On-line Modeling and Control via T-S Fuzzy Models for Nonaffine Nonlinear Systems Using A Second Type Adaptive Fuzzy Approach

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Abstract

This paper proposes a novel method for on-line modeling and robust adaptive control via Takagi–Sugeno (T-S) fuzzy models for nonaffine nonlinear systems, with external disturbances. The T-S fuzzy model is established to approximate the nonaffine nonlinear dynamic system in a linearized way. The so-called second type adaptive law is adopted, where not only the consequent part (the weighting factors) of fuzzy implications but also the antecedent part (the membership functions) of fuzzy implications are adjusted. Fuzzy B-spline membership functions (BMFs) are used for on-line tuning. Furthermore, the effect of all the unmodeled dynamics, BMF modeling errors and external disturbances on the tracking error is attenuated by a fuzzy error compensator which is also constructed from the T-S fuzzy model. In this paper, we can prove that the closed-loop system which is controlled by the proposed controller is stable and the tracking error will converge to zero. Three examples are simulated in order to confirm the effectiveness and applicability of the proposed methods in this paper.

1. Introduction

In model-based control design [1], a systematic way to construct a model mapping the inputs to the outputs is needed. Fuzzy models are usually used in the case where the model structure and parameters are unknown [5]. There are two fuzzy model structures, Takagi–Sugeno (T-S) and Mamdani. T-S fuzzy systems have a linear consequent part described by a set of IF-THEN rules. This model can approximate a wide class of nonlinear systems. In [2-4], the authors proved that the T–S fuzzy system can approximate any continuous function at any precision.

By using well-known, off-line tuning algorithms for unknown fuzzy model parameters, an initial fuzzy model can be constructed. However, the derived fuzzy model with these tuned parameters cannot cope with parameter changes arising from some external disturbance [6]. In these situations, the parameters must be tuned on-line during operation to compensate for undesirable effects. The objective of adaptive control is to maintain consistent performance of a system in the presence of uncertainties. Therefore, fuzzy controllers should be adaptive [6-9], [22], [24], [27], [28]. A further issue is the stability of T–S fuzzy systems [31], [32]. This has been extensively investigated in the literature. The existence of a common positive definite matrix for a set of Lyapunov inequalities is a sufficient condition for stabilization [10]-[12]. However, this is very difficult to achieve, even using the well-known linear matrix inequalities (LMIs) method. Therefore, in this paper, adaptive schemes are used for online modeling, controller design, and stability analysis of the T-S fuzzy systems.

The stabilization problem for the systems represented in T–S fuzzy models has been well addressed, e.g., [13], [14], and [15], but studies concerning tracking controller design based on T–S fuzzy models are relatively few. Tracking control designs are important issues for practical applications; for example, in robotic tracking control and missile tracking control, and are more difficult than stabilization control design. In [6], the authors only consider the stabilization problem for the controlled system. In this paper, we will apply the T–S fuzzy modeling approach to the design of robust tracking controllers for nonlinear systems.

In this paper, this so-called second type adaptive fuzzy approach is adopted. In this type of approach, not only the consequent part (the weighting factors) of fuzzy implications, but also the antecedent part (the membership functions) of fuzzy implications are adjusted. In fuzzy set theory, the selection of appropriate membership functions is also an important issue for engineering problems [22]. It is important that the fuzzy membership functions are updated, because a change in fuzzy membership functions may alter the performance of the fuzzy logic system. The fuzzy B-spline membership functions (BMFs) constructed in [21] possess the property of local control and have been successfully applied to fuzzy-neural control [24]. This is mainly due to the local control property of the B-spline curve, i.e., the BMF has the elegant property of being locally tuned in a learning process. Several learning algorithms have been proposed

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in [21], [23-26] to deal with the tuning of BMFs.

This paper deals with Takagi–Sugeno (T–S) fuzzy models because of their capability to approximate dynamic nonlinear systems [16-20]. We propose an on-line identification algorithm for T–S fuzzy models with the robust tracking controller design using adaptive schemes.

The rest of this paper is organized as follows. Section II describes the T-S fuzzy model for nonaffine nonlinear systems. Section III gives details of the on-line modeling and robust tracking controller design. The simulation results are shown in section IV. Finally, conclusions are drawn in section V.

2. System Formulation

Figure 1 shows the configuration of the BMF fuzzy neural network, which is typically a fuzzy inference system constructed from a neural network structure, and has a total of four layers [21], [23-25]. Nodes at layer I are input nodes (linguistic nodes) that represent input linguistic variables. Nodes at layer II are term nodes which act as membership functions to represent the terms of the respective linguistic variables. Each node at layer III is a fuzzy rule. Layer IV is the output layer.

A. *B-Spline Membership Functions (BMFs)*

We adopt B-spline functions as the fuzzy membership functions [21], [23-25]. For δ order and r control points, the B-spline basis functions have the knot vector $\mathbf{T}=\{t_i, i=1,2,\ldots,r+\delta\}$ with $t_1 < t_2 < \cdots < t_{r+\delta}$. We choose that the order is three or above, and that the type of the knot vector is set to open uniform. An element t_i of the knot vector is defined as

$$t_i = \begin{cases} \bar{z}_1 & \text{if } i \le \delta \\ t_{i-1} + \dfrac{z_{\bar{n}} - z_1}{r-\delta+2} & \text{if } \delta < i \le r \\ \bar{z}_{\bar{n}} & \text{if } i > r \end{cases} \tag{1}$$

where $\bar{\mathbf{z}}=\{\bar{z}_q, q=1,2,\cdots,\bar{n}\,;\, \bar{n}>r\}$ is the data vector of input. For r control points, $\{c_1, c_2, \cdots, c_r\}$, the ith B-spline blending function of δ order is denoted by $N_{i,\delta}(\bar{z}_q)$. Hence, the B-spline curve $B(\bar{z}_q)$ is defined as follows:

$$B(\bar{z}_q) = \sum_{i=1}^{r} c_i N_{i,\delta}(\bar{z}_q), \qquad 1 \le \delta \le r \tag{2}$$

$$N_{i,1}(\bar{z}_q) = \begin{cases} 1, & \text{if } t_i \le \bar{z}_q < t_{i+1} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

and

$$N_{i,\delta}(\bar{z}_q) = \left(\frac{\bar{z}_q - t_i}{t_{i+\delta-1} - t_i}\right) N_{i,\delta-1}(\bar{z}_q) + \left(\frac{t_{i+\delta} - \bar{z}_q}{t_{i+\delta} - t_{i+1}}\right) N_{i+1,\delta-1}(\bar{z}_q). \tag{4}$$

In this paper, the B-spline membership function (BMF) $\mu_F(\bar{z}_q)$ introduced in [21], [23] is modified to satisfy the condition $0 \le \mu_F \le 1$, as follows:

$$\mu_F(\bar{z}_q) = S\Big(\sum_{i=1}^{r} c_i N_{i,\delta}(\bar{z}_q)\Big)$$

where F is a fuzzy set, and

$$S(\xi) = \begin{cases} 1, & \text{if } \xi > 1, \\ \xi, & \text{if } 0 \le \xi \le 1, \\ 0, & \text{if } \xi < 0. \end{cases} \tag{5}$$

We adopt BMFs as the fuzzy membership functions and use adaptive update laws (to be introduced in Section III) to obtain a set of adjusted control points for the BMFs.

