Experimental and analytical study of the internal recycle-effect on the heat transfer for the power-law fluid in a double-pass flat-plate heat exchanger with constant wall temperature

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Abstract

A conjugated Graetz problem of the double-pass flat-plate heat exchanger with internal recycle at uniform wall temperature was solved analytically using the orthogonal expansion technique for the power-law fluid. The mathematical formulation was derived for a fully developed laminar flow through the flat-plate channels by ignoring axial conduction and assuming temperature-independent fluid properties. A constant wall temperature, and both the continuous temperature and the same heat flux at the interface of the two adjacent subchannels made by inserting an impermeable sheet in between, were considered as the thermal boundary conditions. Experiments were carried out in order to validate the proposed mathematical formulation and the results can be very satisfactory. It is found that the recycle ratio and the impermeable-sheet position play significant influences on the efficiencies of this double-pass flat-plate heat exchanger. But, if the power consumption is also evaluated, the performance declines for the double-pass heat exchanger with large reflux ratios. The heat-transfer efficiency enhancement for the power-law fluid with a smaller power-law index is found to be less than that with a larger one, however, if both the heat transfer efficiency and the power consumption increment are considered together, the fluid with a smaller index would have a higher performance.

Keywords: Power-law fluids; Orthogonal expansion technique; Double-pass Operation; Heat transfer efficiency; Recycle.

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Nomenclature

B conduit width, m

c_p specific heat at constant pressure, $kJ/kg\cdot K$

d_{mn} coefficient in the eigenfunction $F_{a,m}$

e_{mn} coefficient in the eigenfunction $F_{b,m}$

F_m eigenfunction associated with eigenvalue λ_m

Gz Graetz number, $VW/\alpha BL$

G function defined in the separation of variables

$\bar{h}$ average heat transfer coefficient, $kW/m^2\cdot K$

I_h percentage improvement in heat-transfer rate

I_p power consumption increment

k thermal conductivity, $kW/m\cdot K$

L conduit length, m

m consistency in the power-law model, $Pa\cdot s^b$

$\overline{Nu}$ average Nusselt number

R recycle ratio

S_m expansion coefficient associated with eigenvalue λ_m

T temperature, K

V input volumetric flow rate, m^3/s

v velocity distribution of fluid, m/s

$\bar{v}$ average velocity of fluid, m/s

W distance between two parallel plates, m

x transversal coordinate, m

z longitudinal coordinate, m

Greek symbols

α thermal diffusivity, m^2/s

$\dot{\gamma}$ shear rate, $1/s$

Δ subchannel thickness ratio, W_a/W

η dimensionless transversal coordinate, x/W

ξ dimensionless longitudinal coordinate, z/L

ψ dimensionless temperature, $(T-T_s)/(T_i-T_s)$

θ dimensionless temperature, $(T-T_i)/(T_s-T_i)$

λ_m eigenvalue

ρ density, kg/m^3

τ shear stress, N/m^2

ω power-law index

Subscripts

0 for the single-pass device

a in forward(lower) flow subchannel

b in backward(upper) flow subchannel

F at the outlet of a double-pass device

i at the inlet

L at ξ being egual to unity

s at the wall surface

1. Introduction

Concerning the heat-transfer of a fluid at steady state in a bounded conduit with ignoring axial conduction is a well-known Graetz problem [1,2], while the applicability of the single-stream problem to the more flowing streams such as evaporators, condensers and heat exchangers is expected to enhance the heat transfer efficiency, which are the so called conjugated Graetz problems associated with mutual conditions at the boundaries [3-5]. Many analytical solutions for the double-pass device of such conjugated Graetz problems for the Newtonian fluid were achieved by means of an orthogonal expansion technique [6,7]. However, many industrial materials such as food, polymeric systems, biological process fluid, pulp and paper suspensions, etc. exhibit a range of non-Newtonian fluid features in displaying shear-thinning and/or shear thickening behavior [8,9]. These materials are generally processed in laminar flow conditions along with the high viscosity levels. For the sake of pursuing analytical solutions, it seems to be reasonable to begin with the purely viscous power-law type fluids in order to investigate the effect of the shear-thinning on the convective heat transfer.

