



Thermal diffusion in a countercurrent-flow Frazier scheme inclined for improved performance

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Abstract

The separation theory of thermal diffusion in inclined flat-plate columns of countercurrent-flow Frazier scheme is developed and investigated. The equations for the optimum angles of inclination as well as for maximum separation, maximum output and minimum column length, are derived. The author has previously shown that considerable improvement in performance is obtained when a concurrent Frazier scheme is operated at the optimum angles of inclination: it is shown here that further improvement is achieved if the feed streams are conducted in countercurrent flow. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Thermal diffusion; Frazier scheme; Inclination; Countercurrent flow

1. Introduction

The phenomenon of thermal diffusion consists of the fact that a temperature gradient in a binary mixture of fluid gives rise to a gradient of the relative concentration of the two constituents. It was first discovered theoretically by Enskog (1911), and independently by Chapman six years later, and first demonstrated experimentally by Chapman and Dootson (1917). If the mixture as a whole is at rest, the equilibrium concentration gradient is such that the effect of thermal diffusion is just counterbalanced by the opposite effect of ordinary diffusion. The separation of a mixture obtainable from such a static thermal-diffusion method is generally so small that it is of theoretical interest only.

It was the great achievement of Clusius and Dickel (1938) to point out that in addition to the effect of thermal diffusion in the horizontal direction, application of temperature gradient to the mixture of fluid in the horizontal direction also creates convective currents, flowing up in the hot region and down at the cold part, which could be utilized to produce a cascading effect analogous to multistage processes, yielding relatively large separation. In industrial applications, the Clusius–

Dickel columns are connected in series such as that shown in Fig. 1, called the Frazier scheme (Frazier, 1962; Grasselli & Frazier, 1962). The first complete presentation of the theory of the thermal diffusion in a thermogravitational thermal-diffusion column (C-D column) was prepared by Furry et al. (Furry, Jones, & Onsager (1939) and Jones & Furry (1946)), while that in the Frazier scheme was given by Rabinovich (1976), Sovorov and Rabinovich (1981).

Actually, the convective currents in C-D columns have two conflicting effects: the desirable cascading effect and the undesirable remixing effect. The remixing effect is due to the fact that the convective currents bring down the fluid at the top of the column, where it is rich in one component, to the bottom of the column, where it is rich in the other component, and vice versa. It appears, therefore, that proper control of the convective strength might effectively suppress this undesirable remixing effect while still preserving the desirable cascading effect and thereby lead to improved separation.

Powers and Wilke (1957) first demonstrated that the strength of convective currents, as well as the undesirable remixing effect in a flat-plate thermogravitational thermal-diffusion column, could be effectively reduced and controlled by tilting the column, resulting in substantial improvement of separation efficiency. The equations for the optimal angle of inclination and the best performance were later obtained by Chueh and Yeh (1967), and by

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Yeh and Ward (1971), and their theories were in good agreement with the experimental results.

The separation theory of thermal diffusion in inclined flat-plate columns of concurrent-flow Frazier scheme has been developed (Yeh, 1994a). The equations for the best angles of inclination, as well as for maximum separation, maximum production rate and minimum column length, have been derived (Yeh, 1996). Considerable improvement in performance is obtained when a concurrent-flow Frazier scheme is operated at the best corresponding angles of inclination, especially for the schemes of large column number. It is the purpose of the present work to develop and investigate the separation theory of thermal diffusion in inclined flat-plate columns of the Frazier scheme under countercurrent-flow operations, instead of operating under concurrent flow.

2. Analysis

2.1. Equations of separation

Fig. 1 is the schematic diagram of an inclined Frazier scheme operating under countercurrent flow, while Fig. 2 illustrates the flows and fluxes prevailing in the $(i + 1)$ th thermogravitational thermal-diffusion column of the Frazier scheme. The transport equation for thermal diffusion in each one of such columns is (Yeh, 1994a)

$$\tau_{i+1} = C(1 - C)H \cos \theta - K \cos^2 \theta \frac{dC_{i+1}}{dz}, \quad (1)$$

where

$$H = \frac{\alpha \rho \beta g (2\omega)^3 B (\Delta T)^2}{6! \mu T_m}, \quad (2)$$

$$K = \frac{\rho \beta^2 g^2 (2\omega)^7 B (\Delta T)^2}{9! D \mu^2}. \quad (3)$$