B. *T-S Fuzzy Model for Nonaffine Nonlinear Systems*

Suppose that the nonaffine uncertain dynamic system is

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(\mathbf{x},u) + d_d \\ y &= x_1 \end{aligned} \tag{6}$$

where $\mathbf{x} = [x_1\ x_2 \cdots x_n]^T = [x\ \dot{x}\ \ddot{x} \ldots x^{(n-1)}]^T$ is a vector of states, $u \in R$ and $y \in R$ are the control input and system output, respectively, and d_d represents external disturbances. $f(\mathbf{x},u): R^{n+1} \to R$ is an uncertain function and smooth mapping defined on an open set of R^{n+1}.

Assumption 1: The nonlinear system (6) can be piece-wise linearized.

Using Taylor series expansion of the nonlinear system in (6) around $[\mathbf{x}_o, u_o]$, we have

$$\begin{aligned} \dot{\mathbf{x}}_\delta &= \mathbf{A}\mathbf{x}_\delta + \mathbf{B}u_\delta + \mathbf{b}_e d \\ d &= d_h + d_d + f(\mathbf{x}_0, u_0) \end{aligned} \tag{7}$$

where d_h stands for high order terms, u_o is an operating input, $u_\delta = u - u_o$ is an input deviation, $\mathbf{x}_o = [x_{o1}\ x_{o2} \ldots x_{on}]^T$ are operating states, $\mathbf{x}_\delta = \mathbf{x} - \mathbf{x}_o$ are state deviations, $\mathbf{b}_e = [0\ \ 0 \ldots\ 1]^T$,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 \\ a_1(\mathbf{x}_0,u_0) & a_2(\mathbf{x}_0,u_0) & a_3(\mathbf{x}_0,u_0) & \ldots & a_n(\mathbf{x}_0,u_0) \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix}0\\0\\0\\\vdots\\a_{n+1}(\mathbf{x}_0,u_0)\end{bmatrix}$$

$\mathbf{a}^T=[a_1\ a_2\ ...\ a_n]=[\partial f/\partial x_1\,|_{(\mathbf{x}_0,u_0)}\ \partial f/\partial x_2\,|_{(\mathbf{x}_0,u_0)}\ ...\ \partial f/\partial x_n\,|_{(\mathbf{x}_0,u_0)}]$, and $b=a_{n+1}=\partial f/\partial u\big|_{(\mathbf{x}_o,u_o)}$.

The T-S fuzzy model defined in [18] is

$$\begin{aligned}&R^{(i)}:\text{If } z_1 \text{ is } F_1^i \text{ and}\ldots\text{and } z_n \text{ is } F_n^i \text{ and } z_{n+1} \text{ is } F_{n+1}^i\\&\quad\text{Then } y=p_1^i z_1+p_2^i z_2+\ldots+p_{n+1}^i z_{n+1}\end{aligned}\tag{8}$$

where p_k^i, $(i=1,2,\ldots,h,\ k=1,2,\ldots,n+1,)$ are adjustable parameters. The T-S fuzzy model can be described by the fuzzy-neural network shown in Fig. 1. The output, p_k, of the fuzzy-neural network is

$$p_k=\sum\nolimits_{i=1}^{h}p_k^i\Big(\prod\nolimits_{j=1}^{n+1}\mu_{F_j^i}(z_j)\Big)\Big/\sum\nolimits_{i=1}^{h}\Big(\prod\nolimits_{j=1}^{n+1}\mu_{F_j^i}(z_j)\Big)\tag{9}$$

where $\mu_{F_j^i}(z_j)$ is the value of the membership function. p_k is used to approximate a_k of the linearized system (7).

In this paper, this so-called second type adaptive fuzzy approach is adopted. In this type of approach, not only the consequent part (the weighting factors) of fuzzy implications is adjusted, but also the antecedent part (the membership functions) of fuzzy implications is also adjusted. For this purpose, we define the adjusted vector $\mathbf{\Phi}_k$ $(k=1,2,\ldots,n+1)$:

$$\begin{aligned}\mathbf{\Phi}_k{}^T&=[\phi_k^1\ \ \phi_k^2\ldots\ \phi_k^{h(n+2)}]\\&=[p_k^1\ \ p_k^2\ldots\ p_k^h\ \ \mu_{F_1^1}\ldots\ \mu_{F_1^h}\ \ \mu_{F_2^1}\ldots\mu_{F_2^h}\ldots\mu_{F_{n+1}^1}\ldots\mu_{F_{n+1}^h}]\end{aligned}\tag{10}$$

where h is the number of fuzzy rules. In [24], the vector of the truth values w_k^i of the antecedent part of the ith implication is calculated by

$$\mathbf{w}_k^T=[w_k^1\ \ w_k^2\ \ \ldots w_k^h\ \ w_k^{h+1}\ldots w_k^{h(n+2)}]\tag{11}$$

where

$$w_k^i=\begin{cases}\dfrac{\prod_{j=1}^{n+1}\mu_{F_j^i}}{\varpi}, & \text{for } \phi_k^i,\ i=1,2,\ldots,h;\ k=1,2,\ldots,n+1\\[2ex]\dfrac{(p_k^{i-h}-p_k)\prod_{j=1}^{n+1}\mu_{F_j^{i-h}}}{\mu_{F_q^{i-h}}\varpi}, & \text{for } \phi_k^i,\ i=h+1,h+2,\ldots,2h;\ k=1,2,\ldots,n+1;\ q=1\\\vdots & \vdots\\\dfrac{(p_k^{i-(n+1)h}-p_k)\prod_{j=1}^{n+1}\mu_{F_j^{i-(n+1)h}}}{\mu_{F_q^{i-(n+1)h}}\varpi}, & \text{for } \phi_k^i,\ i=(n+1)h+1,\ldots,(n+2)h;\ k=1,2,\ldots,n+1;\ q=n+1\end{cases}$$

and $\varpi=\sum_{i=1}^{h}\prod_{j=1}^{n+1}\mu_{F_j^i}$. Note that, for $i=1,2,\ldots,h$, w_k^i is independent of k. Thus, we define

$$w_1^i=w_2^i=\cdots=w_{n+1}^i=w^i\equiv\prod\nolimits_{j=1}^{n+1}\mu_{F_j^i}\Big/\varpi\,.\tag{12}$$

The fact $p_k=\mathbf{w}_k^T\mathbf{\Phi}_k$ was proved in Theorem 1 of [22]. It is the goal of the fuzzy neural network in Fig. 1, that p_k is used to approximate a_k of the linearized system (7).

Assumption 2: The antecedent part of the fuzzy implication describes the conditions of the operation states $[\mathbf{x_o}^T,u_o]^T$.

