Emotional Fuzzy Sliding-Mode Control for Unknown Nonlinear Systems

Chun-Fei Hsu & Tsu-Tian Lee

International Journal of Fuzzy Systems

ISSN 1562-2479
Volume 19
Number 3

Int. J. Fuzzy Syst. (2017) 19:942-953
DOI 10.1007/s40815-016-0216-7

Int. J. Fuzzy Syst. (2017) 19(3):942–953
DOI 10.1007/s40815-016-0216-7

Emotional Fuzzy Sliding-Mode Control for Unknown Nonlinear Systems

Chun-Fei Hsu[1] · Tsu-Tian Lee[1]

Received: 12 January 2016 / Revised: 24 April 2016 / Accepted: 13 June 2016 / Published online: 24 June 2016

Abstract The brain emotional learning model can be implemented with a simple hardware and processor; however, the learning model cannot model the qualitative aspects of human knowledge. To solve this problem, a fuzzy-based emotional learning model (FELM) with structure and parameter learning is proposed. The membership functions and fuzzy rules can be learned through the derived learning scheme. Further, an emotional fuzzy sliding-mode control (EFSMC) system, which does not need the plant model, is proposed for unknown nonlinear systems. The EFSMC system is applied to an inverted pendulum and a chaotic synchronization. The simulation results with the use of EFSMC system demonstrate the feasibility of FELM learning procedure. The main contributions of this paper are (1) the FELM varies its structure dynamically with a simple computation; (2) the parameter learning imitates the role of emotions in mammalians brain; (3) by combining the advantage of nonsingular terminal sliding-mode control, the EFSMC system provides very high precision and finite-time control performance; (4) the system analysis is given in the sense of the gradient descent method.

Keywords Brain emotional learning model · Fuzzy control · Structure learning · Parameter learning · System sensitivity term

✉ Chun-Fei Hsu
fei@ee.tku.edu.tw

Tsu-Tian Lee
ttlee@ee.tku.edu.tw

[1] Department of Electrical Engineering, Tamkang University, No. 151, Yingzhuan Rd., Tamsui Dist., New Taipei 25137, Taiwan

1 Introduction

It is known that sliding-mode control provides robust control for nonlinear systems even in the presence of system uncertainties and external disturbances. On the other hand, fuzzy control inherits many attractive features including easy incorporation of expert knowledge into the control law. During the last two decades, there are many studies on the combination of sliding-mode control and fuzzy control to design a fuzzy sliding-mode control (FSMC) system [1–3]. It is easily implemented while only one variable (sliding surface) is defined as the input variable of fuzzy rules. However, there are two disadvantages in the FSMC design. One is the convergence speed of tracking error and the other is the requirement of expert knowledge.

The most commonly used sliding surface is the linear sliding surface, which can guarantee the system stability and desired control performance of the closed-loop control systems [1]. However, the system states cannot reach the equilibrium point in a finite time [4]. To overcome this drawback, a nonsingular terminal sliding-mode control (NTSMC) [4–6] system with nonsingular sliding surface has been successfully proposed for nonlinear systems. Although the NTSMC system can reach zero steady-state tracking error in a limited time, it cannot be implemented as the plant model is unknown or perturbed. For this reason, the NTSMC system cannot be applied in real applications.

On the other hand, the adaptive fuzzy sliding-mode control (AFSMC) systems have been widely adopted to treat the problem of nonlinear systems with unknown system dynamics [7–10]. The stability of the AFSMC systems can be analyzed using Lyapunov stability theory. In general, there are two types of AFSMC systems, namely direct and indirect. For direct scheme, only one fuzzy approximator is used to approximate the ideal controller. But two fuzzy approximators are required for the indirect scheme. Although the AFSMC systems are robust and capable of online learning, they cannot provide very high precision performance and finite-time control.

In addition, the number of fuzzy rules used in the AFSMC systems is fixed. When the chosen rule number is too large, the learning performance has little improvement by further increasing the rule number. To determine the optimal rule number, several dynamic structure learning methods that can vary its fuzzy rules dynamically have been studied [11–14]. It focuses on generating an optimal rules number and investigates the self-organizing method of adding and pruning fuzzy rules in addition to parameter update.

The AFSMC systems usually require a long-time training sequence. To solve this problem, Moren and Balkenius [15] proposed a brain emotional learning model (BELM), which is a structural model based on the limbic system of mammalian brains. Recently, many studies for the design of BELM-based intelligent control (BIC) systems have been reported [16–20]. Specifically, Sharbafi et al. [16] proposed the motion control of an omnidirectional robot by the BIC system. Roshanaei et al. [17] used the BELM to optimize the beamforming and the direction of arrival estimation. In [18, 19], the BIC system was used for motor speed control to achieve favorable control performance even under motor parameter changes and operating point changes. Khalghani and Khooban [20] proposed the BIC system for controlling the DVR compensator. All these results indicate that the BIC systems do not require any training sequence because its learning algorithm is only to imitate the role of emotions in mammalians brain. Nevertheless, the BIC systems cannot handle the qualitative aspects of human knowledge and reasoning processes due to the sensory input and emotional signal of the BELM which are chosen as a function of tracking error.

To attack the mentioned drawbacks, this paper develops a fuzzy-based emotional learning model (FELM) that comprised a fuzzy amygdale system (FAS) and a fuzzy orbitofrontal system (FOS). The FELM can model the qualitative aspects of human knowledge based on the emotional learning of human brain. Further, this paper proposes an emotional fuzzy sliding-mode control (EFSMC) system to cope with a class of unknown nonlinear systems. The EFSMC system can either increase or decrease the number of fuzzy rules over time based on tracking performance and a full-tuned parameter learning law is developed to upgrade the learning capability. Finally, the proposed EFSMC system is applied to an inverted pendulum and a chaotic synchronization. The simulation results show that not only the EFSMC system can achieve robust characteristics but also the structure learning ability enables the FELM to evolve its structure online.

2 Design of the Ideal NTSMC System

Consider a second-order nonlinear system as

$$x = f(x,\dot{x}) + g(x,\dot{x})u, \tag{1}$$

where x is the system state, $f(x,\dot{x})$ and $g(x,\dot{x}) > 0$ are the system dynamics, and u is the control input. The control objective is to find an appropriate controller so that the system state x can track a system command x_c closely. Define a tracking error

$$e = x_c - x \tag{2}$$

From (2), the error dynamic equation can be rewritten as [21]

$$e = z(x,\dot{x}) - u, \tag{3}$$

where $z(x,\dot{x}) = x_c - \left(1 - \frac{1}{g(x,\dot{x})}\right)x - \frac{f(x,\dot{x})}{g(x,\dot{x})}$. Then a nonsingular sliding surface is defined as [4–6]

$$s = e + \frac{1}{\lambda}\dot{e}^{\frac{p}{q}}, \tag{4}$$

where λ is a positive constant, and p and q are both positive odd integers which satisfy the condition $q < p < 2q$. If the nonlinear term $z(x,\dot{x})$ is known, there exits an ideal NTSMC system as

$$u^* = z(x,\dot{x}) + \lambda\frac{q}{p}\dot{e}^{2-\frac{p}{q}} + k\,\mathrm{sgn}(s), \tag{5}$$

where k is a positive constant and $\mathrm{sgn}(\cdot)$ is a sign function. Differentiating (4) with respect to time and using the control law $u = u^*$, we can obtain that

$$\begin{aligned}\dot{s} &= \dot{e} + \frac{1}{\lambda}\frac{p}{q}\dot{e}^{\frac{p}{q}-1}\ddot{e}\\ &= \dot{e} + \frac{1}{\lambda}\frac{p}{q}\dot{e}^{\frac{p}{q}-1}(z(x,\dot{x}) - u)\\ &= \dot{e} + \frac{1}{\lambda}\frac{p}{q}\dot{e}^{\frac{p}{q}-1}\left(-\lambda\frac{q}{p}\dot{e}^{2-\frac{p}{q}} - k\,\mathrm{sgn}(s)\right)\\ &= -\frac{k}{\lambda}\frac{p}{q}\dot{e}^{\frac{p}{q}-1}\mathrm{sgn}(s)\end{aligned} \tag{6}$$

Consider the candidate Lyapunov function as

$$V_1 = \frac{1}{2}s^2 \tag{7}$$

Differentiating (7) with respect to time and using (6) yield

$$\dot{V}_1 = s\dot{s} = -\frac{k}{\lambda}\frac{p}{q}\dot{e}^{\frac{p}{q}-1}|s| \leq 0. \tag{8}$$

Since p and q are both positive odd integers and $1 < \frac{p}{q} < 2$, then $\dot{e}^{\frac{p}{q}-1} > 0$ for $\dot{e} \neq 0$ [4–6]. When the nonsingular sliding surface $s = 0$ is reached, the system dynamic can be determined by the following nonlinear differential equation

$$\dot{e} = -\lambda e^{\frac{q}{p}}. \tag{9}$$

It can be found that both of tracking error e and its derivative $\dot{e}$ will converge to zero in a limited time [4]. The system stability of the ideal NTSMC system in (5) is proven by Lyapunov theory; however, the ideal NTSMC system cannot be implemented due to the nonlinear term $z(x, \dot{x})$ is unknown in real applications.