In obtaining the above equations, since the degree of separation obtained in a thermal-diffusion column is generally small, the product of the concentration terms, $C(1 - C)$, is taken as constant, i.e.,

$$C(1 - C) = A(\text{constant}). \quad (4)$$

Making material balances for the top and the bottom of the column, one obtains, respectively

$$\begin{aligned} \tau_{i+1} &= AH \cos \theta - K \cos^2 \theta \frac{dC_{i+1}}{dz} \Big|_{z=L} \\ &= \sigma(C_{T,i+1} - C_{T,i}), \end{aligned} \quad (5)$$

$$\begin{aligned} \tau_{i+1} &= AH \cos \theta - K \cos^2 \theta \frac{dC_{i+1}}{dz} \Big|_{z=0} \\ &= \sigma(C_{B,i+2} - C_{B,i+1}). \end{aligned} \quad (6)$$

Further, if a material balance is taken for the whole $(i + 1)$ th column, the result is

$$C_{T,i+1} - C_{T,i} = C_{B,i+2} - C_{B,i+1}. \quad (7)$$

Combination of Eqs. (5)–(7) results in

$$\frac{dC_{i+1}}{dz} \Big|_{z=L} = \frac{dC_{i+1}}{dz} \Big|_{z=0} = \frac{dC_{i+1}}{dz} = \text{constant}. \quad (8)$$

By integrating Eq. (5) through the $(i + 1)$ th column from $z = 0$ ($C_{i+1} = C_{B,i+1}$) to $z = L$ ($C_{i+1} = C_{T,i+1}$), the following equation is obtained:

$$\begin{aligned} C_{T,i+1} - C_{B,i+1} &= [AH \cos \theta - \sigma(C_{T,i+1} - C_{T,i})]L/K \cos^2 \theta \end{aligned} \quad (9)$$

or

$$\begin{aligned} C_{B,i+1} &= C_{T,i+1} \\ &\quad - [AH \cos \theta - \sigma(C_{T,i+1} - C_{T,i})]L/K \cos^2 \theta. \end{aligned} \quad (10)$$

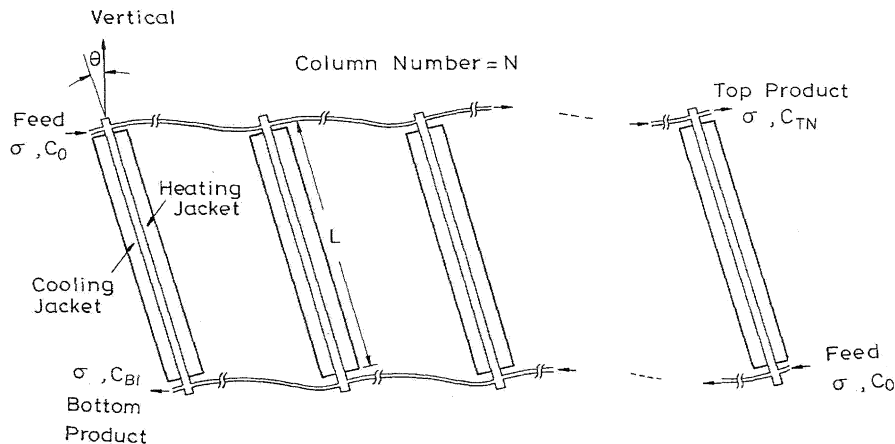


Fig. 1. Schematic diagram of an inclined countercurrent-flow Frazier scheme.