Alternative configuration of a new device for increasing the fluid velocity with the fixed heat transfer area is inserted in parallel an impermeable sheet to conduct a double-pass operation. Furthermore, the recycling operation might play a significant role in enhancing heat and mass transfer rate, and is widely used in the separation processes and the reactor design such as distillation [10], extraction [11], adsorption [12], mass diffusion [13], thermal diffusion [14], loop reactors [15], air-lift reactor [16] and draft-tube bubble column [17]. The understanding of the shear-thinning and/or shear thickening behavior in many substances of multi-phase nature and/or of high molecular weight encountered in the chemical process and food industries is still limited. The simplifying mathematical model mentioned previously for the Newtonian fluid has been investigated theoretically, and could be incorporated in aiming to find the analytical solution for the power-law fluids of the double-pass operations. The purpose of this work is to obtain the analytical solutions by using this method to the heat transfer problem for the power-law fluid flowing through a flat-plate channel with internal recycle and to investigate the effects of the recycle ratio, impermeable-sheet position and the power-law index on the heat-transfer efficiency improvement, as well as on the increment of power consumption.

2. Theoretical formulation

2.1 Double-pass operations with inserting an impermeable sheet

A double-pass flat-plate heat exchanger with thickness *W*, length *L* and width *B* ($>>W$) is demonstrated in Figs. 1 and 2. The flow channel is divided into two subchannels (*a* and *b*) by inserting an impermeable barrier with thickness δ ($\delta << W$). The gaps of the subchannels *a* and *b* are W_a and W_b, respectively, and the impermeable-sheet location is defined as $\Delta = W_a / W$. If the double-pass heat exchanger is operated without recycling, the whole stream leaving from the lower subchannel *a* is recycled back and flowing through the upper subchannel *b* as shown in Fig. 1. The fluid was heated by the device walls kept at constant temperature T_s. In the case with internal recycle, the fresh fluid at temperature T_i premixed, before entering the lower channel *a*, with the fluid recycled from the upper subchannel *b*. Then, at the exit, some part of the stream was pumped back to the upper subchannel *b* as shown in Fig. 2.

The following assumptions are made to simplify the problem: constant physical properties of the fluid; power-law fluid with power index *ω* ($\tau = -m\,\dot{\gamma}^{\varpi}$); fully-developed laminar flow; ignoring the entrance length and longitudinal heat conduction; neglecting the end effects; well-mixing at the inlet and outlet of device. The dimensionless energy balance equations and the corresponding boundary conditions for the double-pass heat exchanger with the uniform wall temperature sketched above can be written as:

$$\frac{\partial^2 \psi_a(\eta_a,\xi)}{\partial \eta_a^2} = \left(\frac{W_a^2 v_a}{L\alpha}\right)\frac{\partial \psi_a(\eta_a,\xi)}{\partial \xi} \tag{1}$$

$$\frac{\partial^2 \psi_b(\eta_b,\xi)}{\partial \eta_b^2} = \left(\frac{W_b^2 v_b}{L\alpha}\right)\frac{\partial \psi_b(\eta_b,\xi)}{\partial \xi} \tag{2}$$

$$\psi_a(0,\xi) = 0 \tag{3}$$

$$\psi_b(0,\xi) = 0 \tag{4}$$

$$\psi_a(1,\xi) = \psi_b(1,\xi) \tag{5}$$

$$-\frac{\partial \psi_a(1,\xi)}{\partial \eta_a} = \frac{W_a}{W_b}\frac{\partial \psi_b(1,\xi)}{\partial \eta_b} \tag{6}$$

where

$$\xi = \frac{z}{L},\ \ \eta_a = \frac{x_a}{W_a},\ \ \eta_b = \frac{x_b}{W_b},\ \ Gz = \frac{V(W_a+W_b)}{\alpha BL} = \frac{VW}{\alpha BL},\ \ W_a = \Delta W,$$
$$W_b = (1-\Delta)W,\qquad \psi_a = \frac{T_a - T_s}{T_i - T_s},\ \ \psi_b = \frac{T_b - T_s}{T_i - T_s} \tag{7}$$

Equations (5) and (6) express that the temperatures and the heat fluxes are identical at the mutual boundary, respectively. The velocity profiles of the fully- developed power-law flow in these two subchannels are

$$v_a = \overline{v_a}\left[\frac{1+2\omega}{1+\omega}\right]\left(1-\left|2\eta_a - 1\right|^{\frac{1}{\omega}+1}\right) \tag{8}$$

$$v_b = \overline{v_b}\left[\frac{1+2\omega}{1+\omega}\right]\left(1-\left|2\eta_b - 1\right|^{\frac{1}{\omega}+1}\right) \tag{9}$$

in which the average flowing velocities,

$$\overline{v_a} = \left[V/W_aB\right],\qquad \overline{v_b} = -\left[V/W_bB\right] \tag{10}$$

for the device without recycling, or

$$\overline{v_a} = \left[V(R+1)/W_aB\right],\qquad \overline{v_b} = -\left[RV/W_bB\right] \tag{11}$$

for the device with recycle ratio *R*.

