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Skewness and leptokurtosis in GARCH-typed VaR estimation of petroleum and metal asset returns

Wan-Hsiu Cheng [a,*], Jui-Cheng Hung [b]

[a] Department of Banking and Finance, Tamkang University, 151 Ying-Chuan Road, Tamsui, Taipei County, 25137, Taiwan
[b] Department of Finance, Lunghwa University of Science and Technology, Taiwan

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ABSTRACT

This paper utilizes the most flexible skewed generalized *t* (SGT) distribution for describing petroleum and metal volatilities that are characterized by leptokurtosis and skewness in order to provide better approximations of the reality. The empirical results indicate that the forecasted Value-at-Risk (VaR) obtained using the SGT distribution provides the most accurate out-of-sample forecasts for both the petroleum and metal markets. With regard to the unconditional and conditional coverage tests, the SGT distribution produces the most appropriate VaR estimates in terms of the total number of rejections; this is followed by the nonparametric distribution, generalized error distribution (GED), and finally the normal distribution. Similarly, in the dynamic quantile test, the VaR estimates generated by the SGT and nonparametric distributions perform better than that generated by other distributions. Finally, in the superior predictive test, the SGT distribution has significantly lower capital requirements than the nonparametric distribution for most commodities.

1. Introduction

Most time series are characterized by leptokurtosis and skewness, not only with regard to financial assets (Bollerslev, 1987; Engle and Gonzales-Rivera, 1991; Ait-Sahalia and Lo, 1998; Theodossiou and Trigeorgis, 2003; Bali and Theodossiou, 2007), but also with regard to energy assets (Solt and Swanson, 1981; Taylor, 1998; Giot and Laurent, 2003a; Chan et al., 2007; Fan et al., 2008). Historically, commodity prices have been the most volatile among all international prices. Often, the volatility of commodity prices has exceeded that of exchange and interest rates (Kroner et al., 1995). However, relatively little research has been conducted on modeling and estimating volatilities in alternative assets using non-normal distributions. For example, Giot and Laurent (2003a), Chan et al. (2007), Fan et al. (2008), and Hung et al. (2008) comprise the limited body of work that calculates the Value-at-Risk (VaR) of commodity assets in oil markets using non-normal distributions. A majority of the studies that measure the volatility of oil returns do so with normal distributions (Sadeghu and Shavvalpour, 2006). In the gold market, the available quantitative literature is very limited, too. Casassus and Collin-Dufresne (2005) recently evaluated the VaR for gold by using a three-factor model. When participating in commodity markets, it is crucial to describe asset prices. However, no appropriate method is available for this purpose. Volatility is the principal factor on which economic and financial models of pricing and hedging can be developed; moreover, estimations made under correct specifications of the conditional distribution are more efficient. In this paper, we utilize the most flexible distribution to describe the petroleum and metal volatilities characterized by leptokurtosis and skewness.

The application of the VaR methodology offers comprehensive and recapitulative advantages for measuring market risk. Portfolio VaR is often calculated on the basis of the variance–covariance approach, and the models that are used most often in this

* Corresponding author. Tel.: +886 2 26215656x2518.
E-mail addresses: whcheng@mail.tku.edu.tw (W.-H. Cheng), hung660804@gmail.com (J.-C. Hung).

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regard are the classical autoregressive conditional heteroscedasticity (ARCH)/generalized ARCH (GARCH) models based on conditional Gaussian innovations (see Engle, 1982; Bollerslev, 1986). However, empirical evidence has demonstrated that the conditional normal time series models are inadequate for estimating the tail quantiles of conditional return distributions. Substantial empirical evidence reveals that the distribution of financial returns is typically skewed, peaked around the mean (leptokurtic), and characterized by fat tails. Bollerslev et al. (1994) proposed that leptokurtosis may be reduced—but not eliminated—when returns are standardized using time-varying estimates for means and variances. This prompts the gradual adoption of models with heavy-tailed innovations in risk modeling. Numerous extensions have been proposed for the classical GARCH models with heavy-tailed innovations.

Student's *t*, generalized error distribution (GED), and a mixture of two normal distributions are frequently used for describing the non-normal characteristics in the VaR literature. With regard to the commodity markets, Giot and Laurent (2003a) compared the performance of the RiskMetrics, skewed Student asymmetric power GARCH (APGARCH), and skewed Student ARCH models for several commodities. They found that the skewed Student ARCH model delivered excellent results and was relatively easy to use. Chan et al. (2007) considered a GARCH model with heavy-tailed innovations and characterized the limiting distribution of an estimator of the conditional VaR, which corresponds to the external quantile of the conditional distribution of the GARCH process. Fan et al. (2008) estimated the VaR of the returns in West Texas Intermediate (WTI) and Brent crude oil spot markets using a GED-GARCH model. They found this approach to be more realistic and comprehensive than the commonly used standard normal distribution-based VaR model, and also more effective than the well-recognized historical simulation with autoregressive moving average (ARMA) forecasts. Hung et al. (2008) investigated the fat-tailed innovation process on the VaR estimates, and the empirical results showed that the GARCH-HT model is quite accurate and efficient in estimating the VaR for energy commodities.

However, because such distributions partially deal with the issues of leptokurtosis and skewness, they cannot fully correct the measurement bias in risk problems (Bali and Theodossiou, 2007). The skew generalized *t* (SGT) distribution, introduced by Theodossiou (1998), is a skewed extension of the generalized *t* distribution, originally proposed by McDonald and Newey (1988). The SGT is a distribution that allows for a very diverse level of skewness and kurtosis, and it has been used to model the unconditional distribution of daily returns for a variety of financial assets (Theodossiou, 1998; Harris and Kucukozmen, 2001). Furthermore, the SGT nests several well-known distributions such as the generalized *t* (GT) of MacDonald and Newey (1988); the skewed *t* (ST) of Hansen (1994); the skewed generalized error distribution (SGED) of Theodossiou (2001); and the normal, Laplace, uniform, GED, and Student *t* distributions. Harris et al. (2004) further found that a conditional SGT distribution offers a substantial improvement in the fit of the GARCH model for stock index assets. Bali and Theodossiou (2007) proposed a conditional technique for estimating the VaR and expected shortfall measures on the basis of the SGT distribution in the S&P 500 index returns. They found that GARCH-type models with the SGT distribution are much superior to the conditional normal distribution for all GARCH specifications and all probability levels. Bali et al. (2008) also used the SGT distribution with time-varying parameters to provide an accurate characterization of the tail of the standardized equity return distributions. To fill in the gap in the inadequate research in which the SGT distribution in non-normal commodity returns has been employed, we use the GARCH-SGT model to model the commodity volatilities. The analytical and empirical results in this paper could provide better approximations of reality.

The remainder of this paper is organized as follows. Section 2 describes the reason for focusing on the petroleum and metal markets. Section 3 presents the methodologies of the GARCH-SGT models and the measurement of the VaR. Section 4 compares the out-of-sample empirical results of the SGT and the normal distribution and GED. Section 5 concludes the paper.

2. Importance of oil and metal markets

Energy is fundamental to the quality of our lives, and it is a key aspect in all sectors of modern economies. Crude oil, gasoline, and heating oil are three of the most important assets in energy markets. One of the characteristics of petroleum market prices is volatility, which is both high and variable over time. In general, oil prices have become more volatile since 1986 (Plourde and Watkins, 1998; Lynch, 2002; Regnier, 2007), and this volatility has had a significant impact on the global economy (Lee et al., 1995; Ferderer, 1996; Sadorsky, 1999, 2006). A traditional demand-based framework was unable to explain the marked deterioration in commodity and oil prices (Chaudhuri, 2001).

