# Identification and damage assessment of real buildings using recursive hybrid genetic algorithm

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ABSTRACT: It is intended to develop a new system identification method which can identify the change of parameters. The new method developed here is called the recursive hybrid Genetic Algorithm. In the development of this new algorithm, the time history of the measurement is divided into a series of time intervals, and then the model of equivalent linear system is employed to identify the modal parameters of the system and the initial displacement and velocity for each time interval.. The new method developed here is called the recursive hybrid Genetic Algorithm. This new algorithm are applied to the identification of the two real building which are the office building of National Center for Research on Earthquake Engineering (NCREE) and the building of Taitung Fire department located in Taiwan. The parameters of these structures are identified using the response records of the structure under the attack of earthquakes of different intensities. By monitoring the variation of the identified parameters, the damage assessment of these structures is performed and the damage states of these structures are evaluated.

### 1 INTRODUCTION

Field of system identification has become an important discipline due to the increasing need to estimate the behavior of a system with partially known dynamics. Identification is basically a process of developing or improving the mathematical model of a dynamic system through the use of measured experimental data. In addition to updating the structural parameters for better response prediction, system identification techniques also made possible to monitor the current state or damage state of the structures.

Most of the identification methods are calculus-based search method. A good initial guess of the parameter and gradient or higher-order derivatives of the objective function are generally required. There is always a possibility to fall into a local minimum. On the other hand, genetic algorithms (GAs) are optimization procedures inspired by natural evolution. They model natural processes, such as selection, recombination, and mutation, and work on populations of individuals instead of a single solution. In this regard, the algorithms are parallel and global search techniques that search multiple points, and they are more likely to obtain a global solution. Many GA applications have been performed on a variety of optimization problems in engineering area. However,

relatively few applications have been on structural identification. Koh et al. (2003) proposed a hybrid strategy of exploiting the merits of GA and local search operator. Two local search methods were studied: an existing SW method and a proposed method called the MV method. The numerical study showed that the hybrid strategy performs better than the GA alone. The author (Wang & Lin 2005) applied the real-coded GA to structural identification problems. The validity and the efficiency of the proposed GA strategy were explored for the cases of systems with simulated input/output measurements. Moreover, the strategy was also applied to the real structure. Genetic algorithms (GAs) are global search techniques for optimization. However, GAs are inherently slow, and are not good at hillclimbing. In order to accelerate the convergence to the optimal solutions, a hybrid GA identification strategy that employs Gauss-Newton method as the local search technique was also proposed and verified by the author (Wang 2009)

Various structural health monitoring method are developed to determine the location and extent of damage. The importance of developing robust monitoring systems that can detect and locate progressive deterioration in structures or abrupt damage induced by extreme loading events is well recognized in the aerospace, mechanical, and civil engineering communities. Identifying the change in the structural stiffness and damping is the most straightforward

approach for damage assessment. However, detecting structural damage and identifying damaged elements in a large complex structure are challenging problems since the measured data of large civil engineering structures are noise contaminated and often incomplete. Chase et.al. (2005) directly identified changes in structural stiffness due to modeling error, using a structural health monitoring method based on adaptive least mean square (LMS) filtering theory. Ratcliffe (1997) developed and presented a technique for identifying the location of structural damage in a beam. The procedure operates solely on the mode shape from the damaged structures and does not require a priori knowledge of the undamaged structure. The procedure is developed using a one-dimensional finite element model of a beam and demonstrated by experiment. The procedure is best suited to the mode shape obtained from the fundamental natural frequency. The mode shapes from higher natural frequencies can be used to verify the location of damage but they are not as sensitive as the lower modes. Another approach of damage assessment proposed by Yuen et. al. (2004) is the twostage damage assessment method. In the first stage, the modal parameters were identified using the measured structural response from the undamaged system and then from the possibly damaged system. In the second stage, these data were used to update a parameterized structural model of the system using Bayesian system identification. The approach allowed one to obtain not only estimates of the stiffness parameters but also the probability that damage in any substructure exceeds any specified threshold expressed in terms of a fractional stiffness loss. It successfully identified the location and severity of damage in all cases of the benchmark problem.