2.2 Single-pass operation

For the single-pass device of the same working dimensions without internal recycle, the impermeable sheet in Fig. 1 is removed and thus $W_a = W$. The dimensionless equation of heat transfer and the velocity distribution equation of the fully-developed power-law flow can be expressed as

$$\frac{\partial^2 \psi_0(\eta_0, \xi)}{\partial \eta_0^2} = \left(\frac{W^2 v_0(\eta_0)}{L\alpha} \right) \frac{\partial \psi_0(\eta_0, \xi)}{\partial \xi} \tag{12}$$

$$v_0 = \overline{v_0} \left[\frac{1+2\omega}{1+\omega} \right] \left(1 - \left| 2\eta_0 - 1 \right|^{\frac{1}{\omega}+1} \right) \tag{13}$$

where

$$\overline{v_0} = \frac{V}{WB}, \quad \eta_0 = \frac{x}{W}, \quad \xi = \frac{z}{L} \tag{14}$$

$$\psi_0(0, \xi) = 0 \tag{15}$$

$$\psi_0(1, \xi) = 0 \tag{16}$$

2.3 Calculation procedures

By following the similar mathematical formulation developed in our previous works [18], the analytical solution to this type of problem will be obtained by using an orthogonal expansion technique. Thus using the method of separation of variables, we have,

$$\psi_a(\eta_a, \xi) = \sum_{m=0}^{\infty} S_{a,m} F_{a,m}(\eta_a) G_m(\xi) \tag{17}$$

$$\psi_b(\eta_b, \xi) = \sum_{m=0}^{\infty} S_{b,m} F_{b,m}(\eta_b) G_m(\xi) \tag{18}$$

in which the eigenfunctions $F_{a,m}(\eta_a)$ and $F_{b,m}(\eta_b)$ were assumed to be polynomials to avoid the loss of generality.

$$F_{a.m}(\eta) = \sum_{n=0}^{\infty} d_{m,n} \eta_a{}^n, \quad d_{m,0} = 0, \quad d_{m,1} = 1 \quad \text{(selected)} \tag{19}$$

$$F_{b,m}(\eta) = \sum_{n=0}^{\infty} e_{m,n} \eta_b{}^n, \quad e_{m,0} = 0, \quad e_{m,1} = 1 \quad \text{(selected)} \tag{20}$$

By substituting Eqs. (17)-(18) and the velocity profiles (8)-(9) into the governing equations, Eqs. (1)-(2), we have, for the case without recycle,

$$G_m(\xi) = e^{-\lambda_m(1-\xi)} \tag{21}$$

$$F''_{a,m}(\eta_a) - \lambda_m \mathrm{Gz}\Delta\left(\frac{1+2\omega}{1+\omega}\right)\left(1-\left|2\eta_a - 1\right|^{\frac{1}{\omega}+1}\right)F_{a,m}(\eta_a) = 0 \tag{22}$$

$$F''_{b,m}(\eta_b) + \lambda_m Gz(1-\Delta)\left(\frac{1+2\omega}{1+\omega}\right)\left(1-\left|2\eta_b - 1\right|^{\frac{1}{\omega}+1}\right)F_{b,m}(\eta_b) = 0 \tag{23}$$

and, for the case with internal recycle,

$$F''_{a,m}(\eta_a) - \lambda_m \mathrm{Gz}\Delta(R+1)\left(\frac{1+2\omega}{1+\omega}\right)\left(1-\left|2\eta_a - 1\right|^{\frac{1}{\omega}+1}\right)F_{a,m}(\eta_a) = 0 \tag{24}$$

$$F''_{b,m}(\eta_b) + \lambda_m Gz(1-\Delta)R\left(\frac{1+2\omega}{1+\omega}\right)\left(1-\left|2\eta_b - 1\right|^{\frac{1}{\omega}+1}\right)F_{b,m}(\eta_b) = 0 \tag{25}$$

