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CONTACT FORCE ANALYSIS IN STATIC TWO-FINGERED ROBOT GRASPING

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ABSTRACT

Static grasping of a spherical object by two robot fingers is studied in this paper. The fingers may be rigid bodies or elastic beams, they may grasp the body with various orientation angles, and the tightening displacements may be linear or angular. Closed-form solutions for normal and tangential contact forces due to tightening displacements are obtained by solving compatibility equations, force-displacement relations based on Hertz contact theory, and equations of equilibrium. Solutions show that relations between contact forces and tightening displacements depend upon the orientation of the fingers, the elastic constants of the materials, and area moments of inertia of the beams.

1. INTRODUCTION

Robot grasping refers to the state that an object loses its mobility and moves along with robot fingers. Certain geometric shapes can only be grasped by frictional contacts [1]. But the presence of friction causes some difficulties in dynamic modeling and simulation. When Coulomb's law of friction is imposed on a system of rigid bodies, a dynamic problem may have no solution, or multiple solutions [2-3]. Dupond [4] showed that this unreasonable phenomenon can happen even in a single degree-of-freedom system, and he suggested imposing compliance to avoid this difficulty [4-5]. Howard and Kumar [6] hence include compliance and local deformations in their study of finger-object elastic contact. Normal and tangential contact forces are distributed in contact regions. Frictional grasping is expressed as a problem of variational calculus, and when the contact region is discretized into a series of cells, contact force distribution may be obtained by a numerical technique. It is uncertain if the model may always provide a unique solution. But when the contact is modeled as a system of lumped springs, a unique solution may always be obtained. A rigorous proof of existence and uniqueness of solution for a lumped-spring formulation is given by Kraus, et al [7].

Therefore, from the aspect of dynamic modeling, it is necessary to include compliance and deformation for frictional grasping; even only one finger is in contact. In solid mechanics, if the number of unknown forces exceeds the number of equilibrium equations, the problem is statically indeterminate, and solution can only be obtained by considering deformation. Cutkosky [8] treated the object as a deformable body and estimated its stiffness. Later, Cutkosky and Kao [9] estimated the compliance of grasp in terms of various parameters. Nguyen [10] developed a method to grasp objects using compliance, with which a planar polygon of any shape can always be grasped firmly. Nguyen [11] also showed that a firm force-closure grasp can always be carried out by elastic springs. Howard and Kumar [12] discussed conditions for stable grasps under static equilibrium, and derived conditions for a finger with compliance to firmly hold a planar object. Donoghue et al [13] and Howard and Kumar [14] obtained stability conditions for spatial work piece fixtures.

In many of the above-mentioned studies compliance is produced by linear springs, but Lin et al [15] pointed out that the linear spring model is not supported by experiments. Rimon and Burdick [16] showed this model may even produce erroneous results since curvature effects of the two bodies in contact can not be included. They used Hertz contact theory to produce nonlinear compliance, and obtained the stiffness matrix during a compliance grasping. In dealing with problems of frictional grasping, Sinha and Abel [17] imposed nonlinear force-displacement relations also in the tangential direction. They used a nonlocal friction law so that system potential energy may be defined, and by minimizing this potential energy, both normal and tangential contact forces were determined. Xydas and Kao [18-20], Li and Kao [21], and Kao and Yang

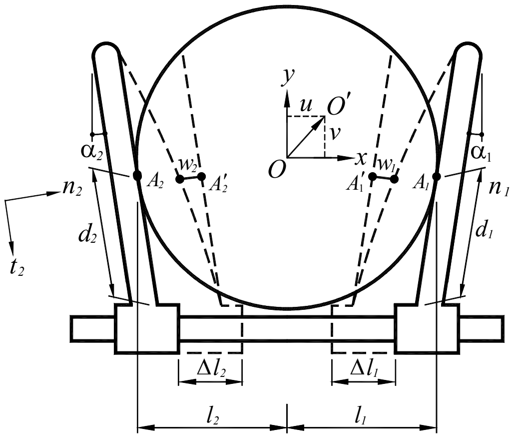

Figure 1 A Two-fingered gripper that may impose linear tightening displacement.

[22] estimated stiffness and compliance for a soft finger. Lin et al [23] established mathematical models grasping flexible objects. Ghafoor et al [24] used line springs based on screw theory to obtain stiffness which includes rotational effects. Khurshid et al [25] included viscoelastic material behavior in modeling two fingered gasping. Wu and Dong [26] performed two dimensional finite element analysis of frictionless contact of human fingers, and later Wu [27] performed the three dimensional analysis. Zivanovic and Vukobratovic [28] established dynamic models for multi-arm cooperative grasping, from which simulation results were obtained.

In cases with enveloping grasps, force-closures are often enforced by tightening displacements [29]. In such cases force-displacement relations are useful tools to estimate magnitudes of grasping forces during gripper design and simulation. The purpose of this study is to determine solutions for static contact forces produced by tightening displacements of two-fingered grippers. Instead of using simplified models with springs and lumped mass, we use Hertz contact theory for elastic bodies. The fingers may be soft or rigid, and the gripper may impose linear or angular tightening displacements. As the first attempt to this problem, we treat the case of a spherical object in planar motion with no accelerations. Solutions are presented in closed-form, which may be used in real time simulation, or may be used as test cases to check numerical solutions.

