Stat Papers (2011) 52:419–429
DOI 10.1007/s00362-009-0239-3

REGULAR ARTICLE

Exponentially weighted moving average control charts for three-level products

Tzong-Ru Tsai · Wen-Pin Yen

Received: 1 September 2008 / Revised: 25 May 2009 / Published online: 5 June 2009

Abstract In this paper, exponentially weighted moving average (EWMA) control charts for multinomial data are developed with a three-level classification scheme. The lower and upper control limits of the proposed EWMA control chart are evaluated using Markov chain approximation. Compared with the three-level Shewhart control chart, numerical results indicate that the proposed EWMA control chart is relatively sensitive to small shifts in a three-level multinomial process. A figure and a table are provided for practitioners to select the value of chart limit coefficient that gives the desired in-control average run length.

Keywords Average run length · EWMA control chart · Markov Chain · Quality value function · Shewhart control chart

1 Introduction

When consumers select competing products and services, quality has become one of the most concern factors. A good understanding and keep improving quality key factors are essential for leading to business success, growth and enhanced competitiveness. Statistical process control (SPC) consists of a collection of statistical methods for monitoring and improving the quality and productivity of industrial processes and service operations. The control chart is one of the most commonly used methods in SPC applications, which is used to determine whether a production process is in a statistical control or not. The general theory of typical control charts was first proposed by

T.-R. Tsai (✉) · W.-P. Yen
Department of Statistics, Tamkang University, Tamsui, Taipei, 251 Taiwan, ROC
e-mail: trtsai@stat.tku.edu.tw

Dr. Walter A. Shewhart while working for Bell Labs in 1920s. Control charts developed according to these principles of Dr. Shewhart are call the Shewhart control charts.

Shewhart control charts often are constructed for a variables or an attributes quality characteristic of interest, and they are commonly used to monitor and detect shifts in a process. However, a quality characteristic may be measured using three or more discrete levels in some situations. For example, a glass container product can be classified into one of the three categories called "conforming", "marginal" or "nonconforming", depends on its bursting-strength and surface-finish defects. The problem of SPC with some discrete-level classification schemes rather than a variables or an attributes measure have been discussed by Shapiro and Zahedi (1990), Bray et al. (1973), Clements (1978), Newcombe and Allen (1988), Bray et al. (1973), Shah and Phatak (1977), Clements (1983).

In decision-making applications, a multi-level quality measure does not adequately quantify the quality of products. Marcucci (1985) had considered the problem of three-level SPC. Cassady and Nachlas (2003) proposed a three-level classification scheme which classifies the quality of a product into one of the three categories called "conforming", "marginal" or "nonconforming". The problems of constructing acceptance sampling plans and Shewhart control charts based on the three-level classification scheme have been discussed by Cassady and Nachlas (2003, 2006).

Three-level Shewhart control charts are a quite good tool at detecting process shifts for three-level products, and they are easy to operate for practitioners. However, the three-level Shewhart control chart is insensitive at detecting small process shifts for three-level products. The Shewhart-type control chart only used the sample information contained in the last plotted test statistic, and it ignores any information about the entire sequence of points. Quality engineers may consider to construct a three-level control chart using other charting schemes, such as the EWMA control scheme or cumulative sum (CUSUM) control scheme to enhance the ability at detecting small shifts in a three-level multinomial process. Compared with the CUSUM control scheme, the performance of the EWMA control scheme is approximately equivalent to that of the CUSUM control scheme in most situations, and the EWMA control scheme is easier to operate. These reasons motivate us to develop a control chart based on an EWMA control scheme to enhance the ability at detecting small shifts in a three-level multinomial process. The developed control chart is called the three-level EWMA control chart.

Roberts (1959) introduced the EWMA control chart and evaluated the chart performance by simulation. Crowder (1989), Ng and Case (1989), Borror et al. (1999) and Borror et al. (1998) have studied the properties of the EWMA control chart. In this paper, the design of three-level EWMA control chart is developed using a zero-state Markov chain approximation. Average run lengths (ARLs) of the proposed three-level EWMA control charts are also investigated.