The exponential term on the right-hand side in Eqs. (22)-(25) was approximated using the fifth polynomial of Eq. (26) in the same manner as in our previous work [18]

$$1-\left|2\eta_b - 1\right|^{\frac{1}{\omega}+1} \cong X_1\eta^1 + X_2\eta^2 + X_3\eta^3 + X_4\eta^4 + X_5\eta^5 \tag{26}$$

in which $X_1, X_2 \ldots X_5$ are five curve-fitted constants dependent on the power law index. All the coefficients $d_{m,n}$ and $e_{m,n}$ for $F_{a,m}(\eta_a)$ and $F_{b,m}(\eta_b)$ in Eqs. (19) and (20), respectively, may be given in terms of eigenvalue λ_m as follows:

$$d_{m,0} = 0,\ d_{m,1} = 1,\ d_{m,2} = 0,\ d_{m,3} = 0,$$

$$d_{m4} = \frac{\lambda_m \mathrm{Gz}\Delta(1+2\omega)}{12(1+\omega)} X_1$$

$$d_{m5} = \frac{\lambda_m \mathrm{Gz}\Delta(1+2\omega)}{20(1+\omega)} X_2$$

$$d_{m6} = \frac{\lambda_m \mathrm{Gz}\Delta(1+2\omega)}{30(1+\omega)} X_3$$

$$\vdots$$

$$d_{mn}=\frac{\lambda_m \mathrm{Gz}\Delta}{n(n-1)}\frac{1+2\omega}{1+\omega}\left(X_1\times d_{m(n-7)}+X_2\times d_{m(n-6)}+X_3\times d_{m(n-5)}+X_4\times d_{m(n-4)}+X_5\times d_{m(n-3)}\right)$$

(27a)

$$e_{m,0}=0,\ \ e_{m,1}=1,\ \ e_{m,2}=0,\ \ e_{m,3}=0$$

$$e_{m4}=\frac{-\lambda_m Gz(1-\Delta)(1+2\omega)}{12(1+\omega)}X_1$$

$$e_{m5}=\frac{-\lambda_m Gz(1-\Delta)(1+2\omega)}{20(1+\omega)}X_2$$

$$e_{m6}=\frac{-\lambda_m Gz(1-\Delta)(1+2\omega)}{20(1+\omega)}X_3$$

$$\vdots$$

$$e_{mn}=\frac{\lambda_m Gz(1-\Delta)}{n(n-1)}\frac{1+2\omega}{1+\omega}\left(X_1\times e_{m(n-7)}+X_2\times e_{m(n-6)}+X_3\times e_{m(n-5)}+X_4\times e_{m(n-4)}+X_5\times e_{m(n-3)}\right)$$

(27b)

for the case without recycling, or

$$d_{m,0}=0,\ \ d_{m,1}=1,\ \ d_{m,2}=0,\ \ d_{m,3}=0,$$

$$d_{m4}=\frac{\lambda_m \mathrm{Gz}(\mathrm{R}+1)\Delta(1+2\omega)}{12(1+\omega)}X_1$$

$$d_{m5}=\frac{\lambda_m \mathrm{Gz}(\mathrm{R}+1)\Delta(1+2\omega)}{20(1+\omega)}X_2$$

$$d_{m6}=\frac{\lambda_m \mathrm{Gz}(\mathrm{R}+1)\Delta(1+2\omega)}{30(1+\omega)}X_3$$

$$\vdots$$

$$d_{mn}=\frac{\lambda_m \mathrm{Gz}(\mathrm{R}+1)\Delta}{n(n-1)}\frac{1+2\omega}{1+\omega}\left(X_1\times d_{m(n-7)}+X_2\times d_{m(n-6)}+X_3\times d_{m(n-5)}+X_4\times d_{m(n-4)}+X_5\times d_{m(n-3)}\right)$$

(28a)

$$e_{m,0}=0,\ \ e_{m,1}=1,\ \ e_{m,2}=0,\ \ e_{m,3}=0$$

$$e_{m4}=\frac{-\lambda_m GzR(1-\Delta)(1+2\omega)}{12(1+\omega)}X_1$$

$$e_{m5}=\frac{-\lambda_m GzR(1-\Delta)(1+2\omega)}{20(1+\omega)}X_2$$

$$e_{m6}=\frac{-\lambda_m GzR(1-\Delta)(1+2\omega)}{20(1+\omega)}X_3$$

$$\vdots$$

$$e_{mn}=\frac{\lambda_m GzR(1-\Delta)}{n(n-1)}\frac{1+2\omega}{1+\omega}\left(X_1\times e_{m(n-7)}+X_2\times e_{m(n-6)}+X_3\times e_{m(n-5)}+X_4\times e_{m(n-4)}+X_5\times e_{m(n-3)}\right)$$

(28b)

for the case with recycle ratio *R*.