2. LINEAR TIGHTENING DISPLACEMENTS

2.1 Problem Formulation

Figure 1 shows an elastic sphere grasped by two elastic fingers that may slide along a link of the hand, and this link may be considered as the fixed link in the following analysis. In the initial state tightening displacements Δl_1 and Δl_2 have not yet been imposed, the fingers touch the sphere at two points A_1 and A_2 but with no contact forces transmitted. In this state the orientation of the two fingers, denoted by α_1 and α_2, are angles between the fingers and a line perpendicular to the fixed link. Unit vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ indicate directions of common normal at A_1 and A_2. The distance OA_1, from the center of the sphere O to the point A_1, has a component parallel to the fixed link, and this component is denoted by l_1. The symbol l_2 is similarly defined, as shown in Fig. 1. Upon imposing tightening displacements Δl_1 and Δl_2, normal forces N_1 and N_2, as well as tangential (or friction) forces T_1 and T_2, develop at A_1 and A_2. In this study we deal with cases with no applied forces. Also, if the two finger surfaces in contact with the sphere are perpendicular to the plane on which figure 1 lies, then both A_1 and A_2 belong to the same great circle of the sphere, and all the contact forces lie on the plane that contains figure 1.

Displacement of each finger can be separated into two parts, as shown in Fig. 1. The first part is the rigid displacements Δl_1 and Δl_2 along the fixed link, obtained by considering the two fingers as rigid bodies. The second part is elastic displacements w_1 and w_2, obtained by modeling the elastic fingers as cantilever beams. According to classical beam theory these displacements are given by

$$w_1 = -\frac{N_1 d_1^3}{3E_f I} \qquad w_2 = -\frac{N_2 d_2^3}{3E_f I}$$

where E_f is the modulus of elasticity of the fingers, and I is the area moment of inertia of the cross section of the beams. Note that normal forces N_1 and N_2 must be compressive, and friction forces T_1 and T_2 on the sphere are positive when they act in the same directions as tangential vectors $\mathbf{t}_1$ and $\mathbf{t}_2$, respectively. When the sphere is free from any applied force, the internal grasping forces are self-equilibrium. Force equilibrium in the x and y directions, as well as moment equilibrium in the z direction, lead to

$$-N_1 \cos\alpha_1 + N_2 \cos\alpha_2 + T_1 \sin\alpha_1 + T_2 \sin\alpha_2 = 0 \quad (1)$$

$$N_1 \sin\alpha_1 + N_2 \sin\alpha_2 + T_1 \cos\alpha_1 - T_2 \cos\alpha_2 = 0 \quad (2)$$

$$T_1 + T_2 = 0 \quad (3)$$

Elastic displacements at O in the x and y directions are denoted by u and v, respectively. Projecting the displacement vector $u\mathbf{i} + v\mathbf{j}$ on $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{t}_1$, and $\mathbf{t}_2$, respectively, we may obtain

$$u_{n1} = u\cos\alpha_1 - v\sin\alpha_1, \quad u_{t1} = -u\sin\alpha_1 - v\cos\alpha_1$$

$$u_{n2} = -u\cos\alpha_2 - v\sin\alpha_2, u_{t2} = -u\sin\alpha_2 + v\cos\alpha_2$$

The displacement of sphere center O relative to A_1 is considered next. The components of this displacement in the direction of $\mathbf{n}_1$ and $\mathbf{t}_1$ are denoted by δ_{n1} and δ_{t1}, respectively, and are given by

$$\begin{aligned}\delta_{n1} &= \Delta l_1 \cos\alpha_1 + u_{n1} + w_1 \\ &= \Delta l_1 \cos\alpha_1 + u\cos\alpha_1 - v\sin\alpha_1 - \frac{N_1 d_1^3}{3E_f I}\end{aligned} \quad (4)$$

$$\begin{aligned}\delta_{t1} &= -\Delta l_1 \sin\alpha_1 + u_{t1} \\ &= -\Delta l_1 \sin\alpha_1 - u\sin\alpha_1 - v\cos\alpha_1\end{aligned} \quad (5)$$

According to Hertz contact theory [30], the relation between relative approach δ_{n1} and normal force N_1 is given by

$$\delta_{n1} = \left(\frac{9N_1^2}{16RE^{*2}} \right)^{1/3} \quad (6)$$

where R is radius of the sphere and E^* is defined later. The relation between relative tangential displacement δ_{t1} and tangential force T_1 is given by ([30], pp. 217-220)

$$\delta_{t1} = \frac{3\mu_1 N_1}{16G^* a_1} \left[1 - \left(1 - \frac{T_1}{\mu_1 N_1} \right)^{2/3} \right] \quad (7)$$

where μ_1 is coefficient of friction between the sphere and the finger at A_1, a_1 denotes the contact length at A_1, given by

$$a_1 = \left(\frac{3N_1 R}{4E^*} \right)^{1/3} \quad (8)$$

E^* and G^* are defined by

$$\frac{1}{E^*} = \frac{1-\nu_s^2}{E_s} + \frac{1-\nu_f^2}{E_f} \quad , \quad \frac{1}{G^*} = \frac{2-\nu_s}{G_s} + \frac{2-\nu_f}{G_f}$$

In the last equation G is shear modulus of elasticity, and ν is Poisson's ratio. Subscripts s and f represent the sphere, and the finger, respectively. Substituting Eqs. (6) and (7) into Eqs. (4) and (5), obtaining

$$(\Delta l_1 + u)\cos\alpha_1 - v\sin\alpha_1 - \frac{N_1 d_1^3}{3E_f I} = \left(\frac{9N_1^2}{16RE^{*2}} \right)^{1/3} \quad (9)$$

$$\Delta l_1 \sin\alpha_1 + u\sin\alpha_1 + v\cos\alpha_1 = \frac{-3\mu_1 N_1}{16G^* a_1} \left[1 - \left(1 - \frac{T_1}{\mu_1 N_1} \right)^{\frac{2}{3}} \right] \quad (10)$$

Similarly, by considering displacement of O relative to the point A_2, we may obtain [31]

$$(\Delta l_2 - u)\cos\alpha_2 - v\sin\alpha_2 - \frac{N_2 d_2^3}{3E_f I} = \left(\frac{9N_2^2}{16RE^{*2}} \right)^{1/3} \quad (11)$$

$$\Delta l_2 \sin\alpha_2 - u\sin\alpha_2 + v\cos\alpha_2 = \frac{3\mu_2 N_2}{16G^* a_2} \left[1 - \left(1 - \frac{T_2}{\mu_2 N_2} \right)^{\frac{2}{3}} \right] \quad (12)$$

where μ_2 is the coefficient of friction at A_2, and a_2 is the contact length at A_2.