The damage of the buildings induced by the Chi-Chi earthquake provides solid evidence that some of the structures have experienced inelastic response. Consequently, the parameters of some real buildings may vary with the change of the amplitude of vibration, so that the identified parameters may not reflect the real state of the structure if we only use a model of a linear system to represent the structure. In this regard, a new system identification method called recursive hybrid genetic algorithm, which can identify the change of parameters, will be proposed and developed for the purpose of identifying the system parameters of the buildings with accelerographs installed. In the development of this new algorithm, the time history of the measurement will be divided into a series of time intervals, and then the model of equivalent linear system will be employed to identify the modal parameters of the system and the initial displacement and velocity for each time interval as well. The process of exploring this new algorithm will be proceeded by using response of the simulated SDOF system and MDOF system considering the effect of noise contamination. Finally, this recursive

hybrid genetic algorithm will be applied to the identification of two real building. The first structure identified will be the office building of National Center for Research on Earthquake Engineering (NCREE) in Taiwan. The parameters of this structure are identified using the response records of the structure under the attack of earthquakes of different intensities. In order to implement the recursive hybrid GA to nonlinear system, the time history of the measurement will be divided into a series of time intervals. Then, the model of equivalent linear system will be employed to identify the modal parameters of the system for each time interval. The other building identified will be the building of Taitung Fire department, which experienced severe damage during the earthquake, occurred on April 1, 2006. The same algorithm will be applied to this structure and the damage indices are then computed according to the identified parameters. By monitoring the variation of the identified parameters, the damage assessment of these structures will be performed and the damage states of these structures will be evaluated.

# 2 HYBRID GENETIC ALGORITHM AND DAMAGE INDEX

#### 2.1 Genetic algorithm

Genetic algorithm is a stochastic search technique based on natural selection and genetics, developed by Holland (1962). Genetic algorithms model natural processes, such as selection, recombination, mutation, migration, and competition. The algorithms work on populations of individuals instead of a single solution. In this way, the search is performed in a parallel manner. However, better results can be obtained by introducing multiple subpopulations. Every subpopulation evolves over a few generation isolated (like the single population GA) before one or more individuals are exchanged between subpopulation using the mechanisms of migration and competition. The multipopulation GA models the evolution of a species in a way more similar to nature than single population

# 2.2 Hybrid GA combining genetic algorithm and Gauss-Newton method

The GA is a parallel and global search technique that searches multiple points and makes no assumption about the search space. However, GAs are inherently slow and are poor at hill-climbing. In order to compensate the computational inefficiency in hill-climbing when the solution yielded by GA approaches the optimal value, a local search operator compatible to GA is merged to the GA strategy. The Gauss-Newton method is the local search operator used in this paper and is performed after completing the evolution process of every 10 generations. Accordingly, a new hybridization of a GA with Gauss-

Newton method is formed. Figure 1 shows the structure for such a hybrid GA.

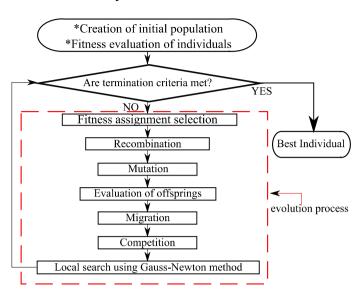


Figure 1. Structure of hybrid GA

#### 2.3 Damage index

Quantitative measurement of structural damage during earthquakes has always been a challenging problem to the structural engineers. Various damage indices have been proposed with the objective of quantifying the structural damage in prototype and model structures subjected to seismic excitation. These indices make use of different parameters such as drift, natural period of structure, energy absorption and cyclic fatigue in estimating the damage level.

Dipasquale and Cakmak (1990)] define the maximum softening for the one-dimensional case, where only the fundamental modal frequency is considered. The index is given by

$$D_M = 1 - \frac{T_{und}}{T_m} \tag{1}$$

where  $T_{und}$  is the undamaged natural period and  $T_m$  is the maximum natural period during the excitation duration. The maximum softening demonstrates a measure of combination of both the stiffness degradation and plasticity effect.