The dimensionless outlet temperature for the double-pass device (θ_F) as well as for the single-pass device $(\theta_{0,F})$ were also obtained in terms of the Graetz number (Gz), eigenvalues $(\lambda_m$ or $\lambda_{0,m})$, expansion coefficients ($S_{a,m}$, $S_{b,m}$ or $S_{0,m}$), location of impermeable sheet (Δ) and eigenfunctions ($F_{a,m}(\eta_a)$, $F_{b,m}(\eta_b)$ or $F_{0,m}(\eta_0)$). The results are

$$\theta_F=1-\psi_F=\frac{T_F-T_i}{T_s-T_i}=1+\left(\frac{1}{(1-\Delta)\mathrm{Gz}}\right)\sum_{m=0}^{\infty}\frac{e^{-\lambda_m}S_{b,m}}{\lambda_m}\left\{F'_{b,m}(1)-F'_{b.m}(0)\right\} \quad (29)$$

for the double-pass device without recycling,

$$\theta_F=1-\psi_F=\frac{T_F-T_i}{T_s-T_i}=1-\frac{1}{(R+1)G_z\Delta}\sum_{m=0}^{\infty}\frac{S_{a,m}}{\lambda_m}[F'_{a,m}(1)-F'_{a,m}(0)] \quad (30a)$$

$$=1+\frac{1}{RG_z(1-\Delta)}\sum_{m=0}^{\infty}\frac{S_{b,m}}{\lambda_m}[F'_{b,m}(1)-F'_{b,m}(0)] \quad (30b)$$

for the double-pass device with recycling, and

$$\theta_{0,F}=1-\psi_{0,F}=\frac{T_{0,F}-T_i}{T_s-T_i}=\frac{1}{Gz}\left[\sum_{m=0}^{\infty}\frac{(1-e^{-\lambda_{0,m}})}{\lambda_{0,m}}S_{0,m}F'_{0,m}(0)+\frac{(1-e^{-\lambda_{0,m}})}{\lambda_{0,m}}S_{0,m}F'_{0,m}(1)\right] \quad (31)$$

for the single-pass device, where the primes on $F'_{a,m}(\eta_a)$, $F'_{b,m}(\eta_b)$ and $F'_{0,m}(\eta_0)$ denote the differentiations with respect to η_a, η_b and η_0, respectively. The eigenvalues (λ_m) for the devices with an impermeable sheet inserted were calculated from the following equations,

$$\frac{S_{a,m}}{S_{b,m}} = \frac{F_{b,m}(1)}{F_{a,m}(1)} = -\frac{W_a}{W_b}\frac{F'_{b,m}(1)}{F'_{a,m}(1)} = -\frac{\Delta}{(1-\Delta)}\frac{F'_{b,m}(1)}{F'_{a,m}(1)} \tag{32}$$

3. Improvement of heat transfer efficiency

The average Nusselt number for either the single- or the double-pass operation may be defined as

$$\overline{Nu} = \frac{\bar{h}W}{k} \tag{33}$$

in which the average heat transfer coefficient is defined as

$$\bar{h}(2BL)(T_s - T_i) = V\rho C_p(T_F - T_i) \tag{34}$$

or

$$\bar{h} = \frac{V\rho C_p}{2BL}\left(\frac{T_F - T_i}{T_s - T_i}\right) = \frac{V\rho C_p}{2BL}(1-\psi_F) \tag{35}$$

Thus, for the double-pass device,

$$\overline{Nu} = \frac{V\rho C_p}{2BL}\left(\frac{T_F - T_i}{T_s - T_i}\right) = \frac{V\rho C_p}{2BL}(1-\psi_F) = 0.5Gz(1-\psi_F) \tag{36}$$

and for a single-pass device,

$$\overline{Nu_0} = \frac{V\rho C_p}{2BL}\left(\frac{T_{0,F} - T_i}{T_s - T_i}\right) = \frac{V\rho C_p}{2BL}(1-\psi_{0,F}) = 0.5Gz(1-\psi_{0,F}) \tag{37}$$

