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SPACECRAFT TRAJECTORY DESIGN WITH PHOTONIC LASER PROPULSION IN THE TWO-BODY PROBLEMS

Shih-Hao Liu*and Fu-Yuen Hsiao †

This paper studies the trajectory design of spacecraft propelled by the photonic laser propulsion (PLP) system under the environment of two-body problem. The PLP system is an innovative technology, and generate continuous and tremendous power by consuming very small energy with repeated reflections of laser beam. Since 2011 trajectory characteristics has been investigated by Hsiao, but trajectory design was still not studied. This paper mainly focuses on the trajectory design. Trajectory design is often modeled as a two-point boundary value problem (2PBVP). However, conventional 2PBVPs may not be suitable for this problem due to certain constraints. In this paper an algorithm is proposed to determine initial conditions in the trajectory design. Theorem of contaction mapping is employed to developed the algorithm of initial-condition determination. Numerical simulations are presented to demonstrate the algorithm and potential applications.

INTRODUCTION

This paper studies the trajectory design of spacecraft propelled by the photonic laser propulsion (PLP) system under the environment of two-body problem. For the past few decades, many researchers has started out to research the photon thruster[1−4]. In 2002 Thomas R. Meyer et.[5] has discussed the idea of laser elevator by momentum transfer using an optical resonator, and six years later, Young K. Bae[6] propose the concept of the photonic laser propulsion. However, those researchers focuses more on the PLP thrust itself but less on the effect to the trajectory. Hence, this paper intends to investigate the trajectory, and to explore the constraints that may affect the use of the PLP system.

Recently, much attention has been focused on the continuous low thrust engine, which has been proofed its efficiency in the Deep Space 1 mission by NASA[8], and a lunar mission by European Space Agency(ESA)[9]. While several interplanetary missions demonstrated the use of low thrust engine, such as electric propulsion, as the main propulsion system of the spacecraft. A new idea which use the power of light has been researched for decades. But for a long time, we didn't see much applications in photon thrusters that because the photon thruster is highly inefficient in generating thrust, and this is the reason the photon thruster has been impractical in most of missions. While Photonic Laser Propulsion(PLP),

*Undergraduate Student; j23456796@hotmail.com.tw
†Corresponding Author; Associate Professor at Department of Aerospace Engineering, Tamkang University; Member AAS; Email: fyhsiao@mail.tku.edu.tw

invented by Dr.Young K. Bae has become one of the most important invention in recent years. While the most different between regular photon thruster and PLP is that with the intracavity arrangement the momentum will be transferred, thus specific thrust, can be multiplied by bouncing photons between high reflectance mirrors so that PLP can generate much more thrust with the same I_{sp}.

In this paper, we first review some simple facts and theories of the PLP system. The studies on the PLP system suggest that the thrust be continuous and constant. Then, based on the Newton's second law the equations of motion are derived. The Jacobi integral is also employed to prove that we are capable of traveling to any place. Moreover, we develop an algorithm to determine the required initial conditions for a specific mission. Since the system is highly nonlinear, our algorithm is inspired by the theorem of contraction mapping. Numerical simulations are presented to verify the algorithm. Missions to the L_2 point and the Mars are simulated. From the simulations, we demonstrate that the algorithm is very robotic. The mission times of these simulations also show that PLP is a very efficient power system for interplanetary traveling.

PHOTONIC LASER PROPULSION

Most conventional spacecraft burn chemical propellant to generate thrust. In this case, a lot of fuel must be carried onboard for an interplanetary mission, increasing the weight of the spacecraft. Many scientists have discussed the concept of using a laser to provide thrust. However, lasers are very inefficient at generating thrust. Transferring the momentum of photons to a spacecraft improves the efficiency of PLP. In this process, photons act as propellant. Although photonic engines have the largest specific impulse compared to conventional ones, they have the smallest thrust-to-power ratio [6]. The specific impulse is approximately $I_{sp} = 3.06 \times 10^7 s$ whereas the thrust-to-power ratio is approximately $T/P = 3.34 \times 10^{-9}$ N/W.

Bae proposed an active resonant optical cavity between two space platforms. In this design, the photon thrust F produced on each mirror is given by [6]

$$F = \frac{E}{ct}, \tag{1}$$

where E is the energy of each photon, c is the light speed, and t is the interaction time.