In this paper, we also examine the volatility behavior of three metal assets: gold, silver, and copper. These commodities, which are among the most traded commodities in the world commodity markets, have different economic uses. Of all the precious metals, gold and silver are the most popular avenues of investment. Silver often tracks gold prices due to store of value demands, although there may be a variation in the ratio. Recently, the prices of gold and silver have risen rather steeply due to credit crunch effects. Copper, which is the third metal asset that is analyzed in this paper, is not a precious metal and it is often referred to as the "metal with a PhD in economics" because its price tends to reflect changes in the business cycle (Lahart, 2004). A majority of studies have reported the relationship between metals and macroeconomic variables (Sherman, 1983; Baker and Van-Tassel, 1985; Kaufmann and Winters, 1989; Sjaastad and Scacciavillani, 1996; Taylor, 1998; Christie-David et al., 2000; Cai et al., 2001; Tully and Lucey, 2006).

To our knowledge, relatively little work has been conducted on modeling and estimating volatilities in alternative assets by using non-normal distributions. In order to address the ambiguous empirical results with regard to measuring the VaR in the petroleum and metal markets, this paper provides a comprehensive analysis using the flexible SGT distribution for modeling volatilities for six assets: crude oil, gasoline, heating oil, gold, silver, and copper.

Existing research in the petroleum and metal markets is extended through this paper in four important ways. First, we calculate the VaR on the basis of the SGT—a distribution that permits for a very diverse level of skewness and kurtosis—for modeling the distribution of commodity returns. The normal distribution and GED are comparable models that are used to assess the robustness of the SGT distribution. Second, considering the behavior of highly volatile assets, we employ GARCH models for estimating the time-varying conditional variance of returns. Third, we analyze the time-varying scaling parameters of the six abovementioned assets. The analysis will reveal why traditional distributions are not appropriate for estimating volatilities and forecasting VaR. Fourth, we investigate the volatility in the prices—both spot and futures—of the aforementioned six assets. In addition, we analyze the performance of out-of-sample forecasting for a long period, encompassing both stable and high-fluctuation periods, including the period of the current global financial crisis. It is found that the VaR in the SGT distribution is significantly superior to that obtained from other distributions.

3. Methodology

3.1. GARCH(1,1) model with skewed generalized t distribution (GARCH-SGT)

This paper investigates GARCH(1,1) model in computing the conditional means and conditional variances for conditional VaR analysis. The GARCH(1,1) model proposed by Bollerslev (1986) is as follows:

$$r_t = \mu_t + \varepsilon_t,\ \varepsilon_t \sim (0, h_t) \tag{1}$$

$$h_t = \beta_0 + \beta_1 h_{t-1} z_{t-1}^2 + \beta_2 h_{t-1} \tag{2}$$

where $\beta_0 > 0, \beta_1 > 0, \beta_2 > 0$ and $\beta_1 + \beta_2 \leq 1$. In the equations, μ_t and h_t are the conditional mean and conditional standard variance of returns r_t based on the information set Ω_{t-1}up to time $t-1$. The standardized error term is $z_t = \varepsilon_t / \sqrt{h_t}$.

Considering the non-normal characteristics of energy assets, the conventional GARCH-normal model fails to capture the behavior of high volatility of petroleum and metal assets. SGT distribution, advanced by Theodossiou (1998), is displaced for well-describing the distribution of asset returns exhibiting skewness and leptokurtosis. The probability density function for the SGT distribution can be represented as follows:

$$f(z_t \mid n, \kappa, \lambda) = C\left(1 + \frac{|z_t + \delta|^{\kappa}}{((n+1)|\kappa)(1 + sign(z_t + \delta)\lambda)^{\kappa}\theta^{\kappa}}\right)^{-\frac{n+1}{\kappa}} \tag{3}$$

$$\text{where } C = 0.5\kappa\left(\frac{n+1}{\kappa}\right)^{-\frac{1}{\kappa}} B\left(\frac{n}{k}, \frac{1}{k}\right)^{-1}\theta^{-1},\ \theta = \frac{1}{\sqrt{g-\rho^2}},\ \delta = \rho\theta,$$

$$\rho = 2\lambda B\left(\frac{n}{\kappa}, \frac{1}{\kappa}\right)^{-1}\left(\frac{n+1}{\kappa}\right)^{\frac{1}{\kappa}} B\left(\frac{n-1}{\kappa}, \frac{2}{\kappa}\right)$$

$$g = (1 + 3\lambda^2) B\left(\frac{n}{\kappa}, \frac{1}{\kappa}\right)^{-1}\left(\frac{n+1}{\kappa}\right)^{\frac{2}{\kappa}} B\left(\frac{n-2}{\kappa}, \frac{3}{\kappa}\right)$$

where λ is a skewness parameter, "sign" is the sign function, $B(\cdot)$ is the beta function, and δ is the Pearson's skewness and mode of $f(z_t)$. The scaling parameters n, κ and λ obey the following constraints: $n > 2$, $\kappa > 0$ and $-1 < \lambda < 1$. The skew parameter λ controls the rate of descent of the density around the mode of z. In the case of positive skewness ($\lambda > 0$), the density function is skewed to the right. In contrary, the density function is skewed to the left with the negative skewness ($\lambda < 0$). The parameters n and κ control the tail and height of the density. Smaller values of κ and n result in larger values for the kurtosis (i.e. more leptokurtosis p.d.f.s) and vice versa. The SGT distribution nests several well-known distributions, as reported in Table 1.

3.2. Measurement and evaluation for distribution-based VaR models

3.2.1. Definition and estimation

Under the framework of the parametric techniques (Jorion, 2000), the conditional VaR estimate for a one-day holding period is obtained as follows:

$$VaR_{t+1} = z_{\alpha} \cdot \hat{\sigma}_t + \mu_t \tag{4}$$

where z_α denotes the corresponding quantile of the distribution of the standardized returns at a given confidence level $1-\alpha$. For example, in the normal distribution, the value of the threshold for confidence level 99% is constant and equal to -2.326. However, in more general case of the SGT, the threshold value is a function of the skewness and kurtosis parameters λ, n and κ. According to

Table 1
The special cases of SGT distributions.

	λ	κ	n	Notes
Skew generalized t (SGT)	Free	Free	Free	$\lambda>0$ skew to the right
Skew t (ST)	Free	2	Free	$\lambda<0$ skew to the left
Skew GED (SGED)	Free	Free	∞	
Skew normal	Free	2	∞	$\kappa>2$ thinner tail than normal
Skew Laplace	Free	1	∞	$\kappa<2$ thicker tail than normal
General t (GT)	0	Free	Free	
Student t	0	2	Free	
GED	0	Free	∞	
Normal	0	2	∞	
Cauchy	0	2	1	
Laplace	0	1	∞	
Uniform	0	∞	∞	

Bali and Theodossiou (2007), the quantiles of the SGT distribution with various combinations of shape parameters are calculated with numerical integration technique[1].

3.3. Nonparametric distribution-based VaR model

In contrast to parametric distribution-based VaR models, we use the filtered historical simulation method (Hull and White, 1998; Christoffersen, 2003) to extend this paper to investigate nonparametric distribution-based VaR[2]. The historical simulation discards particular assumptions regarding the return series and calculates the VaR from the immediate past history of the returns series (Dowd, 1998). The filtered historical simulation method is designed to improve on the shortcomings of historical simulation by augmenting the model-free estimates with parametric models[3]. Even if Pritsker (2006) asserts that filtered historical simulation method compares favorably with historical simulation, the historical simulation method may not avoid the many shortcomings of purely model-free estimation approaches. When historical return series include insufficient extreme outcomes, the simulated VaR may seriously underestimate the actual market risk.