Assumption 3: The consequent part of the fuzzy implication represents the linearization of the nonlinear system (6).

Based on the above assumptions, for the purpose of approximating the linearized system (7), the ith fuzzy implication (8) can be described as

$$\begin{aligned}&R^{(i)}:\text{If } x_{o1} \text{ is } F_1^i \text{ and}\ldots\text{and } x_{on} \text{ is } F_n^i \text{ and } u_o \text{ is } F_{n+1}^i\\&\quad\text{Then } \dot{x}_n=\hat{\mathbf{a}}^{iT}\mathbf{x}_\delta+\hat{b}^i u_\delta\end{aligned}\tag{13}$$

or

$$\begin{aligned}&R^{(i)}:\text{If } x_{o1} \text{ is } F_1^i \text{ and}\ldots\text{and } x_{on} \text{ is } F_n^i \text{ and } u_o \text{ is } F_{n+1}^i\\&\quad\text{Then } \dot{\mathbf{x}}=\hat{\mathbf{A}}^i\mathbf{x}_\delta+\hat{\mathbf{B}}^i u_\delta\end{aligned}$$

where $\hat{\mathbf{a}}^{iT}=[p_1^i\ p_2^i\ ...\ p_n^i]=[\phi_1^i\ \phi_2^i\ ...\ \phi_n^i]$, $\hat{b}^i=p_{n+1}^i=\phi_{n+1}^i$,

$$\hat{\mathbf{A}}^i=\begin{bmatrix}0&1&0&\ldots&0\\0&0&1&\ldots&0\\\ldots&\ldots&\ldots&\ldots&\ldots\\0&0&0&\ldots&1\\\phi_1^i&\phi_2^i&\phi_3^i&\ldots&\phi_n^i\end{bmatrix},\ \text{and}\ \ \hat{\mathbf{B}}^i=\begin{bmatrix}0\\0\\\vdots\\\phi_{n+1}^i\end{bmatrix}.$$

Among the commonly used defuzzification strategies, we can obtain

$$\begin{aligned}\dot{x}_n&=\sum_{i=1}^{h(n+2)}\{[w_1^i\phi_1^i\ w_2^i\phi_2^i\ \ldots\ w_n^i\phi_n^i]\mathbf{x}_\delta+w_{n+1}^i\phi_{n+1}^i u_\delta\}+\tilde{d}\\&=[p_1\ p_2\ \ldots\ p_n]\mathbf{x}_\delta+p_{n+1}u_\delta+\tilde{d}\end{aligned}\tag{14}$$

where p_k is used to approximate a_k of the linearized system (7) and the total error $\tilde{d}=d_h+d_d+f(\mathbf{x}_0,u_0)+d_f$, and d_f is the modeling error.

In fact,

$$\sum_{i=h+1}^{h(n+2)}w_k^i\phi_k^i=0,\ \text{for } k=1,2,\ldots,n+1.\tag{15}$$

Therefore, the T-S fuzzy model in (14) for the non-affine uncertain nonlinear system (1) can be rewritten as

$$\dot{\mathbf{x}}=\sum_{i=1}^{h}w^i\{\hat{\mathbf{A}}^i\mathbf{x}_\delta+\hat{\mathbf{B}}^i u_\delta\}+\mathbf{b}_e\tilde{d}\tag{16}$$

where w^i is defined in (12).

Remark 1: Although (15) is true, it is important to remember that w^i in (12) and (16) is a function of $\mu_{F_j^i}$. It is needed to adjust the membership function to increase the parameter search space. Therefore, the second type adaptive laws are derived from (14), instead of (16).

3. Controller Design for Online Modeling and Robust Tracking

To design a robust controller for (13), the following assumptions are required.

Assumption 4: Let $\mathbf{x_o}$ and u_o belong to compact sets $\mathbf{U_x}$ and $\mathbf{U}_u$, respectively, where

$$\mathbf{U_x} = \{\mathbf{x} \in R^n : \|\mathbf{x}\| \le m_{\mathbf{x}} < \infty\}$$

$$\mathbf{U}_u = \{u \in R : |u| \le m_u < \infty\}$$

and $m_{\mathbf{x}}$, and m_u are design parameters. It is known that the optimal adjustable vectors $\mathbf{\Phi}_k^*$, $k = 1,2,\ldots,n+1$, lie in some convex regions

$$\mathrm{M}_{\mathbf{\Phi}_k} = \{\mathbf{\Phi}_k \in R^{h(n+2)} : \|\mathbf{\Phi}_k\| \le m_{\mathbf{\Phi}_k}\},\quad k = 1,2,\ldots,n+1$$

where the radiuses m_{ϕ_k} are constant,

$$\mathbf{\Phi}_{k'}^* = \arg\min_{\phi_k \in \mathrm{M}_{\phi_k}} [\sup_{\mathbf{x_o}\in \mathbf{U}_x, u_o \in \mathbf{U}_u} |a_{k'}(\mathbf{x_o}, u_o) - \hat{a}_{k'}(\mathbf{x_o}, u_o | \phi_{k'})|]$$

$$k' = 1,2,\ldots,n$$

and

$$\phi_{n+1}^* = \arg\min_{\phi_{n+1}\in \mathrm{M}_{\phi_{n+1}}} [\sup_{\mathbf{x_o}\in \mathbf{U_x}, u_o \in \mathbf{U}_u} |b(\mathbf{x_o}, u_o) - \hat{b}(\mathbf{x_o}, u_o | \phi_{n+1})|].$$

Assumption 5: The adjustable vector ϕ_{n+1} is such that $\hat{b}$ is bounded away from zero.

According to assumption 4, we define the optimal adjustable matrices as

$$\hat{\mathbf{A}}^{i*} = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 \\ \phi_1^{i*} & \phi_2^{i*} & \phi_3^{i*} & \ldots & \phi_n^{i*} \end{bmatrix},\quad \hat{\mathbf{B}}^{i*} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \phi_{n+1}^{i*} \end{bmatrix}$$

$$\hat{\mathbf{a}}^{i*T} = [\phi_1^{i*}\ \ \phi_2^{i*}\ \ \ldots \phi_n^{i*}], \text{ and } \hat{b}^{i*} = \phi_{n+1}^{i*}.$$

Lemma 1 [29]: Suppose that a matrix $\mathbf{\Lambda} \in R^{n\times n}$ is given. For every symmetric positive definite matrix $\mathbf{Q} \in R^{n\times n}$, the Lyapunov matrix equation $\mathbf{\Lambda}^T\mathbf{\Gamma} + \mathbf{\Gamma}\mathbf{\Lambda} = -\mathbf{Q}$ has a unique solution for $\mathbf{\Gamma} = \mathbf{\Gamma}^T > 0$ if and only if $\mathbf{\Lambda} \in R^{n\times n}$ is a Hurwitz matrix.

Lemma 2 [30]: If both $\mathbf{e}(t)$ and $\dot{\mathbf{e}}(t) \in L_\infty^n$, and $\mathbf{e}(t) \in L_p^n$, for some $p \in [1,\infty)$, then $\lim_{t\to\infty}\|\mathbf{e}(t)\| = 0$.