If $E/t = P$, where P is the laser output power through the output mirror, then $F = P/c$. Because the laser is bounced back and forth between highly reflective mirrors, the thrust is given by

$$F = \frac{2PR_mS}{c}, \tag{2}$$

where R_m is the mirror reflectance, and S is the apparent photon thrust amplification factor, defined as the ratio of the intracavity laser power to the extracavity laser power P. The term S is approximated by

$$S = \frac{1}{1 - R_m}. \tag{3}$$

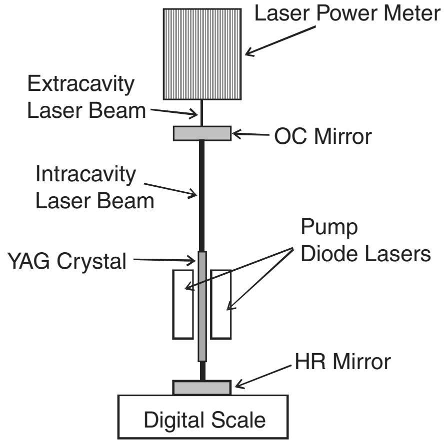

Figure 1. Schematic diagram of prototype PLT demonstration setup [6].

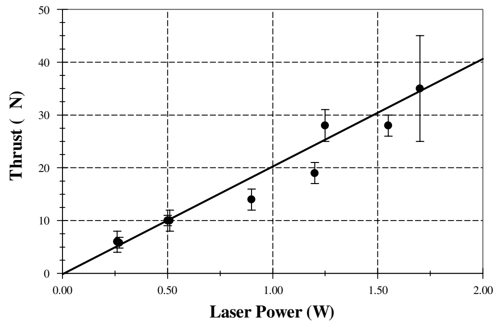

Figure 2. Photon thrust data obtained with an output coupler mirror with a reflectance of 0.99967 [6].

For a laser of constant power, the thrust F also remains constant. Figure 1 provides a proof-of-concept demonstration, and Fig. 2 presents some experimental data. According to Eqs. (2) and (3), the thrust is a continuous and stable force. Even though the force is very small, the continuous force keeps driving the spacecraft until it reaches the desired velocity. Figure 3 illustrates the application of PLP to a spacecraft. The launching process starts with a mother ship, which emits a laser beam to the mission ship to generate thrust. Because of the conservation of momentum, however, the mother ship moves in the opposite direction of the mission ship. Thus, a conventional thruster installed on the mission ship must act against the momentum caused by PLP to prevent the mother ship from falling out of orbit.

EQUATIONS OF MOTION

Figure 4 shows the relative locations of the central body, mother ship, and mission spacecraft. Let $\mathbf{r}$ be the position vector of the mission spacecraft, $\mathbf{R}$ the position vector of the

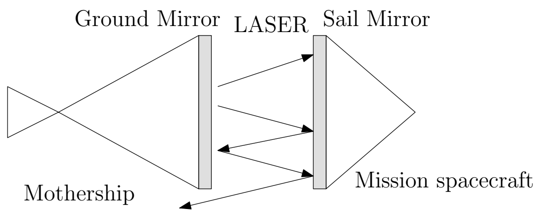

Figure 3. Diagram of the photonic laser propulsion system on a spacecraft. [5].

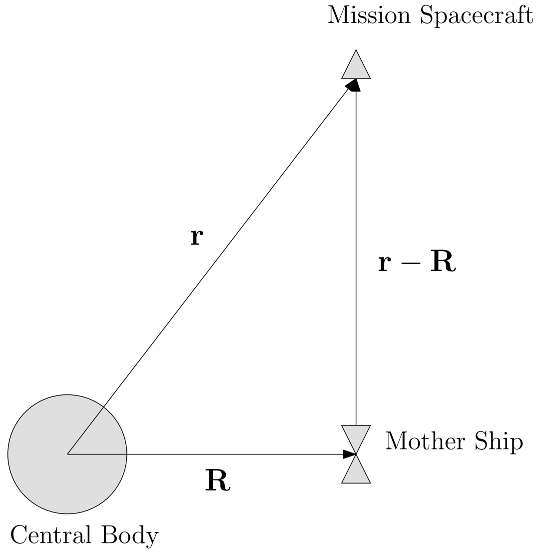

Figure 4. Relative positions between the central body, mother ship, and mission spacecraft.