In Sect. 2 we review the design of the three-level Shewhart control chart proposed by Cassady and Nachlas (2006). The construction of the three-level EWMA control chart is developed in Sect. 3. Performance comparison of the three-level EWMA control chart with the three-level Shewhart control chart is conducted based on the length of out-of-control ARL numerically in Sect. 4. Finally, concluding remarks are given in Sect. 5.

2 Three-level Shewhart control charts

Assume that a random sample of n products are collected form a process, and the quality of each product is quantified as a quality value according to the following three-level quality value function (QVF):

$$\text{QVF}: V = \begin{cases} v_1, & \text{if the item is conforming,} \\ v_2, & \text{if the item is marginal,} \\ v_3, & \text{if the item is nonconforming,} \end{cases} \tag{1}$$

where $0 < v_1 < v_2 < v_3$. Larger quality values imply lower quality of product. Let $V_1, V_2, \ldots, V_n$ denote the sample quality values, and let N_k be the number of products which are classified into the category k, $k = 1, 2, 3$. It follows that observations $\boldsymbol{N} = (N_1, N_2, N_3)$ has a multinomial distribution with classification probabilities $\boldsymbol{p} = (p_1, p_2, p_3)$, where p_1, p_2 and p_3 are probabilities of that a product is classified as "conforming", "marginal" and "nonconforming", respectively, and $p_1 + p_2 + p_3 = 1$. When the process is in control, $\boldsymbol{p} = \boldsymbol{p}_0 = (p_{01}, p_{02}, p_{03})$, where p_{01}, p_{02} and p_{03} are specified probabilities such that the quality of products can meet the desired level. Accordingly, the joint probability of $\boldsymbol{N}$, given $\boldsymbol{p}_0$ is given by

$$\begin{aligned} G_a\left(\boldsymbol{n}|\boldsymbol{p}_0\right) &= P\left(N_1 = n_1, N_2 = n_2, N_3 = n_3 | p_1 = p_{01}, p_2 = p_{02}, p_3 = p_{03}\right) \\ &= \frac{n!}{n_1!n_2!n_3!} p_{01}^{n_1} p_{02}^{n_2} p_{03}^{n_3}, \quad \text{if } \sum_{k=1}^{3} n_k = n, \end{aligned} \tag{2}$$

and 0 otherwise. Let $\overline{V} = \sum_{i=1}^{n} V_i/n$ denote the sample mean, and let μ_0 and σ_0 be the mean and standard deviation of quality values of products, respectively, when the process is in control. It follows that $\mu_0 = \sum_{k=1}^{3} v_k p_{0k}$ and $\sigma_0^2 = \sum_{k=1}^{3} v_k^2 p_{0k} - \mu_0^2$. The lower control limit (LCL) and upper control limit (UCL) of the three-level Shewhart control chart are given below (see Cassady and Nachlas 2006):

$$\begin{aligned} \text{UCL} &= \mu_0 + \ell \frac{\sigma_0}{\sqrt{n}}, \\ \text{LCL} &= \mathtt{max}\left\{\mu_0 - \ell \frac{\sigma_0}{\sqrt{n}}, 0\right\}, \end{aligned}$$

where $\ell(>)0$ is a positive constant. We conclude that the process is in control if $\text{LCL} < \overline{V} < \text{UCL}$.