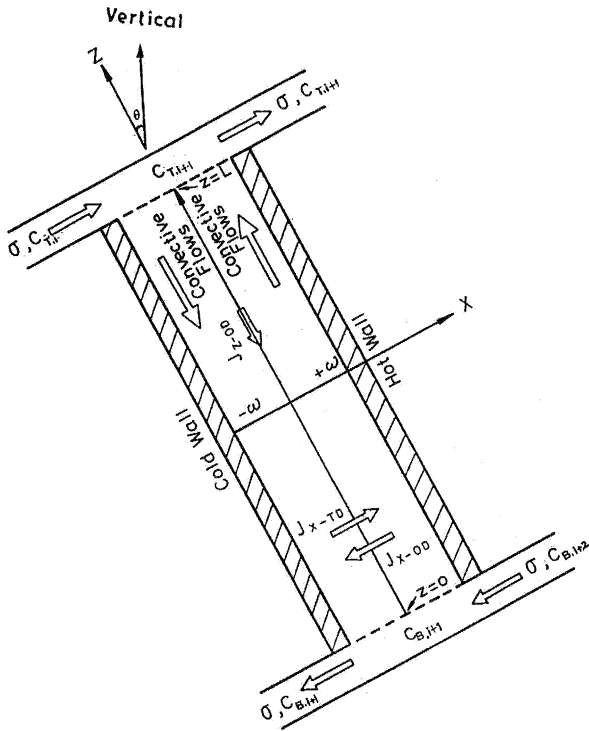


Fig. 2. Flows and fluxes prevailing in the $(i + 1)$ th thermal diffusion column of the Frazier scheme.

If i is replaced by $(i + 1)$, Eq. (10) becomes

$$C_{B,i+2} = C_{T,i+2} - [AH \cos \theta - \sigma(C_{T,i+2} - C_{T,i+1})]L/K \cos^2 \theta. \quad (11)$$

Substitution of Eqs. (10) and (11) into Eq. (7) yields a second-order difference equation as

$$(C_{T,i+2} - 2C_{T,i+1} + C_{T,i})(1 + \sigma L/K \cos^2 \theta) = 0. \quad (12')$$

Since $(1 + \sigma L/K \cos \theta) \neq 0$, the above equation reduces to

$$C_{T,i+2} - 2C_{T,i+1} + C_{T,i} = 0. \quad (12)$$

The general solution is

$$C_{T,i} = a_1 + a_2 i, \quad 0 \leq i \leq N + 1 \quad (13)$$

which, after applying the inlet condition at the top:

$$i = 0, \quad C_{T,i} = C_0 \quad (14)$$

becomes

$$C_{T,i} = C_0 + a_2 i. \quad (15)$$

Substitution of Eq. (15) into Eq. (10) with i in Eq. (10) replaced by $(i - 1)$ results in

$$C_{B,i} = C_0 - (AHL/K \cos \theta) + a_2[(\sigma L/K \cos^2 \theta) + i]. \quad (16)$$

Again, applying the inlet condition at the bottom,

$$i = N + 1, \quad C_{B,i} = C_0, \quad (17)$$

one obtains from Eq. (16)

$$a_2 = \frac{AHL/K \cos \theta}{N + 1 + (\sigma L/K \cos^2 \theta)}. \quad (18)$$

Finally, the equation for the degree of separation Δ is readily obtained from Eqs. (15) and (16) with the use of Eq. (18):

$$\Delta = C_{T,N} - C_{B,1} = \frac{2ALNH \cos \theta}{(N + 1)K \cos^2 \theta + \sigma L}. \quad (19)$$

In obtaining the above solution, the assumptions are made that dimensions B , L and ω of the columns are the same, and that all mass flow rates are the same as σ .

Furry et al. (1939) suggested that the constant A in Eq. (19) may be taken as 0.25 for the equifraction solution ($0.3 < C < 0.7$). Thus, the equation of separation for the equifraction solution is

$$\bar{\Delta} = \frac{(LNH/2) \cos \theta}{(N + 1)K \cos^2 \theta + \sigma L} \quad (20)$$

and Eq. (19) may be rewritten as

$$\Delta = 4A\bar{\Delta}. \quad (21)$$

For the whole range of concentration ($0 < C < 1$), the appropriate choice of constant A was determined by the method of least squares, and the result is (Yeh & Yeh, 1982; Yeh, 1994a)

$$A = C_0(1 - C_0) - \Delta^2/12. \quad (22)$$

The explicit form of separation equation for whole range of concentration is then obtained by substituting Eq. (22) into Eq. (21) and rearranging:

$$\Delta = \left[\left(\frac{1.5}{\bar{\Delta}} \right)^2 + 12C_0(1 - C_0) \right]^{1/2} - \frac{1.5}{\bar{\Delta}}. \quad (23)$$