For the gripper shown in figure 1, solutions depend on the total tightening displacement Δl, given by

$$\Delta l = \Delta l_1 + \Delta l_2 \quad (13)$$

not on individual values of Δl_1 or Δl_2. In other words, only the relative displacement Δl between the two fingers matters. Equation (13) may be called compatibility equation since it requires that displacements of the two fingers must equal to the imposed tightening displacement.

Hence there are 8 equations [Eqs. (1) to (3), and (9) to (13)] for 8 unknowns, namely, contact forces N_1, N_2, T_1, T_2, elastic displacements u, v, and Δl_1, Δl_2.

2.2 Solution Procedure

To solve for these unknowns we first notice that equations (1) to (3) contain 4 unknown forces, and we may express $T_2 (= -T_1)$ in terms of normal forces N_1 and N_2, as follows

$$T_2 = \frac{-N_1\cos\alpha_1 + N_2\cos\alpha_2}{\sin\alpha_1 - \sin\alpha_2} = \frac{N_1\sin\alpha_1 + N_2\sin\alpha_2}{\cos\alpha_1 + \cos\alpha_2}$$

From this equation we may obtain

$$N_1[1 - \cos(\alpha_1 + \alpha_2)] = N_2[1 - \cos(\alpha_1 + \alpha_2)]$$

Hence

$$N_1 = N_2 \equiv N \quad (14)$$

Substituting this result into Eqs. (1) and (2), one may find

$$T_1\sin\alpha_1 + T_2\sin\alpha_2 = N(\cos\alpha_1 - \cos\alpha_2)$$

$$T_1\cos\alpha_1 - T_2\cos\alpha_2 = -N(\sin\alpha_1 + \sin\alpha_2)$$

Solving these two equations for T_1 and T_2, one finds

$$T_1 = -\frac{1-\cos(\alpha_1+\alpha_2)}{\sin(\alpha_1+\alpha_2)} N = -\tan\left(\frac{\alpha_1+\alpha_2}{2} \right) N \quad (15)$$

$$T_2 = \frac{1-\cos(\alpha_1+\alpha_2)}{\sin(\alpha_1+\alpha_2)} N = \tan\left(\frac{\alpha_1+\alpha_2}{2} \right) N \quad (16)$$

Substituting Eq. (14) into Eqs. (9) and (11), obtaining

$$(\Delta l_1 + u)\cos\alpha_1 - v\sin\alpha_1 - \frac{Nd_1^3}{3E_f I} = \left(\frac{9N^2}{16RE^{*2}} \right)^{1/3} \quad (17)$$

$$(\Delta l_2 - u)\cos\alpha_2 - v\sin\alpha_2 - \frac{Nd_2^3}{3E_f I} = \left(\frac{9N^2}{16RE^{*2}} \right)^{1/3} \quad (18)$$

Displacements u and v may be solved from these two equations, giving

$$u = \frac{\sin\alpha_2 - \sin\alpha_1}{\sin(\alpha_1+\alpha_2)} \left(\frac{9}{16RE^{*2}} \right)^{1/3} N^{\frac{2}{3}} + \frac{d_1^3\sin\alpha_2 - d_2^3\sin\alpha_1}{(3E_f I)\sin(\alpha_1+\alpha_2)} N + \frac{\Delta l_2\sin\alpha_1\cos\alpha_2 - \Delta l_1\sin\alpha_2\cos\alpha_1}{\sin(\alpha_1+\alpha_2)} \quad (19)$$

$$v = -\frac{\cos\alpha_1 + \cos\alpha_2}{\sin(\alpha_1+\alpha_2)} \left(\frac{9}{16RE^{*2}} \right)^{1/3} N^{\frac{2}{3}} - \frac{d_2^3\cos\alpha_1 + d_1^3\cos\alpha_2}{(3E_f I)\sin(\alpha_1+\alpha_2)} N + \frac{(\Delta l_1 + \Delta l_2)\cos\alpha_1\cos\alpha_2}{\sin(\alpha_1+\alpha_2)} \quad (20)$$

The result N_1=N_2 implies that the two contact lengths are equal [see Eq. (14)], i.e.

$$a_1 = a_2 \equiv a = \left(\frac{3NR}{4E^*} \right)^{1/3} \quad (21)$$

Substituting this relation, together with Eqs. (15) and (16), into Eqs. (10) and (12), we obtain

$$-(\Delta l_1 + u)\sin\alpha_1 - v\cos\alpha_1 = \frac{1}{8G^*} \left(\frac{9E^*}{2R} \right)^{1/3} \left(\frac{\mu_1}{\bar{\mu}_1} \right) N^{\frac{2}{3}} \quad (22)$$

$$(\Delta l_2 - u)\sin\alpha_2 + v\cos\alpha_2 = \frac{1}{8G^*} \left(\frac{9E^*}{2R} \right)^{1/3} \left(\frac{\mu_2}{\bar{\mu}_2} \right) N^{\frac{2}{3}} \quad (23)$$

where

$$\frac{1}{\bar{\mu}_1} = 1 - \left[1 + \frac{1}{\mu_1}\tan\left(\frac{\alpha_1+\alpha_2}{2}\right)\right]^{2/3} \quad (24a)$$

$$\frac{1}{\bar{\mu}_2} = 1 - \left[1 - \frac{1}{\mu_2}\tan\left(\frac{\alpha_1+\alpha_2}{2}\right)\right]^{2/3} \quad (24b)$$

