. We employ a sample of WTI and Brent crude oil prices that are subject to the effect of supply/demand shocks. The sample contains daily data for WTI and Brent during the years 1989 and 2005. The crude oil price data are from the Commodity Research Bureau database, the Wall Street Journal, the NYMEX and IPE. Inventory data are obtained from the U.S. Energy Information Administration (EIA) and the Economagic database.

Because the study involves many years of data with varying levels of inflation, all prices are translated using the producer price index (PPI) with the base adjusted to January 1990. We take the three-month U.S. Treasury bill rate as the risk-free rate, and discount all crude oil futures prices at the risk-free rate to the beginning month of the business cycle using the following method:

$$F_{t,T} = f_{t,T} \times e^{-r_f \times (T-t)/365}, \quad (12)$$

where f is the pre-discounted futures price. In this way, all futures prices become free of carrying charges. Brennan (1986) and Milonas and Thomadakis (1997) also adopted the same discounting procedure.

According to the EIA report,[2] the storage cost of crude oil is approximately 30 cents per barrel/month during this period, which accounts for about 1% of the crude oil spot price, and is a fixed cost for holders of storable commodities. Thus we set the monthly storage cost at a fixed percentage of 1% of the spot price to estimate

[2] Energy Information Administration, *Petroleum Supply Monthly*. DOE/EIA-0109, Washington DC, March 1996.

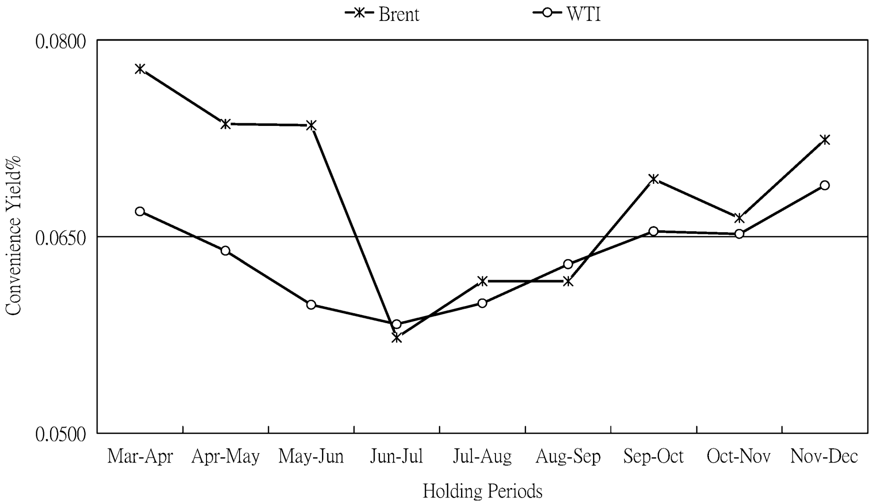

Fig. 2 Term structure of convenience yields

convenience yields. Finally, to estimate the means and volatilities of these variables, we apply the observed spot and futures prices for all trading days during the beginning month.

The convenience yield is calculated on a daily basis throughout the observation month and then averaged over the observation month. In addition to calculating the monthly convenience yields from March to November, we also use September, when spot prices peak, as the shock month in order to estimate of convenience yields. Table 2 presents the convenience yields calculated based on the call option and cost-of-carry models. The results show that the values of the convenience yields estimated from the options model are higher than those from the cost-of-carry model, implying the strategic and management flexibility of the options approach. Moreover, the cost-of-carry model could produce results that contradict the options theory and yield negative estimates.

Figure 2 draws the term structure of the convenience yield. We find that the behavior of crude oil convenience yields exhibits seasonality. This seasonality is more regular with WTI crude oil, where its convenience yields are the highest during the November-December holding period, and then the yields gradually fall to reach the lowest point in June-July. The convenience yields in relation to Brent crude oil are at their highest during the March-April holding period, and then they decline gradually to reach their lowest point in June-July. Based on this seasonal behavior, the convenience yields of Brent are higher than those of WTI in the winter, while the opposite is the case in the summer, suggesting that greater benefits accrue to the holder of Brent crude in the winter when it is subject to supply shocks, and to the holder of WTI crude in the summer when it is subject to demand shocks. Moreover, the convenience yields for Brent crude are more volatile than those for WTI.

We find that the inventory of a commodity tends to decrease as the end of the cycle year draws near and the probability of a stock-out increases, which means that the benefit of holding inventory also rises, resulting in higher convenience yields. Thus