Let μ_v and σ_v denote the mean and standard deviation of V_i, respectively. They can be expressed in terms of μ_0 and σ_0 as follows:

$$\begin{aligned} \mu_v &= \delta_1 \mu_0, \\ \sigma_v &= \delta_2 \sigma_0. \end{aligned}$$

If the process is in control, then $\delta_1 = \delta_2 = 1$. Let $Y = \sqrt{n}(\overline{V} - \mu_v)/\sigma_v$. The joint probability of $\boldsymbol{N}$ in (2) can be rewritten as

$$G_a\left(\boldsymbol{n}|\boldsymbol{p}\right) = P\left(\frac{\sqrt{n}\mu_0(1-\delta_1)}{\delta_2\sigma_0} - \frac{\ell}{\delta_2} < Y < \frac{\sqrt{n}\mu_0(1-\delta_1)}{\delta_2\sigma_0} + \frac{\ell}{\delta_2}\right).$$

Since $V_1, V_2, \ldots, V_n$ are independent and identically distributed random variables, it follows that Y converges in distribution to the standard Normal by using a Central Limit Theorem approximation. Let α denote the probability of a false alarm given that the process is in control. The equation $\Phi(\ell) - \Phi(-\ell) = 1 - \alpha$ can be used to select the value of ℓ that gives the desired in-control ARL$= 1/\alpha$, where $\Phi(\cdot)$ is the cumulative distribution function of the standard Normal.

3 Three-level EWMA control charts

Assume that samples each of size n are collected every $h(> 0)$ units time from a three-level multinomial process which has classification probabilities $\boldsymbol{p} = (p_1, p_2, p_3)$. When the process is in control, $\boldsymbol{p} = \boldsymbol{p}_0$. Let $\overline{V}_1, \overline{V}_2, \ldots$ be the average of each sample, and let

$$Y_t = \frac{\overline{V}_t - \mu_0}{\sigma_0/\sqrt{n}}, \quad t = 1, 2, \ldots$$

The EWMA statistic is defined at tth sample as

$$Z_t = \lambda\, Y_t + (1 - \lambda\,)\, Z_{t-1},$$

where $\lambda(0 < \lambda \leq 1)$ is the smoothing value. The EWMA statistic Z_t can be expressed as a moving average of the current and past observations by

$$Z_t = \lambda \sum_{i=0}^{t-1} (1 - \lambda\,)^i\, Y_{t-i} + (1 - \lambda\,)^t\, Z_0$$

If the process is in control, the starting value $Z_0 = 0$. It is straightforward to show that $E(Z_t) = 0$ and $Var(Z_t) = \frac{\lambda(1-(1-\lambda)^{2t})}{2-\lambda}$, $t = 1, 2, \ldots$. For large values of t, the variance of the EWMA statistic is approximately

$$Var(Z_t) \approx Var(Z_\infty) = \frac{\lambda}{(2-\lambda)}. \tag{3}$$

Accordingly, the asymptotic LCL and UCL of the three-level EWMA control chart can be defined, respectively, as follows:

$$h_U = E(Z_t) + A_U \times \sqrt{Var(Z_\infty)} = A_U \times \sqrt{\frac{\lambda}{(2-\lambda)}},$$
$$h_L = E(Z_t) - A_L \times \sqrt{Var(Z_\infty)} = -A_L \times \sqrt{\frac{\lambda}{(2-\lambda)}}, \quad (4)$$

where A_U and A_L are positive constants. The values of A_U and A_L are often selected to be equal although it sometimes may be advantageous to establish asymmetric control charts. Quality personnel can select values of A_U and A_L those give the desired in-control ARL. The paper uses the condition $A_U = A_L = A$ to develop the three-level EWMA control chart.

Two objectives are specified for employing control charts: (1) if the production process is in control, we require the control chart to signal (false alarm) as we have planned it to do; (2) if the control chart is out of control, we require the control chart to signal as soon as possible. The ARL has long been the benchmark characteristic used to evaluate the performance of a control chart. In this paper, computation of the ARL for the three-level EWMA control chart is developed based on a zero-state Markov chain approximation, which is proposed by Lucas and Saccucci (1987, 1990).