3.4. Unconditional and conditional coverage tests

A "failure" occurred if the return on day $t+1$ is less the VaR computed on day t, that is, $r_{t+1}<\hat{\upsilon}_t$. Intuitively, a "good" VaR estimator $\hat{\upsilon}_t$would be such that $\Pr(r_{t+1}<\hat{\upsilon}_t)$ is close to p. The indicator variable is set as follows,

$$I_t = \begin{cases} 1, & \text{if} \quad r_{t+1}<\hat{\upsilon}_t \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

The stochastic process $\{I_t\}$ is called the failure process. The VaR forecasts are said to be efficient if they display correct conditional coverage, that is, $E(I_{t|t-1})=p\,\forall t$. Kupeic (1995) develops a test for correct unconditional coverage in the likelihood ratio (LR) framework. The likelihood ratio statistics are as follows:

$$LR_{uc} = -2log\left[\frac{p^{n_1}(1-p)^{n_0}}{\hat{\pi}^{n_1}(1-\hat{\pi})^{n_0}}\right] \sim \chi^2_{(1)} \tag{6}$$

where p is the tolerance level where VaR measures are estimated, n_1 (n_0) is the number of 1 (0) in the indicator series, and $\hat{\pi} = n_1/(n_1+n_0)$, the MLE estimate of p. The null hypothesis of the failure probability p is tested against the alternative hypothesis that the failure probability is different from p.

Although the LR_{uc} test can reject a model that either overestimates or underestimates the true but unobservable VaR, it cannot examine whether the exceptions are randomly distributed. In a risk management framework, it is of paramount importance that VaR exceptions be uncorrelated over time, which prompts independence and conditional coverage tests based on the evaluation of interval forecasts. Christoffersen (1998) developed a 'conditional coverage' test (LR_{cc}) that jointly investigates whether the total number of failures is equal to the expected one, and the VaR exceptions are independently distributed. In particular, the advantage of Christoffersen's procedure is that it can reject a model that generates either too many or too few clustered exceptions. Since accurate VaR estimates exhibit the property of correct conditional coverage, the I_t series must exhibit both correct unconditional

[1] See footnote 17 in Bali and Theodossiou (2007).
[2] The authors would like to thank an anonymous referee for suggesting the motivation for using historical simulation method to calculate the nonparametric distribution-based VaR.
[3] The GARCH(1,1) model is chosen as the filter while implementing filtered historical simulation.

coverage and serial independence. The LR_{cc} test is a joint test of these two properties, and the corresponding test statistics are $LR_{cc} = LR_{uc} + LR_{ind}$ as we condition on the first observation. Consequently, under the null hypothesis that the failure process is independent and the expected proportion of exceptions equals p, the appropriate likelihood ratio is represented as follows:

$$LR_{cc} = -2log\frac{(1-p)^{n_0}p^{n_1}}{(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}} \sim \chi^2_{(2)} \quad (7)$$

where $n_{i,j}$ = the number of observations with value i followed by value j $(i,j=0,1)$, $\pi_{ij}=P\{I_t=j|I_{t-1}=i\}$ $(i,j=0,1)$, $\hat{\pi}_{01}=n_{01}/(n_{00}+n_{01})$, $\hat{\pi}_{11}=n_{11}/(n_{10}+n_{11})$.

3.5. Dynamic quantile test

Engle and Manganelli (2004) provide the dynamic quantile (DQ) test to correct for the inefficiency in the conditional coverage test of Christoffersen (1998). We define a sequence of indicator variables as

$$Hit_t = I(r_t < \hat{\upsilon}_t(\theta)) - \theta \quad (8)$$

where Hit_t is an indicator function, and $\theta = 1-\alpha$ is a given confidence level. Engle and Manganelli (2004) suggest to test jointly that: (1) $E(Hit_t)=0$; (2) Hit_t is uncorrelated with variables included in the information set. These two tests can be done using artificial regression $Hit_t = XB + \varepsilon_t$, where X is an $N \times k$ matrix whose first column is a column ones, and the remaining columns are additional explanatory variables. We include five lags of Hit_t and the current VaR as the explanatory variables. Engle and Manganelli (2004) show that under the null hypothesis, the dynamic quantile test statistic $DQ = \frac{\hat{B}'X'X\hat{B}}{\theta(1-\theta)}$, where $\hat{B}$ is the OLS estimate of B. The DQ test statistic has an asymptotic Chi-square distribution with seven degrees of freedom $\chi^2_{(7)}$.

3.6. Regulatory loss function

Under the 1996 Market Risk Amendment (MRA) to the Basel Capital Accord, regulatory capital for the trading positions of commercial banks is set according to the banks' own internal VaR estimates. Given its actual use by market participants, the regulatory loss function implied in the MRA is a natural way to evaluate the relative performance of VaR estimates within an economic framework; see Lopez (1999) for further discussion.

The market risk capital (MRC) loss function expressed in the MRA is specified as

$$MRC_t = max\left[VaR_t(10,\ 99\%), \frac{S_t}{60}\sum_{k=0}^{59} VaR_t(10,\ 99\%)\right] \quad (9)$$

where $VaR_t(10, 99\%)$ is the VaR estimate of 99% confidence level generated on day t for a 10-day holding period and expressed in return, S_t is the MRA's multiplication factor (i.e., from 3 to 4 depending on the number of exceptions over the past 250 days[4]). That is, MRC_t is the amount of regulatory capital a bank must hold with respect to its market risk exposure. The MRA capital loss function has several elements that reflect the bank regulators' concerns.

3.7. Superior predictive ability (SPA) test

Consider $l+1$ different portfolios M_k for $k=1,\ldots,l$ and which are discussed in the previous section. M_0 is the benchmark portfolio and the null hypothesis is that none of the portfolio $k=1,\ldots,l$ outperforms the benchmark in terms of the regulatory loss function chosen. For each portfolio M_k, we generate n VaR forecast $VaR_{k,t}$ for $t=1, 2,\ldots, n$. For every forecast, we generate the loss function $L_{k,t}$ describing as follows. Let $L_{k,t} \equiv MRC_{k,t}$ denote the function as defined in Eq. (9). The performance of model k relative to the benchmark model (at time t), can be defined as:

$$f_{k,t} = L_{0,t} - L_{k,t} \text{ for } k = 1, 2, \ldots, \ell;\ t = 1, 2, \ldots, n. \quad (10)$$

Assuming stability for $f_{k,t}$, we can define the expected relative performance of model k relative to the benchmark as $\mu_k = E[f_{k,t}]$ for $k=1,2,\ldots,\ell$. If models outperform the benchmark one, then the value of μ_k will be positive. Therefore, we can analyze whether

[4] In the green zone, the multiplier value is 3 for exceptions from 0 to 4; in the yellow zone, the multiplier values for five through nine exceptions are 3.4, 3.5, 3.75 and 3.85, respectively; in the red zone, the multiplier value is 4 while exceptions are above 10.

Table 2
Descriptive statistics of spot and futures returns.