3 Design of the EFSMC System

It is highly desirable to propose a model-free controller without knowing the system dynamics. Based on the bio-inspired BELM [16–20], the control output of the EFSMC system as shown in Fig. 1 is designed as

$$u_{ec} = u_{fa} - u_{fo}, \tag{10}$$

where u_{fa} is the output of FAS and u_{fo} is the output of FOS. The FAS is used for providing attention signals of the FELM and the FOS is designed to inhibit inappropriate responses from FAS. At the sampling time t, assuming that there are $n(t)$ fuzzy rules in FAS, each fuzzy rule is described as follows:

$$\text{Rule } i: \text{ IF } s \text{ is } F_i, \text{ THEN } u_{fa} \text{ is } V_i, \quad \text{for } i = 1, 2, \ldots, n(t), \tag{11}$$

where, in the ith rule, F_i represents fuzzy sets of s and V_i is the consequent part with initial value zero. Let fuzzy set F_i be the Gaussian functions described as

$$\phi_i = \exp\left(-\frac{(s - c_i)^2}{(\sigma_i)^2}\right), \tag{12}$$

where σ_i and c_i are the deviation and mean, respectively, of the Gaussian function in the ith term. Thus, the defuzzification of the FAS is accomplished as

$$u_{fa} = \sum_{i=1}^{n(t)} V_i \phi_i, \tag{13}$$

where ϕ_i is the fuzzy firing weight of the ith fuzzy rule. Similarly, each fuzzy rule in the FOS is described as follows:

$$\text{Rule } i: \text{ IF } s \text{ is } F_i, \text{ THEN } u_{fo} \text{ is } W_i, \quad \text{for } i = 1, 2, \ldots, n(t) \tag{14}$$

where F_i is the fuzzy set of s and W_i is the consequent part with initial value zero. Thus, the FOS output is obtained as

$$u_{fo} = \sum_{i=1}^{n(t)} W_i \phi_i. \tag{15}$$

This process is based on the fact that the Orbitofrontal Cortex receives the same signals as the Amygdala does [15].

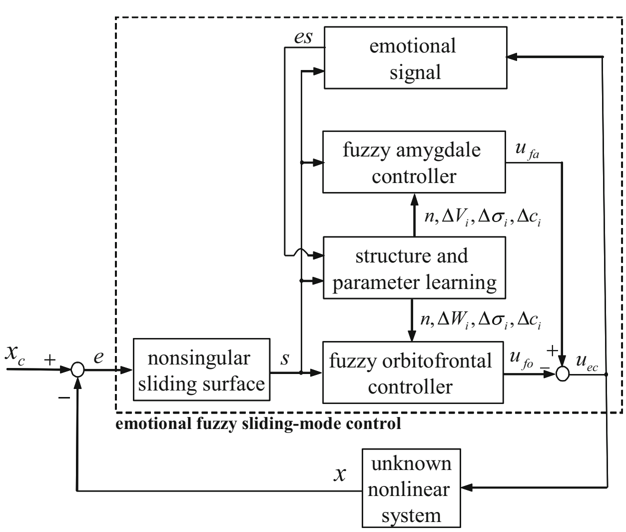

Fig. 1 Block diagram of the EFSMC system

3.1 Online Structure Learning

There is no fuzzy rule in the FELM initially. To solve the problem of rule number determination, a structure learning algorithm for generation and removal of fuzzy rules is developed. An initial fuzzy rule is generated with parameters given as [11–14]

$$V_1 = W_1 = 0, \tag{16}$$

$$\sigma_1 = \bar{\sigma}, \tag{17}$$

$$c_1 = s, \tag{18}$$

where $\bar{\sigma}$ is a parameter specified by designers. For each subsequent piece of input data, the FAS and FOS will not generate a new fuzzy rule but update parameters of the existing fuzzy rules if the new input data falls within the existing fuzzy sets. The fuzzy firing weight in (12) is used as the degree measure and the maximum degree ϕ_{max} is defined as

$$\phi_{max} = \max_{1 \leq k \leq n(t)} \phi_k. \tag{19}$$

Let $\phi_{th} \in (0, 1)$ be a given growing threshold. If $\phi_{max} \leq \phi_{th}$ is satisfied, a new fuzzy rule is generated. For more complex learning problems, a large rule number is

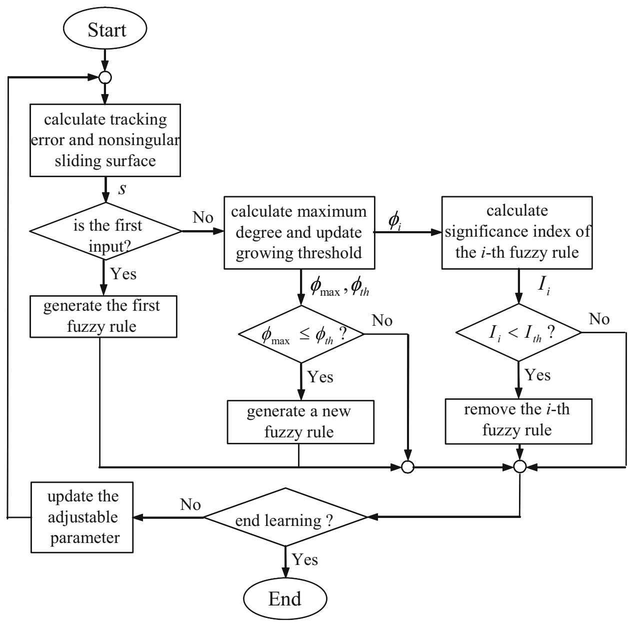

Fig. 2 The learning scheme

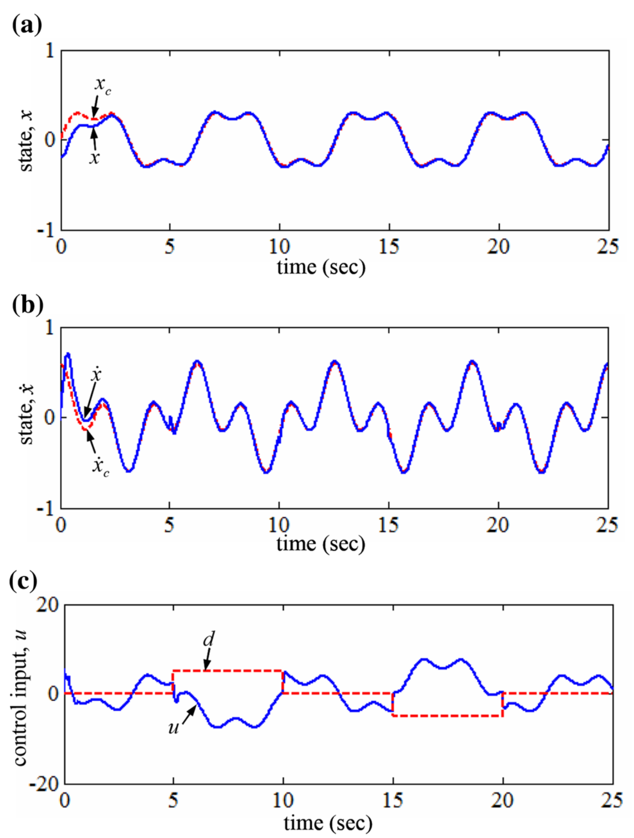

Fig. 3 Simulation results of using AFSMC system with delta adaptation law

required and a large threshold value should be set. However, a large threshold value will lead to an overtraining problem. In this paper, a time-varying growing threshold is proposed as