Let $c = (h_U - h_L)/N$, $U_0 = h_L$, $U_j = h_L + jc$, $j = 1, 2, \ldots, N-1$ and $U_N = h_U$. Partition the in-control region (h_L, h_U) into N equal-spaced subintervals as in Fig. 1, and denote the jth subinterval as $I_j = (U_{j-1}, U_j]$, $j = 1, 2, \ldots, N$. Let

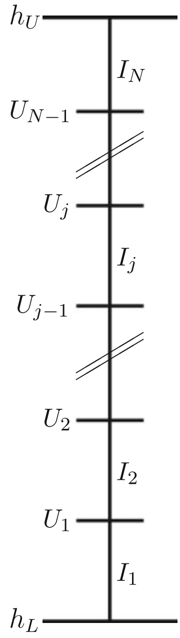

Fig. 1 Dividing the in-control region into N subintervals

$w = c/2$, and let $I_{N+1} = (-\infty, h_L] \bigcup [h_U, \infty)$ denote the absorbing state. The midpoint m_i of the ith subinterval can be represented as

$$m_i = h_L + \frac{(2i-1)}{2} c = h_L + (2i-1)\, w, \quad i = 1, 2, \ldots, N.$$

When the process is in control, the EWMA statistic Z_t is in state j at time t if $U_{j-1} < Z_t < U_j$, $j = 1, 2, \ldots, N$. The transition probability, P_{ij}, is the probability that the next step will be j given that the current state is i, and is given by

$$P_{ij} = P\left\{Z_t \in I_j | Z_{t-1} \in I_i\right\} \quad i, j = 1, 2, \ldots, N.$$

It is straightforward to show that $U_{j-1} = m_j - w$ and $U_j = m_j + w$, and P_{ij} can be rewritten as

$$P_{ij} = P\left\{\frac{(m_j - w) - (1-\lambda)m_i}{\lambda} \le Y_t \le \frac{(m_j + w) - (1-\lambda)m_i}{\lambda}\right\}, \quad i, j = 1, 2, \ldots, N. \tag{5}$$

For $i = N+1$ or $j = N+1$, $P_{N+1,j} = 0$, $j = 1, 2, \ldots, N$, $P_{i,N+1} = 1 - \sum_{j=1}^{N} P_{ij}$, $i = 1, 2, \ldots, N$ and $P_{N+1,N+1} = 1$. The one-step transition probability from the initial state to state j is given by

$$\begin{aligned} P_j &= P\left\{Z_1 \in I_j | Z_0 = 0\right\} \\ &= P\left\{\frac{(m_j - w)}{\lambda} \le Y_1 \le \frac{(m_j + w)}{\lambda} | Z_0 = 0\right\}, \quad j = 1, 2, \ldots, N. \end{aligned} \tag{6}$$

The ARL of the control chart is the average time of the Markov chain to absorption. Let $\mathbf{Q} = (P_{ij})$ be the matrix obtained from the transition matrix by deleting row $N+1$ and column $N+1$. That is, $\mathbf{Q}$ is the transition matrix among the in-control state. The zero-state ARL of the three-level EWMA control chart can be found by

$$\text{ARL} = \boldsymbol{r}^{\mathrm{T}}(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1}, \tag{7}$$

where $\boldsymbol{r}^{\mathrm{T}} = (P_1, P_2, \ldots, P_N)$, $\mathbf{I}$ is an $N \times N$ identity matrix and $\mathbf{1}$ is an $N \times 1$ vector with entries 1.

For a given value of λ, practitioners can select the value of A that gives the desired in-control ARL. In general, an EWMA control scheme is good at detecting small to moderate process shifts with small values of λ and is good at detecting large process shift with larger values of λ. A good rule of thumb is to select $\lambda = 0.05$, $\lambda = 0.1$ and $\lambda = 0.2$ to detect small to moderate shifts in a parameter in practical applications (see Montgomery 2005).

Figure 2 shows plots of the in-control ARLs as a function of A, in which the in-control region is divided into $N = 101$ subintervals for the Markov chain approximation. The determination of N is discussed in Sect. 4. The computation of ARLs based on the Markov chain approximation was implemented using programs written in R (see The R Project for Statistical Computing; http://www.r-project.org/).