	Mean	S.D.	Skewness	Excess kurtosis	J–B test
Panel A. Spot					
WTI	0.011	2.661	−0.808*	14.719*	53049.959*
Gasoline	0.027	2.857	0.036	2.773*	1209.054*
Heating	0.026	2.660	−1.630*	36.558*	331507.245*
Gold	0.017	1.614	−0.218*	20.335*	100085.930*
Silver	0.016	1.721	−0.536*	8.525*	18132.181*
Copper	0.004	1.724	−0.088*	4.974*	5297.468*
Panel B. Futures					
WTI	0.011	2.587	−0.843*	14.682*	52841.263*
Gasoline	0.016	2.553	−0.550*	8.811*	20502.307*
Heating	0.011	2.355	−1.467*	19.369*	119903.726*
Gold	0.017	1.017	0.016	7.694*	14321.981*
Silver	0.017	1.731	−0.649*	8.055*	13392.968*
Copper	0.014	1.645	−0.217*	5.331*	5756.973*

Note: J–B test is Jarque–Bera normality test. * represents significance at the 5% significance level.

any of the competing models significantly outperform the benchmark, testing the null hypothesis that $\mu_k \leq 0$, for $k = 1, 2, \ldots, \ell$. Consequently, the null hypothesis that none of models is better than the benchmark (i.e. no predictive superiority over the benchmark itself) can be formulated as:

$$H_0 : \mu_{max} \equiv \max_{k=1,\cdots,\ell} \mu_k \leq 0. \tag{11}$$

The associate test statistic proposed by Hansen (2005) is given by

$$T = \max_{k=1,\cdots,\ell} \frac{\sqrt{n}\bar{f}_k}{\hat{\omega}_{kk}} \tag{12}$$

with $\hat{\omega}_{kk}^2$ as a consistent estimate of ω_{kk}^2, and where $\bar{f}_k = n^{-1}\sum_{t=1}^{n} f_{k,t}$, $\omega_{kk}^2 = \lim_{N\to\infty} var\left(\sqrt{n}\,\bar{f}_k\right)$. A consistent estimator of ω_{kk} and p-value of test statistic T can be obtained via a stationary bootstrap procedure of Politis and Romano (1994). More details of this procedure are detailed in Hansen (2005) and Hansen and Lunde (2005).

4. Empirical results

4.1. Data and descriptive statistics

This paper calculates the daily VaR for spot and futures returns on West Texas Intermediate (WTI) crude oil, gasoline, heating oil, gold, silver, and copper for the period January 2002 to March 2009. All the spot prices for WTI crude oil, New York harbor reformulated regular gasoline, and #2 heating oil as well as the relative futures price trading on the NYMEX were obtained from the U.S. Energy Information Administration (EIA). Moreover, the spot prices for New York gold, silver, and

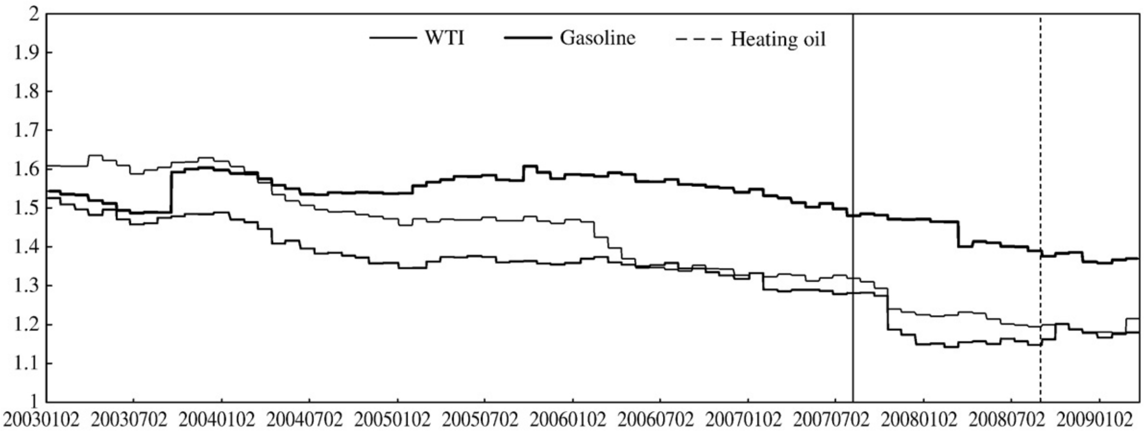

Fig. 1. Kurtosis parameter for petroleum futures in the GED distribution.

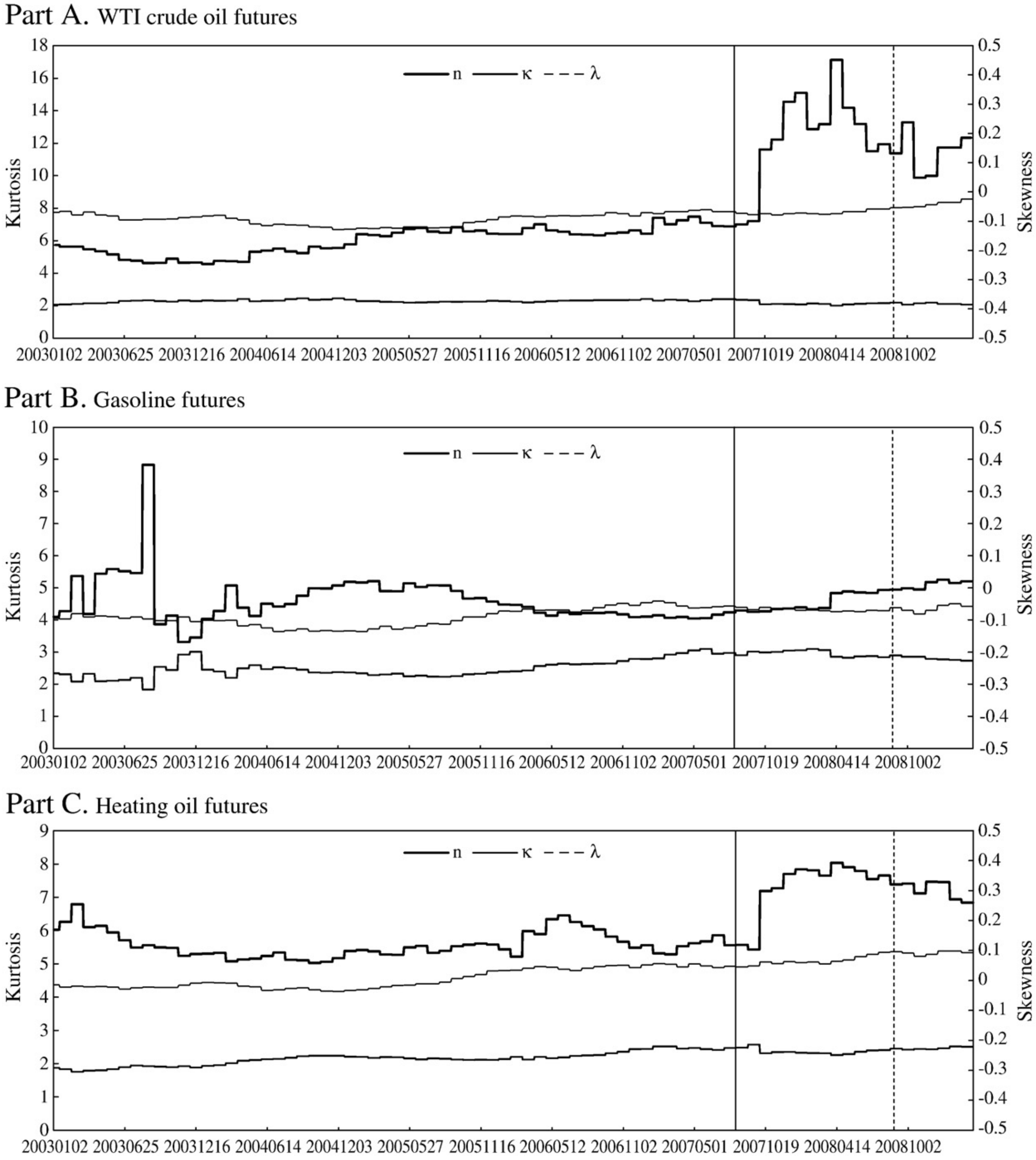

Fig. 2. Kurtosis and skewness parameters for petroleum futures in the SGT distribution.

copper and the relative futures prices trading on the COMEX were obtained from the ShareLynx database. The asset returns are logarithmic returns. The descriptive statistics for the six assets are presented in Table 2. First, we observed that the unconditional standard deviations of the oil assets were higher than those of the metal assets. For example, in the spot market, the standard deviations of WTI, gasoline, and heating oil are 2.661, 2.857, and 2.660, respectively; these values are relatively higher than the standard deviation of the metal assets at 1.614, 1.720, and 1.724 for gold, silver, and copper, respectively. The same results are found for the futures markets: except for gasoline spot and gold futures, the skewness statistics are negative and significant at the 5% level, thereby indicating that the assets' returns are significantly skewed to the left. With respect to the excess kurtosis statistics, all the values are significantly positive, thereby implying that the distribution of returns has larger, thicker tails than the normal distribution. Similarly, the Jarque–Bera statistic is large and significant, thereby implying that the assumption of normality is rejected.