Solving u and v from Eqs. (22) and (23), one yields

$$u = \frac{-N^{\frac{2}{3}}}{8G^*\sin(\alpha_1+\alpha_2)}\left(\frac{9E^*}{2R}\right)^{\frac{1}{3}}\left(\frac{\mu_1}{\bar{\mu}_1}\cos\alpha_2 + \frac{\mu_2}{\bar{\mu}_2}\cos\alpha_1\right) - \frac{\Delta l_1\sin\alpha_1\cos\alpha_2 - \Delta l_2\sin\alpha_2\cos\alpha_1}{\sin(\alpha_1+\alpha_2)} \quad (25)$$

$$v = \frac{-N^{\frac{2}{3}}}{8G^*\sin(\alpha_1+\alpha_2)}\left(\frac{9E^*}{2R}\right)^{\frac{1}{3}}\left(\frac{\mu_1}{\bar{\mu}_1}\sin\alpha_2 - \frac{\mu_2}{\bar{\mu}_2}\sin\alpha_1\right) - \frac{\Delta l_1\sin\alpha_1\sin\alpha_2 + \Delta l_2\sin\alpha_1\sin\alpha_2}{\sin(\alpha_1+\alpha_2)} \quad (26)$$

Equating two expressions of u, given by Eqs. (19) and (25), and two expressions of v, shown in Eqs.(20) and (26), one finds

$$\left[\frac{\sin\alpha_1 - \sin\alpha_2}{2E^*} - \frac{1}{8G^*}\left(\frac{\mu_1}{\bar{\mu}_1}\cos\alpha_2 + \frac{\mu_2}{\bar{\mu}_2}\cos\alpha_1\right)\right]\left(\frac{9E^*}{2R}\right)^{1/3} N^{\frac{2}{3}} + \frac{d_2^3\sin\alpha_1 - d_1^3\sin\alpha_2}{3E_f I}N = \Delta l\sin(\alpha_1-\alpha_2) \quad (27)$$

$$\left[\frac{\cos\alpha_1 + \cos\alpha_2}{2E^*} - \frac{1}{8G^*}\left(\frac{\mu_1}{\bar{\mu}_1}\sin\alpha_2 - \frac{\mu_2}{\bar{\mu}_2}\sin\alpha_1\right)\right]\left(\frac{9E^*}{2R}\right)^{1/3} N^{\frac{2}{3}} + \frac{d_2^3\cos\alpha_1 + d_1^3\cos\alpha_2}{3E_f I}N = \Delta l\cos(\alpha_1-\alpha_2) \quad (28)$$

We have two equations for the normal force N. The two equations are in general different, leading to multiple solutions for N. Note that if the problem is symmetric, namely if $\alpha_1 = \alpha_2 \equiv \alpha$, $d_1 = d_2 \equiv d$, $-\mu_1 = \mu_2 \equiv \mu$, and

$$\frac{1}{\bar{\mu}} = 1 - \left(1 - \frac{\tan\alpha}{\mu}\right)^{2/3} \quad (29)$$

then both sides of Eq. (27) reduces to zero, and Eq. (28) takes the form

$$\left[\frac{\cos\alpha}{E^*} + \frac{\sin\alpha}{4G^*}\left(\frac{\mu}{\bar{\mu}}\right)\right]\left(\frac{9E^*}{2R}\right)^{\frac{1}{3}} N^{\frac{2}{3}} + \frac{2d^3\cos\alpha}{3E_2 I_2}N = \Delta l \quad (30)$$

from which N can be solved. The reason for existing two different equations for N in an asymmetric case is unclear. The authors suspect that in such a case a small amount of rolling of the sphere occurs, and rolling has not been included in the formulation.

Equation (30) may be recast into a cubic equation [31]

$$N^3 + b_1 N^2 + b_2 N + b_3 = 0 \quad (31)$$

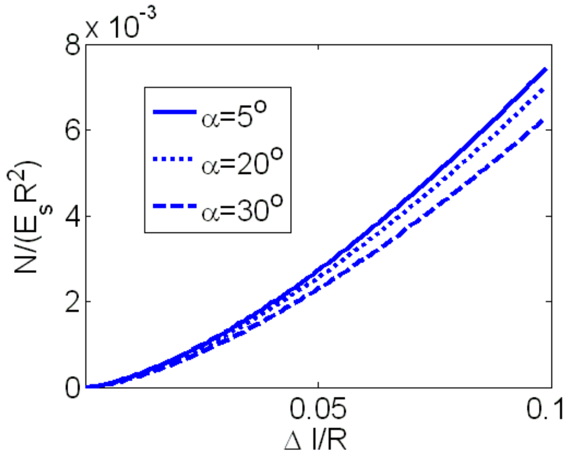

Figure 2 N to Δl relations for various values of α ($E_f = E_s$, $I/(Rd^3)=1$).

whose coefficients are given by

$$b_1 = -\frac{9E_f I(\Delta l)}{2d^3\cos\alpha} + \frac{27E_f^3 I^3}{8d^9}\left(\frac{9E^*}{2R}\right)\left[\frac{1}{E^*} + \frac{\tan\alpha}{4G^*}\left(\frac{\mu}{\bar{\mu}}\right)\right]^3 \quad (32a)$$

$$b_2 = \frac{27E_f^2 I^2(\Delta l)^2}{4d^6\cos^2\alpha} \qquad b_3 = -\frac{27E_f^3 I^3(\Delta l)^3}{8d^9\cos^3\alpha} \quad (32b,c)$$

Closed-form solutions for N is obtained from Eq. (31).

