

# Annual Maximum Wind Simulation Based on Moment Parameters of Parent Distributions

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Polynomial Translation Method      Moment Parameters  
Extreme Value Distribution            Moment Method

## 1. Introduction

The tail characteristics of extreme value distributions for natural hazards have to be examined to provide convenient mathematical models for engineering decision making regarding the long term policy for the structural safety in a society. When the mean and the standard deviation of four moments of parent distribution are properly estimated, by the application of the polynomial translation method, the simulation of annual maximum wind speeds provides a good correspondence to the distribution of observed annual maximum wind speeds. This research aims at examining the effects of yearly and regional variation of four moments, with the consideration of correlation characteristics, on the estimation of annual maximum wind speeds.

## 2. Characteristics of Parent Distribution

10 minute mean wind speeds of 42 years for 155 sites in Japan are utilized to examine the yearly and regional variations of four moments. Correlation coefficients between four moments for 155 sites are listed in Table 1 ( $\mu$ : mean,  $\sigma$ : standard deviation,  $\gamma_1$ : skewness,  $\gamma_2$ : kurtosis). Yearly variation of four moments of each site indicates that high correlated skewness and kurtosis values exist in most sites.

Table 1 The mean and the standard deviation values of correlation coefficients between four moments for 155 sites

	$\mu$ - $\sigma$	$\mu$ - $\gamma_1$	$\mu$ - $\gamma_2$	$\sigma$ - $\gamma_1$	$\sigma$ - $\gamma_2$	$\gamma_1$ - $\gamma_2$
mean	0.5894	-0.0853	-0.0581	0.1681	0.0846	0.9088
s.d.	0.3328	0.3007	0.2598	0.3164	0.2748	0.0453

※ mean : mean of 155 sites      s.d. : s.d. value of 155 sites

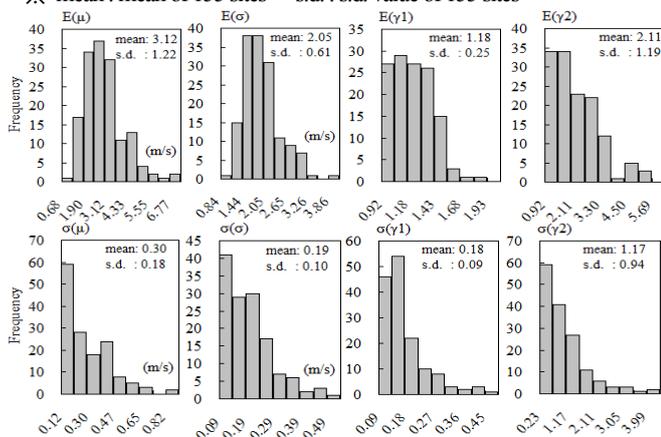


Figure 1 Histograms of mean and standard deviation of four moments for 155 sites

Then the mean and the standard deviation of four moments of each site are calculated as moment parameters indicating the characteristics of regional variations of four moments. The histograms of the moment parameters for 155 sites are plotted as Figure 1. In some sites the standard deviation of four moments is close to zero which suggests the identical nature for parent distributions, but in most sites the standard deviation is significant so that the identical hypothesis would not be applied. Table 2 shows the correlation coefficients between these 8 moment parameters of 155 sites, from which it can be clearly observed that a high correlation exists between  $E(\mu)$  and  $E(\sigma)$ ,  $E(\gamma_1)$  and  $E(\gamma_2)$ ,  $\sigma(\mu)$  and  $\sigma(\sigma)$ , and  $\sigma(\gamma_1)$  and  $\sigma(\gamma_2)$ .

Table 2 Correlation coefficients between moment parameters for 155 sites

	$E(\mu)$	$E(\sigma)$	$E(\gamma_1)$	$E(\gamma_2)$	$\sigma(\mu)$	$\sigma(\sigma)$	$\sigma(\gamma_1)$	$\sigma(\gamma_2)$
$E(\mu)$	1.00	0.92	-0.53	-0.22	0.40	0.42	0.15	0.05
$E(\sigma)$	--	1.00	-0.38	-0.29	0.37	0.41	-0.03	-0.12
$E(\gamma_1)$	--	--	1.00	0.72	-0.18	-0.17	0.11	0.18
$E(\gamma_2)$	--	--	--	1.00	-0.02	-0.04	0.70	0.74
$\sigma(\mu)$	--	--	--	--	1.00	0.85	0.15	0.07
$\sigma(\sigma)$	--	--	--	--	--	1.00	0.15	0.08
$\sigma(\gamma_1)$	--	--	--	--	--	--	1.00	0.94
$\sigma(\gamma_2)$	--	--	--	--	--	--	--	1.00

## 3. Effects of Regional and Yearly Variation of Four Moments on the Estimation of Annual Maximum Wind Speeds

Through examining the characteristics of four moments, effects of the moment parameters of parent distribution on the probabilistic distributions of annual maximum wind speeds are then examined by a parametric study based on the aforementioned tendencies. The simulation process of annual maximum wind speeds

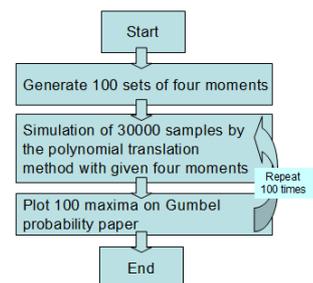


Figure 2 Simulation process

based on the polynomial translation method has been introduced in Choi and Kanda (2003). Figure 2 shows the simulation flowchart. When the simulation process is carried out, the moment method is then applied to obtain the Gumbel parameters from the median estimates of annual

maxima and considered as the characteristics of the extreme value distribution.