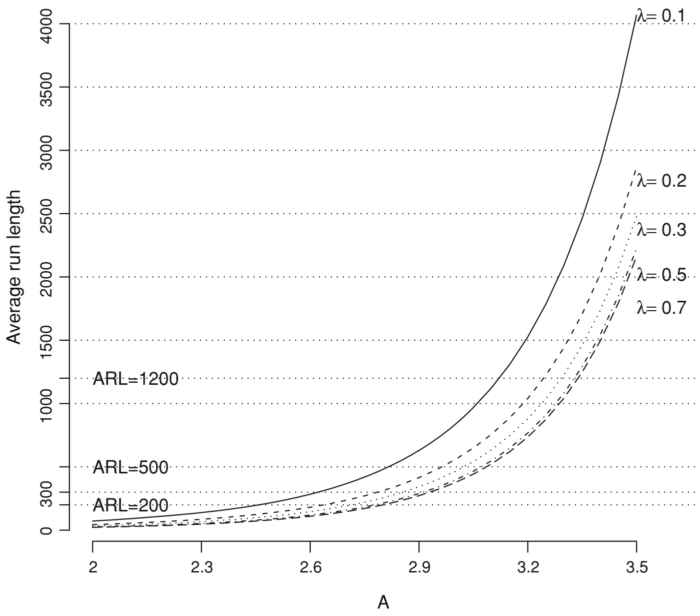

Fig. 2 In-control ARLs for various values of A with $\lambda = 0.1, 0.2, 0.3, 0.5, 0.7$

Suppose for illustration that $\lambda = 0.2$ and a desired in-control ARL 370. What should the value of A be to make the in-control ARL equal to the desired level 370? Figure 2 indicates that $A = 2.861$ yields an in-control ARL around 370 for $\lambda = 0.2$. Values of A for in-control ARLs 200, 250, 370 and 500 with several EWMA control schemes $\lambda = 0.1, 0.2, 0.3, 0.5, 0.7$ and various sizes of N are given in Table 1. Practitioners can select the value of A from Fig. 2 or Table 1 to construct the three-level EWMA control chart.

In a similar way, we can combine a three-level Shewhart control chart and a three-level EWMA control chart to develop a three-level Shewhart-EWMA control chart following the procedure of Lucas and Saccucci (1990). The LCL and UCL of the three Shewhart-EWMA control chart are given below:

$$\mathrm{UCL}_{\mathrm{SEWMA}} = \mathtt{min}\left\{\mathrm{SCL}_{\mathrm{U}}, \mathtt{max}\{\mathrm{SCL}_{\mathrm{L}}, h_L\}\right\},$$
$$\mathrm{LCL}_{\mathrm{SEWMA}} = \mathtt{max}\left\{\mathrm{SCL}_{\mathrm{L}}, \mathtt{min}\{\mathrm{SCL}_{\mathrm{U}}, h_U\}\right\}.$$

The combination makes the selection of A depend on the value of ℓ, and makes the selection of ℓ become complicated and subjective. Some authors recommended using slightly wider than usual limits on the Shewhart-type control chart, for example, $\ell = 3.25$ or 3.5 to develop the Shewhart-EWMA control chart. However, practitioners need to select the value of ℓ carefully. If the main propose in the charting procedure is to detect small process shifts, the EWMA control schemes is a better choice and is easier to operate than the combined Shewhart-EWMA control chart.

