Heat Transfer Enhancement of Power-Law Fluids in a Parallel-Plate Channel for Improved Device Performance under Asymmetric Wall Temperatures

(何啟東) Chii-Dong Ho*, (林國賡) Gwo-Geng Lin, (林承毅) Cheng-Yi Lin and (林立邦) Li-Pang Lin

Energy and Opto-Electronic Materials Research Center, Department of Chemical and Materials Engineering, Tamkang University, Tamsui, New Taipei, Taiwan 251

(淡江大學化學工程與材料工程學系、能源與光電材料研究中心)

Abstract —The heat transfer of a Graetz problem to a double-pass parallel-plate heat exchanger under uniform wall temperatures to enhance the device performance improvement is investigated theoretically. The theoretical mathematical model is solved analytically using the separation of variables, superposition principle and an orthogonal expansion technique in extended power series. The theoretical results show that the power-law fluids results in the significant heat-transfer efficiency improvement as compared with those in an open conduit (without an impermeable resistless sheet inserted), especially when the double-pass device was operated in large Graetz number. The results show that the good agreement between the experimental results and theoretical prediction. The effects of impermeable-sheet position and power consumption increment for power-law fluids have also been presented.

Keywords: Power-law fluids; Uniform wall temperatures; Double-pass operations; Nusselt number; Performance improvement.

*Corresponding author. Tel: 886-26215656 ext. 2724 Fax: 886-2-26209887;

E-mail: cdho@mail.tku.edu.tw

1. Introduction

The study of the thermal response of the conduit wall and fluid temperature distributions to the Newtonian fluid in laminar heat transfer problems with negligible axial conduction is known as the Graetz problem [1,2]. Other engineering applications were devoted to the non-Newtonian materials with the power-law constants in the appropriate shear rate range due to their high viscosity levels. The purposes of the present study are to extend the heat transfer problem in a double-pass with recycle design parallel-plate heat exchanger to asymmetrical wall boundary conditions [3] and to non-Newtonian fluids [4] in aiming to solve analytically by means of orthogonal expansion techniques [5].

2. Theoretical treatments

The design of parallel-plate heat exchanger with height *W*, length *L*, and width *B* (*B*>>*W*), the system of open duct be inserted an impermeable sheet, which thickness can be neglected, to divide two channels, subchannel *a* and subchannel *b*, with thickness ΔW and $(1-\Delta)W$,with temperature T_1 and T_2, as shown in Figure 1.

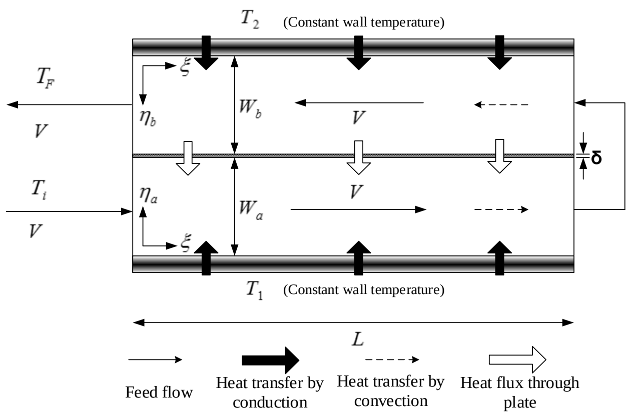

Fig. 1 Double-pass heat exchangers

The following assumptions were made to simplify the system analysis: fully-developed laminar flow with power law index ω in each subchannel, constant physical properties of fluid, neglecting end effects, impermeable sheet thermal resistance and axial heat conduction, and well-mixing at the inlet and outlet of fluid. The dimensionless velocity distributions may be written as

$$v_a = \frac{V}{W_a B}\left[\frac{1+2\omega}{1+\omega}\right]\left(1-\left|2\eta_a - 1\right|^{\frac{1}{\omega}+1}\right) \quad (1)$$

$$v_b = \frac{-V}{W_b B}\left[\frac{1+2\omega}{1+\omega}\right]\left(1-\left|2\eta_b - 1\right|^{\frac{1}{\omega}+1}\right) \quad (2)$$

In this study, using the power series to expansion the third term of velocity profile are

$$\left(1-\left|2\eta - 1\right|^{\frac{1}{\omega}+1}\right) = X_1\eta + X_2\eta^2 + X_3\eta^3 + X_4\eta^4 + X_5\eta^5 \quad (3)$$