# 3 RECURSIVE HYBRID GENETIC ALFGORITHM

The parameters of real buildings may vary with the change of the amplitude of vibration, so that the identified parameters may not reflect the real state of the structure if we only use a model of a linear system to represent the structure. In this regard, a new system identification method which can identify the change of parameters will be proposed and devel-

oped for the purpose of identifying the system parameters of the buildings with accelerographs installed. The new method developed here is called the recursive hybrid Genetic Algorithm. In the development of this new algorithm, the time history of the measurement is divided into a series of time intervals, and then the model of equivalent linear system is employed to identify the modal parameters of the system and the initial displacement and velocity for each time interval as well. The process of exploring this new algorithm is proceeded by using response of the simulated single degree of freedom (SDOF) system and multiple degree of freedom (MDOF) system considering the effect of noise contamination.

#### 3.1 Identification model for SDOF system

The motion equation of a single degree of freedom linear system when excited by a uni-directional earthquake ground acceleration is

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = -\ddot{u}_{\sigma} \tag{2}$$

where  $\xi$  = damping ratio,  $\omega$  = natural frequency, and  $\ddot{u}_g$  = ground acceleration in one direction. The measured response is the relative acceleration and can be represented as

$$y = \ddot{u} = -\ddot{u}_g - 2\xi\omega\dot{u} - \omega^2 u = -\ddot{u}_g - A_1\dot{u} - A_3u$$
 (3)

where  $A_1=2\xi\omega$  and  $A_3=\omega^2$ . Actually,  $A_1$  and  $A_3$  are the parameters needed to be identified in this case. In order to optimize the system, error function is defined in such a way that it is quadratic in terms of the parameters and is denoted as the error index , E.I., for the system

$$E.I = \left[\frac{\sum_{i=1}^{N} (y_i - v_i)^2}{\sum_{i=1}^{N} y_i^2}\right]^{1/2}$$
 (4)

Equation (4) is also used to define the objective function in GA.

In the new method, the excitation measurement and the response measurement are automatically divided into non-overlapping time intervals or time windows by setting the number of time instants or sampling points, na, in the beginning. In order to account for the effect of initial condition of each interval, the initial velocity and displacement are also implemented as the parameters to be identified in addition to the system parameters  $A_1$  and  $A_3$ . Then the model of equivalent linear system and the hybrid GA is employed to identify the modal parameters of the system and the initial displacement and velocity for each time interval. The best five individuals or system parameters are replicated and used as the individuals of the initial population of

next interval. This process will be continued until the measurement of all the rest of the time interval are implemented. Figure 2 shows the procedure of the proposed recursive hybrid genetic algorithm where the red dashed box represented the evolution process of hybrid genetic algorithm illustrated in Figure 1.

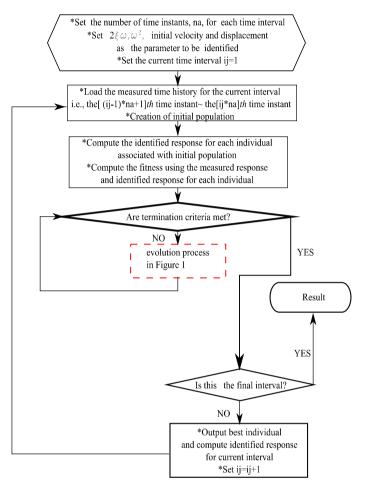


Figure 2. Structure of recursive hybrid GA for SDOF system

## 3.2 Identification of a SDOF linear system

In order to verify the accuracy of the proposed recursive hybrid GA strategy to system identification problem, we apply the new strategy to the SDOF linear systems with  $2\xi\omega = 0.232 \cdot \omega^2 = 44.256$ . Set the number of time instants for each interval to be 500. Identified parameters associated  $\omega^2$  are shown in Figure 3. The identified values are the same as the true values. Figure 4 illustrates the comparison of the true acceleration with the identified one. From the error index in the figure, the same conclusion can be drawn, too.