Table 2 Estimated sample means for convenience yields

Period	WTI Crude Oil ($/Spot Price)			Brent Crude Oil ($/Spot Price)		
	OCY[1]	CCY[1]	(1)–(2)	OCY[1]	CCY[1]	(1)–(2)
Mar–Apr	0.0669	0.0244	0.0435	0.0778	0.0235	0.0543
	(10.19)***	(5.60)***	(7.22)***	(4.98)***	(5.97)***	(3.74)***
Mar–May	0.1005	0.0468	0.0551	0.1131	0.0451	0.0680
	(10.37)***	(5.94)***	(6.40)***	(5.63)***	(5.91)***	(3.82)***
Mar–Jun	0.1274	0.0675	0.0616	0.1463	0.0641	0.0847
	(10.51)***	(6.14)***	(5.88)***	(5.87)***	(6.00)***	(4.01)***
Mar–Jul	0.1525	0.0878	0.0666	0.1470	0.0795	0.0722
	(10.70)***	(6.39)***	(5.56)***	(12.69)***	(5.70)***	(6.17)***
Mar–Aug	0.1841	0.1068	0.0793	0.1804	0.0946	0.0899
	(11.24)***	(6.65)***	(5.88)***	(12.24)***	(5.82)***	(6.35)***
Mar–Sep	0.2102	0.1243	0.0880	0.1989	0.1057	0.0931
	(11.51)***	(6.88)***	(6.18)***	(11.72)***	(5.43)***	(6.19)***
Mar–Oct	0.2354	0.1413	0.0963	0.2585	0.1210	0.1374
	(11.12)***	(7.14)***	(6.26)***	(8.73)***	(5.57)***	(4.57)***
Mar–Nov	0.2598	0.1572	0.1049	0.2478	0.1391	0.1087
	(11.61)***	(7.37)***	(6.03)***	(10.79)***	(5.53)***	(5.81)***
Mar–Dec	0.2860	0.1728	0.1158	0.2926	0.1706	0.1220
	(11.86)***	(7.60)***	(5.97)***	(11.96)***	(5.76)***	(5.47)***
Apr–May	0.0639	0.0222	0.0422	0.0736	0.0215	0.0521
	(10.29)***	(6.32)***	(7.27)***	(4.94)***	(5.74)***	(3.81)***
May–Jun	0.0598	0.0205	0.0396	0.0735	0.0186	0.0553
	(10.36)***	(6.56)***	(7.15)***	(4.44)***	(6.02)***	(3.50)***
Jun–Jul	0.0583	0.0199	0.0386	0.0573	0.0176	0.0401
	(10.51)***	(7.30)***	(7.10)***	(12.06)***	(6.13)***	(7.94)***
Jul–Aug	0.0599	0.0186	0.0415	0.0616	0.0153	0.0462
	(11.11)***	(8.17)***	(7.69)***	(12.05)***	(5.86)***	(8.49)***
Aug–Sep	0.0629	0.0169	0.0461	0.0616	0.0148	0.0468
	(11.88)***	(8.78)***	(8.74)***	(11.53)***	(5.98)***	(8.89)***
Sep–Oct	0.0654	0.0163	0.0492	0.0964	0.0148	0.0546
	(11.86)***	(9.87)***	(9.05)***	(13.05)***	(6.87)***	(9.76)***
Oct–Nov	0.0652	0.0152	0.0501	0.0664	0.0144	0.0521
	(11.37)***	(10.23)***	(8.74)***	(10.79)***	(6.98)***	(8.70)***
Nov–Dec	0.0689	0.0148	0.0542	0.0724	0.0156	0.0568
	(11.07)***	(11.09)***	(8.52)***	(11.08)***	(7.35)***	(8.62)***

Notes: [1]"OCY" indicates estimation of convenience yield by option model, and "CCY" is by the cost-of-carry model.

***Significant at the 0.01 levels. *T*-statistic is in parentheses.

the term structure of the convenience yields is upward sloping. Figure 3 supports this conclusion, which also shows that the convenience yield behavior of WTI is more stable. The empirical results above indicate that WTI possesses a price advantage over Brent in the spot market, but its convenience yields are not necessarily higher than those of Brent.

We apply simple regression analysis to examine whether the convenience yield is negatively related to the inventory level. The test results of Model 1 in Tables 3 and 4 show that the WTI inventory coefficient is not statistically significant, but generally

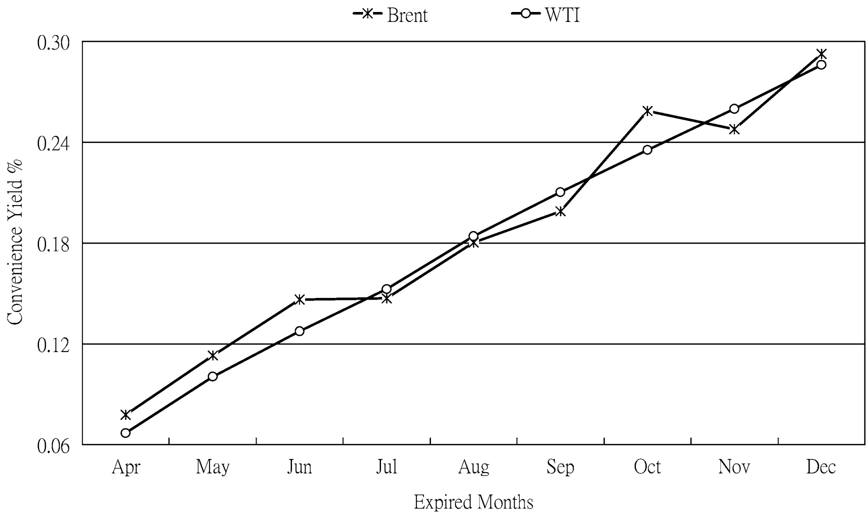

Fig. 3 Convenience yields: March as shock month

negative, while the Brent inventory coefficient is significantly negative. These results imply that WTI crude oil is a more strategic commodity with weaker correlation between the inventory and the spot price.