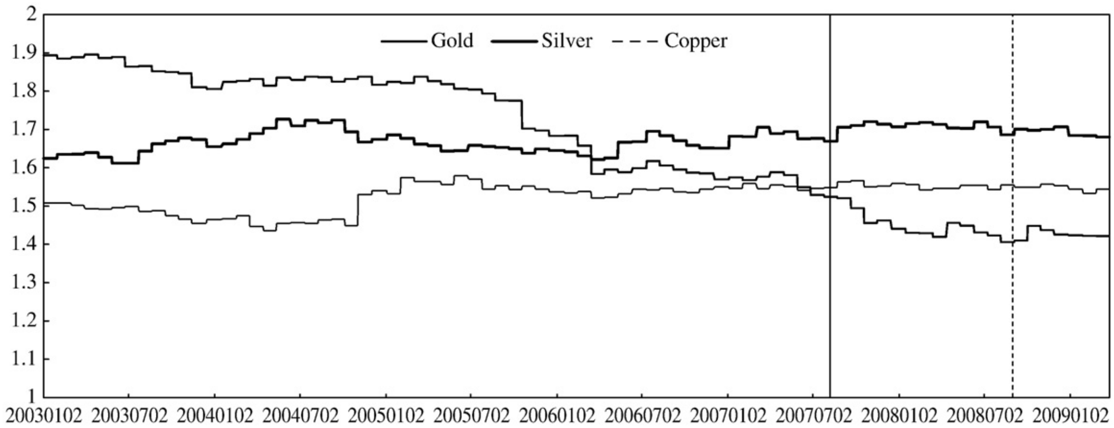

Fig. 3. Kurtosis parameter for metal futures in the GED distribution.

4.2. Time-varying scaling parameters

In this paper, the predicted one-day-ahead VaR is based on a rolling window out-of-sample procedure. Figs. 1 to 4 illustrate the time-varying scaling parameters in the GED and SGT distribution for futures returns.[5] Two vertical lines are drawn: The solid line indicates the values as on August 9, 2007, and the dotted line indicates the values as in September 2008. The former date indicates the beginning of the global financial crisis, which resulted in a liquidity crisis that prompted a substantial injection of capital into the financial markets by the United States Federal Reserve, the Bank of England, and the European Central Bank. In September 2008, the crisis deepened, as stock markets worldwide crashed and entered a period of high volatility, and a large number of banks, mortgage lenders, and insurance companies failed in the subsequent weeks. In the part of Fig. 1 indicating the GED for petroleum futures, we can see that the fat-tail parameter (κ) is below 2 and downward gradually in the forecasting period, indicating that the fat-tail exists in the oil returns. However, the fluctuation of the parameter κ is not large, except in the global financial crisis period when it is comparatively low. In comparison, an observation of the scaling parameters of the SGT distribution in Fig. 2 shows that the skewness parameter λ is negative for WTI crude oil and gasoline and positive for heating oil; however, the values are smooth around 0, thus indicating that the skewness is not very important. The two kurtosis parameters (κ and n) perform differently in the forecasting period. The first parameter, κ, is very smooth and the average value is close to that of the normal distribution (i.e., 2), indicating no peakness for the empirical distribution. The second parameter controlling the fat-tail characteristic, n, on the other hand, is more volatile, but the value is not large. By definition, the smaller values of κ and n result in larger values for the kurtosis, and vice versa, and the SGT distribution is close to the normal distribution, while $\lambda = 0$, $\kappa = 2$, and $n = \infty$. We can therefore say that the normal distribution is not appropriate for oil returns. For the WTI and heating oil futures, the parameter n is upward for the starting year of the global financial crisis, then downward along with the crisis period. For the gasoline futures, we cannot observe a significant trend within the crisis period. Yet we still confirm that the fat-tail distribution is more appropriate for the reason that the parameter n has low values.

Fig. 3 illustrates the kurtosis parameter in the GED distribution for metal futures returns. Similarly, all three parameters κ are below 2, indicating that the fat-tail exists in the metal returns. However, the trends of the silver and copper assets are very smooth within the whole empirical period, whereas that of the gold assets is obviously downward. More specifically, the parameter κ in gold returns is steady and close to 2 in the beginning, and then goes down gradually after 2005. The scaling parameters of the SGT distribution are shown in Fig. 4. Similar to the oil assets, the skewness parameter λ and the peakness parameter κ are very smooth and close to 0 and 2, respectively; relatively, the fat-tail parameter n fluctuates much more than the others. Among them, the trend of the gold assets is similar to the WTI and heating oil returns; that is, the parameter n is upward for the starting year of the global financial crisis, then downward along with the crisis period. For the silver assets, the lowest value appears in the crisis period; in contrast, the parameter n for copper returns is upward slightly, similar to gasoline. The same occurs with the oil assets, and we confirmed that the fat-tail distribution is more appropriate to metal assets for the reason that the parameter n has low values. This is also why the forecasting performance with the normal distribution was not better than that with the alternative distributions.

4.3. The results of VaR performance assessment

With regard to the empirical results, we only assess the out-of-sample performance of the VaR models because it is, in practice, a routine procedure for a qualified VaR model. In this paper, the predicted one-day-ahead VaR is based on a rolling window out-of-

[5] Similar results are shown in terms of spot returns.