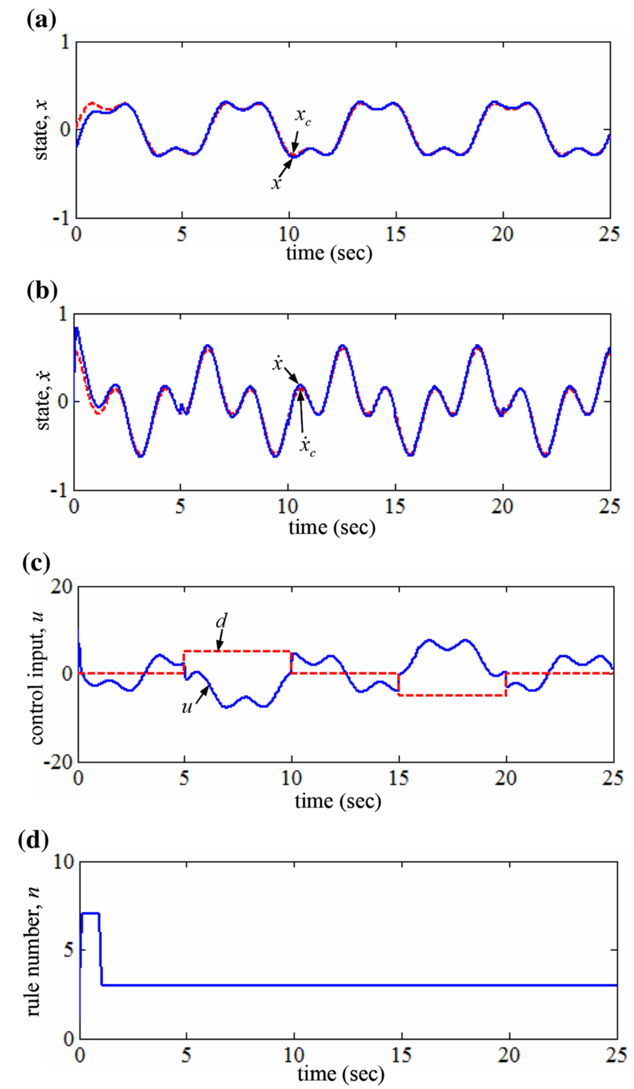

Fig. 4 Simulation results of using EFMSC system with smaller signal gain

$$\phi_{\mathrm{th}} = \phi_{\mathrm{f}} + (\phi_{\mathrm{s}} - \phi_{\mathrm{f}}) \exp(-\tau_1 t), \tag{20}$$

where ϕ_{s} and ϕ_{f} are positive constants which satisfy the condition $\phi_{\mathrm{s}} > \phi_{\mathrm{f}}$, and τ_1 is a constant that controls the decay speed. We can see that the time-varying growing threshold allows the threshold value to decay from ϕ_{s} to ϕ_{f}. Thus, the FELM can easily generate a new fuzzy rule at the initial learning phase and avoid the overtraining problem while the growing threshold decays during the learning process. Once a new fuzzy rule is generated, the next step is to assign the initial adjustable parameters as follows:

$$V_{n(t)+1} = W_{n(t)+1} = 0 \tag{21}$$

$$\sigma_{n(t)+1} = \bar{\sigma} \tag{22}$$

$$c_{n(t)+1} = s. \tag{23}$$

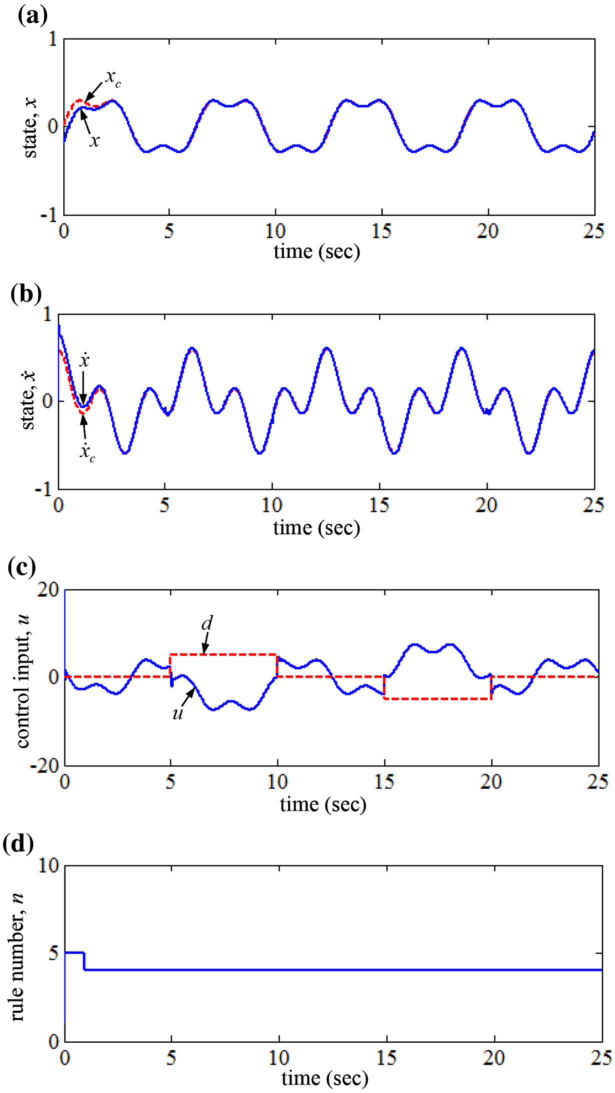

Fig. 5 Simulation results of using EFMSC system with larger signal gain

In order to remove inappropriate fuzzy rules, a significance index of the *i*th fuzzy rule given in [22] is introduced

$$I_i(t+1) = \begin{cases} I_i(t)\exp(-\tau_2), & \text{if } \phi_i \le \phi_r \\ I_i(t)[2-\exp(-\tau_3(1-I_i(t)))], & \text{if } \phi_i > \phi_r \end{cases} \tag{24}$$

where $\phi_r \in (0,1)$ is a given threshold, and τ_2 and τ_3 are constants that control the decay speed of significance index. The initial value of significance index is set to 1 for each fuzzy rule. If $I_i \le I_{\text{th}}$ is satisfied, where I_{th} is a pre-given pruning threshold, then the *i*th fuzzy rule will be removed. The flow chart of the learning scheme is shown in Fig. 2. It shows that the fuzzy rules can be created and adapted as online learning proceeds via simultaneous structure and parameter learning.

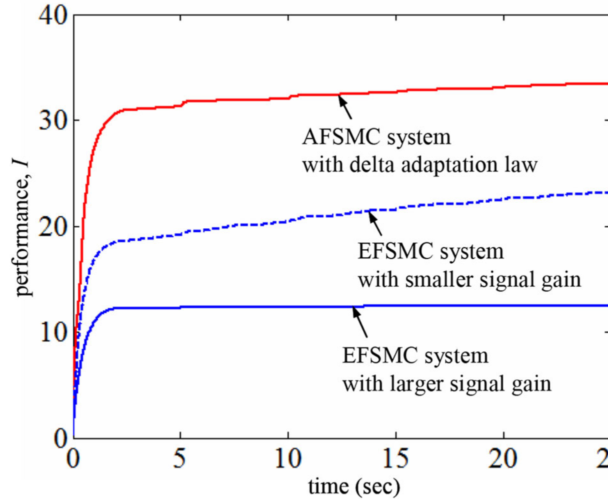

Fig. 6 Performance comparison of the inverted pendulum example

3.2 On-line Parameter Learning

There has not been any defined mathematical method to tune the BELM until now. The parameter learning that is inspired by emotional learning in mammalians brain [16] can be applied as

$$\Delta V_i(t) = \eta_v \max(0, \phi_i(\text{es} - u_{\text{fa}})) \tag{25}$$

$$\Delta W_i(t) = \eta_w \phi_i(u_{\text{ec}} - es), \tag{26}$$

where $i = 1, 2, \ldots, n(t)$, η_v and η_w are the learning rates for V_i and W_i, respectively, and *es* is the emotional signal which is a function of several parameters. In order to represent the incapability of forgetting the previous emotion signals, the FAS uses monotonic weight-adjusting functions with a maximum term. Thus, the parameter weights $V_i(t)$ cannot be decreased. When the fuzzy firing weights $\phi_i = 0$, which mean there is no learning signal input to the FAS, thus the parameter weights $V_i(t)$ are certainly stable. In [16–20], the emotional signal is selected as a complex function of other signals such as plant output, model output, and tacking error. To explain the perception ability of the EFSMC system from the environment, this paper designs the emotional signal as

$$\text{es} = k_1 s + k_2 u_{\text{ec}}, \tag{27}$$

where k_1 and k_2 are the constant gains. Further, the effectiveness of EFSMC system will be limited while only the fuzzy rules V_i and W_i are online tuned. To upgrade the learning capability of EFSMC system, define a cost function as

$$C = \frac{1}{2}s^2. \tag{28}$$

Based on the gradient descent method [23, 24], the parameter adaptation law can be represented as