We define the reference signal vector $\mathbf{r} = [r\ \dot{r}\ \ddot{r} \ldots r^{(n-1)}]^T$, so the error vector is $\mathbf{e} = \mathbf{x} - \mathbf{r}$. Let $\omega = r^{(n)} + \lambda_n(r^{(n-1)} - x^{(n-1)}) + \cdots + \lambda_1(r - x)$. Using (14), a fuzzy controller can be chosen as

$$u_\delta = \frac{x^{(n)} - \sum_{i=1}^{h(n+2)} [w_1^i\phi_1^i\ w_2^i\phi_2^i \ldots w_n^i\phi_n^i]\mathbf{x}_\delta - \tilde{d}}{\sum_{i=1}^{h(n+2)} w_{n+1}^i\phi_{n+1}^i}$$

$$\equiv \frac{-\sum_{i=1}^{h(n+2)} [w_1^i\phi_1^i\ w_2^i\phi_2^i \ldots w_n^i\phi_n^i]\mathbf{x}_\delta + \omega - u_s}{\sum_{i=1}^{h(n+2)} w_{n+1}^i\phi_{n+1}^i} \tag{17}$$

where u_s is an error compensator. Therefore, the error dynamic equation becomes:

$$\dot{\mathbf{e}} = \mathbf{\Lambda}\mathbf{e} + \mathbf{b}_e(\tilde{d} - u_s) \tag{18}$$

where

$$\mathbf{\Lambda} = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 1 \\ -\lambda_1 & -\lambda_2 & -\lambda_3 & \ldots & -\lambda_n \end{bmatrix}$$

and $\mathbf{\lambda} = [\lambda_1\ \ \lambda_2\ \cdots\ \lambda_n] \in R^n$ is a vector of the control parameters designed by the designer, and chosen such that the polynomial $e^{(n)} + \lambda_n e^{(n-1)} + \ldots + \lambda_1 e = 0$ is Hurwitz. From (17), we have

$$u = \frac{-\sum_{i=1}^{h(n+2)} [w_1^i\phi_1^i\ \ w_2^i\phi_2^i\ \ldots\ w_n^i\phi_n^i]\mathbf{x}_\delta + r^{(n)} - \mathbf{\lambda}^T\mathbf{e} - u_s}{\sum_{i=1}^{h(n+2)} w_{n+1}^i\phi_{n+1}^i} + u_o. \tag{19}$$

Now, using $e^{(n)} = \dot{x}_n - r^{(n)}$, and the actual linearized system (7), the error dynamic equation of the actual linearized system becomes

$$\dot{\mathbf{e}} = \mathbf{\Lambda}\mathbf{e} + b_e\{\sum_{i=1}^{h(n+2)} \{[w_1^i\tilde{\phi}_1^i\ w_2^i\tilde{\phi}_2^i \ldots w_n^i\tilde{\phi}_n^i]\mathbf{x}_\delta + w_{n+1}^i\tilde{\phi}_{n+1}^i u_\delta\} - u_s + \tilde{d}\} \tag{20}$$

where $\tilde{\phi}^i = \phi^{i*} - \phi^i$. We define the error compensator

$$u_s = \mathrm{sign}(\mathbf{e}_\Delta)\hat{d} \tag{21}$$

where $\mathbf{e}_\Delta \equiv \mathbf{e}^T\mathbf{\Gamma}\mathbf{b}_e$, $\mathbf{\Gamma} > 0$ is a Lyapunov matrix. The fuzzy implications are defined to obtain $\hat{d}$, which is the estimate of $\tilde{d}$, as follows:

$$R_d^{(i)}: \text{If } \mathbf{e}_\Delta \text{ is } F_{\mathbf{e}_\Delta}^i \text{ and } \|\mathbf{x}\| \text{ is } F_{\|\mathbf{x}\|}^i \text{ Then } \hat{d}^i = q^i. \tag{22}$$

Among the commonly used defuzzification strategies, we have

$$\hat{d} = \frac{\sum_{i=1}^{3h_{\hat{d}}} q^i(\mu_{F_{\mathbf{e}_\Delta}^i}(\mathbf{e}_\Delta)\mu_{F_{\|\mathbf{x}\|}^i}(\|\mathbf{x}\|))}{\sum_{i=1}^{3h_{\hat{d}}}(\mu_{F_{\mathbf{e}_\Delta}^i}(\mathbf{e}_\Delta)\mu_{F_{\|\mathbf{x}\|}^i}(\|\mathbf{x}\|))} = \sum_{i=1}^{3h_{\hat{d}}} \tau^i\varphi^i \tag{23}$$

where

$$\mathbf{\varphi}^T = [\varphi^1\ \ \varphi^2 \ldots \varphi^{3h_{\hat{d}}}]$$

$$= [q^1\ \ q^2 \ldots q^{h_{\hat{d}}}\ \ \mu_{F_{\mathbf{e}_\Delta}^1}\ \ \mu_{F_{\mathbf{e}_\Delta}^2} \cdots \mu_{F_{\mathbf{e}_\Delta}^{h_{\hat{d}}}}\ \ \mu_{F_{\|\mathbf{x}\|}^1}\ \ \mu_{F_{\|\mathbf{x}\|}^2} \cdots \mu_{F_{\|\mathbf{x}\|}^{h_{\hat{d}}}}]$$

and

$$\boldsymbol{\tau}^T = [\tau^1 \ \ \tau^2 \ldots \tau^{3h_{\hat{d}}}]$$
$$= [\frac{\mu_{F^1_{\mathbf{e}_\Delta}} \mu_{F^1_{\|\mathbf{x}\|}}}{\sum_{i=1}^{h_{\hat{d}}} (\mu_{F^i_{\mathbf{e}_\Delta}} \mu_{F^i_{\|\mathbf{x}\|}})} \cdots \frac{\mu_{F^{h_{\hat{d}}}_{\mathbf{e}_\Delta}} \mu_{F^{h_{\hat{d}}}_{\|\mathbf{x}\|}}}{\sum_{i=1}^{h_{\hat{d}}} (\mu_{F^i_{\mathbf{e}_\Delta}} \mu_{F^i_{\|\mathbf{x}\|}})} \ \frac{(q^1 - \hat{d})\mu_{F^1_{\|\mathbf{x}\|}}}{\sum_{i=1}^{h_{\hat{d}}} (\mu_{F^i_{\mathbf{e}_\Delta}} \mu_{F^i_{\|\mathbf{x}\|}})} \cdots \frac{(q^{h_{\hat{d}}} - \hat{d})\mu_{F^{h_{\hat{d}}}_{\|\mathbf{x}\|}}}{\sum_{i=1}^{h_{\hat{d}}} (\mu_{F^i_{\mathbf{e}_\Delta}} \mu_{F^i_{\|\mathbf{x}\|}})}$$
$$\frac{(q^1 - \hat{d})\mu_{F^1_{\mathbf{e}_\Delta}}}{\sum_{i=1}^{h_{\hat{d}}} (\mu_{F^i_{\mathbf{e}_\Delta}} \mu_{F^i_{\|\mathbf{x}\|}})} \cdots \frac{(q^{h_{\hat{d}}} - \hat{d})\mu_{F^{h_{\hat{d}}}_{\mathbf{e}_\Delta}}}{\sum_{i=1}^{h_{\hat{d}}} (\mu_{F^i_{\mathbf{e}_\Delta}} \mu_{F^i_{\|\mathbf{x}\|}})}]$$