mother ship, and let $\mathbf{r} - \mathbf{R}$ be the relative position of the spacecraft with respect to the mother ship. Assume the masses of the mother ship and the spacecraft are negligibly small so that they do not produce any gravitational force. According to Newton's gravitation law and Newton's second law of motion, the equations of motion (EOM) of the mother ship and spacecraft are given by

$$\ddot{\mathbf{R}} = -\frac{\mu}{R^3}\mathbf{R}, \tag{4}$$

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r} + F\hat{\mathbf{L}}, \tag{5}$$

where μ is the gravitational parameter of the central body, $\mathbf{L} = \mathbf{r} - \mathbf{R}$, $\hat{\mathbf{L}} = \mathbf{L}/|\mathbf{L}|$, and F is the PLP force given by Eq. (2). Note that Eqs. (4) and (5) are described in the inertial frame.

This study does not consider the reaction force by the PLP to the mother ship because in practical applications, this reaction can be counteracted by a traditional propulsion system.This study also assumes that F is constant and acts along the relative position of the mission spacecraft and the mother ship.

Normalization

Normalization can be used to enable a wider application of the EOM. Define

$$\begin{aligned}
\bar{\mathbf{r}} &= \frac{\mathbf{r}}{R} \\
&= \frac{X}{R}\mathbf{i} + \frac{Y}{R}\mathbf{j} + \frac{Z}{R}\mathbf{k} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \qquad (6) \\
\tau &= nt, \qquad (7)
\end{aligned}$$

and denote the derivative with respect to τ as $(\cdot)'$. The normalized Eq. (**??**) can then be written as

$$\begin{aligned}
x'' - 2y' &= x - \frac{1}{\bar{r}^3}x + \bar{F}\frac{x-1}{l}, \qquad (8) \\
y'' + 2x' &= y - \frac{1}{\bar{r}^3}y + \bar{F}\frac{y}{l}, \qquad (9) \\
z'' &= \frac{-1}{\bar{r}^3}z + \bar{F}\frac{z}{l}, \qquad (10)
\end{aligned}$$

where $\bar{r} = |\bar{\mathbf{r}}|$, $\mathbf{l} = (x-1)\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and $l = |\mathbf{l}|$. Moreover,

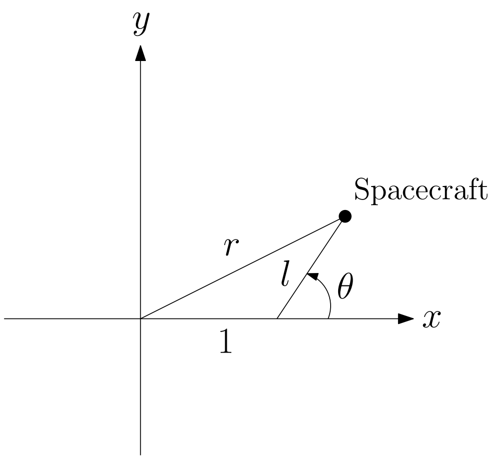

Figure 5. The normalized geometric relationship between the central body, mother ship, and mission spacecraft.

$$\bar{F} = F\frac{R^2}{\mu} \qquad (11)$$

is the ratio of thrust force per unit mass to the gravitational force exerted on the mother ship. The PLP thrust force is then given by $\bar{\mathbf{F}} = \bar{F}\hat{\mathbf{l}}$

JACOBI INTEGRAL

It is straightforward to show that $\nabla \times \bar{\mathbf{F}} = \mathbf{0}$ where $\nabla = \partial/\partial\bar{\mathbf{r}} = \partial/\partial x\mathbf{i} + \partial/\partial y\mathbf{j} + \partial/\partial z\mathbf{k}$. Hence, $\bar{\mathbf{F}}$ is conservative and can be represented as a force potential. Let

$$
\begin{aligned}
V(x, y, z) &= \frac{1}{2}(x^2 + y^2) + \frac{1}{\bar{r}} + \bar{F}l \qquad (12) \\
&= U_c + U_p, \qquad (13)
\end{aligned}
$$

where $U_c = (x^2 + y^2)/2 + 1/\bar{r}$ is the conventional force potential commonly used in astrodynamics, and $U_p = \bar{F}l$ is the pseudo force potential generated by the PLP system. This study claims that the right hand sides of Eqs. (8) to (10) are the gradient of $V(x, y, z)$.