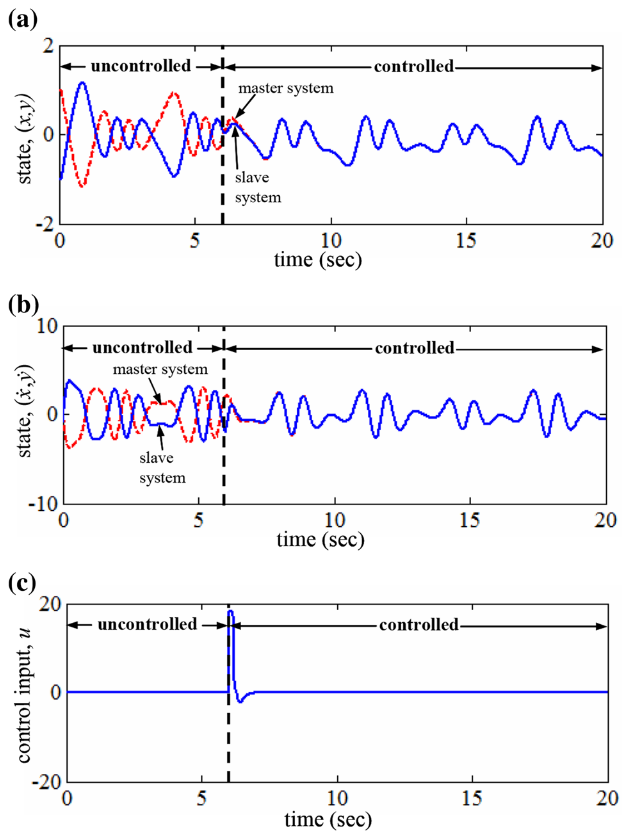

Fig. 7 Simulation results of the FSMC system for scenario 1

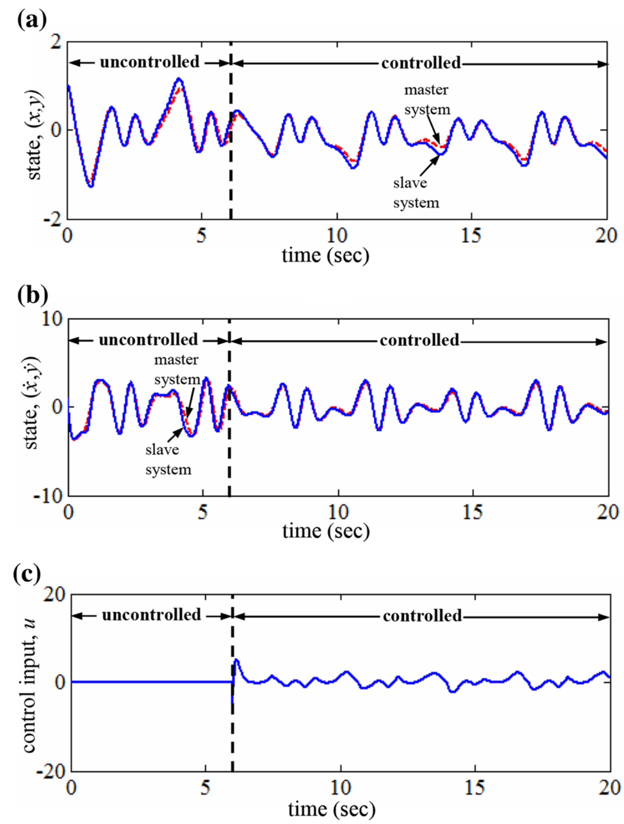

Fig. 8 Simulation results of the FSMC system for scenario 2

$$\begin{aligned}\Delta W_i(t) &= -\eta_w \frac{\partial C}{\partial W_i} \\ &= -\eta_w \frac{\partial C}{\partial u_{\mathrm{ec}}} \frac{\partial u_{\mathrm{ec}}}{\partial u_{\mathrm{fo}}} \frac{\partial u_{\mathrm{fo}}}{\partial W_i} \\ &= \eta_w \frac{\partial C}{\partial u_{\mathrm{ec}}} \phi_i.\end{aligned} \tag{29}$$

Here $\frac{\partial C}{\partial u_{\mathrm{ec}}}$ cannot be determined exactly due to the uncertainties of the plant dynamics. To overcome this problem, a delta adaptation law [25, 26] has been proposed. However, its convergence property cannot be proven. In this paper, the sensitivity term of plant model can be obtained by comparing (26) with (29) to yield

$$\frac{\partial C}{\partial u_{\mathrm{ec}}} = u_{\mathrm{ec}} - \mathrm{es}. \tag{30}$$

In order to train the EFSMC system effectively, the full-tuned parameter adaptive laws $\Delta\sigma_i$ and Δc_i for the ith fuzzy rules can be obtained as

$$\begin{aligned}\Delta\sigma_i(t) &= -\eta_\sigma \frac{\partial C}{\partial u_{\mathrm{ec}}} \frac{\partial u_{\mathrm{ec}}}{\partial u_{\mathrm{fo}}} \frac{\partial u_{\mathrm{fo}}}{\partial \phi_i} \frac{\partial \phi_i}{\partial \sigma_i} \\ &= \eta_\sigma (u_{\mathrm{ec}} - \mathrm{es}) W_i \frac{2(s - c_i)^2}{(\sigma_i)^3} \phi_i\end{aligned} \tag{31}$$

$$\begin{aligned}\Delta c_i(t) &= -\frac{\partial C}{\partial u_{\mathrm{ec}}} \frac{\partial u_{\mathrm{ec}}}{\partial u_{\mathrm{fo}}} \frac{\partial u_{\mathrm{fo}}}{\partial \phi_i} \frac{\partial \phi_i}{\partial c_i} \\ &= \eta_c (u_{\mathrm{ec}} - \mathrm{es}) W_i \frac{2(s - c_i)}{(\sigma_i)^2} \phi_i,\end{aligned} \tag{32}$$

where η_σ and η_c are positive learning rates. The controller parameters of the EFSMC system are updated as follows:

$$V_i(t+1) = V_i(t) + \Delta V_i(t) \tag{33}$$

$$W_i(t+1) = W_i(t) + \Delta W_i(t) \tag{34}$$

$$\sigma_i(t+1) = \sigma_i(t) + \Delta\sigma_i(t) \tag{35}$$

$$c_i(t+1) = c_i(t) + \Delta c_i(t). \tag{36}$$

It is shown that the fuzzy sets and the fuzzy rules can be tuned to increase the online learning ability of the EFSMC system. Since the modification of each fuzzy rule in the FELM is based on the fuzzy firing weights and the emotional signal, the parameter learning algorithm is reasonable.