Table 1 Values of A for various sizes of N under the in-control ARL of 200, 250, 370 and 500

N	ARL = 200					ARL = 250				
	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$
5	2.717	2.812	2.819	2.818	2.813	2.830	2.916	2.909	2.897	2.886
11	2.524	2.671	2.734	2.786	2.803	2.626	2.759	2.815	2.861	2.875
31	2.463	2.640	2.716	2.779	2.800	2.556	2.724	2.795	2.853	2.872
51	2.458	2.637	2.714	2.778	2.800	2.550	2.721	2.793	2.852	2.872
81	2.456	2.637	2.713	2.778	2.800	2.548	2.720	2.792	2.852	2.872
101	2.455	2.636	2.713	2.778	2.800	2.548	2.720	2.792	2.852	2.872
151	2.455	2.636	2.713	2.778	2.800	2.547	2.719	2.792	2.852	2.872
201	2.455	2.636	2.713	2.778	2.800	2.547	2.719	2.792	2.852	2.872
N	ARL = 370					ARL = 500				
	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$
5	3.019	3.095	3.066	3.031	3.012	3.156	3.225	3.181	3.130	3.105
11	2.800	2.908	2.954	2.989	2.999	2.927	3.016	3.055	3.083	3.090
31	2.714	2.866	2.929	2.980	2.996	2.828	2.969	3.027	3.073	3.087
51	2.707	2.862	2.927	2.979	2.996	2.820	2.965	3.025	3.072	3.087
81	2.704	2.861	2.926	2.979	2.996	2.817	2.964	3.024	3.072	3.086
101	2.704	2.861	2.926	2.979	2.996	2.816	2.963	3.024	3.072	3.086
151	2.703	2.860	2.926	2.979	2.995	2.815	2.963	3.024	3.072	3.086
201	2.703	2.860	2.926	2.979	2.995	2.815	2.963	3.024	3.072	3.086

4 Numerical studies

When constructing a three-level EWMA control chart, the size of N needed to be determined in advance. In general, N should be large enough to support a good Markov chain approximation. A numerical study is conducted to evaluate the effect of N on the Markov chain approximation. When the value of N is large enough to support a good Markov chain approximation, an adequate value of A can be selected which gives the desired in-control ARL.

Table 1 gives values of A for several EWMA control schemes and various sizes of N under the in-control ARL of 200, 250, 370 and 500, respectively. Based on the numerical experience, dividing the in-control region into at least 81 subintervals is enough to give an adequate value of A. We choose N to be odd here so that there is a unique middle value.

A further numerical study is conducted for evaluating the performance of the three-level EWMA control chart and doing performance comparison of the three-level EWMA control chart and three-level Shewhart control chart. All numerical results are given in Table 2. Assume that products are produced by a company, and the quality of each product is classified into three levels, "conforming", "marginal" and "nonconforming". The quality values are measured according to the QVF in (1) with $v_2 = 0.2$,

Table 2 Comparison of the three-level EWMA control chart and the three-level Shewhart control chart