As a result,

$$
\frac{\partial V}{\partial \bar{\mathbf{r}}} = \frac{\partial U_c}{\partial \bar{\mathbf{r}}} + \frac{\partial U_p}{\partial \bar{\mathbf{r}}} = x\mathbf{i} + y\mathbf{j} - \frac{1}{\bar{r}^3}\bar{\mathbf{r}} + \frac{\bar{F}}{l}\mathbf{l}. \qquad (14)
$$

For subsequent derivations, it is possible to write Eqs. (8) to (10) in brief notation by

$$
\bar{\mathbf{r}}'' - 2\mathbf{J}\bar{\mathbf{r}}' = V_{\bar{\mathbf{r}}}, \qquad (15)
$$

where $V_{\bar{\mathbf{r}}} = \partial V/\partial\bar{\mathbf{r}}$ and

$$
\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \qquad (16)
$$

Consequently, the Jacobi integral can be defined as

$$
J(\bar{\mathbf{r}}', \bar{\mathbf{r}}) = \frac{1}{2}(x'^2 + y'^2 + z'^2) - V(x, y, z). \qquad (17)
$$

It is then easy to show that

$$
\begin{aligned}
\frac{dJ}{d\tau} &= \frac{\partial J}{\partial \bar{\mathbf{r}}'} \cdot \frac{d\bar{\mathbf{r}}'}{d\tau} + \frac{\partial J}{\partial \bar{\mathbf{r}}} \cdot \frac{d\bar{\mathbf{r}}}{dt} \\
&= \bar{\mathbf{r}}' \cdot \bar{\mathbf{r}}'' - \frac{\partial V}{\partial \bar{\mathbf{r}}} \cdot \bar{\mathbf{r}}' \\
&= 0. \qquad (18)
\end{aligned}
$$

Equation (17) gives a constraint on the range within which a spacecraft can move using the proposed PLP system. If a spacecraft has an initial Jacobi integral of C_0, then the range within which it can travel is constrained by the zero-velocity surface embedded in $\mathbb{R}^3$. This surface is described by

$$
\frac{1}{2}\mathbf{r}' \cdot \mathbf{r}' = C_0 + V(x, y, z) = 0. \qquad (19)
$$

If $C_0 > 0$, the motion of the spacecraft has no spatial restriction. If $C_0 \leq 0$, the spacecraft can only move in the region satisfying $V(x, y, z) = -C_0 > 0$.

Unlike the traditional three body problem, the initial value of C_0 is not arbitrary. One of the most likely scenarios is that the mission spacecraft departs from the mother ship with negligible velocity and initial offset. It is natural to assume $\mathbf{r}_0 = 1\mathbf{i}$ and $\mathbf{r}'_0 = \mathbf{0}$. Thus,

$$C_0 = -\left(\frac{1}{2} + \frac{1}{1}\right) = -\frac{3}{2}. \tag{20}$$

SMALLEST PROPULSION TO LAUNCH

When a spacecraft travels along a trajectory, its Jacobi integral remains constant. Therefore, a spacecraft gains velocity by changing its position. Consider the planar motion(i.e., $z = 0$ and $z' = 0$). Equations (17) and (20) impose a constraint on the motion of the spacecraft by

$$\frac{1}{2}(x'^2 + y'^2) - V(x, y) = -\frac{3}{2}. \tag{21}$$

Hence,

$$\frac{1}{2}(x'^2 + y'^2) = -\frac{3}{2} + V(x, y). \tag{22}$$

Ref. 12 shows that $V(x, y) \geq 3/2$, leading to $(x'^2 + y'^2)/2 \geq 0$. Consequently, Eq. (22) is always satisfied, implying that the spacecraft can travel anywhere in space by providing a non-zero PLP force.

This statement is true if traveling time is not considered. In practical situations, however, traveling time is critical. Suppose no thrust is applied. A spacecraft with negative energy is confined by the gravity of the central body and never escapes. After thrust is applied, the larger the thrust, the faster the spacecraft travels. As a result, spacecraft propelled by small thrust takes longer to escape from gravity.