3.3 Stability Analysis

Substituting $u = u_{\mathrm{ec}}$ (10) into (3) yields

$$e = z(x, \dot{x}) - u_{\mathrm{fa}} + u_{\mathrm{fo}}. \tag{37}$$

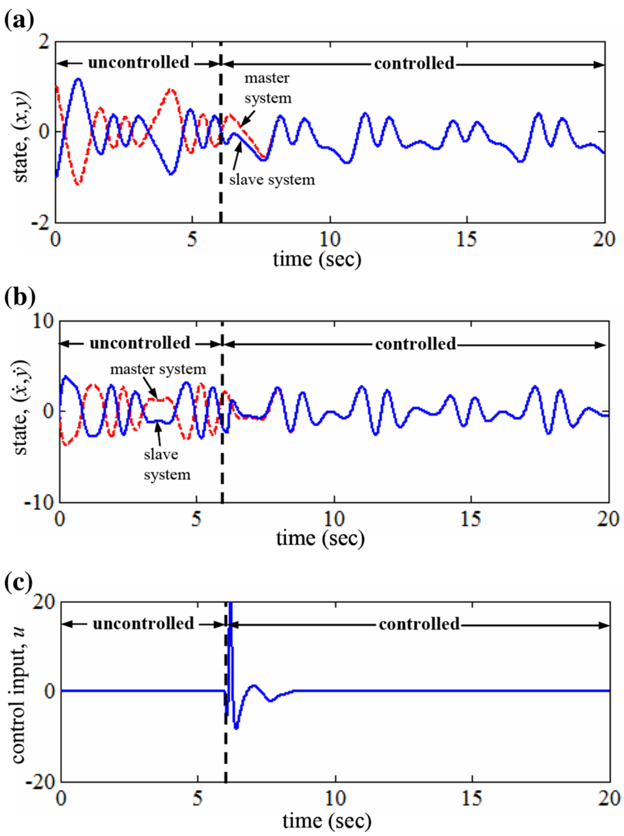

Fig. 9 Simulation results of the BIC system for scenario 1

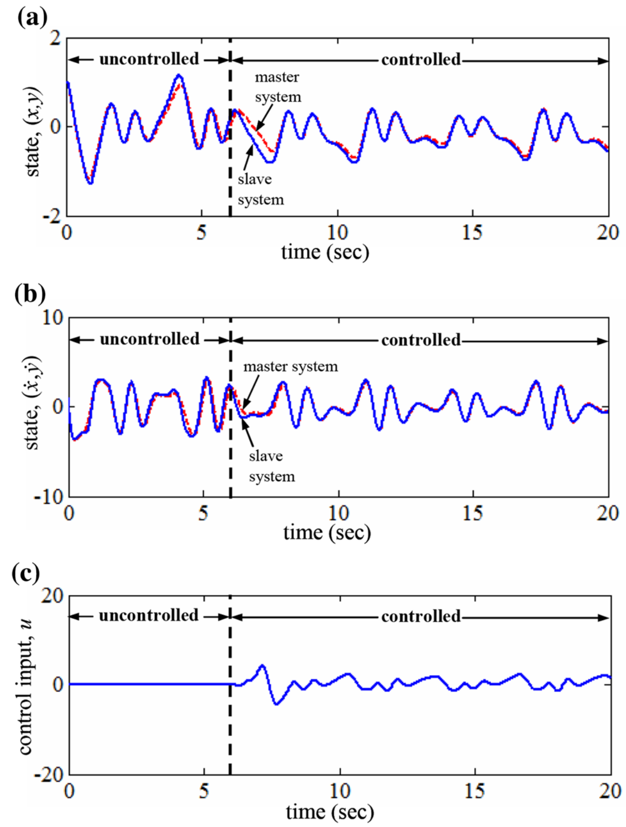

Fig. 10 Simulation results of the BIC system for scenario 2

Differentiating (4) with respect to time and using (5) and (37), we can obtain that

$$\dot{s} = \frac{1}{\lambda}\frac{p}{q}\dot{e}^{\frac{p}{q}-1}(-k\mathrm{sgn}(s) + u^{*} - u_{\mathrm{fa}} + u_{\mathrm{fo}}). \tag{38}$$

Multiplying both sides by s, gives

$$s\dot{s} = \frac{1}{\lambda}\frac{p}{q}\dot{e}^{\frac{p}{q}-1}(-k|s| + s(u^{*} - u_{\mathrm{fa}} + u_{\mathrm{fo}})). \tag{39}$$

According to the gradient descent method, the weights in the FOS are updated by the following equation [27, 28]

$$\Delta W_i(t) = -\eta' s\phi_i, \tag{40}$$

where η is the learning rate and $\eta' = \eta\frac{1}{\lambda}\frac{p}{q}\dot{e}^{\frac{p}{q}-1}$. Due to $\dot{e}^{\frac{p}{q}-1} > 0$ for all $\dot{e} \neq 0$ [4–6], η' is taken as new learning rate. On the other hand, while the gains in the emotional signal are chosen $k_1 > 0$ and $k_2 = 1$, the emotional parameter learning (26) can be rewritten as

$$\Delta W_i(t) = -\eta_w k_1 s\phi_i = -\eta'' s\phi_i, \tag{41}$$

where $\eta'' = \eta_w k_1$ is taken as new learning rate. We can find that the emotional parameter learning (41) is the same as the parameter learning (40) using the gradient descent method. Therefore, the stability of the proposed EFSMC system can be guaranteed.

The design procedure of the EFSMC system is summarized as follows:

Step 1: Define the tracking error e and the nonsingular sliding surface s as (2) and (4), respectively.
Step 2: Determine the structure of the FELM according to Fig. 2, where the parameters are tuned by (33)–(36).
Step 3: Calculate the control law as (10), where the output of FOS is given as (13) and the output of FAS is given as (15).

4 Simulation Results

4.1 Inverted Pendulum Example

Consider an inverted pendulum problem to demonstrate the robustness of the EFSMC scheme. The dynamics of the inverted pendulum system is given as [29]