The improvement of device performance by employing a double-pass operation either with or without recycle, relative to a single-pass operation with the same working dimension and operating parameters, is best illustrated by calculating the percentage increase in heat-transfer rate as follows:

$$I_h = \frac{\overline{Nu} - \overline{Nu_0}}{\overline{Nu_0}} \times 100\% \tag{38}$$

4. Experimentation

4.1 Preparation of working fluid

Two aqueous polymer solutions with different concentrations were prepared in the same manner as our previous work [19], which were used as the working fluids in the heat transfer module to verify the theoretical analysis in this work. The first one is a polyacrylic acid (Carbopol 934, Lubrizol Corporation) aqueous solution at the concentration of 1000ppm and the other is at 2000ppm. At room temperature, both of them possess the shear-thinning property with the power law indices $\omega = 0.8$ and 0.4, respectively, obtained from the rheometer (MCR 101, Anton Paar, Austria). Their heat capacities were measured using the DSC (Perkin Elmer Diamond) and their thermal conductivities were acquired by use of the instrument (Hot Disk TPS 2500, Sweden). The experimental values are tabulated in Table 1.

4.2 Experimental procedure

All the experiments in this work were performed using the heat transfer equipment depicted in Figs. 1 and 2 either with or without recycling. A parallel conduit made of two aluminum sheets screwed together with working dimensions of 61×10×1 cm^3 was constructed as shown. A very thin aluminum sheet with dimensions of 61×10×0.2 cm^3 was also screwed between the plates at the central position, say $\Delta = 0.5$, to carry out the double-pass operations in this flat-plate heat exchanger. Both the upper and lower aluminum plates were heated by circulated water through a bath equipped with a thermostat maintained at constant temperature, T_s = 40℃. The working fluid firstly flowed into the lower channel *a* with the aid of a conventional pump, and then recycled back through the upper channel *b*, and finally returned to the thermostatic storage tank kept at a preset temperature, T_i = 20℃, as shown in Fig. 1. In case of the heat exchanger with recycling, a three-way valve was installed immediately behind the exit of the subchannel *a*, and thereby a preset portion (reflux ratio) of the outlet stream were pumped back to the subchannel *b*, and then returned to the subchannel *a* along with the fresh inlet fluid. The fluid flow rate was regulated by means of adjusting the pumping power and was measured using flow meters. The pipelines outside the heat-exchanger module were wrapped with 1.0 cm thick foamed plastic to reduce the undesirable heat loss.

5. Results and discussions

5.1 Enhancement of heat transfer efficiency in a double-pass device

The average Nusselt numbers, defined by Eq. (36), versus Graetz numbers are presented in Fig. 3 for the double-pass plate heat exchanger (without recycle) with the power-law index as the parameter, in which $\overline{Nu}$ for a single-pass exchanger was also shown for comparison. It is very clear that the double-pass heat exchanger has larger $\overline{Nu}$ than the single-pass one, and the fluid with a low power-law index has larger $\overline{Nu}$ than the fluid with a high index, irrespective of whether double- or single-pass device. The relation between the experimental $\overline{Nu}$ and the fluid's power index for the double-pass heat exchanger also show the same trend, although which are fairly comparable with the simulated results. It could be inferred that the velocity profile levels off near the central region for the fluid with a small power-law index; on the other hand, in the region adjacent to the wall, it may flow faster than the fluid with a larger power-law index. In other words, the fluid with smaller index may transport more heat from the wall than that with a larger one, which results in higher Nusselt number for the heat transfer. For the double-pass device with recycle ratio R=1.0, the average Nusselt number was found to increase due to recycling of fluid from subchannel *b* back into subchannel *a* as shown in Fig. 4. The experimental data for the double-pass device are also presented and match well with the simulated results. However, the fluid with a small power-law index doesn't have large Nusselt number, as in the case of without recycling shown in Fig. 3. It can be speculated that the hotter fluid with a smaller index recycling from subchannel *b*, as stated above, would mixed with the fresh fluid just before the entrance to the device, and leads to a reduction of the heat-transfer efficiency in subchannel *a*. Figure 5 showed $\overline{Nu}$ vs. Gz with the reflux ratio as the parameter for $\Delta=$