The equations of energy in dimensionless form for double-pass heat exchangers with uniform wall fluxes may be obtained as:

$$\frac{\partial^2\psi_a(\eta_a,\xi)}{\partial\eta_a^2} = \left[\frac{W_a^2 v_a(\eta_a)}{\alpha GzL}\right]\frac{\partial\psi_a(\eta_a,\xi)}{\partial\xi} \quad (4)$$

$$\frac{\partial^2\psi_b(\eta_b,\xi)}{\partial\eta_b^2} = \left[\frac{W_b^2 v_b(\eta_b)}{\alpha GzL}\right]\frac{\partial\psi_b(\eta_b,\xi)}{\partial\xi} \quad (5)$$

The definition of dimensionless groups in the Eqs. (1) to (5) are

$$\xi = \frac{z}{LGz},\ Gz = \frac{VW}{\alpha BL},\ W_a = \Delta W,\ W_b = (1-\Delta)W,$$

$$\frac{W_a}{W_b} = \frac{\Delta}{1-\Delta},\ \psi_a = \frac{T_a - T_i}{T_1 - T_i},\ \psi_b = \frac{T_b - T_i}{T_1 - T_i},\ \eta_a = \frac{x_a}{W_a},$$

$$\eta_b = \frac{x_b}{W_b},\ \sigma = \frac{T_2 - T_i}{T_1 - T_i} \quad (6)$$

The corresponding boundary conditions for solving Eq. (4) and Eq. (5) are

$$\psi_a(0,\xi) = 1 \quad (7)$$

$$\psi_b(0,\xi) = \sigma \quad (8)$$

$$\psi_a(1,\xi) = \psi_b(1,\xi) \quad (9)$$

$$\frac{\partial\psi_a(1,\xi)}{\partial\eta_a} = -\frac{\Delta}{1-\Delta}\frac{\partial\psi_b(1,\xi)}{\partial\eta_b} \quad (10)$$

The complete solutions of Eq. (4) and Eq. (5) can be separated into the linear superposition of an asymptotic solution, $\theta(\eta,\xi)$, and a homogeneous solution, $\phi(\eta,\xi)$, as follows:

$$\psi_a(\eta_a,\xi) = \theta_a(\eta_a,\xi) + \phi_a(\eta_a,\xi) \quad (11)$$

$$\psi_b(\eta_b,\xi) = \theta_b(\eta_b,\xi) + \phi_b(\eta_b,\xi) \quad (12)$$

The asymptotic solutions are

$$\theta_a(\eta_a) = A_1\eta_a + A_2 \quad (13)$$

$$\theta_b(\eta_b) = B_1\eta_b + B_2 \quad (14)$$

Using the separation of variables to express the solutions of $\phi_a(\eta_a,\xi)$ and $\phi_b(\eta_b,\xi)$ as follows

$$\phi_a(\eta_a,\xi) = \sum_{m=0}^{\infty} S_{a,m}F_{a,m}(\eta_a)G_m(\xi) \quad (15)$$

$$\phi_b(\eta_b,\xi) = \sum_{m=0}^{\infty} S_{b,m}F_{b,m}(\eta_b)G_m(\xi) \quad (16)$$

Substituting Eqs. (15) and (16) into the governing equations of $\phi_a(\eta_a,\xi)$ and $\phi_b(\eta_b,\xi)$ gives

$$G_m(\xi) = e^{-\lambda_m(\frac{1}{Gz}-\xi)}$$

$$F''_{a,m}(\eta_a) - \lambda_m\Delta\left(\frac{1+2\omega}{1+\omega}\right)\left(1-\left|2\eta_a - 1\right|^{\frac{1}{\omega}+1}\right)F_{a,m}(\eta_a) = 0 \quad (17)$$

$$F''_{b,m}(\eta_b) + \lambda_m(1-\Delta)\left(\frac{1+2\omega}{1+\omega}\right)\left(1-\left|2\eta_b - 1\right|^{\frac{1}{\omega}+1}\right)F_{b,m}(\eta_b) = 0 \quad (18)$$

In which the eigenfunctions $F_{a,m}(\eta_a)$ and $F_{b,m}(\eta_b)$ were assumed to be polynomials to avoid the loss of generality.

$$F_{a.m}(\eta) = \sum_{n=0}^{\infty} d_{m,n}\eta_a{}^n,\ d_{m,0} = 1,\ d_{m,1} = 0 \quad (19)$$

$$F_{b,m}(\eta) = \sum_{n=0}^{\infty} e_{m,n}\eta_b{}^n,\ e_{m,0} = 1,\ e_{m,1} = 0 \quad (20)$$