2.3 Special Cases

In some special cases Eq.(31) can be further simplified, as follows.

a). Very large fingers: The case of two fingers grasping a very small sphere can be modeled by letting $I \to \infty$ in Eq. (30), and the normal force is found to be

$$N = (\Delta l)^{\frac{3}{2}}\left(\frac{9E^*}{2R}\right)^{\frac{-1}{2}}\left[\frac{\cos\alpha}{E^*} + \frac{\sin\alpha}{4G^*}\left(\frac{\mu}{\bar{\mu}}\right)\right]^{\frac{-3}{2}} \quad (33)$$

b). Rigid fingers: Letting E_f and G_f approach ∞ in Eq. (30), one obtains

$$N = (\Delta l)^{\frac{3}{2}}\left[\frac{9(1-\nu_s^2)^2}{2RE_s^2}\right]^{\frac{-1}{2}}\left[\cos\alpha + \frac{\sin\alpha}{2}\left(\frac{2-\nu_s}{1-\nu_s}\right)\left(\frac{\mu}{\bar{\mu}}\right)\right]^{\frac{-3}{2}} \quad (34)$$

c). Rigid sphere: By letting $E_s \to \infty$ and $G_s \to \infty$, Eq. (30) reduces to

$$\left[\cos\alpha + \frac{\sin\alpha}{2}\left(\frac{2-\nu_f}{1-\nu_f}\right)\left(\frac{\mu}{\bar{\mu}}\right)\right]\left[\frac{9(1-\nu_f^2)^2}{2RE_f^2}\right]^{\frac{1}{3}} N^{\frac{2}{3}} + \frac{2d^3\cos\alpha}{3E_f I}N = \Delta l \quad (35)$$

d). A rigid sphere grasped by very large fingers: By setting $E_s \to \infty$ in Eq. (33), one finds

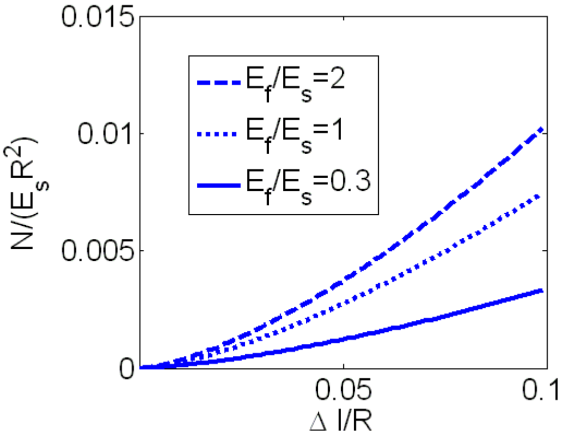

Figure 3 N to Δl relations for various values of E_f/E_s (α=5°, $I/(Rd^3)$=1).

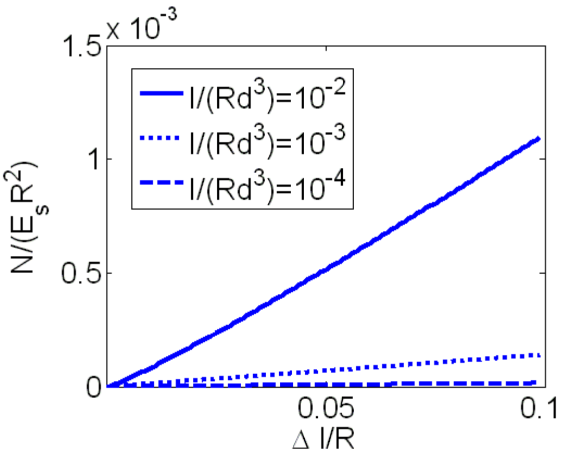

Figure 4 N to Δl relations for various values of I (α=5°, $I/(Rd^3)$=1).

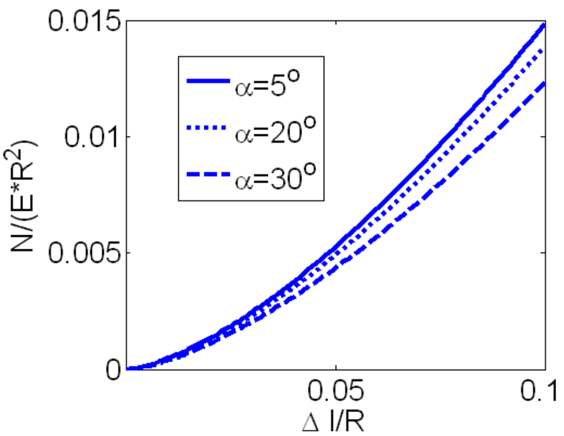

Figure 5 N to Δl relations for very large fingers ($E_f = E_s$).

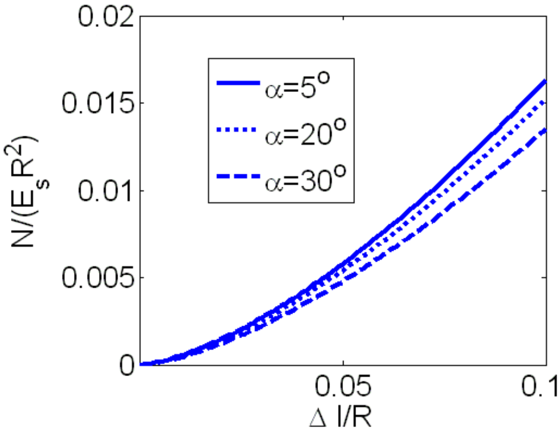

Figure 6 N to Δl relations for rigid fingers ($E_f \to \infty$).