2.2. Maximum separation

The optimum angle of inclination θ_A for maximum separation Δ_{\max} is obtained by partially differentiating Eq. (23) with respect to θ and setting $\partial \Delta / \partial \theta = 0$. It is known that this also leads to $\partial \bar{\Delta} / \partial \theta = 0$ (Yeh, 1994a,b). In other words, the optimum angle of inclination is independent of the solution concentration. After differentiating Eq. (20) with respect to θ and setting $\partial \bar{\Delta} / \partial \theta = 0$, one obtains the equations for calculating the optimum angle of inclination and maximum separation as

$$\theta_A = \cos^{-1} \left[\frac{\sigma L}{K(N + 1)} \right]^{1/2}, \quad (24)$$

$$\Delta_{\max} = \left[\left(\frac{1.5}{\bar{\Delta}_{\max}} \right)^2 + 12C_0(1 - C_0) \right] - \frac{1.5}{\bar{\Delta}_{\max}}, \quad (25)$$

where

$$\bar{A}_{\max} = \left(\frac{H^2 L}{16\sigma K} \right)^{1/2} \left[\frac{N}{(N+1)^{1/2}} \right]. \quad (26)$$

A condition for the existence of θ_A is determined from Eq. (24) by setting $\cos \theta_A < 1$; the result is

$$\sigma < (K/L)(N+1). \quad (27)$$

2.3. Maximum output

The equation of output can be derived by rewriting Eq. (20) as

$$\sigma = (HN/2\bar{A})\cos\theta - (N+1)K\cos^2\theta/L, \quad (28)$$

where \bar{A} can be calculated from Eq. (23) with known Δ , i.e.,

$$\bar{A} = \frac{3\Delta}{12C_o(1-C_o) - \Delta^2}. \quad (29)$$

The optimum angle of inclination θ_σ required to obtain the maximum production rate σ_{\max} for a given scheme which is to give a specified degree of separation Δ , is ready to obtain by partially differentiating Eq. (28) with respect to θ and setting $\partial\sigma/\partial\theta = 0$. The results are

$$\theta_\sigma = \cos^{-1} \left[\left(\frac{HL}{4K\bar{A}} \right) \left(\frac{N}{N+1} \right) \right], \quad (30)$$

$$\sigma_{\max} = \left(\frac{H^2 L}{16K\bar{A}^2} \right) \left(\frac{N^2}{N+1} \right) \quad (31)$$

and the restriction on the existence of θ_σ is

$$\left(\frac{HL}{4K\bar{A}} \right) \left(\frac{N}{N+1} \right) < 1. \quad (32)$$

2.4. Minimum column length

Eq. (20) can be rewritten to obtain the expression for column length as

$$L = \frac{(N+1)K\cos^2\theta}{[(NH/2\bar{A})\cos\theta - \sigma]}. \quad (33)$$

To find the minimum column length L_{\min} required to accomplish a specified degree of separation Δ (or \bar{A}) and production rate σ , we minimize L from Eq. (33) with respect to θ . The results are

$$\theta_L = \cos^{-1}(4\sigma\bar{A}/HN), \quad (34)$$

$$L_{\min} = \left(\frac{16\sigma\bar{A}^2 K}{H^2} \right) \left(\frac{N+1}{N^2} \right). \quad (35)$$

Note that the restriction on the existence of θ_L is, from Eq. (34):

$$\sigma < HN/4\bar{A}. \quad (36)$$

3. The improvement in performance

The improvement in performance resulting from operating at the optimum angles of inclination may be illustrated numerically by using the experimental data of Chueh and Yeh (1967). The conditions are: benzene and *n*-heptane system; $\Delta T = 69^\circ\text{F} = 38.3^\circ\text{C}$; L (or L_0 for $\theta = 0$) = 185 cm; $B = 10$ cm; $(2\omega) = 0.09$ cm; $H = 0.845$ g/min; $K = 419$ g cm/min.

With these numerical values, the maximum separation Δ_{\max} , the maximum output σ_{\max} and the minimum column length L_{\min} in inclined countercurrent-flow Frazier schemes were calculated from the appropriate equations, and the results were compared with those obtained in countercurrent-flow Frazier schemes without inclination ($\theta = 0$), by the following definitions of improvement:

$$I_A = \frac{\Delta_{\max} - \Delta_0}{\Delta_0} = \frac{\bar{A}_{\max} - \bar{A}_0}{\bar{A}_0}, \quad (37)$$

$$I_\sigma = \frac{\sigma_{\max} - \sigma_0}{\sigma_0}, \quad (38)$$

$$I_L = \frac{L_0}{L_{\min}} = \frac{185 \text{ cm}}{L_{\min}}, \quad (39)$$

where Δ_0 and σ_0 denote the degree of separation and production rate, respectively, obtained in the vertical columns, and are readily calculated from Eqs. (20) and (23) as well as from Eqs. (28) and (29), by setting $\theta = 0$. The results of the numerical examples are given in Tables 1–3.