TRAJECTORY DESIGN ALGORITHM

Design Procedure

Suppose the location of destination is $\mathbf{r}_{des}$. Suppose the departure distance is r_0 and departure velocity is $\mathbf{v}_0 = 0$. Define

$$\Theta_{des} = \cos^{-1}\left(\frac{\mathbf{r}_{des} \cdot \hat{x}}{||\mathbf{r}_{des}||}\right) \tag{23}$$

$$l = ||\mathbf{r}(t) - 1\hat{x}|| \tag{24}$$

$$l_{des} = ||\mathbf{r}_{des} - 1\hat{x}|| \tag{25}$$

1. Let $\theta_0 = \Theta_{des}$, the departure coordinate will be $(r_0 \cos\theta_0, r_0 \sin_0)$.

2. Integrate trajectory until $l = l_{des}$. Suppose $\mathbf{r} = \mathbf{r}_i$ and $t = t_i$ at this instant.
3. Compute

$$\Delta\theta_i = \cos^{-1}\left(\frac{\mathbf{r}_i \cdot \mathbf{r}_{des}}{||\mathbf{r}_i||||\mathbf{r}_{des}||}\right). \tag{26}$$

4. Define $\theta_{i+1} = \theta_i - \Delta\theta_i$, repeat the iteration until $\Delta\theta_i \leq \epsilon$.

Since the final time is unknown, a reasonable trial is to set $t_{f,i+1} = 1.5t_i$. After the integration, reset $t_{f,i+1}$ by the criteria of $l = l_{des}$.

Numerical Simulations

Several simulations are demonstrated in Figs. 6 to 13. Figures 6 and 7 demonstrate trajectory design for missions to the L_2 point under different thrust level. Figures 8 to 13 demonstrate trajectory design for missions to Mars under different thrust level. Moreover, the Mars is assumed to locate at different place.

CONCLUSION

This paper studies the trajectory design of spacecraft propelled by the photonic laser propulsion (PLP) system under the environment of two-body problem. The PLP system is an innovative technology, and generate continuous and tremendous power by consuming very small energy with repeated reflections of laser beam. Since 2011 trajectory characteristics has been investigated by Hsiao, but trajectory design was still not studied. This paper mainly focuses on the trajectory design. Trajectory design is often modeled as a two-point boundary value problem (2PBVP). In this paper, we develop an algorithm to determine the required initial conditions for a specific mission. Since the system is highly nonlinear, our algorithm is inspired by the theorem of contraction mapping. Numerical simulations are presented to verify the algorithm. Missions to the L_2 point and the Mars are simulated. From the simulations, we demonstrate that the algorithm is very robotic. The mission times of these simulations also show that PLP is a very efficient power system for interplanetary traveling.

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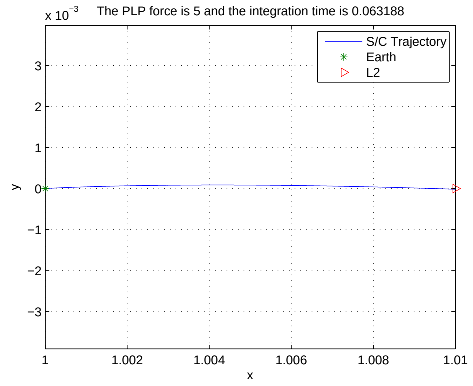

Figure 6. Mission to L_2 point with normalized PLP force of 5.