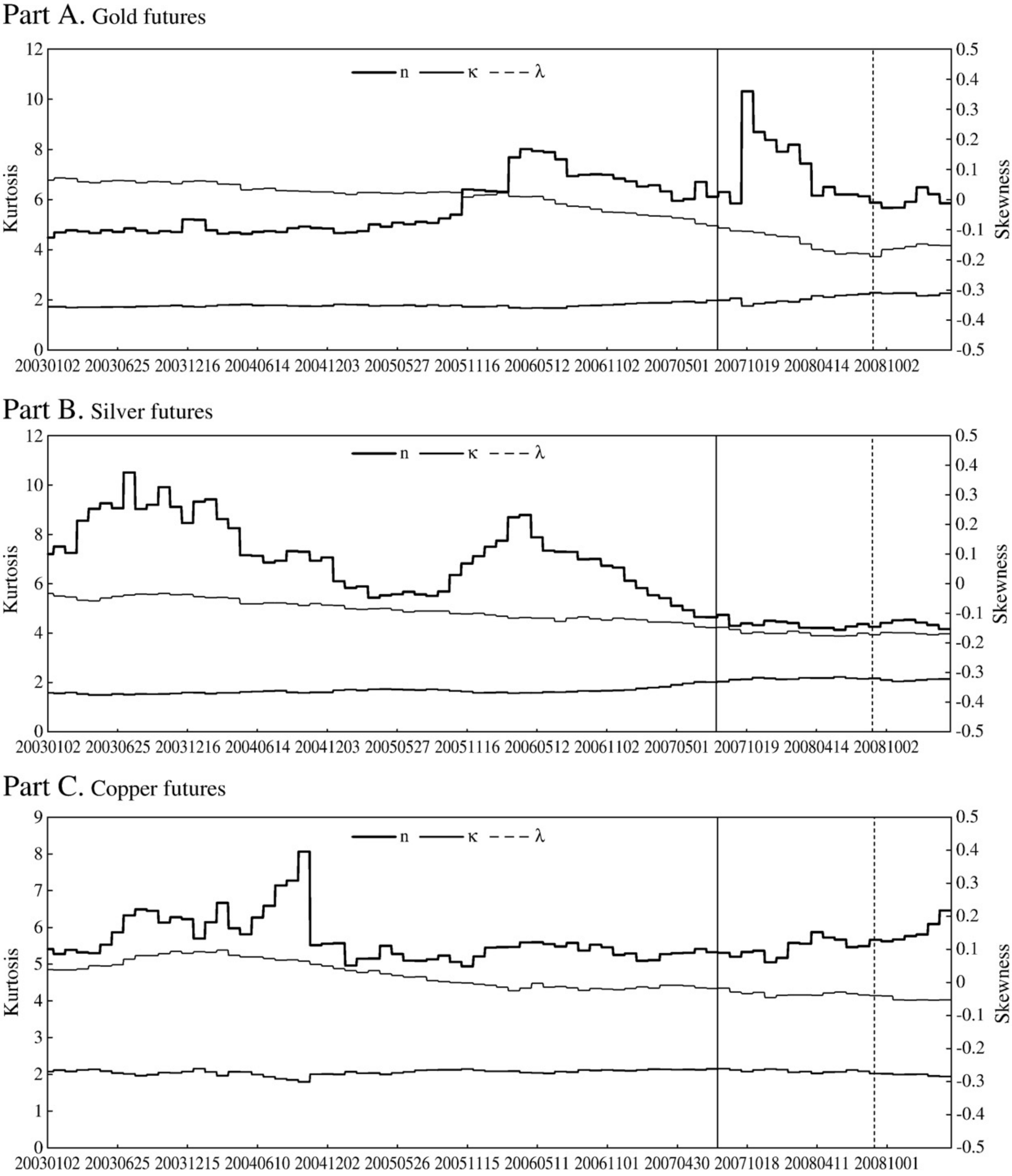

Fig. 4. Kurtosis and skewness parameters for metal futures in the SGT distribution.

sample procedure.[6] The window size is fixed at 1500 observations. More specifically, the procedure was conducted in the following manner: The first rolling sample included the returns from December 18, 1996 to December 31, 2002. Following Giot and Laurent (2003b) and Bali et al. (2008), the model was re-estimated every 20 trading days in order to update the distribution parameters of each model. The predicted one-step-ahead was computed for the next 20 days based on the model estimates;

[6] The authors would like to thank an anonymous referee for suggesting the use of the recursive estimation window. However, we found that there is no significant difference between the empirical results of these two predicting procedures.

Table 3
Mean VaR and the failure rates.

	Normal distribution			Generalized error distribution			Skewed generalized *t* distribution			Nonparametric distribution		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
Panel A. Spot												
WTI	−3.291	−4.254	−6.062	−3.033	−4.156	−6.510	−2.887	−3.952	−6.565	−3.147	−4.158	−7.321
	(0.083)	(0.042)	(0.017)	(0.097)	(0.046)	(0.010)	(0.105)	(0.056)	(0.012)	(0.092)	(0.046)	(0.008)
Gasoline	−3.795	−4.902	−6.979	−3.599	−4.881	−7.528	−3.587	−4.838	−7.685	−3.683	−5.016	−7.336
	(0.098)	(0.052)	(0.017)	(0.106)	(0.053)	(0.012)	(0.108)	(0.053)	(0.009)	(0.102)	(0.050)	(0.016)
Heating	−3.268	−4.222	−6.013	−3.132	−4.193	−6.338	−3.058	−4.079	−6.269	−2.991	−4.071	−6.384
	(0.079)	(0.041)	(0.010)	(0.087)	(0.039)	(0.009)	(0.092)	(0.040)	(0.010)	(0.096)	(0.046)	(0.008)
Gold	−2.400	−3.097	−4.403	−1.889	−2.715	−4.612	−1.745	−2.513	−4.625	−1.937	−2.811	−4.674
	(0.069)	(0.048)	(0.015)	(0.096)	(0.055)	(0.010)	(0.102)	(0.056)	(0.009)	(0.099)	(0.056)	(0.014)
Silver	−2.511	−3.227	−4.571	−2.239	−3.179	−5.308	−2.316	−3.294	−5.503	−2.254	−3.135	−5.498
	(0.084)	(0.055)	(0.025)	(0.108)	(0.058)	(0.019)	(0.101)	(0.054)	(0.016)	(0.108)	(0.060)	(0.017)
Copper	−2.155	−2.784	−3.964	−2.118	−2.815	−4.211	−2.000	−2.612	−4.024	−2.269	−3.204	−5.757
	(0.108)	(0.074)	(0.029)	(0.112)	(0.073)	(0.020)	(0.123)	(0.083)	(0.021)	(0.108)	(0.059)	(0.016)
Panel B. Futures												
WTI	−3.193	−4.124	−5.871	−3.004	−4.044	−6.162	−2.882	−3.891	−6.209	−3.025	−4.204	−6.469
	(0.090)	(0.044)	(0.008)	(0.107)	(0.048)	(0.007)	(0.110)	(0.058)	(0.009)	(0.101)	(0.045)	(0.005)
Gasoline	−3.689	−4.754	−6.752	−3.303	−4.544	−7.176	−3.299	−4.491	−7.506	−3.360	−4.622	−7.385
	(0.091)	(0.045)	(0.010)	(0.111)	(0.053)	(0.008)	(0.111)	(0.055)	(0.009)	(0.108)	(0.050)	(0.009)
Heating	−3.230	−4.169	−5.929	−2.914	−3.941	−6.056	−2.880	−4.049	−6.198	−2.708	−3.709	−6.029
	(0.077)	(0.037)	(0.011)	(0.086)	(0.030)	(0.005)	(0.101)	(0.038)	(0.007)	(0.101)	(0.038)	(0.006)
Gold	−1.481	−1.910	−2.714	−1.367	−1.898	−3.049	−1.353	−1.871	−3.132	−1.368	−1.873	−3.191
	(0.100)	(0.061)	(0.025)	(0.117)	(0.060)	(0.017)	(0.114)	(0.064)	(0.015)	(0.117)	(0.065)	(0.016)
Silver	−2.500	−3.215	−4.557	−2.197	−3.084	−5.025	−2.368	−3.337	−5.749	−2.269	−3.204	−5.757
	(0.092)	(0.059)	(0.028)	(0.116)	(0.066)	(0.020)	(0.101)	(0.055)	(0.015)	(0.108)	(0.059)	(0.016)
Copper	−2.427	−3.125	−4.436	−2.221	−3.04	−4.788	−2.174	−2.951	−4.830	−2.249	−2.983	−4.908
	(0.101)	(0.057)	(0.023)	(0.122)	(0.058)	(0.019)	(0.121)	(0.063)	(0.018)	(0.119)	(0.060)	(0.019)

Note: The table reports mean VaR estimates and the failure rates (in parentheses) for the alternative VaR models under 90%, 95%, and 99% confidence levels.

thereafter it was compared with the observed return and both results were recorded for subsequent evaluation using statistical tests. Next, the estimated sample was rolled forward by omitting the returns for the oldest 20 days and adding the returns for the latest 20 days. This procedure was repeated until the sample was exhausted.