For realistic simulation, the time histories of the applied excitation as well as the acceleration response of the mass were noise contaminated. Consequently, the identification strategy should preferably be not too sensitive to the noise of the input and output measurements. The proposed recursive hybrid strategy is also explored for noise level of 5%. Fig-

ure 5 shows true parameter versus identified parameter for each interval. Since the identified parameters fluctuate, the average value of identified parameter  $\omega^2$  is also computed as 44.118 and plotted in the same figure for comparison. The average identified parameter is close to the true one. In order to demonstrate the relation between the fluctuations with the noise level, the noise level for each interval is plotted in Figure 6. From the above two figures, we can see that the parameter value deviates from the true value more if the noise level is higher. Figure 7 illustrates the comparison of the true acceleration with the identified one. The error index computed for this case is consistent with the noise level and the average identified parameters are close to the true ones. Consequently, the strategy is demonstrated to be able to identify the system parameters even though the signals are contaminated.

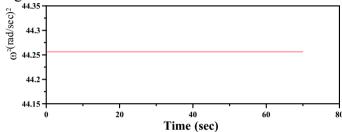


Figure 3. Identified parameter for a SDOF system (na = 500)

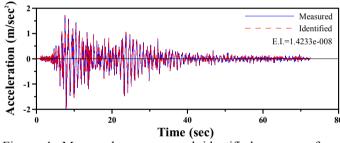


Figure 4. Measured response and identified response for a SDOF system (na = 500)

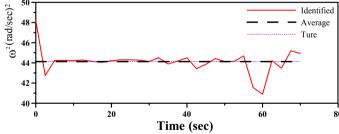


Figure 5. Identified parameters for SDOF system with 5% noise level ( na = 500)

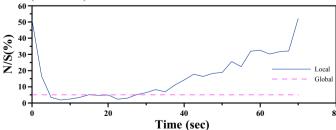


Figure 6. Variation of noise level for each time interval (na = 500)

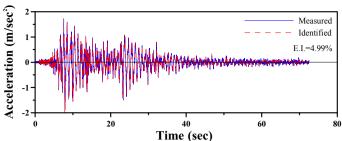


Figure 7. Measured response and identified response for a SDOF system with 5% noise level (na = 500)

#### 3.3 Identification of a nonlinear SDOF system

In order to demonstrate the accuracy of the proposed strategy to nonlinear SDOF system problem, the excitation is divided into four segments and the response for each segment is computed using different parameter values.

For this case, the number of time instants for each interval is set to be 200. The identified parameters associated with the frequency are shown in Figure 8. Figure 9 illustrates the comparison of true acceleration measurement with the identified response for this case. The error index computed for this case is extremely small and the identified parameters are exactly the values we set. Consequently, the strategy is demonstrated to be able to identify the system parameters even though the system is nonlinear.

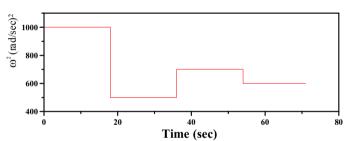


Figure 8. Identified parameters for a SDOF nonlinear system

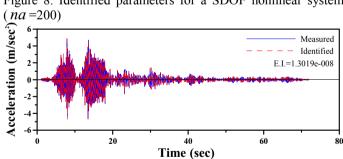


Figure 9. Measured response and identified response for SDOF nonlinear system (na = 200)

#### 3.4 Identification of a MDOF linear system

In this section, we will apply the proposed strategy to the identification of a multiple degree of freedom system with single output or measurement. The motion equation of a MDOF system subjected to single excitation can be transformed into modal equation through mode superposition as

$$\ddot{u}_{sm} + 2\xi_m \omega_m \dot{u}_{sm} + \omega_m^2 u_{sm} = -P_{sm} \ddot{u}_g \tag{5}$$

where  $u_{sm}$  is the modal displacement in mode m at the s<sup>th</sup> DOF, and  $P_{sm}$  the effective participation factor in mode m at the s<sup>th</sup> DOF associated with the ground motion  $\ddot{u}_{o}$ .

The identified parameters associated with the first modal frequency are also shown in Figure 10 for the case of 5% noise level. Figure 11 illustrates the comparison of true acceleration measurement with the identified response for this case. The error index computed for this case is 4.98% which is consistent with the noise level. In summary, it can be concluded that the proposed recursive hybrid GA strategy is not sensitive to the noise involved in the input and output measurements for MDOF system, too.