The theory of storage also suggests that holding inventory becomes more costly in periods of high interest rates. Furthermore, the convenience yield can be treated as a call option, in theory it is positively correlated with the covariance (σ_p) of two futures contracts and the risk-free rate (r_f). We use a multiple regression to test the relationship between the convenience yield estimated on the basis of the cost-of-carry model (CCY) and the independent variables:

$$CCY_{t,T} = \beta_0 + \beta_1 I_{t-1} + \beta_2 \sigma_{pt}^2 + \beta_3 r_{ft} + \varepsilon_t, \tag{13}$$

where β and ε are the regression coefficient and the error term, respectively. We find from Model 2 in Tables 3 and 4 that the sign of the coefficient is consistent with the theory. β_2 is, however, statistically insignificant. Following the theory of storage, we may expect a significant non-zero coefficient of β_3. In fact, to the extent that holding inventory becomes more costly in period of high interest rates, we may expect negative correlation between interest rates and inventory, and positive correlation between interest rates and convenience yields, and hence a positive β_3. In testing the risk-free rate coefficient, β_3 of WTI is statistically significant, while that of Brent is more statistically significant for short-term holding.

To observe the effect of a variable explaining the convenience yield, we use the spot price spread and futures price spread in the two crude oil markets as explanatory variables and perform regression on the following equation,

$$OCY_{t,T} = \alpha_0 + \alpha_1 SPF_{t,T}^{n} + \alpha_2 SPF_{t,T}^{d} + \alpha_3 SPS_{t,T}^{WTI\text{-}Brent} + \varepsilon_t, \tag{14}$$

Table 3 Linear regression of convenience yields on inventory, volatility and risk-free rate: WTI crude oil[1,2]

Period	Model	β_0	β_1	β_2	β_3	R^2	F
Mar–Apr	I	0.1430	– 0.0001			0.01	0.18
		(0.79)	– (0.42)				
	II	0.0806	– 0.0001	3.9235	0.3873	0.72	5.39**
		(0.68)	– (0.67)	(0.91)	(2.01)**		
Mar–May	I	0.2248	– 0.0001			0.02	0.22
		(0.85)	– (0.47)				
	II	0.1687	– 0.0002	7.3824	0.5978	0.70	6.11**
		(0.77)	– (0.72)	(0.90)	(2.05)**		
Mar–Jun	I	0.2983	– 0.0002			0.02	0.27
		(0.90)	– (0.52)				
	II	0.2549	– 0.0003	10.3423	0.7324	0.61	6.59**
		(0.81)	– (0.74)	(0.87)	(2.00)**		
Mar–Jul	I	0.3711	– 0.0002			0.02	0.32
		(0.95)	– (0.56)				
	II	0.3462	– 0.0003	12.7209	0.8122	0.60	6.01**
		(0.87)	– (1.78)*	(0.83)	(1.98)*		
Mar–Aug	I	0.5062	– 0.0004			0.04	0.53
		(1.14)	– (0.73)				
	II	0.4097	– 0.0004	14.2944	0.8852	0.59	5.61**
		(0.87)	– (1.77)*	(0.80)	(1.97)*		
Mar–Sep	I	0.5592	– 0.0004			0.03	0.50
		(1.13)	– (0.71)				
	II	0.4878	– 0.0005	17.6198	0.9519	0.62	5.98**
		(0.93)	– (1.82)*	(0.92)	(1.89)*		
Mar–Oct	I	0.7630	– 0.0006			0.06	0.87
		(1.35)	– (0.93)				
	II	0.5194	– 0.0005	19.2246	0.9428	0.62	6.10**
		(0.91)	– (1.79)*	(1.03)	(1.82)*		
Mar–Nov	I	0.7704	– 0.0006			0.05	0.72
		(1.28)	– (0.85)				
	II	0.6291	– 0.0006	15.6610	1.0332	0.59	5.51**
		(1.01)	– (1.87)*	(0.82)	(1.83)*		
Mar–Dec	I	0.9431	– 0.0007			0.07	1.05
		(1.47)	– (1.02)				
	II	0.6712	– 0.0006	13.7013	1.1395	0.51	5.41**
		(1.00)	– (1.86)*	(0.69)	(1.84)*		
Apr–May	I	0.1215	– 0.0001			0.01	0.11
		(0.71)	(– 0.34)				
	II	0.0871	– 0.0001	3.4189	0.2071	0.51	5.80**
		(0.86)	(– 1.77)*	(0.89)	(2.15)**		
May–Jun	I	0.1028	– 0.0000			0.01	0.07
		(0.65)	(– 0.27)				
	II	0.0851	– 0.0001	2.7484	0.1284	0.50	4.91**
		(0.91)	(– 1.79)*	(0.76)	(1.82)*		
Jun–Jul	I	0.0943	– 0.0000			0.00	0.06
		(0.62)	(– 0.24)				
	II	0.0890	– 0.0001	2.1736	0.0718	0.49	4.20**
		(1.08)	(– 1.91)*	(0.66)	(1.83)*		

(*Continued on next page*)

Table 3 (*Continued*)

Period	Model	β_0	β_1	β_2	β_3	R^2	F
Jul–Aug	I	0.1135	− 0.0001			0.01	0.13
		(0.77)	(− 0.36)				
	II	0.0794	− 0.0001	1.5279	0.0606	0.51	4.21**
		(1.16)	(− 1.96)*	(0.56)	(1.85)*		
Aug–Sep	I	0.1240	− 0.0001			0.01	0.18
		(0.85)	(− 0.42)				
	II	0.0701	− 0.0001	1.3764	0.0555	0.52	4.31**
		(1.22)	(− 1.61)*	(0.65)	(1.73)*		
Sep–Oct	I	0.1579	− 0.0001			0.03	0.38
		(1.05)	(− 0.62)				
	II	0.0674	− 0.0001	0.9878	0.0495	0.32	3.85**
		(1.38)	(− 1.13)	(0.64)	(0.51)		
Oct–Nov	I	0.1773	− 0.0001			0.04	0.52
		(1.14)	(− 0.72)				
	II	0.0668	− 0.0001	0.5447	0.0426	0.52	3.73**
		(1.52)	(− 1.75)*	(0.45)	(1.74)*		
Nov–Dec	I	0.1859	− 0.0001			0.03	0.48
		(1.10)	(− 0.69)				
	II	0.0648	− 0.0001	0.1937	0.0478	0.51	3.71**
		(1.65)	(− 1.75)*	(0.19)	(1.70)*		