Table 3 presents mean VaR estimates and the failure rates for each of the four models under 90%, 95%, and 99% confidence levels over the entire out-of-sample period from January 2, 2003, to March 30, 2009. These basic statistics can be regarded as the preliminary understanding of average performance during forecasting period before the implementation of more rigorous VaR evaluation tests. The results of VaR evaluation tests introduced in the preceding section will be reported in Tables 4–7. These tables and the discussion below are framed with regard to the distribution-based and nonparametric distribution-based VaR estimates.

As observed in Table 3, except for copper, the mean VaR estimate of spot is higher than that of futures. This implies that holding the spot position of these commodities will bear higher average market risk than holding the futures position, which is dissimilar

Table 4
Unconditional coverage test.

	Normal distribution			Generalized error distribution			Skewed generalized *t* distribution			Nonparametric distribution		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
Panel A. Spot												
WTI	×	O	×	O	O	O	O	O	O	O	O	O
Gasoline	O	O	×	O	O	O	O	O	O	O	O	×
Heating	×	O	O	O	O	O	O	O	O	O	O	O
Gold	×	O	×	O	O	O	O	O	O	O	O	O
Silver	×	O	×	O	O	×	O	O	×	O	O	×
Copper	O	×	×	O	×	×	×	×	×	O	O	×
Panel B. Futures												
WTI	O	O	O	O	O	O	O	O	O	O	O	×
Gasoline	O	O	O	O	O	O	O	O	O	O	O	O
Heating	×	×	O	O	×	O	O	×	O	O	O	O
Gold	O	×	×	×	O	×	O	×	×	×	×	×
Silver	O	O	×	×	×	×	O	O	O	O	O	×
Copper	O	O	×	×	O	×	×	×	×	×	O	×

Note: The symbol × (O) indicates that the null hypothesis was rejected (accepted) at the 5% significance level for the LR_{uc} statistics under 90%, 95%, and 99% confidence levels. The LR_{uc} statistics are asymptotically distributed $\chi^2(1)$.

Table 5
Conditional coverage test.

	Normal distribution			Generalized error distribution			Skewed generalized *t* distribution			Nonparametric distribution		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
Panel A. Spot												
WTI	×	×	×	×	×	O	×	×	O	×	×	O
Gasoline	O	×	×	O	×	O	O	O	O	O	O	×
Heating	×	O	O	O	O	O	O	O	O	O	O	O
Gold	×	O	O	O	O	O	O	O	O	O	O	O
Silver	×	×	×	O	×	×	O	×	×	O	×	×
Copper	O	×	×	×	×	×	×	×	×	×	×	×
Panel B. Futures												
WTI	×	×	O	O	×	O	O	×	O	O	×	×
Gasoline	O	O	O	×	O	O	×	O	×	O	O	O
Heating	×	×	O	O	×	×	O	O	O	O	O	O
Gold	O	O	×	O	O	×	O	×	O	O	×	×
Silver	×	×	×	×	×	×	O	×	×	×	×	×
Copper	O	O	×	×	O	×	×	×	×	×	O	×

Note: The symbol × (O) indicates that the null hypothesis was rejected (accepted) at the 5% significance level for the LR_{uc} statistics under 90%, 95%, and 99% confidence levels. The LR_{cc} statistics are asymptotically distributed $\chi^2(2)$.

with the general recognition that futures are riskier than spot. Moreover, for both spot and futures, the mean VaR estimates of oil commodities are relatively higher than that of metal commodities, where gasoline and silver have the highest market risk among oil and metal commodities, respectively. At 90% and 95% confidence levels, the normal distribution almost generates the highest mean VaR estimates among the four VaR models, and normal distribution tends to overestimate real market risk at a 90% confidence level because the failure rate is lower than 10% for most cases. On the contrary, at a 99% confidence level, the normal distribution produces the lowest VaR estimates and the failure rate points out the tendency of underestimation for real market risk. The generalized error distribution underestimates for the spot and the futures of copper and also the futures of silver. Similarly, the skewed generalized *t* distribution underestimates both spot and futures of copper, and does so in the nonparametric distribution for the gold futures. Besides the spot of copper, GED, SGT, and nonparametric distributions perform quite well under all confidence levels.

4.4. Unconditional and conditional coverage tests

The results for the unconditional (LR_{uc}) and conditional coverage (LR_{cc}) tests are reported in Tables 4 and 5, respectively. The symbol "×" indicates that the null hypothesis was rejected at the 5% significance level, and the symbol "O" indicates that the test did not reject the null hypothesis at the 5% significance level. If LR_{uc} is statistically insignificant, it implies that the expected and the actual number of observations falling below the VaR estimates are statistically the same. Further, rejection of the null hypothesis indicates that the computed VaR estimates are not sufficiently accurate. According to the LR_{uc} test statistics, VaR models based on GED, SGT, and nonparametric distributions perform relatively better than the normal distribution for all spot commodities of since

Table 6
Dynamic quantile test.

	Normal distribution			Generalized error distribution			Skewed generalized *t* distribution			Nonparametric distribution		
	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
Panel A. Spot												
WTI	×	×	×	×	×	×	×	×	×	×	×	×
Gasoline	×	×	×	×	×	×	×	×	×	×	×	×
Heating	×	×	O	×	×	O	×	O	O	O	O	O
Gold	×	O	O	×	O	O	O	O	O	O	×	O
Silver	×	×	×	O	×	×	O	×	×	O	×	×
Copper	×	×	×	×	×	×	×	×	×	×	×	×
Panel B. Futures												
WTI	×	×	×	O	×	×	×	×	×	O	×	×
Gasoline	×	O	O	×	×	×	×	×	×	O	O	O
Heating	×	O		O	×	×	×	O	×	O	O	×
Gold	O	×	×	O	×	×	O	×	×	O	×	×
Silver	O	×	×	×	×	×	O	×	×	×	×	×
Copper	×	×	×	×	×	×	×	×	×	×	×	×

Note: The symbol × (O) indicates that the null hypothesis was rejected (accepted) at the 5% significance level for the DQ statistics under 90%, 95%, and 99% confidence levels. The DQ statistics are asymptotically distributed $\chi^2(7)$.

Table 7
Superior predictive ability test for market risk capital.

	WTI	Gasoline	Heating	Gold	Silver	Copper
Panel A. Spot						
Skewed generalized *t* distribution						
%trading	73.923	55.230	38.019	44.119	72.903	95.227
Mean	75.855	86.472	67.719	51.515	70.816	51.511
(*p*-value)	(0.511)	(0.501)	(0.000)	(0.527)	(0.219)	(0.479)
Nonparametric distribution						
%trading	26.077	44.770	61.981	55.881	27.097	4.773
Mean	81.964	89.247	65.418	53.377	70.495	74.849
(*p*-value)	(0.000)	(0.000)	(0.499)	(0.005)	(0.781)	(0.000)
Panel B. Futures						
Skewed generalized *t* distribution						
%trading	59.230	22.693	31.345	66.256	36.490	69.361
Mean	64.380	78.756	57.502	39.674	74.898	61.873
(*p*-value)	(0.499)	(0.002)	(0.001)	(0.509)	(0.470)	(0.498)
Nonparametric distribution						
%trading	40.770	77.307	68.655	33.744	63.510	30.639
Mean	65.825	77.703	55.788	40.731	74.849	63.029
(*p*-value)	(0.000)	(0.478)	(0.481)	(0.000)	(0.530)	(0.000)

Note: This table shows that the percentage of trading days of the benchmark model has a lower capital charge than the competing model. Mean and *p*-value denote the average market risk capital and the reality check *p*-value of the Hansen's consistent test for the market risk capital-based loss function. In SPA test, each competing model has to take turns to be the benchmark model, and the null hypothesis is that none of the models is better than the benchmark. The number of bootstrap replications to calculate the *p*-values is 1000 and the dependency parameter *q* is 0.5.