0.5 and $\omega = 0.8$. It can be found that the heat-transfer efficiencies are considerably enhanced with increasing reflux ratio in the double-pass heat exchanger. Again, the parallelism between the experimental and the theoretical results for *Gz* over the range of 7~ 40 is satisfactory. It is reasonable to put conjecture that for the device with a large reflux ratio *R*, the high flow rates in both the subchannels *a* and *b* may give rise to high convective heat-transfer coefficients. In addition, for the double pass model, the fluid is heated twice through the heat exchanger, while only once for the single pass model. Therefore, the heat transfer efficiency should be much higher for the double-pass model than for the single-pass one. The average Nusselt number $\overline{Nu}$ versus the impermeable-sheet position Δ was presented in Fig. 6 for the double-pass devices, with ω=0.8, *R*=1.0 and *Gz*=100, to show the effect of impermeable-sheet position on the heat transfer. It is clear that the double-pass device with recycling can have very great heat-transfer efficiency, and it becomes much high as the Δ is deviating from the value of 0.5.

The heat-transfer enhancement efficiency I_h by employing a double-pass operation without recycling was calculated with the power-law index as a parameter, and the results are demonstrated in Fig. 7. For the case with internal recycle, the calculated I_h are demonstrated in Figs. 8 and 9 with the power-law index and recycle ratio as parameters, respectively. It appears that the heat transfer efficiency is improved as Graetz number or recycle ratio increases, but would approach constant values as G_Z is raised to several hundreds. However, for the fluid with a small power index, the enhancement of the heat-transfer efficiency is less prominent in comparison with that for the fluid with a large index. It is worthy of notice that the present device with internal recycling has very high heat-transfer improvement efficiency relative to our previous device with external recycling [18], as shown in Fig. 9.

5.2 Power consumption increment

The heat transfer enhancement in association with the power-consumption increment due to the reduction of cross-sectional area from single- to double-pass will be considered and evaluated at this point. At first, the factor for the power consumption increment, I_p , is defined as,

$$I_p = \frac{(\ell w_{f,a} + \ell w_{f,b}) - (\ell w_{f,0})}{\ell w_{f,0}} \tag{39}$$

where $\ell w_{f,a}$, $\ell w_{f,b}$ are the friction losses in subchannels *a* and *b* for double-pass and $\ell w_{f,0}$ is that for single-pass devices, with the laminar flow condition being assumed. Then, it can readily be derived as in the previous work [20], for a power law fluid [21] flowing in the double-pass device without recycling,

$$I_p = \left(\frac{1}{\Delta^{2\omega+1}} + \frac{1}{(1-\Delta)^{2\omega+1}} \right) - 1 \tag{40}$$

and with internal recycling,

$$I_p = \left(\frac{(R+1)^{\omega+1}}{\Delta^{2\omega+1}} + \frac{R^{\omega+1}}{(1-\Delta)^{2\omega+1}} \right) - 1 \tag{41}$$

From Eqs. (40)-(41), it is seen that I_p does not depend on the Graetz number, but does grow up with the increasing *R* as well as with the Δ value deviating from 0.5. And, it can be dramatically reduced as the power-law index ω diminishes. On considering simultaneously both I_h and I_p, their ratios are plotted versus Graetz number with the power index and the recycle ratio as parameters, as shown in Figs. 10-12. The trend of the I_h/I_p ratio relative to either ω or *R* is found to be opposite to that of I_h shown in Figs. 7-9. This is because high recycle ratios would bring about high flow rates and thus high power consumptions in the conduits, but the fluid with a small power-law index could, on the contrary, significantly decrease the power expense owing to the shear-thinning effect. In Fig. 11, it is shown again that the ratio of I_h/I_p for the present device with internal recycling is more favorable than that for the previous device with external recycling.

6. Conclusions

A mathematical formulation of the heat transfer problem of a power-law fluid flowing through a double-pass flat-plate heat exchanger with (or without) recycling has been developed in this study. This so-called conjugated Graetz problem has been investigated analytically by using the orthogonal expansion technique with the eigenfunctions expanded in terms of power series. Experiments were carried out and the results are found to fit very well with the simulated data. The average Nusselt number, percentage improvement of heat-transfer, as well as the induced power-consumption increment have been presented graphically with the recycle ratio, power-law index, and the subchannel thickness ratio as parameters. It indicates that the heat-transfer efficiencies are considerably enhanced with increasing recycle ratio in the double-pass heat exchanger, but if the power consumption is also considered, the performance declines for the device with large recycle ratios (comparing the Figs. 9 and 12). The heat-transfer enhancement for the power-law fluid with a smaller index is found to be less than that with a larger one, but if both the heat-transfer efficiency and the power-consumption increment are taken into account together, the fluid with a smaller index would have a higher performance.