We design the fuzzy error compensator using (23) as follows:

$$u_s = \text{sign}(\mathbf{e}_\Delta)\hat{d} = \text{sign}(\mathbf{e}_\Delta) \ \boldsymbol{\tau}^T \boldsymbol{\varphi} \,. \tag{24}$$

Assumption 6: $\|\tilde{d}\| \le \boldsymbol{\tau}^T \boldsymbol{\varphi}^*$, where $\boldsymbol{\varphi}^*$ is the optimal adjustable vector and $\boldsymbol{\varphi}$ represents the estimate of $\boldsymbol{\varphi}^*$. On the basis of the above discussion, the following theorem can be obtained.

Theorem 1: Consider the nonlinear system (1) that satisfies Assumptions 1 and 3-6. If the controller is designed as (19) with update laws

$$\dot{\phi}^i_k = \eta w^i_k x_{k\delta} \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_\mathbf{e} \tag{25}$$

$$\dot{\phi}^i_{n+1} = \eta w^i_{n+1} u_\delta \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \,, \ \ i = 1, 2, ..., h(n+2) \,, \ \ k = 1, 2, \ldots, n \tag{26}$$

and

$$\dot{\varphi}^i = \gamma \tau^i \left\| \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \right\| , \ \ i = 1, 2, \cdots 3h_{\hat{d}} \tag{27}$$

where η and γ are positive constants, then the closed-loop system is robust stable and $\lim_{t\to\infty} \|\mathbf{e}(t)\| = 0$.

Proof:

Consider the Lyapunov-like function candidate

$$v = \frac{1}{2}\mathbf{e}^T \boldsymbol{\Gamma} \mathbf{e} + \frac{1}{2\eta} \sum_{i=1}^{h(n+2)} \tilde{\mathbf{a}}^{iT} \tilde{\mathbf{a}}^i + \frac{1}{2\eta} \sum_{i=1}^{h(n+2)} (\tilde{b}^i)^2 + \frac{1}{2\gamma} \tilde{\boldsymbol{\varphi}}^T \dot{\tilde{\boldsymbol{\varphi}}} \tag{28}$$

where $\tilde{\varphi} = \varphi^* - \varphi$. Differentiating (28) with respect to time, we get

$$\begin{aligned} \dot{v} &= \frac{1}{2}\dot{\mathbf{e}}^T \boldsymbol{\Gamma} \mathbf{e} + \frac{1}{2}\mathbf{e}^T \boldsymbol{\Gamma} \dot{\mathbf{e}} \\ &- \frac{1}{2\eta} \sum_{i=1}^{h(n+2)} (\dot{\tilde{\mathbf{a}}}^{iT} \tilde{\mathbf{a}}^i + \tilde{\mathbf{a}}^{iT} \dot{\tilde{\mathbf{a}}}^i) - \frac{1}{\eta} \sum_{i=1}^{h(n+2)} \tilde{b}^i \dot{\tilde{b}}^i - \frac{1}{\gamma} \tilde{\boldsymbol{\varphi}}^T \dot{\boldsymbol{\varphi}} \,. \end{aligned} \tag{29}$$

Inserting (20) in the above equation yields

$$\begin{aligned} \dot{v} &= \frac{1}{2}\mathbf{e}^T (\boldsymbol{\Lambda}^T \boldsymbol{\Gamma} + \boldsymbol{\Gamma} \boldsymbol{\Lambda}) \mathbf{e} \\ &+ \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e (\sum_{i=1}^{h(n+2)} [w^i_1 \tilde{\phi}^i_1 \ \ w^i_2 \tilde{\phi}^i_2 \ \ \ldots \ \ w^i_n \tilde{\phi}^i_n] \mathbf{x}_\delta + \sum_{i=1}^{h(n+2)} w^i_{n+1} \tilde{\phi}^i_{n+1} u_\delta) \\ &+ \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \tilde{d} - \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e u_s - \sum_{i=1}^{h(n+2)} \frac{\dot{\tilde{\mathbf{a}}}^{iT} \tilde{\mathbf{a}}^i}{\eta} - \sum_{i=1}^{h(n+2)} \frac{\tilde{b}^i \dot{\tilde{b}}^i}{\eta} - \frac{1}{\gamma} \tilde{\boldsymbol{\varphi}}^T \dot{\boldsymbol{\varphi}} \,. \end{aligned} \tag{30}$$

From Lemma 1, substituting $\boldsymbol{\Lambda}^T \boldsymbol{\Gamma} + \boldsymbol{\Gamma} \boldsymbol{\Lambda} = -\mathbf{Q}$ for (30), we have

$$\begin{aligned} \dot{v} &= -\frac{1}{2}\mathbf{e}^T \mathbf{Q} \mathbf{e} + \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \sum_{i=1}^{h(n+2)} w^i_k \mathbf{x}^T_\delta \tilde{\mathbf{a}}^i + \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \sum_{i=1}^{h(n+2)} w^i_{n+1} \tilde{\phi}^i_{n+1} u_\delta \\ &- \sum_{i=1}^{h(n+2)} \frac{\dot{\tilde{\mathbf{a}}}^{iT} \tilde{\mathbf{a}}^i}{\eta} - \sum_{i=1}^{h(n+2)} \frac{\dot{\tilde{b}}^i \tilde{b}^i}{\eta} + \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \tilde{d} - \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e u_s - \frac{1}{\gamma} \tilde{\boldsymbol{\varphi}}^T \dot{\boldsymbol{\varphi}} \end{aligned}$$

and

$$\dot{v} \le \Delta + \left\| \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \right\| \left\| \tilde{d} \right\| - \left\| \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \right\| \hat{d} - \frac{1}{\gamma} \tilde{\boldsymbol{\varphi}}^T \dot{\boldsymbol{\varphi}} \tag{31}$$

where

$$\begin{aligned} \Delta &= -\frac{1}{2}\mathbf{e}^T \mathbf{Q} \mathbf{e}^T + \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \sum_{i=1}^{h(n+2)} \mathbf{x}^T_\delta [w^i_1 \tilde{\phi}^i_1 \ \ w^i_2 \tilde{\phi}^i_2 \ \ \ldots \ \ w^i_n \tilde{\phi}^i_n]^T \\ &- \sum_{i=1}^{h(n+2)} \frac{\dot{\tilde{\mathbf{a}}}^{iT} \tilde{\mathbf{a}}^i}{\eta} + \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \sum_{i=1}^{h(n+2)} w^i_{n+1} \tilde{\phi}^i_{n+1} u_\delta - \sum_{i=1}^{h(n+2)} \frac{\dot{\tilde{b}}^i \tilde{b}^i}{\eta} \,. \end{aligned} \tag{32}$$