Notes: [1]Model I: $OCY_{t,T} = \beta_0 + \beta_1 I_{t-1} + \varepsilon_t$; Model II: $CCY_{t,T} = \beta_0 + \beta_1 I_{t-1} + \beta_2\sigma^2_{pt} + b_3 r_t + \varepsilon_t$.
[2] Standard errors are analyzed with the correction of heteroscedasticity and results differ only marginally.
***, **, * Significant at the 0.01, 0.05, and 0.1 levels, respectively. *T*-statistic is in parentheses.

where α and ε are the regression coefficient and the error term, respectively, and SPF^n, SPF^d, and $SPS^{WTI-Brent}$ are the price spreads of a nearby contract, the price spreads of a distant contract, and the spot price spreads, respectively. All price spreads are computed by the difference between WTI and Brent prices. Table 5 presents the test results for WTI, which shows that signs of α_1 and α_2 are inconsistent during the holding periods of October-November and November-December. In addition, the greater the spreads between the nearby contract and the spot price, the higher is the convenience yield of WTI, while the spreads in relation to the distant contracts are negatively related to the convenience yield. The test results for Brent crude as depicted in Table 6 exhibit inconsistent coefficient signs.

Since Eq. (13) could lead to multicollinearity among explanatory variables, which in turn might lead to insignificant coefficient test results or inconsistency in the signs of the coefficients, we then use the convenience yields of WTI and Brent as explanatory variables and run the following equation:

$$SPS^{WTI\text{-}Brent}_{t,T} = \gamma_0 + \gamma_1 OCY^{WTI}_{t,T} + \gamma_2 OCY^{Brent}_{t,T} + \varepsilon_t. \quad (15)$$

The results presented in Table 7 show that γ_1 is significantly positive, while γ_2 is significantly negative in the holding periods of March-April, March-May, March-June, March-October, April-May and May-June, indicating that the higher the convenience yield for WTI, the lower the convenience yield for Brent and the greater the spot price

Table 4 Linear regression of convenience yields on inventory, volatility and risk-free rate: brent crude oil[1,2]

Period	Model	β_0	β_1	β_2	β_3	R^2	F
Mar–Apr	I	0.0780	0.0000			0.00	0.00
		(0.18)	(0.00)				
	II	0.1283	− 0.0001	0.5635	0.3483	0.39	2.51*
		(1.37)	(− 1.30)	(1.04)	(1.80)*		
Mar–May	I	0.1803	− 0.0001			0.00	0.01
		(0.33)	(− 0.12)				
	II	0.2335	− 0.0002	1.6715	0.5725	0.35	2.18
		(1.25)	(− 1.17)	(1.28)	(1.49)		
Mar–Jun	I	0.1960	− 0.0001			0.00	0.01
		(0.29)	(− 0.07)				
	II	0.4598	− 0.0005	2.9344	0.3053	0.32	1.71
		(1.56)	(− 1.45)	(1.40)	(0.45)		
Mar–Jul	I	0.6492	− 0.0006			0.21	3.32*
		(2.35)**	(− 1.82)*				
	II	0.7012	− 0.0007	− 17.9120	0.0263	0.23	1.00
		(1.70)	(− 1.56)	(− 0.64)	(0.03)		
Mar–Aug	I	0.8152	− 0.0007			0.21	3.26*
		(2.32)**	(− 1.81)*				
	II	0.8231	− 0.0008	− 12.9874	0.0550	0.21	0.9
		(1.65)	(− 1.52)	(− 0.48)	(0.05)		
Mar–Sep	I	0.9823	− 0.0009			0.25	3.91*
		(2.48)**	(− 1.98)*				
	II	0.9577	− 0.0009	− 12.3513	− 0.0140	0.20	0.83
		(1.60)	(− 1.48)	(− 0.36)	(− 0.01)		
Mar–Oct	I	0.9674	− 0.0008			0.07	0.85
		(1.26)	(− 0.92)				
	II	0.9843	− 0.0010	− 2.0630	0.0979	0.19	0.80
		(1.59)	(− 1.44)	(− 0.30)	(0.07)		
Mar–Nov	I	1.1805	− 0.0011			0.22	3.13*
		(2.24)**	(− 1.77)*				
	II	1.0623	− 0.0010	− 3.4442	0.0549	0.18	0.65
		(1.38)	(− 1.27)	(− 0.08)	(0.03)		
Mar–Dec	I	1.2355	− 0.0011			0.26	3.19*
		(2.34)**	(− 1.79)*				
	II	1.1739	− 0.0011	− 22.3119	− 0.0370	0.15	0.41
		(1.24)	(− 1.09)	(− 0.42)	(− 0.18)		
Apr–May	I	0.0773	− 0.0000			00	0.00
		(0.19)	(− 0.01)				
	II	0.1061	− 0.0001	0.8960	0.2154	0.32	1.91
		(1.13)	(− 1.03)	(1.53)	(1.12)		
May–Jun	I	− 0.0291	0.0001			0.00	0.05
		(− 0.06)	(0.23)				
	II	0.1492	− 0.0001	0.4878	− 0.0078	0.23	1.09
		(1.60)	(− 1.46)	(1.11)	(− 0.04)		
Jun–Jul	I	0.1750	− 0.0001			0.07	0.91
		(1.42)	(− 0.96)				
	II	0.1406	− 0.0001	− 2.3942	− 0.0325	0.17	0.67
		(1.52)	(− 1.37)	(− 0.35)	(− 0.15)		