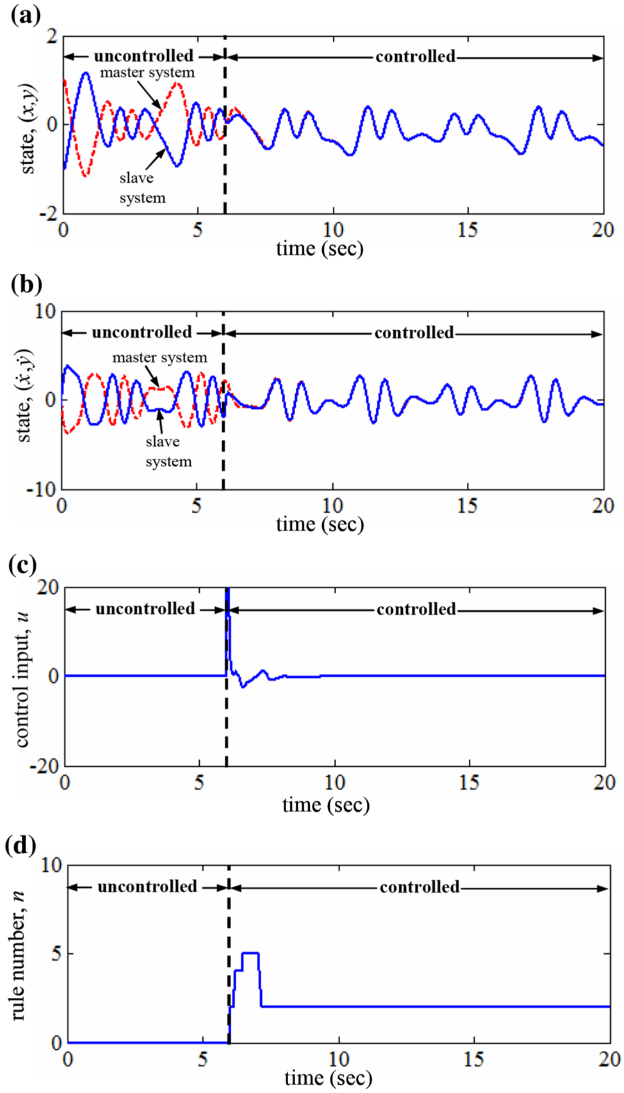

Fig. 11 Simulation results of the EFSMC system for scenario 1

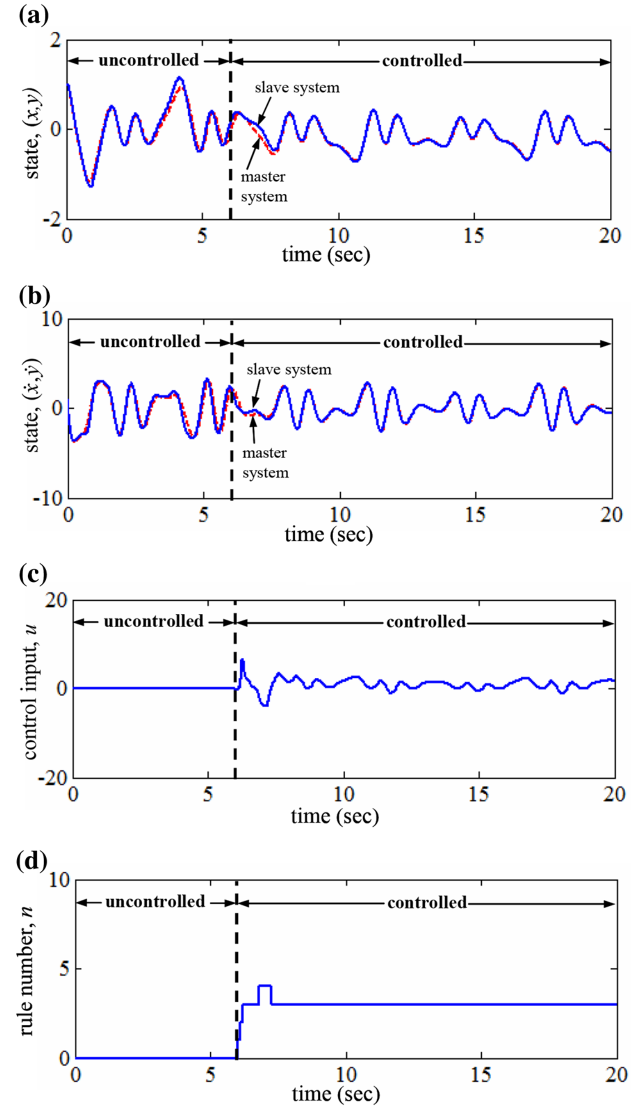

Fig. 12 Simulation results of the EFSMC system for scenario 2

$$x = f(x,\dot{x}) + g(x,\dot{x})u + d, \tag{37}$$

where x is the angle of the pendulum, $f(x,\dot{x}) = \frac{ml\dot{x}\sin(x)\cos(x)-(M+m)g_0\sin(x)}{ml\cos^2(x)-\frac{4}{3}l(M+m)}$ and $g(x,\dot{x}) = \frac{-\cos(x)}{ml\cos^2(x)-\frac{4}{3}l(M+m)}$ are the system dynamics, u is the control input, $M = 1$ is the mass of cart, $m = 0.1$ is the mass of rod, $g_0 = 9.81$ is the gravity, $l = 0.5$ is the length of rod, and d is the external disturbance $-5 \le d \le 5$. To investigate the effectiveness of the EFSMC system, an AFSMC system with delta adaptation law [25, 26] is applied for comparison. A fuzzy approximator was used to approximate the ideal controller and the sensitivity term was determined as $\frac{\partial C}{\partial u} = \dot{e} + e$. The simulation results of the AFSMC system with delta adaptation law are shown in Fig. 3. The response of x is shown in Fig. 3a; the response of $\dot{x}$ is shown in Fig. 3b; and the control input is shown in Fig. 3c. The simulation results show that the accurate tracking control performance can be obtained after the parameter learning. However, the convergence speed of tracking error is slow at the initial learning phase and only parameter learning is considered. There is a trade off between the approximation accuracy and computational loading.

Then, the EFSMC system is applied again. The control parameters of the EFSMC system are selected as $\lambda = 0.5$, $p = 7$, $q = 5$, $\eta_v = \eta_w = 0.4$, $\eta_\sigma = \eta_c = 0.2$, $k_1 = k_2 = 1$, $\bar{\sigma} = 0.4$, $\phi_s = 0.6$, $\phi_f = 0.2$, $\phi_r = 0.01$, $I_{th} = 0.01$, $\tau_1 = 1$, and $\tau_2 = \tau_3 = 0.01$. Generally, these control parameters require some trial-and-error tuning procedures to determine. The simulation results of the EFSMC system with smaller gain constants ($k_1 = 1$ and $k_2 = 1$) are shown in Fig. 4. The response of x is shown in Fig. 4a; the response of $\dot{x}$ is shown in Fig. 4b; the control input is shown in Fig. 4c; and the number of fuzzy rules is shown in Fig. 4d. The simulation results show that the favorable tracking responses can be provided and the concise system size can also be observed since the structure learning and the parameter learning are applied simultaneously. Since the FELM does not process the emotional signals and determine its significance well, the tracking errors will exist after learning.

As the emotional signal influences the convergence speed of tracking error, the set of larger gain constants ($k_1 = 10$ and $k_2 = 1$) is applied to the EFSMC system. The simulation results of the EFSMC system with larger gain constants are shown in Fig. 5. The response of x is shown in Fig. 5a; the response of $\dot{x}$ is shown in Fig. 5b; the control input is shown in Fig. 5c; and the number of fuzzy rules is shown in Fig. 5d. Figure 5 shows that the EFSMC system with larger gain constants is robust to the external disturbance. Results show that more favorable tracking performance with faster convergence speed of the tracking error can be obtained if the EFSMC system with larger gain constants in emotional signal are selected.

For further performance comparison, a performance index $I = \frac{1}{2}\sum e^2 + \dot{e}^2$ is considered. The performance comparison between the AFSMC system and the EFSMC system is shown in Fig. 6. It is observed that the performance index of the EFSMC system with larger gain constants is smaller than other methods. This is due to the fact that the tracking errors converge the most quickly using larger gain constants. The drawback is that it will require large control signal at initial control phase.

4.2 Chaotic Synchronization Problem

Chaos synchronization can be applied in the vast areas of engineering community such as physics, chemistry, biology, ecology, and secure communication [30–32]. Consider a master–slave coupled chaotic gyros system as follows [30]:

$$x = f_x \sin\omega t \sin x - \alpha^2 \frac{(1-\cos x)^2}{\sin^3 x} + \beta \sin x - c_1\dot{x} - c_2\dot{x}^3 = g_x(x,\dot{x}) \tag{38}$$

$$y = f_y \sin\omega t \sin y - \alpha^2 \frac{(1-\cos y)^2}{\sin^3 y} + \beta \sin y - c_1\dot{y} - c_2\dot{y}^3 + u = g_y(x,\dot{x}) + u, \tag{39}$$

where $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $\omega = 2$, $g_x(t)$, and $g_y(t)$ are the dynamic of the master and slave gyros, respectively, and u is the control input. The control objective is that the two coupled chaotic gyros should be synchronized by an appropriate control input u. Two test scenarios are considered in this paper. The settings of scenario 1 (initial variation case) are $(x,\dot{x},y,\dot{y}) = (1,1,-1,-1)$, $f_x = 32$ and $f_y = 32$, and those of scenario 2 (parameter variation case) are $(x,\dot{x},y,\dot{y}) = (1,1,1,1)$, $f_x = 32$ and $f_y = 37$. To evaluate the performance of the proposed EFSMC system, the results are compared with those obtained by the FSMC system [1] and the BIC system [19].

First, the FSMC system [1] is applied to the chaos synchronization. Assuming that there are five fuzzy rules in the FSMC system, each fuzzy rule is described as follows:

$$\text{Rule } i: \text{ IF } s \text{ is } F_i, \text{ THEN } u_{fc} \text{ is } \alpha_i, \quad \text{for } i = 1,2,\ldots,5, \tag{40}$$

where α_i is the singleton control actions and F_i are the labels of the fuzzy sets which are fixed in this study. The fuzzy rules can be constructed by the sense that s will