4. Results and discussion

The comparison of separations, \bar{A}_{\max} and \bar{A}_0 , as well as Δ_{\max} and Δ_0 , obtained at the optimum angle of inclination θ_A and at the vertical ($\theta = 0$), respectively, in countercurrent-flow devices under various flow rates, is shown in Table 1. It is found in this table that the optimum angle of inclination for maximum separation increases when the flow rate decreases, or as the column number increases. The improvement in separation I_A is obtained, especially for low flow-rate operation, or for the scheme of large column number. The same results for concurrent operation were obtained in the previous work (Yeh, 1996).

The comparison of production rate, σ_{\max} and σ_0 , obtainable at the best corresponding angle of inclination θ_σ and at the vertical ($\theta = 0$), respectively, under various feed concentrations and degrees of separation is presented in Table 2. It is seen in this table that the optimum angle of inclination for maximum production rate increases when the specified degree of separation or the column number increases. The improvement in

Table 1
Comparison of separations (Δ_{\max} , Δ_0 and Δ_0) obtained in inclined and vertical columns as well as in countercurrent-flow and concurrent-flow columns with $L = 185$ cm and various σ : (a) $N = 10$, in this case $\sigma < 24.91$ g/min for θ_i and $\sigma < 0.8904$ g/min for θ_j ; (b) $N = 20$, in this case $\sigma < 47.56$ g/min for θ_i and $\sigma < 72.9$ g/min for θ_j ; (c) $N = 40$, in this case $\sigma < 92.86$ g/min for θ_i and $\sigma < 145$ g/min for θ_j

σ or $\dot{\sigma}$ (g/min)	Concurrent flow (Yeh, 1996)				Countercurrent flow						
	$\dot{\Delta}_0$ (%)	θ_A (deg)	$\dot{\Delta}_{\max}$ (%)	\dot{I}_A (%)	$\dot{\Delta}_0$ (%)	θ_A (deg)	$\dot{\Delta}_{\max}$ (%)	I_A (%)	$\dot{\Delta}_{\max} - \dot{\Delta}_0/\dot{\Delta}_0$ (%)	$(\dot{\Delta}_{\max} - \dot{\Delta}_{\max})/\dot{\Delta}_{\max}$ (%)	$(\dot{\Delta}_0 - \dot{\Delta}_{\max})/\dot{\Delta}_{\max}$ (%)
(a)											
8	9.22	62.21	13.74	49.02	12.84	55.55	14.96	16.51	39.26	8.88	− 6.55
16	8.55	48.75	9.71	13.57	10.33	36.87	10.58	2.42	20.82	8.96	6.39
32	6.84	21.19	6.87	0.44	7.42	—	—	—	8.48	—	8.01
64	4.62	—	—	—	4.75	—	—	—	2.81	—	—
(b)											
8	9.32	70.64	19.72	111.59	15.21	65.79	21.66	42.41	63.20	9.84	− 22.87
16	9.27	62.05	13.94	50.38	13.29	54.55	15.32	15.27	43.37	9.90	− 4.66
32	8.67	48.48	9.86	13.73	10.62	34.89	10.83	1.98	22.49	9.84	7.71
64	6.95	20.37	6.97	0.29	7.57	—	—	—	8.92	—	8.61
(c)											
8	9.32	76.42	28.10	201.50	16.76	72.93	31.00	84.96	79.83	10.32	− 40.36
16	9.32	70.60	19.87	113.20	15.52	65.47	21.92	41.24	66.52	10.32	− 21.89
32	9.28	61.98	14.05	51.40	13.54	54.05	15.50	14.48	45.91	10.32	− 3.63
64	8.72	48.37	9.94	13.99	10.77	33.88	10.96	1.76	23.51	10.26	8.35

production rate I_σ is obtained, especially for higher degree of separation, or for the scheme of larger column number.