(*Continued on next page*)

Table 4 (*Continued*)

Period	Model	β_0	β_1	β_2	β_3	R^2	F
Jul–Aug	I	0.1993	− 0.0002			0.08	1.10
		(1.51)	(− 1.05)				
	II	0.1406	− 0.0001	− 2.3526	− 0.0757	0.19	0.76
		(1.68)	(− 1.50)	(− 0.47)	(− 0.40)		
Aug–Sep	I	0.2331	− 0.0002			0.12	1.62
		(1.73)*	(− 1.27)				
	II	0.1074	− 0.0001	1.0141	− 0.0447	0.17	0.69
		(1.36)	(− 1.23)	(0.23)	(− 0.26)		
Sep–Oct	I	0.1988	− 0.0001			0.07	0.88
		(1.44)	(− 0.94)				
	II	0.0990	− 0.0001	− 0.2810	− 0.0317	0.15	0.60
		(1.46)	(− 1.29)	(− 0.08)	(− 0.21)		
Oct–Nov	I	0.2308	− 0.0002			0.10	1.17
		(1.51)	(− 1.08)				
	II	0.0687	− 0.0001	1.8061	0.0063	0.19	0.72
		(1.10)	(− 0.96)	(0.65)	(0.05)		
Nov–Dec	I	0.2403	− 0.0002			0.12	1.18
		(1.56)	(− 1.09)				
	II	0.0575	− 0.00004	1.2794	0.0078	0.13	0.35
		(0.81)	(− 0.67)	(0.35)	(0.05)		

Notes: [1]Model I: $OCY_{t,T} = \beta_0 + \beta_1 I_{t-1} + \varepsilon_t$; Model II: $CCY_{t,T} = \beta_0 + \beta_1 I_{t-1} + \beta_2 \sigma^2_{pt} + \beta_3 r_t + \varepsilon_t$.
[2]Standard errors are analyzed with the correction of heteroscedasticity and results differ only marginally.
***, **, * Significant at the 0.01, 0.05, and 0.1 levels, respectively. *T*-statistic is in parentheses.

spread between WTI and Brent. On the contrary, the higher the convenience yield for WTI, the lower the convenience yield for Brent and the lower the spot price spread in the holding periods of March-September, March-November, March-December, June-July, August-September, October-November and November-December.

Table 8 illustrates the convenience yields from the shock month of September till the final month of the cycle. The negative correlation between WTI's convenience yield and the inventory level suggests that it is closely linked to business cycle, as the convenience yield is unrelated to the petroleum stocks in the U.S. or the U.K. In the case of Brent crude, we find that the relationship between the convenience yield and the inventory level are consistent with WTI. Table 9 illustrates the relationship between the convenience yield, inventory levels and production in the regression analysis. The results are consistent with those reported in Table 8.

Samuelson (1965) argues that futures prices are less variable than spot prices. The theory of storage also predicts that, at a low inventory level, futures prices vary less than spot prices; at a high inventory level, spot prices and futures prices have similar variability. Fama and French (1988) supported Samuelson's hypothesis by examining the interest-adjusted basis of industrial metals. Thus it is believed that the convenience yield declines at higher inventory levels, and that spot and futures prices have roughly the same variability. Conversely, the convenience yield rises at a low inventory level, and spot prices are more variable than futures prices. To test the Samuelson (1965) hypothesis, we adopt the same approach as Fama and French (1988) and perform the

Table 5 Regressions of convenience yields on price spreads: WTI crude oil
$OCY_{t,T} = \alpha_0 + \alpha_1 SPF^n_{t,T} + \alpha_2 SPF^d_{t,T} + \alpha_3 SPS_{t,T} + \varepsilon_t$