Combination of Eq. (9) and Eq. (10) results in

$$S_{a,m}F'_{a,m}(1) = -\frac{\Delta}{1-\Delta}S_{b,m}F'_{b,m}(1) \quad (21)$$

All the coefficients $d_{m,n}$ and $e_{m,n}$ for $F_{a,m}(\eta_a)$ and $F_{b,m}(\eta_b)$, can be obtained in terms of eigenvalue λ_m by substituting Eq. (19) and Eq. (20) into Eq. (17) and Eq. (18) Also, the constants of $S_{a,m}$ and $S_{b,m}$ in Eq. (15) and Eq. (16) can be calculated from orthogonality conditions when $n \neq m$:

$$W_b\int_0^1\left[\frac{W_a^2 v_a(\eta_a)}{\alpha LGz}\right]S_{a,m}S_{a,n}F_{a,m}F_{a,n}d\eta_a + W_a\int_0^1\left[\frac{W_b^2 v_b(\eta_b)}{\alpha LGz}\right]S_{b,m}S_{b,n}F_{b,m}F_{b,n}d\eta_b \quad (22)$$

The overall energy balance of whole system

$$V(0-\psi_F) = \int_0^{\frac{1}{Gz}} Gz\frac{\alpha BL}{W_a}\frac{\partial\psi_a(0,\xi)}{\partial\eta_a}d\xi + \int_0^{\frac{1}{Gz}} Gz\frac{\alpha BL}{W_b}\frac{\partial\psi_b(0,\xi)}{\partial\eta_b}d\xi \quad (23)$$

The average dimensionless outlet temperature

ψ_F may be obtained at the end of subchannel b by

$$\psi_F = -\sum_{m=0}^{\infty}\frac{e^{-\frac{\lambda_m}{Gz}}\cdot S_{b,m}}{(1-\Delta)\lambda_m}\left[F_{b,m}^{'}(1)-F_{b,m}^{'}(0)\right]$$
$$-\int_0^1(\frac{1+2\omega}{1+\omega})(X_1\eta_b^1+X_2\eta_b^2+X_3\eta_b^3+X_4\eta_b^4+X_5\eta_b^5)\times$$
$$((\Delta-1)(\sigma-1)\eta_b+\sigma)d\eta_b \quad (24)$$

The average dimensionless temperatures at the end of the subchannels a and b are

$$\psi_{aL} = -\sum_{m=0}^{\infty}\frac{S_{a,m}}{\Delta\lambda_m}\left[F_{a,m}^{'}(1)-F_{a,m}^{'}(0)\right]$$
$$+\int_0^1(\frac{1+2\omega}{1+\omega})(X_1\eta_a^1+X_2\eta_a^2+X_3\eta_a^3+X_4\eta_a^4+X_5\eta_a^5)\times$$
$$(\Delta(\sigma-1)\eta_a+1)d\eta_a \quad (25)$$

$$\psi_{bL} = -\sum_{m=0}^{\infty}\frac{S_{b,m}}{(1-\Delta)\lambda_m}\left[F_{b,m}^{'}(1)-F_{b,m}^{'}(0)\right]$$
$$-\int_0^1(\frac{1+2\omega}{1+\omega})(X_1\eta_b^1+X_2\eta_b^2+X_3\eta_b^3+X_4\eta_b^4+X_5\eta_b^5)\times$$
$$((\Delta-1)(\sigma-1)\eta_b+\sigma)d\eta_b \quad (26)$$

The average Nusselt number for double-pass parallel-plate heat exchangers under uniform wall fluxes may be calculated as:

$$\overline{Nu} = \frac{\bar{h}W}{k} = \frac{VW}{2\alpha BL}\frac{\overline{\psi_F}}{\overline{\psi_w}} = \frac{G_Z}{2}\frac{\overline{\psi_F}}{\overline{\psi_w}} \quad (27)$$

The improvement of the heat-transfer efficiency, I_h, based on the heat transfer of a single-pass device

$$I_h = \frac{\overline{Nu}-\overline{Nu_0}}{\overline{Nu_0}} \times 100\% \quad (28)$$

I_P due to the friction losses ($\ell w_{f,a}$ and $\ell w_{f,b}$ for double pass while $\ell w_{f,0}$ for single pass) in the conduits

$$I_p = \frac{(\ell w_{f,a}+\ell w_{f,b})-(\ell w_{f,0})}{\ell w_{f,0}}$$
$$= \left(\frac{1}{\Delta^{2\omega+1}}+\frac{1}{(1-\Delta)^{2\omega+1}}\right)-1 \quad (29)$$

3. Experimental system

Polymer solutions with two different concentrations were prepared using as the working fluid in the heat transfer module for experimentation to verify the theoretical analysis of this work. The first one is the 1000ppm aqueous poly-acrylic acid (Carbopol 934 from Lubrizol) solution and the other was of 2000ppm. All the experiments conducted in this work were performed using the heat transfer equipment depicted in Fig. 1. Both the upper and lower aluminum plates were heated indirectly by water at different temperature under thermostat to keep constant wall temperature, and the fabrication of the double-pass heat exchanger module was demonstrated in Fig. 2. The whole experimental apparatus consisted of the pumps, temperature-controlled fluid storage tanks, heat exchanger module and measuring units for the liquid flow rate and fluid temperature.