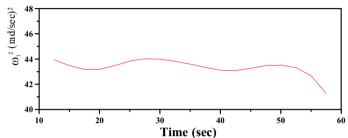


Figure 10. Identified parameters for MDOF system with 5% noise level (na = 500)

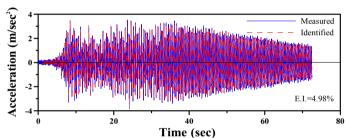


Figure 11. Measured response and identified response for MDOF system with 5% noise level (na = 500)

#### 4 IDENTIFICATION AND DAMAGE ASSESSMENT OF REAL BUILDINGS

### 4.1 *Identification and damage assessment of* NCREE building

The first real building identified here is the office building of National Center for Research on Earthquake Engineering (NCREE) located in Taipei, Taiwan. This building is a 6-story reinforced concrete structure. Thirty channels of seismographs are installed. Seismographs on the basement will be used as the input ground motion, and those on the third floor and the six floor (or the top floor) be used as the measured response. Four sets of strong motion records collected during earthquakes occurred on July 11 of 1998(with magnitude of 5.07), Chi-Chi earthquake on September 21 of 1999 (with magnitude of 7.3), 1022 earthquake or Chia-Yi earthquake on October 22 of 1999 (with magnitude of 6.4) and 614 earthquake on June 14 of 2001 (with magnitude of 6.3) are analyzed. This building seems to experience no visible damages under the attacks of the

earthquakes. However, the dynamic parameters may be altered even though the damage of the structure is slight or invisible.

To get knowledge of this damage state of the structure, single-input-single-output model is utilized to perform the identification of modal parameters of the structure. Through spectral analysis, estimation of frequencies is made and rational ranges of these parameters can be obtained when performing the recursive hybrid GA identification procedure. Modal damping ratio, modal frequencies, and participations factors can be obtained on the basis of either longitudinal (L) or transverse (T) measurements. As a result, by monitoring the variation of the identified parameters, the damage assessment of the structure will be performed and the damage state of the structure will then be evaluated.

We can study the variation of the identified parameters by plotting them in the same figure. Figure 12 shows the change of the modal frequency for the motions recorded on the top floor in the longitudinal direction during these earthquakes. From the plot of the fundamental frequency, we can see that the modal frequency maintained at around 24.5 rad/sec during the struck of the first earthquake (EQ before 921). During the struck of the Chi-Chi earthquake, the fundamental frequency went up to 25 rad/sec and then dropped to 23 rad/sec at the end of the earthquake. One month later, the fundamental frequency during the 1022 earthquake returned to about 25.5 rad/sec and again dropped to 23 rad/sec during the last part of the earthquake. Finally, the fundamental frequency during the 614 earthquake increased to about 23.5 rad/sec, which is about 4.1% lower than the value 24.5 rad/sec yielded during the first earthquake.

Figure 13 shows the change of the modal frequency for the motions recorded on the top floor in the transverse direction during these earthquakes. The modal frequency stayed at around 23 rad/sec during the struck of the first earthquake. The fundamental modal frequency decreased from 23rad/sec to 21.5rad/sec under the struck of 921 earthquake. During the 1022 earthquake, the fundamental frequency started from 23 rad/sec and then went down to 22 rad/sec. Finally, the fundamental frequency increased to 22.5rad/sec at the end of the last earthquake. In conclusions, the modal frequency decreased from 23 rad/sec to 22.5rad/sec.

The measured response of the third floor can also be employed to study the variation of modal parameters in both directions. Similar conclusions as those using the measured response on the top floor can be drawn.

The damage assessment of this building using the maximum softening index will be performed using the results of identification performed above as follows:

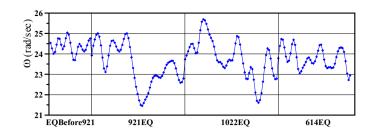


Figure 12. Variation of modal parameters on top floor in the longitudinal direction for NCREE building

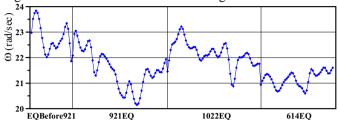


Figure 13. Variation of modal parameters on top floor in the transverse direction for NCREE building