The minimum column length L_{\min} and the corresponding angle of inclination θ_L under various flow rates and specified degrees of separation are given in Table 3. It is shown in this table that the optimum angle of inclination for minimum column length increases when the specified flow rate decreases, and when the specified degree of separation or column number increases. The improvement in column length I_L is obtained, especially for lower flow-rate operation, or for the scheme of large column number.

Table 1 also gives the calculated results in concurrent-flow devices obtained in the previous work (Yeh, 1996) for comparison. It is seen in this table that in a vertical scheme ($\theta = 0$), the performance ($\bar{\Delta}_0$ as well as Δ_0) of countercurrent flow is much better than that ($\dot{\Delta}_0$ as well as $\dot{\Delta}_0$) of concurrent flow, especially for lower flow rate, or for the scheme of larger column number. In an inclined scheme, however, though the countercurrent-flow operation is still superior to the concurrent-flow operation, ($\bar{\Delta}_{\max} - \dot{\Delta}_{\max}$) is much smaller than ($\bar{\Delta}_0 - \dot{\Delta}_0$). In other words, the improvement of separation \dot{I}_A by tilting the scheme in a concurrent-flow device, is much larger than that I_A in countercurrent-flow one. It is also found that with certain flow rates and column numbers (say $N = 2$, $\sigma = 64$ g/min), the performance in a vertical scheme operating under countercurrent flow ($\bar{\Delta}_0 = 7.57\%$) is even better than that ($\dot{\Delta}_{\max} = 6.97\%$) in an inclined device but operating under concurrent flow.

5. Conclusions

It has been shown that the undesirable remixing effect in thermal diffusion columns of the Frazier scheme with countercurrent flow of feed can be effectively reduced and controlled by tilting the columns, resulting in substantial improvement in performance. Further, a Frazier scheme operating with countercurrent flow of feed is better than one operating with concurrent flow of feed.

The equations of the optimal angles of inclination for the best performance (maximum separation, maximum output and minimum column length) have been derived. The regions within which inclination improves the performance have also been delineated.

A numerical example for separation of a mixture of benzene and *n*-heptane is given. In concurrent flow, considerable improvement in performance is obtained by operating at the optimum angle of inclination, as shown in Table 1; in countercurrent flow performance can be further improved, as shown in Tables 1–3.

The expenditure involved in making a separation by thermal diffusion essentially includes two parts: a fixed charge and an operating expense. The fixed charge is roughly proportional to the equipment cost, say the plate

Table 2

Comparison of production rates (σ_{\max} and σ_0) obtained in inclined and vertical countercurrent-flow columns with $L = 185$ cm and various \bar{A} (or \bar{A}_0): (a) $N = 10$, $\bar{A} > 0.0848$; (b) $N = 20$, $\bar{A} > 0.089$; (c) $N = 40$, $\bar{A} > 0.091$

	\bar{A} or \bar{A}_0 (%)	Δ or Δ_0 (%)			σ_0 (g/min)	θ_σ (deg)	σ_{\max} (g/min)	I_σ (%)
		$C_O = 0.1$ or 0.9	$C_O = 0.3$ or 0.7	$C_O = 0.5$				
(a)	12.84	4.61	10.74	12.77	8	48.67	10.86	35.75
	10.33	3.71	8.65	10.29	16	34.82	16.79	4.94
	7.42	2.51	6.22	7.41	32	—	—	—
	4.75	1.70	3.99	4.75	64	—	—	—
(b)	15.21	5.45	12.69	15.09	8	54.27	16.22	102.75
	13.29	4.77	11.11	13.21	16	48.06	21.25	32.81
	10.64	3.82	8.89	10.58	32	33.23	33.28	4.00
	7.59	2.72	6.35	7.56	64	—	—	—
(c)	16.76	6.01	13.97	16.61	8	57.12	27.37	242.13
	15.52	5.57	12.95	15.40	16	54.10	31.92	99.50
	13.54	4.86	11.32	13.46	32	47.77	41.94	31.06
	10.77	3.87	9.02	10.73	64	32.34	66.29	3.58

Table 3

Comparison of column lengths (L_{\min} and $L_0 = 185$ cm) required in inclined and vertical countercurrent-flow columns with various σ (or σ_0) and \bar{A} (or \bar{A}_0): (a) $N = 10$, $\sigma\bar{A} < 2.113$ g/min; (b) $N = 20$, $\sigma\bar{A} < 4.225$ g/min; (c) $N = 40$, $\sigma\bar{A} < 8.450$ g/min