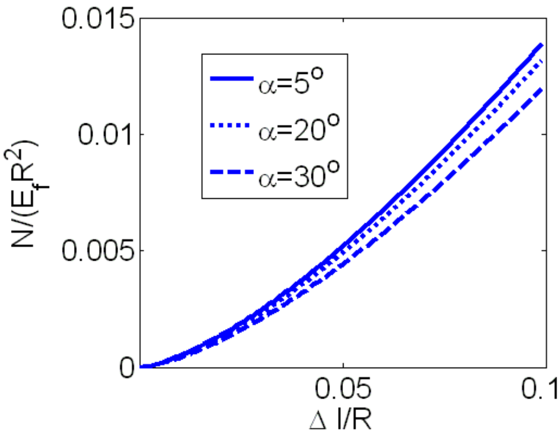

Figure 7 N to Δl relations for a rigid sphere ($E_s \to \infty$).

$$N = (\Delta l)^{\frac{3}{2}} \left[\frac{9(1-\nu_f^2)^2}{2RE_f^2} \right]^{\frac{-1}{2}} \left[\cos\alpha + \frac{\sin\alpha}{2} \left(\frac{2-\nu_f}{1-\nu_f} \right) \left(\frac{\mu}{\bar{\mu}} \right) \right]^{\frac{-3}{2}} \quad (36)$$

Note that the tangential forces may be obtained from Eqs. (15) and (16), as follows

$$-T_1 = T_2 = N \tan\alpha \quad (37)$$

Equation (37), obtained from static equilibrium, predicts that slipping occurs when the angle α reaches the value $\tan^{-1}\mu$, a well-known condition for slipping in statics. In this paper we assume that $\alpha < \tan^{-1}\mu$.

Relations between the normal N and the tightening displacement Δl, given by Eq. (31), are shown in figures 2 for three values of α, in figure 3 for various values of E_f/E_s, and in figure 4 for various values of I. It can be seen from these figures that for a given tightening displacement Δl, a larger normal force N can be produced by a finger with a smaller value of α, by a stiffer finger with larger E_f, or by a larger finger with bigger area moment of inertia I. Special N to Δl relations are shown in

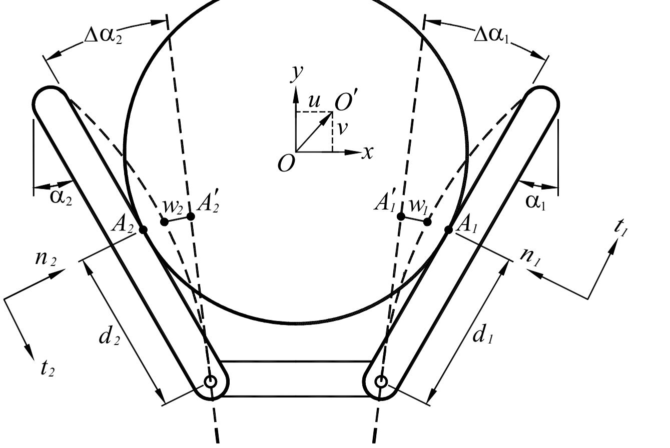

Figure 8 A Two-fingered gripper that may impose angular tightening displacement.

figures 5-7. Figure 5 shows cases with very large fingers, the N to Δl relation of which is governed by Eq. (33). In Fig. 6 the fingers are rigid, and the relation is given by Eq. (34). Finally, in Fig. 7, the relation is Eq. (35) for a rigid sphere.

3. ROTATIONAL TIGHTENING DISPLACEMENTS

3.1 Problem Formulation

Figure 8 shows a two-fingered gripper which may impose angular tightening displacements on a sphere. In the initial state the two fingers touch the sphere with no force transmission, whose angular positions are α_1 and α_2. We first note that the results given by equations (14)-(16) are still valid since they were obtained from equilibrium equations (1)-(3), and Eq. (21) is a direct result of Eq. (14). Then we consider displacement of O relative to initial contact point A_1 when angular tightening displacements $\Delta\alpha_1$ and $\Delta\alpha_2$ are imposed. The projection of this relative displacement onto the vector $\mathbf{n}_1$, denoted by δ_{n1}, is shown on the left hand side of Eq. (9), with $\Delta l_1\cos\alpha_1$ replaced by $d_1\Delta\alpha_1$, namely,

$$d_1\Delta\alpha_1 + u\cos\alpha_1 - v\sin\alpha_1 - \frac{N d_1^{\,3}}{3E_f I} = \left(\frac{9N^2}{16RE^{*2}}\right)^{1/3} \tag{38}$$

The component of the displacement of O relative to A_1 in the direction of $\mathbf{t}_1$, which we call δ_{t1}, is obtained by deleting the term $\Delta l_1\sin\alpha_1$ in Eq. (10), i.e.

$$-u\sin\alpha_1 - v\cos\alpha_1 = \frac{3\mu_1 N}{16G^* a}\left[1-\left(1-\frac{T_1}{\mu_1 N_1}\right)^{2/3}\right] \tag{39}$$

Similarly, the displacements of O relative to A_2 has two components δ_{n2} and δ_{t2}, in the directions of $\mathbf{n}_2$ and $\mathbf{t}_2$, respectively, given by

$$d_2\Delta\alpha_2 - u\cos\alpha_2 - v\sin\alpha_2 - \frac{N_2 d_2^{\,3}}{3E_f I} = \left(\frac{9N_2^2}{16RE^{*2}}\right)^{1/3} \tag{40}$$

$$-u\sin\alpha_2 + v\cos\alpha_2 = \frac{3\mu_2 N_2}{16G^* a}\left[1-\left(1-\frac{T_2}{\mu_2 N_2}\right)^{2/3}\right] \tag{41}$$

Finally, the angular displacements of the two fingers must be compatible with total angular tightening displacement $\Delta\alpha$, namely

$$\Delta\alpha_1 + \Delta\alpha_2 = \Delta\alpha \tag{42}$$

We hence have 7 equations, i.e. Eqs. (15), (16), and (38)-(42) for the following 7 unknowns: N, T_1, T_2, u, v, $\Delta\alpha_1$, and $\Delta\alpha_2$.