The maximum softening index  $D_M$  is evaluated using the results of the identified modal frequency of the top floor in the longitudinal direction. The parameter identified at the beginning of the earthquake on July 11, 1998 is implemented as the undamaged frequency or the baseline frequency. The maximum softening index of the first mode,  $D_M$ , is then evaluated and plotted in Figure 14. To compare the results with the intensities of the earthquake, Figure 15 shows the measured response of the top floor in the longitudinal direction. The result seems to follow the usual trend of increasing maximum softening index (damage) with increasing excitation amplitude since that the structural stiffness will decrease with the increase of the measured response amplitude. The index increased to 13% during the strong motion part of the Chi-Chi earthquake. Finally, this index returned to 5%.

The result in the transverse direction is illustrated in Figure 16. Figure 17 also shows the measured response of the top floor in the associated direction. The index of the first mode increased to 13% during the major part of the Chi-Chi earthquake. Finally, this index returned to 6%.

The results using the identified modal frequency of the third floor are similar to those using the modal parameters of the top floor. The trend of these indices is similar to that of the indices using the parameters of the top floor.

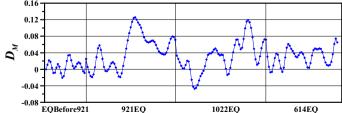


Figure 14. Variation of maximum softening index in the longitudinal direction for NCREE building

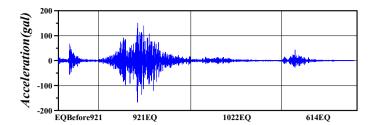


Figure 15. Measured response on top floor in the longitudinal direction for NCREE building

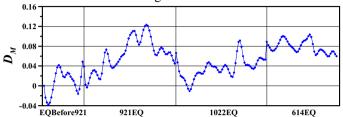


Figure 16. Variation of maximum softening index in the transverse direction for NCREE building

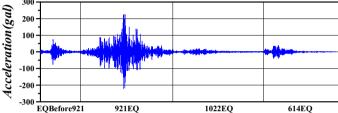


Figure 17. Measured response on top floor in the transverse direction for NCREE building

## 4.2 Identification and damage assessment of Taitung Fire Department building

The second real building identified here is the Taitung Fire Department building located in Taitung, Taiwan. This four-story reinforced concrete building experienced severe damage during the 2006 Taitung Beinan Earthquake (or 401 Earthquake with magnitude M=6.2). Three-component seismographs are installed on the first floor and the top floor of the building. Accelerograms collected during earthquakes can be utilized to monitor the change of the structural parameters. Three sets of strong motion records collected during 912 earthquake, 1022 earthquake, and 401 earthquake. To get knowledge of the damage state of the structure, single-input-singleoutput model is also utilized to perform the identification of modal parameters of the structure. Figures 18 and 19 show the variation of the modal frequency for the motions recorded on the top floor in the longitudinal and transverse directions during these earthquakes. During the first two earthquakes, the modal frequencies maintained at about 23.5 rad/sec and 32 rad/sec, respectively. Under the attack of the 401 earthquake, the modal frequencies for both directions dropped to 7 rad/sec and returned to 12 rad/sec and 11 rad/sec, respectively, at the end of attack.

The maximum softening index of the first mode,  $D_M$ , and the measured response of the top floor in the longitudinal direction are plotted in Figure 20 & 21. The index increased to 68 % during the strong

motion part of the 401 earthquake and returned to 55% at the end. Figures 22 and 23 show the results associated with the transverse direction. The index moved up to 75 % during the strong motion part of the 401 earthquake and dropped d to 65% at the end.

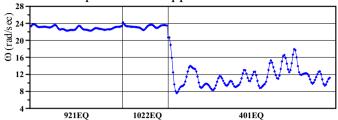


Figure 18. Variation of modal parameters on top floor in the longitudinal direction for Taitung Fire Department



Figure 19. Variation of modal parameters on top floor in the transverse direction for Taitung Fire Department

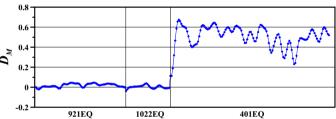


Figure 20. Variation of maximum softening index in the longitudinal direction for Taitung Fire Department