v	p_1	p_2	p_3	μ	σ	EWMA control schemes					Shewhart Chart
						$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$	
	0.87	0.10	0.03	0.050	0.031	49.402	51.947	53.937	55.394	57.402	63.547
	0.85	0.12	0.03	0.054	0.031	23.558	26.442	28.813	33.130	36.111	43.557
0.2	0.88	0.08	0.04	0.056	0.040	15.293	15.772	16.435	17.723	19.914	24.607
	0.83	0.14	0.03	0.058	0.032	12.276	13.505	15.002	18.541	22.309	29.818
	0.81	0.16	0.03	0.062	0.032	8.314	8.355	9.104	11.085	13.369	19.694
	0.87	0.08	0.05	0.066	0.048	6.195	5.652	5.555	5.991	6.583	8.253
	0.85	0.08	0.07	0.086	0.065	3.123	2.624	2.414	2.254	2.265	2.548
	0.84	0.09	0.07	0.088	0.065	2.977	2.502	2.283	2.106	2.107	2.339
	0.83	0.09	0.08	0.098	0.074	2.463	2.057	1.862	1.691	1.644	1.733
	0.82	0.09	0.09	0.108	0.081	2.159	1.783	1.600	1.445	1.396	1.415
	0.88	0.09	0.03	0.075	0.046	50.607	54.074	57.983	64.112	72.315	85.954
	0.87	0.10	0.03	0.080	0.048	34.487	37.682	40.459	46.529	51.686	61.414
0.5	0.89	0.06	0.05	0.080	0.058	27.051	27.765	29.056	31.984	35.829	44.567
	0.85	0.12	0.03	0.090	0.051	10.627	11.014	12.161	14.798	17.860	25.570
	0.83	0.14	0.03	0.100	0.055	5.764	5.266	5.345	5.981	7.296	11.196
	0.81	0.16	0.03	0.110	0.057	4.058	3.531	3.363	3.432	3.869	5.417
	0.85	0.08	0.07	0.110	0.077	4.082	3.541	3.313	3.274	3.480	4.403
	0.84	0.09	0.07	0.115	0.079	3.615	3.079	2.855	2.707	2.858	3.465
	0.83	0.09	0.08	0.125	0.086	2.985	2.508	2.288	2.113	2.125	2.388
	0.82	0.09	0.09	0.135	0.094	2.558	2.136	1.919	1.730	1.686	1.790
	0.88	0.08	0.04	0.096	0.070	42.247	45.646	48.910	53.980	58.501	67.386
	0.87	0.10	0.03	0.100	0.069	27.345	30.369	33.682	39.290	45.099	56.466
0.7	0.87	0.08	0.05	0.106	0.078	14.607	15.459	16.462	19.351	22.532	29.425
	0.85	0.12	0.03	0.114	0.075	8.994	8.856	9.253	10.757	13.267	18.988
	0.85	0.08	0.07	0.126	0.093	5.176	4.625	4.496	4.744	5.362	7.343
	0.83	0.14	0.03	0.128	0.082	4.914	4.398	4.345	4.547	5.279	7.552
	0.84	0.09	0.07	0.133	0.096	4.278	3.741	3.574	3.623	3.951	5.218
	0.81	0.16	0.03	0.142	0.088	3.468	2.958	2.766	2.665	2.839	3.632
	0.83	0.09	0.08	0.143	0.103	3.412	2.904	2.696	2.583	2.675	3.203
	0.82	0.09	0.09	0.153	0.110	2.901	2.447	2.229	2.043	2.035	2.324

0.5, 0.7, respectively. The larger value of v, the worse quality of the marginal products. Assume that if the process is in control, the probabilities of "conforming", "marginal" and "nonconforming" are $p_{10} = 0.89$, $p_{20} = 0.08$, $p_{30} = 0.03$, respectively. The corresponding mean and standard deviation are $\mu_0 = 0.070$ and $\sigma_0 = 0.212$, respectively. A three-level Shewhart control chart is constructed with $\ell = 3$, and three-level EWMA control charts are constructed with $A = 2.704, 2.861, 2.926, 2.979, 2.996$ for $\lambda = 0.1$, 0.2, 0.3, 0.5, 0.7, respectively based on the desired in-control ARL 370. We divide the in-control region into 101 subintervals for the Markov chain approximation. Those values of A can be found from Table 1 or Fig. 2. Both types of three-level control charts are used to monitor the process of three-level products.

Combinations of p_1, p_2 and p_3 are selected and given in Table 2 to simulate the scenario of process shifts in quality deterioration. When more lower-quality products with levels of "marginal" or "nonconforming" are produced during the process shifts, we expect the control chart has a short ARL to give an out-of-control signal soon. All ARLs in Table 2 are evaluated numerically based on 10,000 replications. Each replication is conducted as follows: Assume that the in-control classification probabilities p_{10}, p_{20} and p_{30} are unknown in practice. They are estimated from a pre-sample of 100 observations generated from a multinomial distribution with classification probabilities p_{10}, p_{20} and p_{30}. Both three-level control charts are established based on the pre-sample. For each combination of probabilities of Table 2, samples each of 100 observations are generated sequentially from the multinomial distribution with the selected combination of probabilities. Test statistics of both three-level control charts are computed based on the same samples and plotted on both control charts, separately. The individual charting procedure continues till the control chart gives an out-of control signal, and the corresponding sample number at the time is recorded as a run length of the control chart. When both three-level control charts give an out-of-control signal, a replication is finished. The out-of-control ARL for each three-level control chart for each combination of probabilities of Table 2 are computed by taking the average of 10,000 numerical values of run length of each control chart. Programs written in R are prepared for the numerical study.