3.2 Solution procedure

Solution procedure is exactly the same as in the previous case with linear tightening displacements [31]. We may first substitute T_1 and T_2 in Eqs. (15) and (16), as well as a in Eq. (21), into Eqs. (39) and (41), to obtain two equations for elastic displacements u and v, from which analytical expressions for u and v may be found. But u and v may also be obtained by solving Eqs. (38) and (40). Equating the two expressions for u and then the two expressions for v, we find

$$\left[\frac{\sin\alpha_1-\sin\alpha_2}{2E^*}-\frac{1}{8G^*}\left(\frac{\mu_1}{\bar{\mu}_1}\cos\alpha_2+\frac{\mu_2}{\bar{\mu}_2}\cos\alpha_1\right)\right]\left(\frac{9E^*}{2R}\right)^{\frac{1}{3}}N^{\frac{2}{3}} + \frac{d_2^3\sin\alpha_1-d_1^3\sin\alpha_2}{3E_f I}N+\left(d_1\sin\alpha_2+d_2\sin\alpha_1\right)\Delta\alpha_1-d_2\sin\alpha_1\Delta\alpha=0 \tag{43}$$

$$\left[\frac{\cos\alpha_1+\cos\alpha_2}{2E^*}-\frac{1}{8G^*}\left(\frac{\mu_1}{\bar{\mu}_1}\sin\alpha_2-\frac{\mu_2}{\bar{\mu}_2}\sin\alpha_1\right)\right]\left(\frac{9E^*}{2R}\right)^{\frac{1}{3}}N^{\frac{2}{3}} + \frac{d_2^3\cos\alpha_1+d_1^3\cos\alpha_2}{3E_f I}N-\left(d_1\cos\alpha_2-d_2\cos\alpha_1\right)\Delta\alpha_1 - d_2\cos\alpha_1\Delta\alpha=0 \tag{44}$$

Again we have two different equations for N. A solution exits only in a case of symmetry, that is, $\alpha_1=\alpha_2\equiv\alpha$, $d_1=d_2\equiv d$, and $-\mu_1=\mu_2\equiv\mu$, then Eq. (44) may be recast into the following cubic equation of N

$$N^3+c_1N^2+c_2N+c_3=0 \tag{45}$$

where

$$c_1=-\frac{9E_f I(\Delta\alpha)}{2d^2}+\frac{27E_f^3I^3}{8d^9}\left(\frac{9E^*}{2R}\right)\left[\frac{1}{E^*}+\frac{\tan\alpha}{4G^*}\left(\frac{\mu}{\bar{\mu}}\right)\right]^3 \tag{46a}$$

$$c_2=\frac{27E_f^2I^2(\Delta\alpha)^2}{4d^4} \tag{46b}$$

$$c_3=-\frac{27E_f^3I^3(\Delta\alpha)^3}{8d^6} \tag{46c}$$

and $\bar{\mu}$ is defined in Eq. (29). Solutions in the extreme cases are given below.

3.3 Special Caese

a). Very large fingers: By letting $I\to\infty$, one may show that [31]

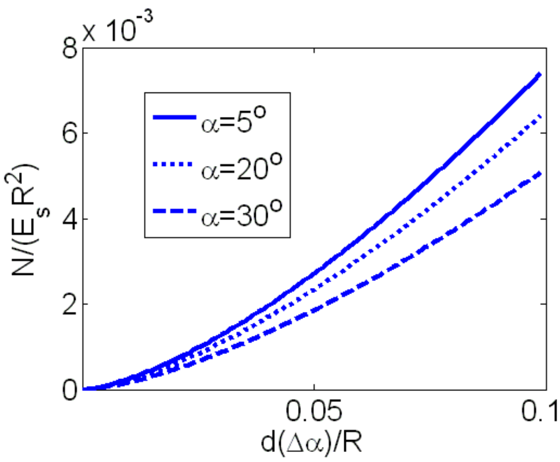

Figure 9 N to $\Delta\alpha$ relations for various values of α (E_f = E_s, $I/(Rd^3)$=1).

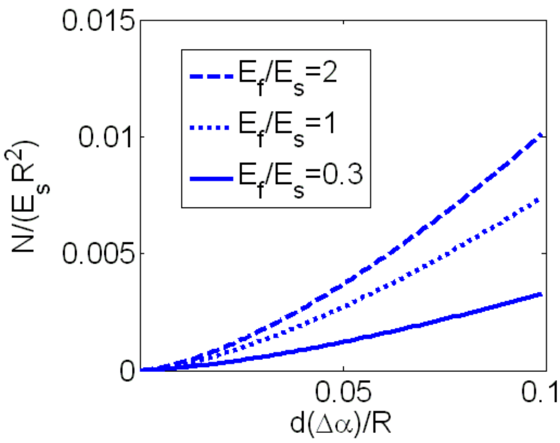

Figure 10 N to $\Delta\alpha$ relations for various values of E_f / E_s (α=5°, $I/(Rd^3)$=1).

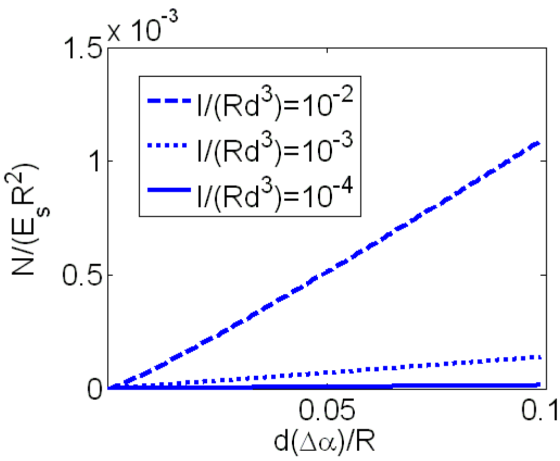

Figure 11 N to $\Delta\alpha$ relations for various values of I (α=5°, $I/(Rd^3)$=1).