When generating 100 sets of four moments in the first step in Figure 2, the following equations are utilized.

$$\mu_i = E(\mu) + R_{1,i}\sigma(\mu) \tag{1}$$

$$\sigma_i = E(\sigma) + R_{2,i}\sigma(\sigma) \tag{2}$$

$$\gamma_{1i} = E(\gamma_1) + R_{3,i}\sigma(\gamma_1) \tag{3}$$

$$\gamma_{2i} = E(\gamma_2) + R_{4,i}\sigma(\gamma_2) \tag{4}$$

where  $R_{1,i}$ ,  $R_{2,i}$ ,  $R_{3,i}$ , and  $R_{4,i}$  are random numbers generated based on the standard normal distribution for  $i$ -th set of four moments.  $R_{3,i}$  and  $R_{4,i}$  are assumed fully correlated based on the high correlation shown in Table 1. Since the non-identical nature is clearly observed in Figure 1, 100 sets (corresponding to 100 years) of four moments are generated with the mean of four moments and the standard deviation of four moments which are parametrically varied referring to the histograms of regional distribution. As shown in Figure 2, the mean and the standard deviation of four moments are regarded as variables for the parametric study. Also according to Table 2, there are four pairs of correlated moment parameters. In order to investigate the effect of each pair on the estimation of annual maximum wind speeds, four different cases are assumed. For example, to observe the effect of the first pair,  $E(\mu)$  and  $E(\sigma)$ , the values of both moment parameters vary as “regional mean + N\*regional standard deviation (N: regional varying integers)”. Since the high correlated feature,  $E(\mu)$  increases when  $E(\sigma)$  increases.

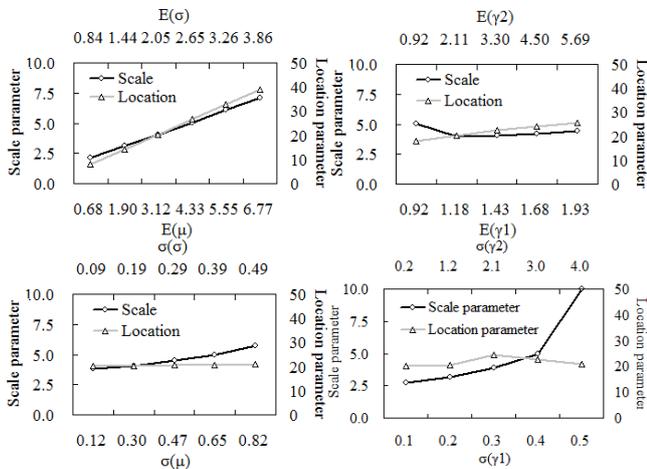


Figure 3 Gumbel parameters with four pairs of moment parameters

For the variation of scale parameters shown in Figure 3, when the values of  $E(\mu)$  and  $E(\sigma)$ ,  $\sigma(\mu)$  and  $\sigma(\sigma)$ , or  $\sigma(\gamma_1)$  and  $\sigma(\gamma_2)$  increase, the scale parameter also increases. Among these three cases, the case of  $\sigma(\gamma_1)$  and  $\sigma(\gamma_2)$ , has rather significant effect than other two cases. For the variation of location parameters shown in Figure 3, location parameter increases when the values of  $E(\mu)$  and  $E(\sigma)$  increase. In other cases, the variation of location parameters is not significant apparently. Once the moment parameters are properly estimated, the tail characteristics of extreme

value distribution can be predicted through the examination of the Gumbel parameters.

#### 4. Comparison of Simulated Maxima with Observed Records and Discussion

The statistical data of Tokyo and Kagoshima are used to demonstrate estimate the simulated annual maximum wind speeds compared with the observed annual maxima. The simulation is carried out in the same manner as described in the previous section. The solid line represents the median estimates of 100 times of simulation process and the two dash lines represent the 95% confidence interval envelopes in Figure 4. Table 3 shows the statistical information about the moment parameters. The agreement between the median estimates and the observed maxima is considered good on the Gumbel probability paper for the case of Tokyo. However, with an extremely high value of  $\sigma(\gamma_2)$ , estimated annual maxima for Kagoshima in the tail part show a large concave tendency, which corresponds to the effects of moment parameters mentioned.

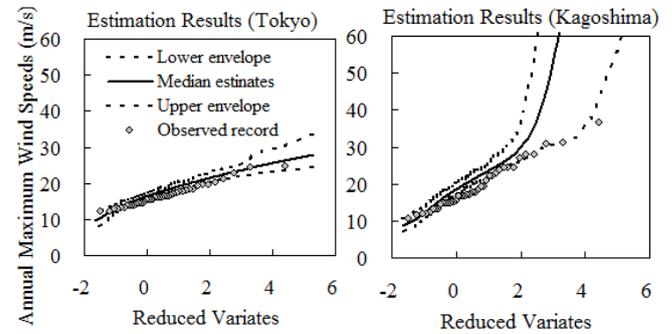


Figure 4 Simulation results based on the 42 years' statistical data and the observed annual maxima

Table 3 Moment parameters for Tokyo and Kagoshima

	$E(\mu)$	$E(\sigma)$	$E(\gamma_1)$	$E(\gamma_2)$	$\sigma(\mu)$	$\sigma(\sigma)$	$\sigma(\gamma_1)$	$\sigma(\gamma_2)$
Tokyo	3.27	1.81	1.15	2.15	0.24	0.17	0.22	1.04
Kagoshima	2.83	1.65	1.35	4.62	0.31	0.15	0.38	4.34

#### 5. Conclusions

The variations of four moment parameters are generally significant and the non-identical nature of the parent distribution is confirmed. When the yearly and regional variation of four moment parameters are considered properly for the simulation of annual maximum wind speeds by the polynomial translation method, the behavior of the tail characteristics could be predicted properly.

#### References

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