Period	α_0	α_1	α_2	α_3	R^2	F
Mar–Apr	0.0773	0.0032	− 0.0202	0.0081	0.12	14.29***
	(19.38)***	(3.01)***	(− 5.62)***	(5.83)***		
Mar–May	0.1199	0.0048	− 0.0329	0.0114	0.13	16.70***
	(18.15)***	(3.27)***	(− 5.87)***	(5.94)***		
Mar–Jun	0.1741	0.0068	− 0.0644	0.0169	0.22	28.16***
	(18.48)***	(3.92)***	(− 7.89)***	(7.16)***		
Mar–Jul	0.1361	0.0080	− 0.0278	0.0169	0.31	41.84***
	(13.24)***	(5.68)***	(− 3.37)***	(8.53)***		
Mar–Aug	0.1677	0.0097	− 0.0387	0.0219	0.36	52.50***
	(13.25)***	(5.82)***	(− 3.80)***	(9.74)***		
Mar–Sep	0.1976	0.0120	− 0.0461	0.0210	0.34	47.25***
	(13.45)***	(6.33)***	(− 3.90)***	(8.47)***		
Mar–Oct	0.2402	0.0127	− 0.0748	0.0285	0.42	67.87***
	(14.97)***	(6.13)***	(− 5.82)***	(10.72)***		
Mar–Nov	0.2608	0.0109	− 0.0555	0.0227	0.30	34.14***
	(15.01)***	(4.72)***	(− 3.95)***	(7.53)***		
Mar–Dec	0.2939	0.0075	− 0.0481	0.0236	0.27	26.78***
	(16.33)***	(3.13)***	(− 3.25)***	(7.52)***		
Apr–May	0.0832	0.0261	− 0.0450	0.0051	0.11	12.85***
	(17.91)***	(2.25)**	(− 3.57)***	(3.85)***		
May–Jun	0.0864	0.0527	− 0.0793	0.0043	0.20	25.70***
	(19.27)***	(5.38)***	(− 7.32)***	(3.72)***		
Jun–Jul	0.0600	0.0438	− 0.0525	0.0029	0.13	13.35***
	(13.44)***	(3.56)***	(− 4.07)***	(2.95)***		
Jul–Aug	0.0606	0.0279	− 0.0377	0.0045	0.12	13.32***
	(12.81)***	(1.61)*	(− 2.06)**	(4.81)***		
Aug–Sep	0.0651	0.0555	− 0.0653	0.0033	0.12	13.31***
	(13.49)***	(2.40)**	(− 2.78)***	(3.32)***		
Sep–Oct	0.0730	0.0826	− 0.0972	0.0031	0.18	20.76***
	(15.71)***	(3.46)***	(− 4.10)***	(3.41)***		
Oct–Nov	0.0776	− 0.0691	0.0523	0.0053	0.13	11.79***
	(14.66)***	(− 1.65)*	(1.30)	(5.00)***		
Nov–Dec	0.0829	− 0.1282	0.1133	0.0042	0.09	7.03***
	(14.29)***	(− 3.05)***	(2.78)***	(3.86)***		

Note: ***, **, * Significant at the 0.01, 0.05, and 0.1 levels, respectively. *T*-statistic is in parentheses.

following regression:

$$\ln\left(\frac{F_{t,T}}{F_{t-1,T-1}}\right) = \eta_0 + \eta_1 \ln\left(\frac{S_t}{S_{t-1}}\right) + \varepsilon_t. \qquad (16)$$

We apply the regression analysis in the event months, and divide the estimated values of the coefficients η_1 into high convenience yields and low convenience yields based on the means of the convenience yields. We find that high convenience yields have smaller average values for coefficients, while low convenience yields have average coefficient values close to one. This implies that at a low inventory level, the spot prices of crude oil vary more than the futures prices and that the convenience

Table 6 Regressions of convenience yields on price spreads: brent crude oil
$OCY_{t,T} = \alpha_0 + \alpha_1 SPF^n_{t,T} + \alpha_2 SPF^d_{t,T} + \alpha_3 SPS_{t,T} + \varepsilon_t$

Period	α_0	α_1	α_2	α_3	R^2	F
Mar–Apr	0.1254	0.0032	− 0.0523	0.0143	0.11	13.68***
	(13.12)***	(1.25)	(− 6.06)***	(4.30)***		
Mar–May	0.1908	0.0025	− 0.0785	0.0185	0.13	16.52***
	(13.84)***	(0.83)	(− 6.72)***	(4.60)***		
Mar–Jun	0.2873	0.0023	− 0.1406	0.0262	0.21	27.50***
	(15.69)***	(0.67)	(− 8.86)***	(5.70)***		
Mar–Jul	0.1501	0.0019	− 0.0300	0.0200	0.29	38.11***
	(15.10)***	(1.40)	(− 3.75)***	(10.18)***		
Mar–Aug	0.1870	0.0012	− 0.0353	0.0215	0.23	27.49***
	(13.48)***	(0.64)	(− 3.16)***	(8.75)***		
Mar–Sep	0.2181	0.0039	− 0.641	0.0308	0.41	64.22***
	(15.38)***	(2.12)**	(− 5.61)***	(12.83)***		
Mar–Oct	0.2762	− 0.0025	− 0.0211	0.0078	0.01	0.86
	(8.63)***	(− 0.60)	(− 0.82)	(1.47)		
Mar–Nov	0.2720	0.0052	− 0.0723	0.0316	0.35	43.83***
	(15.14)***	(2.17)**	(− 4.98)***	(10.16)***		
Mar–Dec	0.2900	0.0009	− 0.0608	0.0363	0.40	49.35***
	(16.25)***	(0.37)	(− 4.14)***	(11.67)***		
Apr–May	0.1379	− 0.0005	− 0.0581	0.0120	0.14	17.92***
	(12.49)***	(− 0.02)	(− 1.94)*	(3.82)***		
May–Jun	0.1785	0.1401	− 0.2361	0.0096	0.27	38.14***
	(15.14)***	(5.44)***	(− 8.28)***	(3.15)***		
Jun–Jul	0.0587	0.0116	− 0.0206	0.0058	0.17	19.27***
	(13.44)***	(0.96)	(− 1.63)*	(6.12)***		
Jul–Aug	0.0589	0.0572	− 0.0623	0.0035	0.13	13.15***
	(11.74)***	(3.11)***	(− 3.20)***	(3.53)***		
Aug–Sep	0.0598	0.0217	− 0.0324	0.0083	0.30	39.63***
	(12.57)***	(0.95)	(− 1.40)	(8.32)***		
Sep–Oct	0.0708	0.0485	− 0.0572	0.0043	0.13	13.60***
	(13.43)***	(1.79)*	(− 2.13)**	(4.07)***		
Oct–Nov	0.0729	− 0.0430	0.0301	0.0062	0.12	11.25***
	(11.81)***	(− 0.88)	(0.64)	(4.96)***		
Nov–Dec	0.0748	− 0.1264	0.1117	0.0086	0.25	24.91***
	(13.76)***	(− 3.21)***	(2.92)***	(8.53)***		

Note: ***, **, * Significant at the 0.01, 0.05, and 0.1 levels, respectively. T-statistic is in parentheses.

yields derived from the options model are higher; at a high inventory level, the spot and futures prices of crude oil have similar variability, while the convenience yields are lower. These results are consistent with the hypothesis of Samuelson (1965) and the findings of Fama and French (1988). However, the test results for Brent crude are not verified.