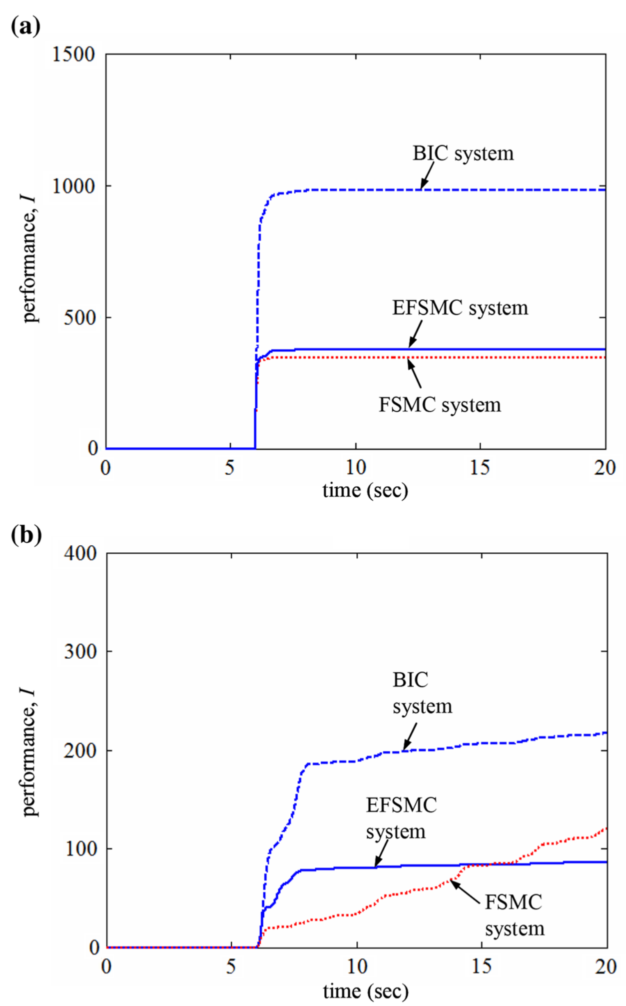

Fig. 13 Performance comparison of the chaos synchronization example

approach to zero. The simulation results of the FSMC system are shown in Figs. 7 and 8 for scenario 1 and scenario 2, respectively. The responses of states (x, y) are shown in Figs. 7a and 8a; the responses of states $(\dot{x}, \dot{y})$ are shown in Figs. 7b and 8b; and the control inputs are shown in Figs. 7c and 8c. The simulation results show that the satisfactory performance can be achieved for scenario 1 but not for scenario 2. The fuzzy rules α_i should be preconstructed by trial-and-error tuning procedure; however, it is difficult to tune a fuzzy rules base that can cope with wide system uncertainties.

Then, the BIC system [19] is applied to the chaos synchronization again. The sensory input of the BIC system is chosen as $\text{es} = \dot{e} + e$ and the controller parameter that is initiated from zero can be online tuned in the sense of the BELM. The simulation results of the BIC system are shown in Figs. 9 and 10 for scenario 1 and scenario 2, respectively. The responses of states (x, y) are shown in Figs. 9a and 10a; the responses of states $(\dot{x}, \dot{y})$ are shown in Figs. 9b and 10b; and the control inputs are shown in Figs. 9c and 10c. All of the tracking errors for both of the test scenarios would converge to zero after controller parameter learning, and it means that the chaos synchronization is stable using the BIC system. Although the simulated results show the effectiveness of the bio-inspired emotional learning approach, the convergence speed of tracking error is slow. In addition, the BELM cannot model the qualitative aspects of human knowledge.

Finally, the EFSMC system is applied to the chaos synchronization again. The control parameters are selected as $\lambda = 0.5, p = 7, q = 5, \eta_v = \eta_w = 0.02, k_1 = k_2 = 1, \bar{\sigma} = 2,$ $\phi_s = 0.4$, $\phi_f = 0.2$, $\phi_r = 0.01$, $I_{th} = 0.01$, $\tau_1 = 1$, and $\tau_2 = \tau_3 = 0.01$. All of the parameters are determined by trial and error in order to guarantee the desired control performance. The simulation results of the EFSMC system are shown in Figs. 11 and 12 for scenario 1 and scenario 2, respectively. The responses of states (x, y) are shown in Figs. 11a and 12a; the responses of states $(\dot{x}, \dot{y})$ are shown in Figs. 11b and 12b; the control inputs are shown in Figs. 11c and 12c; and the numbers of fuzzy rules are shown in Figs. 11d and 12d. The simulation results show that not only the FELM has the admirable property of small fuzzy rules size and high learning accuracy but also more favorable tracking performance with faster convergence speed of the tracking error.

For further performance comparison, a performance index $I = \frac{1}{2}\sum e^2 + \dot{e}^2$ is considered. The performance indexes of the FSMC system, the BIC system, and the EFSMC system are shown in Figs. 13a, b for scenario 1 and scenario 2, respectively. The performance index of the FSMC system is smaller than other methods due to the fuzzy rule constructed by time-consuming trial-and-error tuning procedures. But, the favorable tracking performance cannot be achieved as the fuzzy rules are not adaptive. It implies that the FSMC system is not suitable for chaos synchronization. Nevertheless, the proposed EFSMC system can achieve better design performance than that using BIC system for both the test scenarios.

5 Conclusions

A FELM with dynamic structure learning to handle the emotional learning of human knowledge is presented. The growing or pruning of fuzzy rules relies on the input data (tracking error and nonsingular sliding surface). Further, this paper has successfully demonstrated the design of an EFSMC system based on emotional learning process of the FELM to control an inverted pendulum and a chaotic synchronization. A comparison among the BIC system, the FSMC system and the proposed EFSMC systems is made. The FSMC system cannot cope with wide system uncertainties and the BIC system cannot model the qualitative aspects of human knowledge. Nevertheless, the proposed EFSMC system can achieve robust characteristics but also the structure learning ability enables the FELM to evolve its structure online. However, the EFSMC system requires a priori knowledge about the sign of control gain.

Further works on the EFSMC system include: (1) consider autotuning of the learning rates η_v, η_w, η_σ , and η_c in the parameter learning law to increase the convergence of the closed-loop system [33, 34]; (2) extend the FELM to control a multi-input multi-output unknown nonlinear system such as mobile manipulator robot [35].

Acknowledgments The authors are grateful to the associate editor and the reviewers for their valuable comments. The authors appreciate the partial financial support from the Ministry of Science and Technology of Republic of China under Grant MOST 103-2221-E-032-063-MY2.

References

1. Li, T.S., Chen, C.C., Su, Y.T.: Optical image stabilizing system using fuzzy sliding-mode controller for digital cameras. IEEE Trans. Consum. Electron. **58**(2), 237–245 (2012)
2. Chang, Y.H., Chang, C.W., Chen, C.L., Tao, C.W.: Fuzzy sliding-mode formation control for multirobot systems: design and implementation. IEEE Trans. Syst. Man Cybern. B **42**(2), 444–457 (2012)
3. Londhe, P.S., Patre, B.M., Tiwari, A.P.: Design of single-input fuzzy logic controller for spatial control of advanced heavy water reactor. IEEE Trans. Nucl. Sci. **61**(2), 901–911 (2014)
4. Feng, Y., Yu, X., Han, F.: On nonsingular terminal sliding-mode control of nonlinear systems. Automatica **49**(6), 1715–1722 (2013)