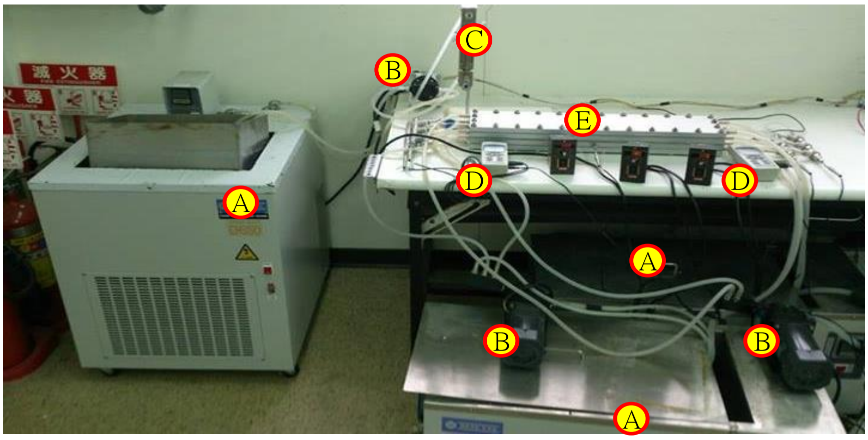

Fig. 2 Experimental setup

4. Results and conclusions

The mathematical modeling equations for counterflow double-pass heat-exchanger equation of power-law fluids with uniform wall temperatures have been formulated and solved analytically using orthogonal expansion technique for the homogeneous part and an asymptotic solution for the non-homogeneous part. As show in Fig. 3, the average Nusselt number $\overline{Nu}$ increases with increasing Gz. Besides, the smaller the power-law index ω is, the higher is the average Nusselt number, especially for the higher volumetric flow rate. Figure 3 shows the relation between the theoretical average Nusselt numbers $\overline{Nu}$ and Graetz number Gz with the wall temperature ratio σ and power-law index ω as parameters. The average Nusselt number of the double-pass device $\overline{Nu}$ increases with increasing wall temperature ratio σ and power-law index ω.

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Nomenclature

Gz Graetz number, $VW/\alpha BL$
I_h heat transfer enhancement
I_p power consumption increment
m consistency in power-law model
$\overline{Nu}$ average Nusselt number
T temperature of fluid, K
T_i inlet temperature of fluid in conduit, K
T_1 wall temperature in lower plate, K
T_2 wall temperature in upper plate, K
V input volume flow rate of conduit, m^3/s
v velocity distribution of fluid, $\mathrm{m/s}$
W conduit height, m
x longitudinal coordinate
z transversal coordinate

Greek symbols

α thermal diffusivity of fluid, m^2/s
$\dot{\gamma}$ shear rate
η longitudinal coordinate, x/W
λ_m eigenvalue
σ wall temperature ratio
δ Impermeable-sheet thickness, m
ξ transversal coordinate, z/LGz
ψ Dimensionless temperature
ω power-law index

Subscripts

0 at the inlet or for the single pass
a in forward flow channel
b in backward flow channel
F at the outlet of a double-pass device
L At the end of the channel
w at the wall surface

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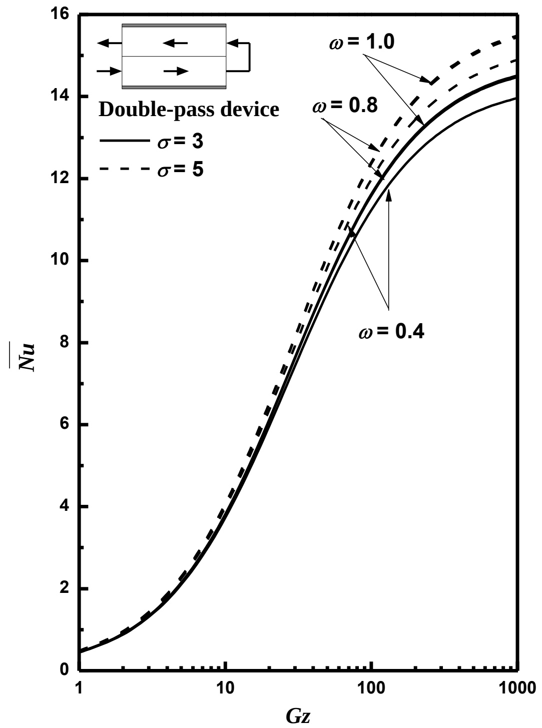

Fig. 3 $\overline{Nu}$ vs. Gz with ω and σ as parameters.