Substituting (23) for (31) and based on Assumption 4, we find

$$\begin{aligned} \dot{v} &\le \Delta + \left\| \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \right\| \boldsymbol{\tau}^T \boldsymbol{\varphi}^* - \left\| \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \right\| \boldsymbol{\tau}^T \boldsymbol{\varphi} - \frac{1}{\gamma} \tilde{\boldsymbol{\varphi}}^T \dot{\boldsymbol{\varphi}} \\ &= \Delta + (\gamma \left\| \mathbf{e}^T \boldsymbol{\Gamma} \mathbf{b}_e \right\| \boldsymbol{\tau}^T - \dot{\boldsymbol{\varphi}}) \frac{1}{\gamma} \tilde{\boldsymbol{\varphi}} \,. \end{aligned} \tag{33}$$

If we select the adaptive laws in (25)-(27), then

$$\dot{v} = -\frac{1}{2}\mathbf{e}^T \mathbf{Q} \mathbf{e} \le 0 \,. \tag{34}$$

Equations (28) and (34) only guarantee that $\mathbf{e}(t) \in L_\infty$, and $\varphi^i \in L_\infty$, $i = 1, 2, ..., 3h_{\hat{d}}$, but not converged. The boundedness of $\mathbf{e}(t)$ implies the boundedness of $\mathbf{x}(t)$. Since the operating states are finite, $\mathbf{x}_\delta$ is bounded. Based on Assumption 1, the boundedness of $\mathbf{x}_\delta$ and φ^i, $i = 1, 2, ..., 3h_{\hat{d}}$, u_δ is bounded. Therefore, $\dot{\mathbf{e}}(t)$ is bounded, i.e., $\dot{\mathbf{e}}(t) \in L_\infty$. Integrating both side of (34) yields

$$v(t) - v(0) \le -\frac{1}{2}\lambda_{\min}(\mathbf{Q}) \int_0^t \left\| \mathbf{e}(\tau) \right\|^2 d\tau \tag{35}$$

where $\lambda_{\min}(\mathbf{Q}) > 0$ is the minimum eigenvalue of $\mathbf{Q}$.

When t tends to infinity, (35) becomes

$$\int_0^\infty \left\| \mathbf{e}(\tau) \right\|^2 d\tau \le \frac{v(0) - v(\infty)}{\frac{1}{2}\lambda_{\min}(\mathbf{Q})} \tag{36}$$

Since the right side of (36) is bounded, $\mathbf{e} \in L_2$. Therefore, by using Lemma 2, $\|\mathbf{e}(t)\| \to 0$ as $t \to \infty$. This completes the proof.

4. Simulation Results

This section presents the simulation results of the proposed controller to illustrate that the tracking error of the closed-loop system can be made arbitrarily small. The adjustable parameters include not only the weighting factors in the consequent part, but also the membership functions in the antecedent part of the fuzzy rule. The simulation results confirm that all the unmodeled

T-S fuzzy system dynamics, modeling errors and external disturbances on the tracking error are attenuated efficiently by the proposed controller.

Example 1: Consider a nonaffine system [28]:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = 0.2(1+e^{x_1x_2})(2+\sin(x_2))(u+e^u-1)+d_d$$
$$y = x_1$$

where u is the control input, and d_d is the external disturbances and is assumed to be a square wave with the amplitude ± 0.1 and the period 2π.

Five fuzzy sets over the interval $[-6, 6]$ are defined for $\mathbf{x}_o = [x_{o1}, x_{o2}]^T = [0,0]^T$ with the term sets (PB, PS, Z, NS, NB) and $[0, 6]$ for $\|\mathbf{x}\|$ with the term set (PB, PS, Z, NS, PS). Similarly, five fuzzy sets over the interval $[-48,48]$ are defined for $\boldsymbol{e}_\Delta$ with the term (PB, PS, Z, NS, NB). We also define three fuzzy sets over the interval $[-10,10]$ for $u_o = 0$. The design parameters are selected as $\eta = 0.01$, $\gamma = 0.008$, $\lambda_1 = 1$, $\lambda_2 = 2$, and $\boldsymbol{Q} = diag[10,10]$. The initial states of system are assumed to be $\mathbf{x} = [0.2, 0]^T$. For solving (25)-(27), we select the initial value of the vectors $\dot{x}_n$, $k = 1,2,3\ldots,n+1$, randomly in the interval $[-0.1, 0.1]$ and φ randomly in the interval $[-0.25, 0.25]$.

We use the proposed control law in (19) to control the state x_1 of the system to track the reference signal r=1.5–e$^{-0.5t}$ (case 1 of example 1 in [28]) and $r = \sin(0.5t) + \cos(t)$ (case 2 of example 1 in [28]). Figures 2 and 3 (case 1) and Figs. 4 and 5 (case 2) show that the curves of the states x_1 and x_2 of the closed-loop system, respectively. The simulation results in Figs. 2-5 indicate that the effect of all the unmodeled dynamics, BMF modeling errors and external disturbances on the tracking error is attenuated efficiently by the proposed controller.

Example 2: Consider the Duffing forced oscillation system:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos t + u + d_d$$

where u is the control input, and d_d is the external disturbances and is assumed to be a square wave with the amplitude ± 0.1 and the period 2π.

Five fuzzy sets over the interval [-6, 6] are defined for $\mathbf{x}_o = [x_{o1}, x_{o2}]^T = [0,0]^T$ with the term set (PB, PS, Z, NS, NB) and [0,6] for $\|\mathbf{x}\|$ with the term set (PB, PS, Z, NS, NB). Similarly, five fuzzy sets over the interval [-48, 48] are defined for $\boldsymbol{e}_\Delta$ with the term set (PB, PS, Z, NS, NB). Three fuzzy sets over the interval [-1400, 1400] are defined for $u_o = 0$. In this case, the design parameters are selected as η=0.5, γ=0.3, λ_1=1, λ_2=2 and $\boldsymbol{Q}$=*diag*[10,10]. The initial states of the system are assumed to be $\mathbf{x}$=[0.3, 0]^T. For solving (25)-(27), we select the initial value of the vectors ϕ_k, k=1,2,3, …, n+1 and φ randomly in the interval [-10, 10].

We use the proposed control law in (19) to control the state x_1 of the system to track the reference signal r=sin$t$. Fig. 6 and Fig. 7 show that the curves of the states x_1 and x_2 of the closed system, respectively. The response of control input u is shown in Fig. 8. The simulation results in Figs. 6-7 indicate that the effect of all the unmodeled dynamics, BMF modeling errors and external disturbances on the tracking error is attenuated efficiently by the proposed controller.