5. Zhu, Z., Yan, Y.: Space-based line-of-sight tracking control of GEO target using nonsingular terminal sliding mode. Adv. Space Res. **54**(6), 1064–1076 (2014)
6. Hsu, C.F., Lee, T.T., Tanaka, K.: Intelligent nonsingular terminal sliding-mode control via perturbed fuzzy neural network. Eng. Appl. Artif. Intell. **45**(10), 339–349 (2015)
7. Hwang, C.L., Chiang, C.C., Yeh, Y.W.: Adaptive fuzzy hierarchical sliding-mode control for the trajectory tracking of uncertain underactuated nonlinear dynamic systems. IEEE Trans. Fuzzy Syst. **22**(2), 286–299 (2014)
8. Li, Q., Zhang, W., Han, G., Yang, Y.: Adaptive neuro-fuzzy sliding mode control guidance law with impact angle constraint. IET Control Theory Appl. **9**(14), 2115–2123 (2015)
9. Boldbaatar, E.A., Lin, C.M.: Self-learning fuzzy sliding-mode control for a water bath temperature control system. Int. J. Fuzzy Syst. **17**(1), 31–38 (2015)
10. Saghafinia, A., Hew, W.P., Uddin, M.N., Gaeid, K.S.: Adaptive fuzzy sliding-mode control into chattering-free IM drive. IEEE Trans. Ind. Appl. **51**(1), 692–701 (2015)
11. Juang, C.F., Hsu, C.H.: Reinforcement interval type-2 fuzzy controller design by online rule generation and Q-value-aided ant colony optimization. IEEE Trans. Syst. Man Cybern. B **39**(6), 1528–1542 (2009)
12. Han, H., Wu, X.L., Qiao, J.F.: Nonlinear systems modeling based on self-organizing fuzzy-neural-network with adaptive computation algorithm. IEEE Trans. Cybern. **44**(4), 554–564 (2014)
13. Lin, C.M., Li, H.Y.: Dynamic petri fuzzy cerebellar model articulation control system design for magnetic levitation system. IEEE Trans. Contr. Syst. Technol. **23**(2), 693–699 (2015)
14. Wang, N., Er, M.J.: Self-constructing adaptive robust fuzzy neural tracking control of surface vehicles with uncertainties and unknown disturbances. IEEE Trans. Contr. Syst. Technol. **23**(3), 991–1002 (2015)
15. Moren, J., Balkenius, C.: A computational model of emotional learning in the amygdala: from animals to animals. In: Proceedings of the 6th International Conference on the Simulation of Adaptive Behaviour (2000)
16. Sharbafi, M.M., Lucas, C., Daneshvar, R.: Motion control of omni-directional three-wheel robots by brain-emotional-learning-based intelligent controller. IEEE Trans. Syst. Man Cybern. C **40**(6), 630–638 (2010)
17. Roshanaei, M., Vahedi, E., Lucas, C.: Adaptive antenna applications by brain emotional learning based on intelligent controller. IET Microw. Antennas Propag. **4**(12), 2247–2255 (2010)
18. Dehkordi, B.M., Parsapoor, A., Moallem, M., Lucas, C.: Sensorless speed control of switched reluctance motor using brain emotional learning based intelligent controller. J. Energy Convers. Manag. **52**(1), 85–96 (2011)
19. Daryabeigi, E., Zarchi, H.A., Markadeh, G.R., Soltani, J., Blaabjerg, F.: Online MTPA control approach for synchronous reluctance motor drives based on emotional controller. IEEE Trans. Power Electron. **30**(4), 2157–2166 (2015)
20. Khalghani, M.R., Khooban, M.H.: A novel self-tuning control method based on regulated bi-objective emotional learning controller's structure with TLBO algorithm to control DVR compensator. Appl. Soft Comput. **24**(11), 912–922 (2014)
21. Hsu, C.F.: Intelligent total sliding-mode control with dead-zone parameter modification for a DC motor driver. IET Control Theory Appl. **8**(11), 916–926 (2014)
22. Lin, P.Z., Hsu, C.F., Lee, T.T., Wang, C.H.: Robust fuzzy-neural sliding-mode controller design via network structure adaptation. IET Control Theory Appl. **2**(12), 1054–1065 (2008)
23. Wang, L.X.: Adaptive fuzzy systems and control: design and stability analysis. Prentice-Hall, Englewood Cliffs (1994)
24. Juang, C.F., Juang, K.J.: Reduced interval type-2 neural fuzzy system using weighted bound-set boundary operation for computation speedup and chip implementation. IEEE Trans. Fuzzy Syst. **21**(3), 477–491 (2013)
25. Lin, F.J., Tan, K.H., Fang, D.Y., Lee, Y.D.: Intelligent controlled three-phase squirrel-cage induction generator system using wavelet fuzzy neural network for wind power. IET Renew. Power Gener. **7**(5), 552–564 (2013)
26. Lin, F.J., Hung, Y.C., Chen, J.M., Yeh, C.M.: Sensorless IPMSM drive system using saliency back-EMF-based intelligent torque observer with MTPA control. IEEE Trans. Ind. Inform. **10**(2), 1226–1241 (2014)
27. Lin, C.M., Li, H.Y.: A novel adaptive wavelet fuzzy cerebellar model articulation control system design for voice coil motors. IEEE Trans. Ind. Electron. **59**(4), 2024–2033 (2012)
28. Hsu, C.F., Lin, C.M., Yeh, R.G.: Supervisory adaptive dynamic RBF-based neural-fuzzy control system design for unknown nonlinear systems. Appl. Soft Comput. **13**(4), 1620–1626 (2013)
29. Hsueh, Y.C., Su, S.F., Tao, C.W., Hsiao, C.C.: Robust L_2-gain compensative control for direct-adaptive fuzzy-control-system design. IEEE Trans. Fuzzy Syst. **18**(4), 661–673 (2010)
30. Chen, H.K.: Chaos and chaos synchronization of a symmetric gyro with linear-plus-cubic damping. J. Sound Vib. **255**(4), 719–740 (2002)
31. Lin, C.M., Lin, M.H., Yeh, R.G.: Synchronization of unified chaotic system via adaptive wavelet cerebellar model articulation controller. Neural Comput. Appl. **23**(3), 965–973 (2013)
32. Hsu, C.F., Chang, C.W.: Intelligent dynamic sliding-mode neural control using recurrent perturbation fuzzy neural networks. Neurocomputing **173**(3), 734–743 (2016)
33. Huang, M.T., Lee, C.H., Lin, C.M.: Type-2 fuzzy cerebellar model articulation controller-based learning rate adjustment for blind source separation. Int. J. Fuzzy Syst. **16**(3), 411–421 (2014)
34. Hung, Y.C., Lin, F.J., Hwang, J.C., Chang, J.K., Ruan, K.C.: Wavelet fuzzy neural network with asymmetric membership function controller for electric power steering system via improved differential evolution. IEEE Trans. Power Electron. **30**(4), 2350–2362 (2015)
35. Mai, T., Wang, Y.: Adaptive force/motion control system based on recurrent fuzzy wavelet CMAC neural networks for condenser cleaning crawler-type mobile manipulator robot. IEEE Trans. Contr. Syst. Technol. **22**(5), 1973–1982 (2014)

Chun-Fei Hsu received the B.S., M.S., and Ph.D. degrees in Electrical Engineering from Yuan-Ze University, Taiwan, Republic of China in 1997, 1999, and 2002, respectively. During 2007–2011, he joined the faculty of the Department of Electrical Engineering, Chung Hua University, Taiwan, Republic of China, where he was appointed as an Associate Professor. In 2012, he joined the faculty of the Department of Electrical Engineering, Tamkang University, Taiwan, Republic of China and he is currently a Professor of Electrical Engineering. His research interests include adaptive control, flight control, and intelligent control using fuzzy system and neural network technologies. The outstanding achievement of his research is for contributions to real-time intelligent control in practical applications. He was a recipient of the Young Automatic Control Engineering Award from the Chinese Automatic Control Society in 2007, and the Outstanding Young Award from the Taiwan Association of Systems Science and Engineering in 2011.

Tsu-Tian Lee is currently the National Endow Chair of Ministry of Education, Tamkang University. He received his Ph.D. degree in Electrical Engineering from the University of Oklahoma, OK, in 1975. Previously, he had served as a Professor and Chairman of the Department of Control Engineering at National Chiao Tung University, as a Visiting Professor (1987), as a Full Professor of Electrical Engineering at the University of Kentucky, KY (1988–1990), USA, as Professor and Chairman of the Department of Electrical Engineering, National Taiwan University of Science and Technology (NTUST), as Dean of the Office of Research and Development, NTUST, and as a Chair Professor of the Department of Electrical and Control Engineering, NCTU. From February 1, 2004 to January 31, 2011, he had served as the President of National Taipei University of Technology (NTUT). He had served as a Chair Professor at the Department of Electrical Engineering, Chung Yuan Christian University during 2011–2013. Since 2014, he has been with the Department of Electrical Engineering, Tamkang University, as a Chair Professor. He received the Distinguished Research Award from the National Science Council, Taiwan, during 1991–1998, the Academic Achievement Award in Engineering and Applied Science from the Ministry of Education, Taiwan, in 1997, the National Endow Chair from the Ministry of Education, Taiwan, in 2003 and 2006, respectively, and the TECO Science and Technology Award from TECO Technology Foundation in 2003. He was elected to the grade of IEEE Fellow in 1997. He became a Fellow of IET in 2000, a Fellow of New York Academy of Sciences (NYAS) in 2002, and a Fellow of Chinese Automatic Control Society (CACS) in 2007. In 2003, he received the IEEE SMC Society Outstanding Contribution Award, and in 2009, IEEE SMC Society Norbert Wiener Award. He had served as Vice President for Membership and Students Activities (two terms), and Vice President for Conferences and Meetings, respectively, for the IEEE Systems, Man, and Cybernetics Society, and General Chair, General Co-Chair, Program Chair, and Invited Session Chair in many IEEE sponsored international conferences.