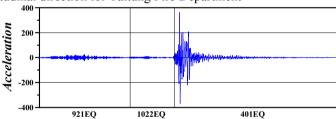


Figure 21. Measured response on top floor in the longitudinal direction for Taitung Fire Department

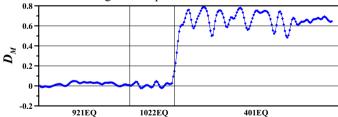


Figure 22. Variation of maximum softening index in the transverse direction for Taitung Fire Department

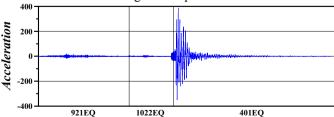


Figure 23. Measured response on top floor in the transverse direction for Taitung Fire Department

#### 5 CONCLUSIONS

This paper developed a new identification strategy called the recursive hybrid genetic algorithm for nonlinear system. The time history of the measurement is divided into a series of time intervals, and then the model of equivalent linear system is employed to identify the modal parameters of the system and the initial displacement and velocity for each time interval. The numerical accuracy can be improved a lot especially for the case of nonlinear behavior and the following conclusions can be made:

- 1. The new recursive hybrid GA identification strategy has been applied to the simulated input/output measurements of SDOF linear and nonlinear dynamic systems as well as a MDOF linear dynamic system. The identified parameters are very close to the true one and the error index is extremely small in each case. Also, the identified response and the measured response are almost overlapped in all the cases. Consequently, the applicability of the propose strategy to structural dynamic parameter identification is proved. Moreover, the strategy is also shown to be not sensitive to the noise contamination. This assures the feasibility of future application to the measurements of real systems.
- 2. From the identification results of National Earthquake Engineering Research Center office building, we can see that the modal frequency reduced under the struck of 921 earthquake and regained a little bit after the struck of the 614 earthquake. The maximum softening index  $D_M$  was alsoevaluated using the results of the identified modal frequency. This index increased to 13% during the strong motion part of the Chi-Chi earthquake and returned to  $5\sim6\%$  after the struck of the 614 earthquake. The damage is quite slight and invisible for this building.
- 3. According to the identification results of Taitung Fire Department building, the change of the modal frequencies in the transverse direction and longitudinal direction can be neglected during the 921 earthquake and 1022 earthquake. However, the reduction of the modal frequencies for both directions is quite significant and can not be neglected during the 401 earthquake. Also from the results of damage assessment, the damage can be concluded to be a serious state under the attack of 401 earthquake, since the maximum softening index increased to 68% and 75% during the strong motion part of the 401 earthquake, and returned to 55% and 65% at the end of the earthquake in the transverse direction and longitudinal direction, respectively.

#### **ACKNOWLEDGEMENT**

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### REFERENCES

- Chase, J. G. & et. al. 2005. A simple LMS-based approach to structural health monitering benchmark problem. *Int. Journal of Earthquake Engineering and Structural Dynamics*. 34: 575-594.
- Dipasquale, E. & Cakmak, A.S. 1990. Seismic damage assessment using linear models. *Soil Dynamics and Earthquake Engineering*. 9(4): 194-215.
- Holland, J.H. 1962. Outline for a logical theory of adaptive systems. *Journal of the association for computing machinery* 3: 297-314
- Koh, C.G. Chen, Y.F. & Liaw, C.Y. 2003. A hybrid computational strategy for identification of structural parameters. *Computers and Structures*. 81: 107-117.
- Ratcliffe, C.P. 1997. Damage detection using a modified Laplacian operator on mode shape data. *Journal of Sound and Vibration*. 204(3): 505-517.
- Wang, G. S. & Lin, H.H. 2005. Structural parameter identification of torsionally-coupled buildings. *Journal of the Chi*nese Institute of Civil Engineering and Hydraulic Engineering 17(2): 281-291.
- Wang, G.S. 2009. Application of hybrid genetic algorithm to system identification. Structural Control and Health Monitoring 16(2): 125-153.
- Yuen, K. V. Siu, K. A. & Beck, J.L. 2004. Two-stage Structural Health Monitoring Approach for Phase I Benchmark Studies. *Journal of Engineering Mechanics*, ASCE. 130(1): 16-33.