5 Conclusion

The crude oil convenience yield calculated by our extended Milonas and Thomadakis (1997) call option model exhibits seasonality in the presence of the business cycle.

Table 7 Regression of spot price spread on WTI and brent convenience yields $SPS_{t,T}^{WTI-Brent} = \gamma_0 + \gamma_1 OCY_{t,T}^{WTI} + \gamma_2 OCY_{t,T}^{Brent} + \varepsilon_t$

Period	γ_0	γ_1	γ_2	R^2	F
Mar–Apr	0.4203	31.5012	− 11.0303	0.10	17.82***
	(1.86)**	(5.94)***	(− 4.97)***		
Mar–May	0.3408	23.8323	− 9.4306	0.11	20.88***
	(1.53)	(6.38)***	(− 5.27)***		
Mar–Jun	0.3853	18.8658	− 7.8140	0.12	21.62***
	(1.78)**	(6.48)***	(− 5.20)***		
Mar–Jul	− 0.4900	− 2.8517	17.6682	0.25	47.22***
	(− 2.09)**	(− 0.70)	(4.15)***		
Mar–Aug	− 0.4276	9.1220	3.0044	0.25	47.18***
	(− 1.83)*	(4.46)***	(1.47)		
Mar–Sep	− 0.3254	− 20.4718	30.6812	0.43	104.01***
	(− 1.66)*	(− 6.87)***	(10.52)***		
Mar–Oct	− 0.1645	9.6028	− 1.0419	0.27	56.21***
	(− 0.71)	(10.15)***	(− 1.68)*		
Mar–Nov	0.5358	− 36.9618	42.4704	0.44	94.19***
	(2.16)**	(− 8.50)***	(10.52)***		
Mar–Dec	0.0433	− 24.1721	31.1115	0.51	116.68***
	(0.16)	(− 8.49)***	(12.00)***		
Apr–May	0.7524	24.0545	− 8.4533	0.06	10.23***
	(3.33)***	(4.52)***	(− 3.85)***		
May–Jun	0.8128	22.3025	− 6.9180	0.05	7.76***
	(3.37)***	(3.94)***	(− 3.37)***		
Jun–Jul	0.1142	− 134.8540	158.3869	0.35	76.57***
	(0.55)	(− 9.40)***	(11.07)***		
Jul–Aug	0.2294	14.0623	10.7306	0.11	17.71***
	(0.89)	(2.33)**	(1.89)*		
Aug–Sep	0.4892	− 94.3116	113.1407	0.53	159.80***
	(2.67)***	(− 12.38)***	(16.34)***		
Sep–Oct	0.1951	8.6441	13.8747	0.11	16.58***
	(0.72)	(1.11)	(1.96)*		
Oct–Nov	0.7072	− 18.3669	34.3403	0.11	14.47***
	(2.36)**	(− 1.34)	(2.93)***		
Nov–Dec	0.7257	− 150.0271	164.8700	0.78	387.40***
	(4.67)***	(− 23.97)***	(27.31)***		

Note: ***, **, * Significant at the 0.01, 0.05, and 0.1 levels, respectively. T-statistic is in parentheses.

Our results show that the negative correlation between the convenience yields for WTI crude oil and the inventory level is weak, but such a negative relationship is significant if examined on the basis of the convenience yields in the shock months during which the spot prices are the highest. This demonstrates that when using our option model the choice of the timing of the business cycle is critical to the calculation of the oil convenience yield. Our results also indicate that the convenience yields of Brent crude oil exhibit a negative relationship with inventories and U.S. petroleum stock, implying that U.S. petroleum stock prices affects the price patterns of Brent crude oil. Moreover, interest rates are affected by economic activity, which in turn affects convenience yields of crude oils.

Table 8 Estimated convenience yields: Holding period from the shock month of september to the final month of cycle[1]