	\bar{A} or \bar{A}_0 (%)	σ or σ_0 (g/min)	θ_L (deg)	L_{\min} (cm)	$I_L = L_0/L_{\min}$
(a)	12.84	8	60.90	136.22	1.36
	10.33	16	38.52	176.33	1.05
	7.42	32	—	—	—
	4.75	64	—	—	—
(b)	15.21	8	73.26	91.22	2.03
	13.29	16	59.78	139.29	1.33
	10.64	32	36.30	178.56	1.04
	7.59	64	—	—	—
(c)	16.76	8	80.87	54.06	3.42
	15.52	16	72.91	92.72	2.00
	13.54	32	59.15	141.14	1.31
	10.77	64	35.34	178.60	1.04

surface area, $S = BL$, while the operating expense is mainly heat. The heat transfer rate is also proportional to the plate surface area if $\Delta T/2w$ or both ΔT and $2w$ are specified. Therefore, the total expenditure is almost fixed as long as the plate surface area is kept unchanged. Values of Δ_{\max} and σ_{\max} , the maximum separation and maximum output, in Tables 1 and 2, are calculated for constant plate surface area, i.e., with total expenditure fixed. Similarly, since the use of minimum plate surface area as well as minimum column length (with column width fixed) will minimize both the fixed charge and

operating expense, the values of L_{\min} and θ_L in Table 3 provide the optimum design for minimum total expenditure with specified separation and output.

Notation

A	constant ($C(1 - C)$)
B	column width, cm
C	fractional mass concentration of component 1
C_F	C in feed streams
$C_{T,i}, C_{B,i}$	C in product streams exiting from i th column, for top and bottom ends, respectively
D	ordinary diffusion coefficient, cm^2/s
g	gravitational acceleration, cm/s^2
H	system constant, defined by Eq. (2), g/s
I	improvement of performance, defined by Eqs. (37)–(39)
J_{X-OD}	mass flux of component 1 in the x -direction due to ordinary diffusion, $\text{g}/\text{cm}^2 \text{ s}$
J_{X-TD}	mass flux of component 1 in the x -direction due to thermal diffusion, $\text{g}/\text{cm}^2 \text{ s}$
J_{Z-OD}	mass flux of component 1 in the z -direction due to ordinary diffusion, $\text{g}/\text{cm}^2 \text{ s}$
K	system constant, defined by Eq. (3), g cm/s
L	column length, cm
L_{\min}	minimum value of L obtained at the optimum angle of inclination θ_L , cm
N	total column number
T_m	mean absolute temperature, K
ΔT	difference in temperature of hot and cold plates, K
x	axis in temperature-gradient direction, cm
z	axis in transport direction, cm

Greek letters

α	thermal-diffusion constant
β	$-(\partial\rho/\partial T)$, evaluated at T_m under constant pressure, g/cm ³ K
Δ	degree of separation in a countercurrent-flow Frazier scheme ($= C_{T,N} - C_{B,1}$)
$\dot{\Delta}$	degree of separation in a concurrent-flow Frazier scheme ($= C_{T,N} - C_{B,N}$)
$\bar{\Delta}, \dot{\Delta}_0$	$\Delta, \dot{\Delta}$ for equifraction solution ($0.3 < C < 0.7$)
$\Delta_0, \dot{\Delta}_0$	$\Delta, \dot{\Delta}$ obtained in a vertical Frazier scheme
$\Delta_{\max}, \dot{\Delta}_{\max}$	maximum value of $\Delta, \dot{\Delta}$ obtained at the optimum angle of inclination, θ_{Δ}
θ	column angle of inclination, deg
$\theta_{\Delta}, \theta_{\sigma}, \theta_L$	the optimum angle of inclination for Δ_{\max} , for σ_{\max} , for L_{\min} , deg
μ	absolute viscosity, g/cm s
ρ	mass density evaluated at T_m , g/cm ³
σ	mass flow rate, g/s
σ_{\max}	maximum value of σ obtained at the optimum angle of inclination θ_{σ} , g/s
σ_0	σ obtained in a vertical Frazier scheme, g/s
τ_i	transport of component 1 along the z-direction in i th column, g/s
ω	half of plate spacing, cm

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