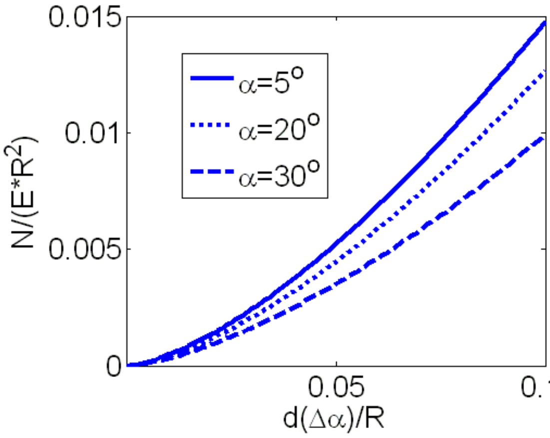

Figure 12 N to $\Delta\alpha$ relations for very large fingers (E_f = E_s).

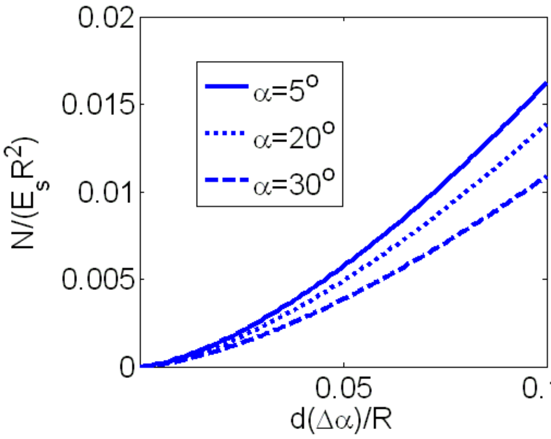

Figure 13 N to $\Delta\alpha$ relations for rigid fingers ($E_f \to \infty$).

$$N = [d(\Delta\alpha)]^{\frac{3}{2}} \left(\frac{9E^*}{2R}\right)^{\frac{-1}{2}} \left[\frac{1}{E^*} + \frac{\tan\alpha}{4G^*}\left(\frac{\mu}{\bar{\mu}}\right)\right]^{\frac{-3}{2}} \quad (47)$$

b). Rigid fingers: Letting E_f and G_f approach ∞, then [31]

$$N = [d(\Delta\alpha)]^{\frac{3}{2}} \left[\frac{9(1-\nu_s{}^2)^2}{2RE_s{}^2}\right]^{\frac{-1}{2}} \left[1 + \frac{\tan\alpha}{2}\left(\frac{2-\nu_s}{1-\nu_s}\right)\left(\frac{\mu}{\bar{\mu}}\right)\right]^{\frac{-3}{2}} \quad (48)$$

c). Rigid sphere: By letting $E_s \to \infty$ and $G_s \to \infty$, one may find

$$\left[1 + \frac{\tan\alpha}{2}\left(\frac{2-\nu_f}{1-\nu_f}\right)\left(\frac{\mu}{\bar{\mu}}\right)\right]\left[\frac{9(1-\nu_f^2)^2}{2RE_f^2}\right]^{\frac{1}{3}} N^{\frac{2}{3}} + \frac{2d^3}{3E_f I} N = d(\Delta\alpha) \quad (49)$$

d). A rigid sphere grasped by very large fingers: By setting $E_s \to \infty$, Eq. (47) reduces to

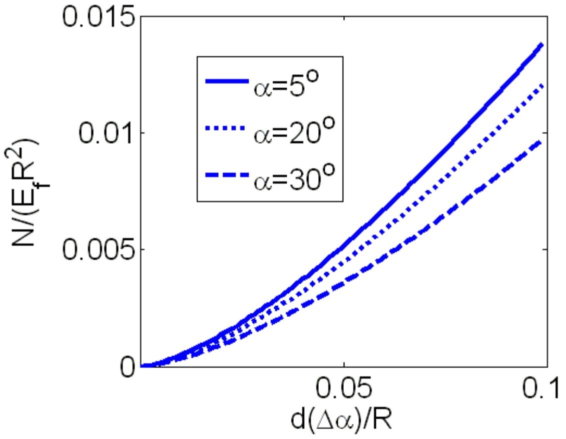

Figure 14 N to $\Delta\alpha$ relations for a rigid sphere ($E_s \to \infty$).

$$N = \left[d(\Delta\alpha)\right]^{\frac{3}{2}} \left[\frac{9(1-\nu_f^2)^2}{2RE_f^2}\right]^{\frac{-1}{2}} \left[1 + \frac{\tan\alpha}{2}\left(\frac{2-\nu_f}{1-\nu_f}\right)\left(\frac{\mu}{\bar{\mu}}\right)\right]^{\frac{-3}{2}} \quad (50)$$

Figures 9 to 11 show N to $\Delta\alpha$ relations for various values of α (Fig. 9), various values of E_f / E_s (Fig. 10), and various values of I (Fig. 11). Again we see that a larger normal force N may be obtained by using grippers with smaller orientation angles α, stiffer fingers, or fingers with larger inertia I. Figure 12 shows N to $\Delta\alpha$ relations predicted by Eq. (47) for very large fingers. Figure 13 shows the same relations obtained from Eq. (48) for rigid fingers, and figure 14 shows the relations governed by Eq. (49) for a rigid sphere.

4. CONCLUSIONS

In this study we present closed-form solutions for contact forces between an elastic sphere and two elastic fingers symmetrically placed. By modeling the fingers as fixed-end beams, and using force-displacement equations based upon Hertz contact theory, we find the normal contact force satisfy a cubic equation, and can be expressed in terms of linear or angular tightening displacements. For static cases the normal force N and tangential friction T are always symmetric about the center axis, and the relation between them is $T = N \tan\alpha$, which implies sliding would occur when the inclination angle α satisfies the relation $\alpha \geq \tan^{-1}\mu$. Both linear and rotational tightening show the same tendency, that is, a larger normal force may be produced by a finger with a smaller inclination angle α, by a stiffer finger, or by a finger with a larger area moment of inertia.

5. FURUTE WORK

The solution procedure can be applied to grasping with three or more than three fingers, to cases with inertia forces, and to unsymmetrical situations.

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