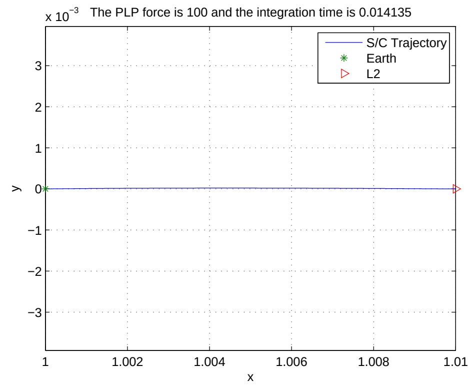

Figure 7. Mission to L_2 point with normalized PLP force of 100

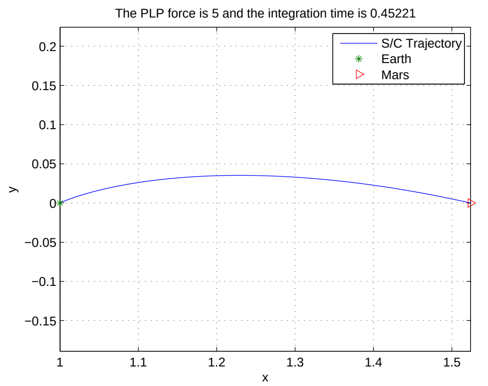

Figure 8. Mission to Mars with normalized PLP force of 5. The Mars is assumed to locate align with the earth.

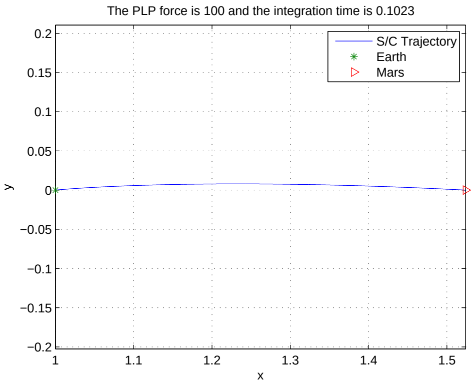

Figure 9. Mission to Mars with normalized PLP force of 100. The Mars is assumed to locate align with the earth.

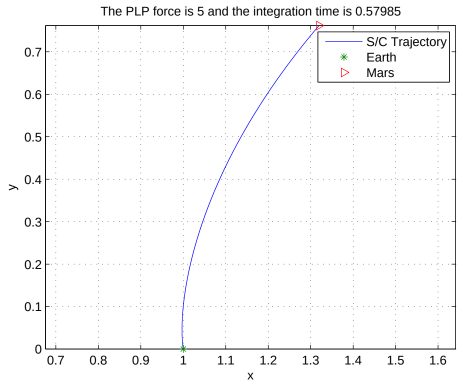

Figure 10. Mission to Mars with normalized PLP force of 5. The Mars is assumed to locate at angle of $30°$.

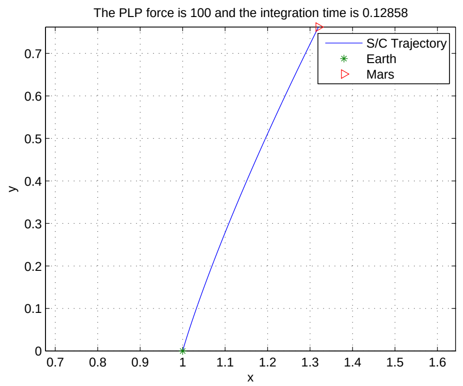

Figure 11. Mission to Mars with normalized PLP force of 100. The Mars is assumed to locate at angle of $30°$..

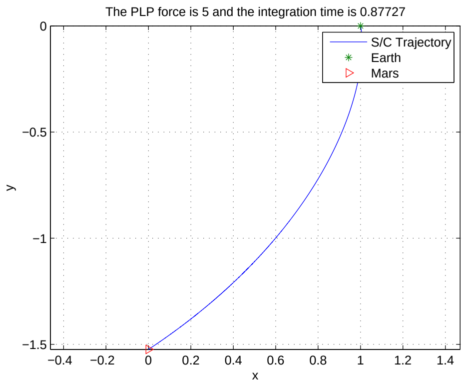

Figure 12. Mission to Mars with normalized PLP force of 5. The Mars is assumed to locate at angle of $-90°$.

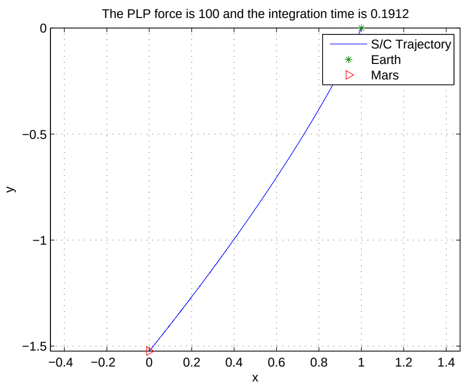

Figure 13. Mission to Mars with normalized PLP force of 100. The Mars is assumed to locate at angle of $-90°$.

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