Example 3: We apply the robust adaptive fuzzy controller proposed in section III to the inverted pendulum stabilization problem. Let x_1 be the angle of the pendulum with respect to the vertical line. The control objective is to track the reference trajectory $r = \frac{\pi}{30}\sin t$. The dynamic equations of the inverted pendulum system are

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{mlx_2^2 \sin x_1 \cos x_1 - (M+m)g\sin x_1 - u\cos x_1}{ml\cos^2 x_1 - \frac{4}{3}l(M+m)} + d_d \end{bmatrix}$$

where M is the mass of the cart, m is the mass of the rod, $g = 9.8\, {}^{m}\!/\!_{sec^2}$ is the acceleration due to gravity, l is the half length of the rod, u is the control input, and d_d is the external disturbance and is assumed to be a square wave with the amplitude ± 0.05 and the period 2π. In this example, we assume that $M = 10kg$, $m = 1kg$, and $l = 3m$.

Five fuzzy sets over the interval $[-\frac{3}{12}\pi, \frac{3}{12}\pi]$ are defined for $\mathbf{x}_o = [x_{o1}, x_{o2}]^T = [x_1(t) \quad x_2(t)]^T$ with the term set (PB, PS, Z, NS, NB) and $[0, \frac{\pi}{3}]$ for $\|\mathbf{x}\|$ with the term set (PB, PS Z, NS, NB). Similarly, five fuzzy sets over the interval [-12, 12] are defined for $\boldsymbol{e}_\Delta$ with the term set (PB, PS, Z, NS, NB). We also define three fuzzy sets over the interval [-1000, 1000] for $u_o = u(t)/2$. In this case, the design parameters are selected as η=0.006, γ=1.6, λ_1=1, λ_2=2 and $\boldsymbol{Q}$=*diag*[10,10]. The initial states of the system are assumed to be $\mathbf{x}$=[$\pi/30$,0]^T. For solving (25)-(27), we select the initial value of the vectors ϕ_k, k=1,2,3, …, n+1 and φ randomly in the interval [-0.01,0.01].

We use the proposed control law in (19) to control the state x_1 of the system to track the reference signal r. Figs. 9 and 10 show that the curves of the states x_1 and x_2 of the closed system, respectively. The simulation results in Figs. 9 and 10 indicate that the effect of all the unmodeled dynamics, BMF modeling errors and external disturbances on the tracking error is attenuated efficiently by the proposed controller.

5. Conclusion

We propose a novel approach of on-line T-S fuzzy modeling and robust adaptive control for uncertain nonlinear systems to achieve the attenuation of the unmodeled dynamics, BMF modeling errors and external disturbances. The update laws (25)-(27) are used to tune the parameters including not only the weighting factors in the consequence part but also the membership functions (BMF's) in the antecedent part of the fuzzy implications. The parameters of the fuzzy model are tuned through update laws and the unknown nonlinear system is assumed to be linearizable and can be approximated to any degree of accuracy by the T-S fuzzy model. The robust control term u_s is used to attenuate the unmodeled dynamics, BMF modeling errors and external disturbances on the tracking error. In Theorem 1, we prove that although the bound of unmodeled dynamics, BMF modeling errors and external disturbances is unknown, the tracking error of the closed-loop system can be made arbitrarily small. Simulation results support the theoretical arguments on the T-S fuzzy modeling and tracking performance of the design algorithms under the adaptive tuning methods.

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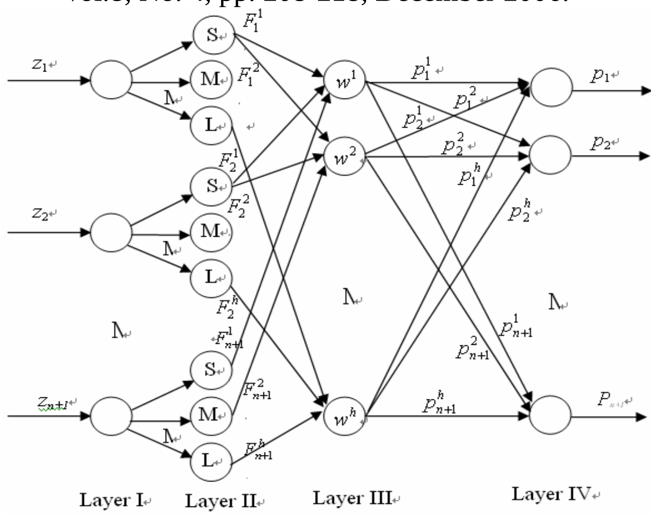

Fig. 1. Configuration of the fuzzy neural network.

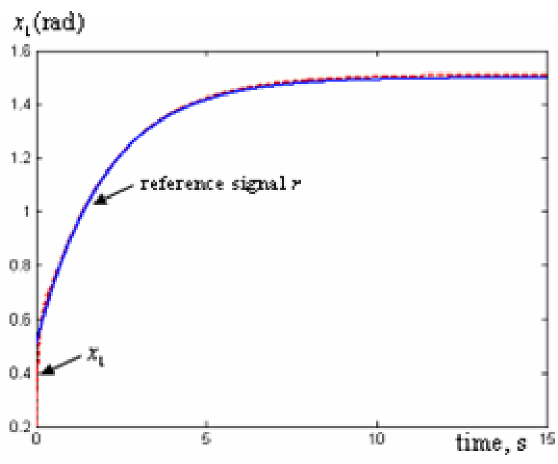

Fig. 2. Curve of x_1 of tracking control (case 1) in example1.

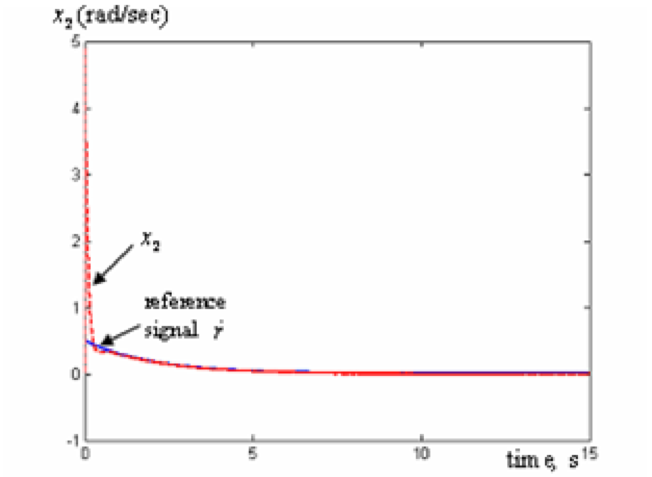

Fig. 3. Curve of x_2 of tracking control (case 1) in example1.

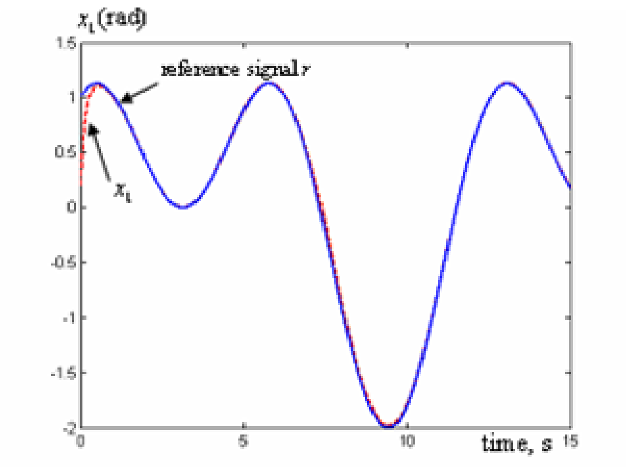

Fig. 4. Curve of x_1 of tracking control (case 2) in example1.