As our expectation, the EWMA control scheme with small λ(=0.1, 0.2) is sensitive to small shifts. They give out-of-control signals quickly with shorter ARLs. Table 2 also indicates that all three-level EWMA control charts have shorter ARLs than those given by the three-level Shewhart control chart if the process shift is small. Otherwise, if process shift is large, both three-level control charts have near ARLs when the three-level EWMA control scheme has a value of λ larger than or equal to 0.2. The three-level EWMA control chart with $\lambda = 0.2$ performs well with short ARLs for nearly all process shifts.

5 Conclusions

The paper presents a way to develop control charts for three-level products using an EWMA scheme with a zero-state Markov chain approximation method. A figure and a table are provided to practitioners to select the value of the chart limit coefficient that gives the desired in-control ARL. Numerical results indicate that the three-level EWMA control chart gives an out-of-control signal quicker than that is given by the three-level Shewhart control chart to small process shifts. The three-level EWMA control chart with $\lambda = 0.2$ is recommended because the control chart performs well with short ARLs to process shifts.

Methods for selecting the QVF needed to be investigated, and the extra cost and time needs is another problem in today's industry. Extending the proposed method to adaptive control schemes or other control schemes, like double EWMA control scheme, or CUSUM control scheme may be fruitful areas in the future study.

Acknowledgments The authors wish to thank the editor and the anonymous referees for their constructive comments on this paper.

References

Bray D, Lyon D, Burr I (1973) Three-class attributes plans in acceptance sampling. Technometrics 15(3):575–585

Brror C, Champ C, Rigdon S (1998) Poisson EWMA control chart. J Qual Technol 30:352–361

Brror C, Montgomery D, Runger G (1999) Robustness of the EWMA control chart to non-normality. J Qual Technol 21:242–250

Clements J (1978) Standards and sampling plans for product safety. ASQC Technical Conference Transactions, 483–490

Clements J (1983) Trinomial sampling plans to match MIL-STD-105D. ASQC Qual Congr Trans 37:256–264

Crowder SV (1989) Design of exponentially weighted moving average schemes. J Qual Technol 21:155–162

Cassady C, Nachlas J (2003) Evaluating and implementing 3-level acceptance sampling plans. Qual Eng 15(3):361–369

Cassady C, Nachlas J (2006) Evaluating and implementing 3-level Control Charts. Qual Eng 18:285–292

Lucas JM, Saccucci MS (1987) Exponentially weighted moving average control schemes: Properties and enhancements. Drexel University Faculty Working Series Paper 87-4, Philadelphia, PA

Lucas JM, Saccucci MS (1990) Exponentially weighted moving average control schemes: properties and enhancements. Technometrics 32:1–12

Marcucci M (1985) Monitoring multinomial processes. J Qual Technol 17(2):86–91

Montgomery DC (2005) Introduction to statistical quality control, 5th edn. Wiley, New York

Newcombe P, Allen O (1988) A three-class procedure for acceptance sampling by variables. Technometrics 30(4):415–421

Ng C, Case K (1989) Development and evaluation of control charts using exponentially weighted moving averages. J Qual Technol 31:309–315

Roberts SW (1959) Control chart tests based on geometric moving averages. Technometrics 1:239–250

Shah D, Phatak A (1973) The maximum likelihood estimation under curtailed three-class attributes plans. Technometrics 19(2):159–166

Shapiro S, Zahedi H (1990) Bernoulli trials and discrete distributions. J Qual Technol 22(3):193–205