	September–December	WTI's CY (USD$/spot price) OCY$^{(1)}$	CCY$^{(2)}$	Brent's CY (US$/spot price) OCY$^{(1)}$	CCY$^{(2)}$
Year	1989	–	0.0512	–	–
	1990	0.0874	0.0558	–	–
	1991	0.2064	0.0493	–	–
	1992	0.0937	0.0355	–	–
	1993	0.0734	0.0368	–	–
	1994	0.1072	0.0126	–	–
	1995	0.0962	0.0439	0.0853	0.0363
	1996	0.0779	0.0469	0.0606	0.0436
	1997	0.1403	0.0735	0.1319	0.0664
	1998	0.0950	0.0313	0.0857	0.0212
	1999	0.1448	0.0106	0.1236	0.0024
	2000	0.1529	0.0768	0.1381	0.0674
	2001	0.1634	0.0652	0.1579	0.0538
	2002	0.1908	0.0332	0.1711	0.0317
	2003	0.1161	0.0622	0.1028	0.0605
	2004	0.1285	0.0536	0.1252	0.0523
	2005	0.1594	0.0527	0.1530	0.0528
	Convenience Yield	0.1267 (12.65)***	0.0465 (10.32)***	0.1308 (10.93)***	0.0478 (7.06)***
	(1) minus (2)	0.0806 (7.85)***		0.0830 (6.86)***	
Means	Stocks of Crude Oil	897.5089 Million barrels (Mb)			
	Stocks of petroleum (Country)	690.6915 Mb (U.S.)		107.1021 Mb (U.K.)	
ρ (with OCY)	With Stocks of Crude Oil	– 0.2050** 0.0762†		– 0.3888* 0.09373†	
	With Stocks of Petroleum, U.S.	0.0322 0.9559†		– 0.1524 0.6546†	
	With Stocks of Petroleum, U.K.	– 0.1796 0.5056†		0.14492 0.6707†	

Notes: [1] "–" indicates no data available; "†" indicate probability value.

***, **, * Significant at the 0.01, 0.05, and 0.1 levels, respectively. *T*-statistic is in parentheses.

We find that the crude oil spot price spreads and the futures price spreads cannot explain the convenience yield because of multicollinearity but explaining the convenience yield in terms of the spot price spreads produces better results. It is thus implied that, the higher the convenience yield in relation to WTI crude oil, the lower the convenience yield in terms of Brent crude oil, and the greater/lower the spot price spread between WTI and Brent crude oils.

Samuelson (1965) proposed that spot and futures price variations will be similar when a supply/demand shock occur during higher inventory levels but that spot prices will be more variable than the futures prices at lower inventory levels. Using

Table 9 Convenience yields regression results: Holding period from the shock month of september to the final month of cycle

Oil	Model[1]	β_0	β_1	β_2	β_3	F	R^2
	I	0.3392	−0.0002			0.62	0.04
		(1.25)	(−1.79)*				
WTI	I	0.1009	0.00003			0.01	0.001
	(U.S.)	(0.47)	(0.12)				
	I	0.2263	−0.0009			0.47	0.03
	(U.K.)	(1.55)	(−0.68)				
	II	0.2014	−0.0002	1.4374	0.1506	4.75**	0.35
		(1.50)	(−1.84)*	(0.35)	(1.77)*		
Brent	I	0.4816	−0.0004			0.16	0.15
		(1.74)	(−1.27)				
	I	0.2445	−0.0002			0.21	0.02
	(U.S.)	(0.99)	(−0.46)				
	I	−0.0129	0.0014			0.19	0.02
	(U.K.)	(−0.04)	(0.44)				
	II	0.1995	−0.0002	1.8379	−0.0548	0.27	0.10
		(0.88)	(−0.72)	(0.16)	(−0.12)		

Notes: [1]Model I: $OCY_{t,T} = \beta_0 + \beta_1 I_{t-1} + \varepsilon_t$; Model II: $CCY_{t,T} = \beta_0 + \beta_1 I_{t-1} + \beta_2\sigma^2_{pt} + \beta_3 r_t + \varepsilon_t$.
**, *Significant at the 0.05 and 0.1 level, respectively. T-statistic is in parentheses.

Table 10 Testing the hypothesis of samuelson[1]

Sample	OCY level	WTI			Brent		
		No	$\overline{CY}$	$\overline{\eta_1}$	No	$\overline{CY}$	$\overline{\eta_1}$
Full	High	100	0.2246	0.4217	71	0.2257	0.1708
			(27.25)***	(15.28)***		(25.92)***	(5.85)***
	Low	188	0.0672	0.5150	135	0.0700	0.1677
			(39.08)***	(22.41)***		(37.18)***	(9.43)***
March as Shock	High	63	0.2698	0.3670	45	0.2672	0.1802
			(30.05)***	(11.26)***		(30.01)***	(4.46)***
	Low	81	0.1107	0.5725	54	0.1189	0.1567
			(25.32)***	(16.62)***		(21.91)***	(6.38)***
March to December as Shock	High	66	0.0836	0.4072	56	0.0815	0.2005
			(39.91)***	(10.69)***		(65.31)***	(5.31)***
	Low	78	0.0464	0.5464	51	0.0481	0.1364
			(44.57)***	(16.71)***		(33.98)***	(9.91)***
Total sample							
WTI and Brent	High	171	0.2251	0.3175			
			(37.46)***	(14.27)***			
	Low	323	0.0684	0.3698			
			(53.69)***	(20.53)***			

Notes: The Estimated model is $ln(\frac{F_{t,T}}{F_{t-1,T-1}}) = \eta_0 + \eta_1 ln(\frac{S_t}{S_{t-1}}) + \varepsilon_t$.
***Significant at the 0.01 level. T-statistic is in parentheses.

a regression analysis method for both futures prices against spot prices and their volatilities as in Fama and French (1988), to verify the Samuelson hypothesis, we find that at a low inventory level, oil spot prices vary more than futures prices. Thus our estimated convenience yields are shown to be higher at a lower inventory level than at a higher inventory level. At a high inventory level, spot and futures prices share roughly the same variability, which leads to a lower convenience yield at a higher inventory level, verifying the Samuelson hypothesis.

Acknowledgment We acknowledge the financial support from National Science Council of R.O.C. (Grant No: NSC 90-2416-H-032-002).

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