GED, SGT, and nonparametric distributions have fewer rejections than a normal distribution. However, for the cases of the 95% and 99% confidence levels, the failure rates of normal, GED, and SGT distributions for the spot price of copper are significantly higher than the expected failure rates, which is evident from Table 3. Moreover, the performance of the normal distribution at the 99% confidence level is the worst; this is because of the rejection of all cases except for heating oil and because all models failed to provide an accurate prediction of the downside risk for silver and copper at the 99% confidence level. On the other hand, with regard to futures, both the SGT and normal distributions have six rejections; the normal distribution performs better than GED and nonparametric distributions. However, the number of rejections among these VaR models is rather close. Based on the results of LR_{uc} test statistics for both spot and futures of all commodities, the SGT and nonparametric distributions both have 10 rejections, followed by GED with 11 rejections, and normal distribution with 16 rejections. Thus, it is evident that the SGT and nonparametric distributions produce better VaR estimates than other distributions in terms of the total number of rejections.

Since LR_{uc} only counts failure in the backtesting period without considering the dependence of each failure, the LR_{cc} test used in this paper is a test of correct conditional coverage that simultaneously takes account of both serial independence of failure series and the correct unconditional coverage. Table 5 presents the results of the conditional coverage test for the 90%, 95%, and 99% confidence levels. The GED, SGT, and nonparametric distributions behave fairly well for heating oil and gold spots at all confidence levels; moreover, the SGT distribution performance rather well for gasoline spots. With regard to futures, the null hypothesis of the LR_{cc} test cannot be rejected when using the normal and nonparametric distributions to calculate VaR estimates of gasoline and when adopting SGT and nonparametric distributions for computing VaR estimates of heating oil. Based on the results of LR_{cc} test statistics for both spot and futures of all commodities, the SGT distribution has 16 rejections, nonparametric distribution has 17 rejections, GED has 18 rejections, and normal distribution has 21 rejections. With respect to the total number of rejections, the SGT distribution produces more favorable VaR estimates as compared with the other distributions.

4.5. Dynamic quantile test

Table 6 presents the results of the out-of-sample VaR performance using the dynamic quantile test statistics given by Engle and Manganelli (2004). As compared with the results of the unconditional and conditional coverage tests, the dynamic quantile test is more severe since the number of rejections is much more than those presented in Tables 4 and 5. The SGT distribution passes the dynamic quantile test at all confidence levels for gold spots, as does the nonparametric distribution for heating oil spots and gasoline futures. In light of the number of rejections, the VaR estimates generated by SGT and nonparametric distributions perform better than that generated by normal and GED distributions.

4.6. Regulatory loss function

The abovementioned tests focused on examining the accuracy of failure frequency and the independence of the failure process for VaR models. However, there may be a large number of VaR models that can pass these statistical evaluation tests. How do risk managers choose among alternative VaR models? Which one will generate fewer regulatory capital requirements and induce less opportunity cost of capital? In this section, we employ the two-stage selection procedure given by Sarma et al. (2003), in which the first stage of model selection involves a statistical accuracy test and the second one includes an efficiency test based on specific loss functions. The loss function used here is based on the regulatory capital that is earmarked for the trading positions of commercial banks according to the banks' internal VaR estimates. The regulatory loss function related to market risk capital requirement is introduced in Eq. (9) and is applied in order to evaluate the relative performance of VaR models within an economic framework.

It must be noted that, based on the results of three abovementioned statistical accuracy tests, the SGT and nonparametric distributions tended to produce more superior VaR estimates than normal and GED distributions. Thus, in the first stage, the VaR models based on SGT and nonparametric distributions were selected for the efficiency evaluation test based on a market risk capital-based loss function. If a certain VaR model has a smaller loss function value, it implies that the likelihood of using this model for computing the VaR-based market risk capital requirement will be lower than other models.

Table 7 reports the percentage of trading days of the benchmark model has a lower capital charge than the competing model. The mean and p-values denote the average market risk capital and the reality check p-value of the superior predictive test (SPA) given by Hansen (2005) for the market risk capital-based loss function. In the SPA test, each competing model is used as the benchmark model by turns, and the null hypothesis is that none of the competing models is better than the benchmark. For both spot and futures of all commodities, the SGT distribution has significantly fewer capital requirements than the nonparametric distribution for seven cases as indicated by the p-values of the SPA test; moreover, the SGT distribution yields lower capital charges on over 50% of the trading days. Further, the nonparametric distribution performs better than the SGT distribution with regard to the heating oil spot and gasoline and heating oil futures. With regard to the cases of both spot and futures of silver, the capital charges generated by these two models are rather close without any significant difference. From the perspective of percentage of trading days, the SGT distribution has lower capital charges than the nonparametric distribution in a greater number of days for silver spot, and the situation is reversed for silver futures. Consequently, the SGT distribution appears to be the most appropriate choice since it enables risk managers to fulfill their purpose of minimizing MRA regulatory capital requirements.

5. Conclusion

This paper provides a comprehensive analysis using the flexible SGT distribution for modeling six commodity volatilities—WTI crude oil, gasoline, heating oil, gold, silver and copper—and analyzing the time-varying scaling parameters, including those in the petroleum and metal markets. It also estimates the VaR within the framework of the GARCH-SGT model. The out-of-sample forecasting period covers a long period, encompassing both stable and high-fluctuation periods, including the most unsteady period of the global financial crisis. The empirical results indicate that the forecasted VaR obtained using the SGT distribution provides the most accurate out-of-sample forecasts for both the petroleum and metal markets. The estimated time-variation of the parameters provides evidence for tail fatness of the six return series. The fat-tail parameter in GED is below 2 (normal distribution) and decreases gradually in the forecasting period. The value is comparatively low during the global financial crisis period. Furthermore, the skewness and peakness parameter in SGT distribution are very smooth and close to that of the normal distribution; however, the fat-tail parameter in the SGT distribution is small for all returns, indicating that the fat-tail distribution is more appropriate than the normal distribution. Except for silver, the fat-tail parameter is upward for the starting year of the global financial crisis, then downward along with the crisis period.

As in the failure rate, the mean VaR estimates of petroleum commodities are relatively higher than those of the metal commodities. The mean VaR estimates of spot are higher than those of the futures, except for copper. This implies that holding the spot position of these commodities will bear higher average market risk than holding the futures position, which is dissimilar with the general recognition that futures are riskier than spot. With regard to the unconditional and conditional coverage tests, the SGT distribution produces the most appropriate VaR estimates in terms of the total number of rejections; this is followed by the nonparametric distribution, generalized error distribution (GED), and finally the normal distribution. Similar to the dynamic quantile test, the VaR estimates generated by SGT and nonparametric distributions perform better than that generated by normal and GED distributions. Finally, for the superior predictive test, the SGT and nonparametric distributions are selected to be the benchmark for the following efficiency evaluation test based on a market risk capital-based loss function. The results show that the SGT distribution has significantly lower capital requirements than the nonparametric distribution for most commodities; however, the nonparametric distribution performs better than the SGT distribution merely for the heating oil spot and gasoline and heating oil futures. Therefore, the SGT distribution appears to be the most appropriate choice since it enables risk managers to fulfill their purpose of minimizing MRA regulatory capital requirements.

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