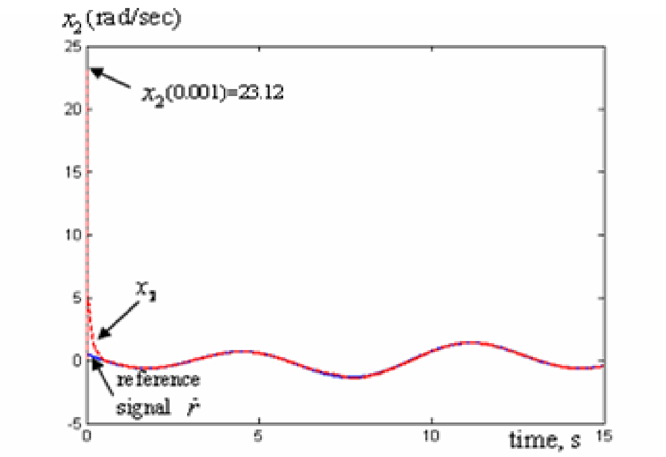

Fig. 5. Curve of x_2 of tracking control (case 2) in example1.

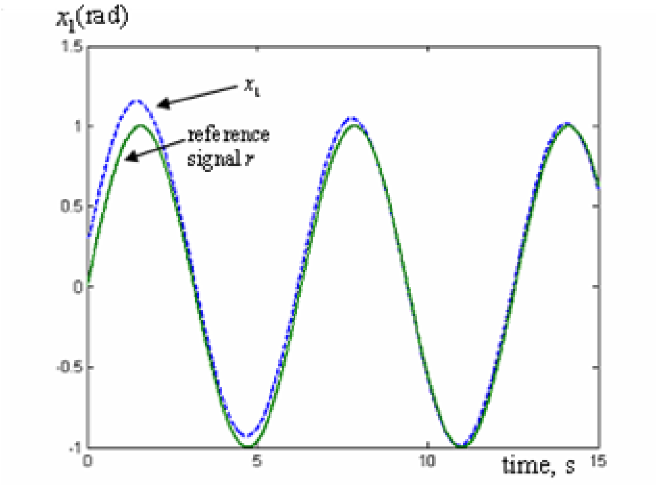

Fig. 6. Curve of x_1 of tracking control in example2.

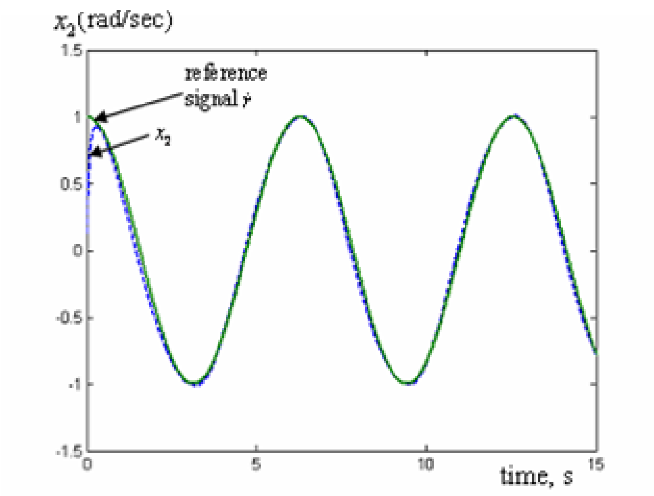

Fig. 7. Curve of x_2 of tracking control in example2.

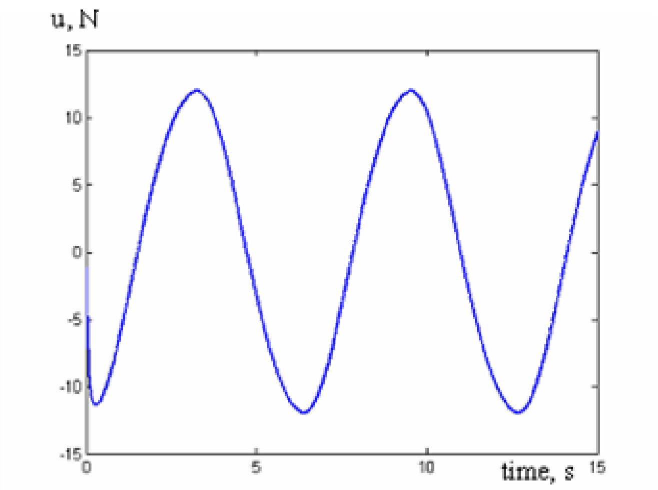

Fig. 8. Response of control input *u* in example2 (0-15 s).

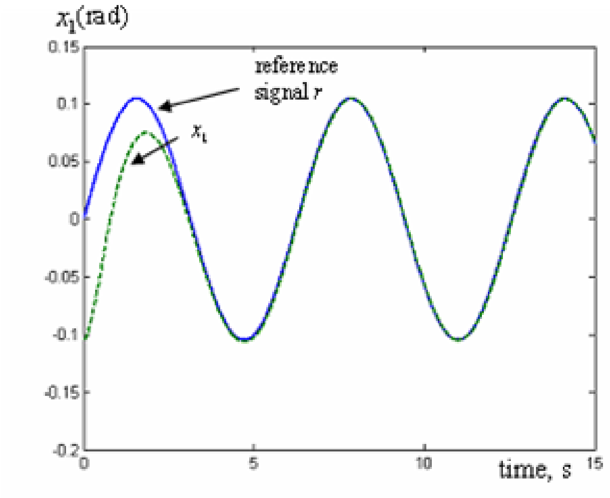

Fig. 9. Curve of x_1 of tracking control in example3.

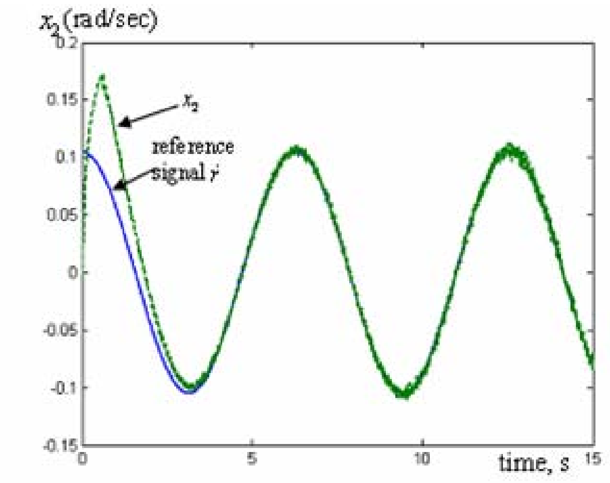

Fig. 10. Curve of x_2 of tracking control in example3.

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