Table 1 Thermal conductivity and heat capacity of the working fluids in this study.

Property	ω=0.4	ω=0.8
k (W/m·K)	0.64	0.65
c_p (J/g·℃)	3.14	2.64

Testing conditions: relative humidity：47%; temperature：22.8℃

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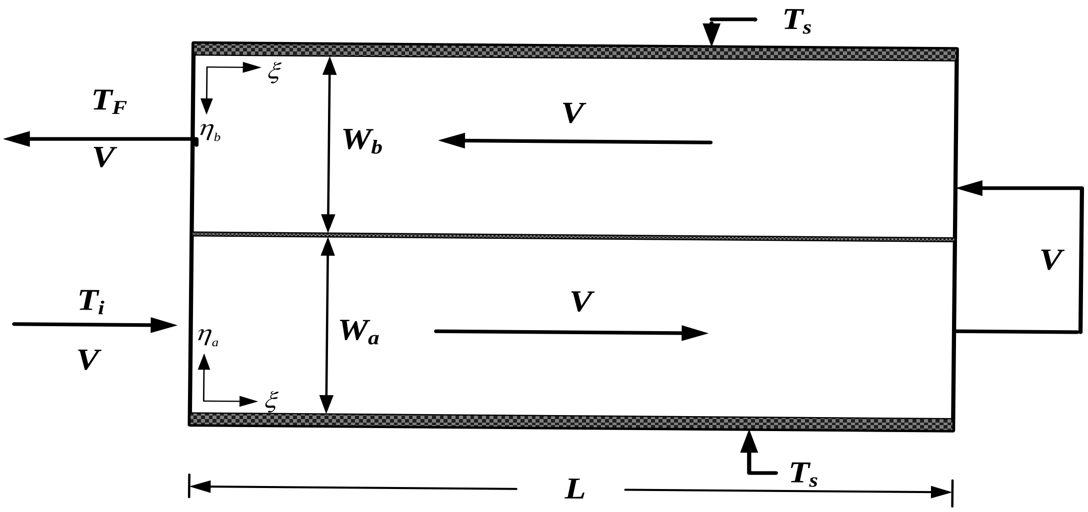

Figure 1. Double-pass parallel-plate conduits without internal recycling

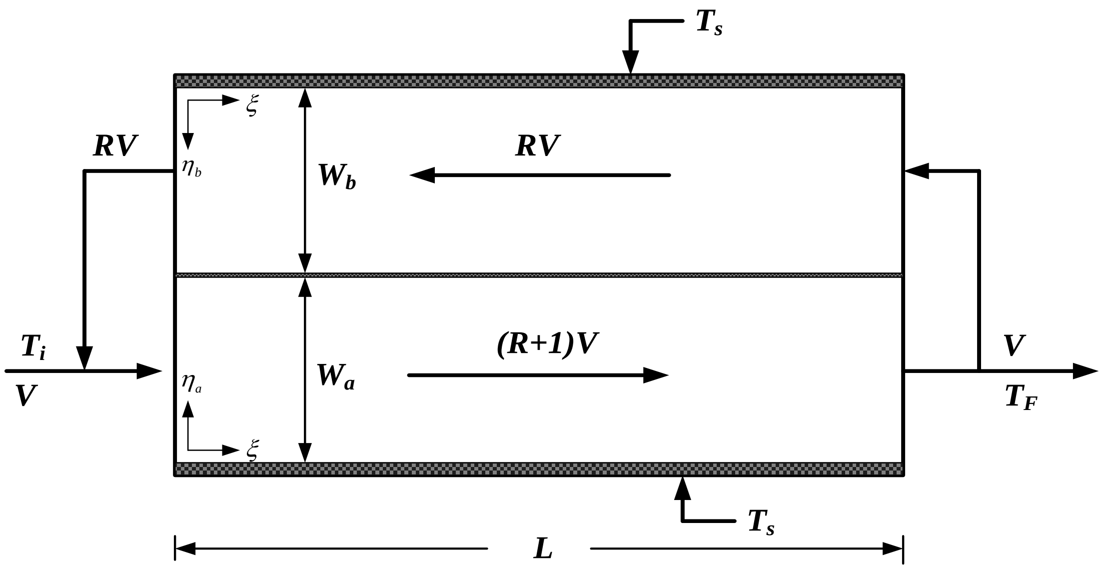

Figure 2. Double-pass parallel-plate conduits with internal recycling

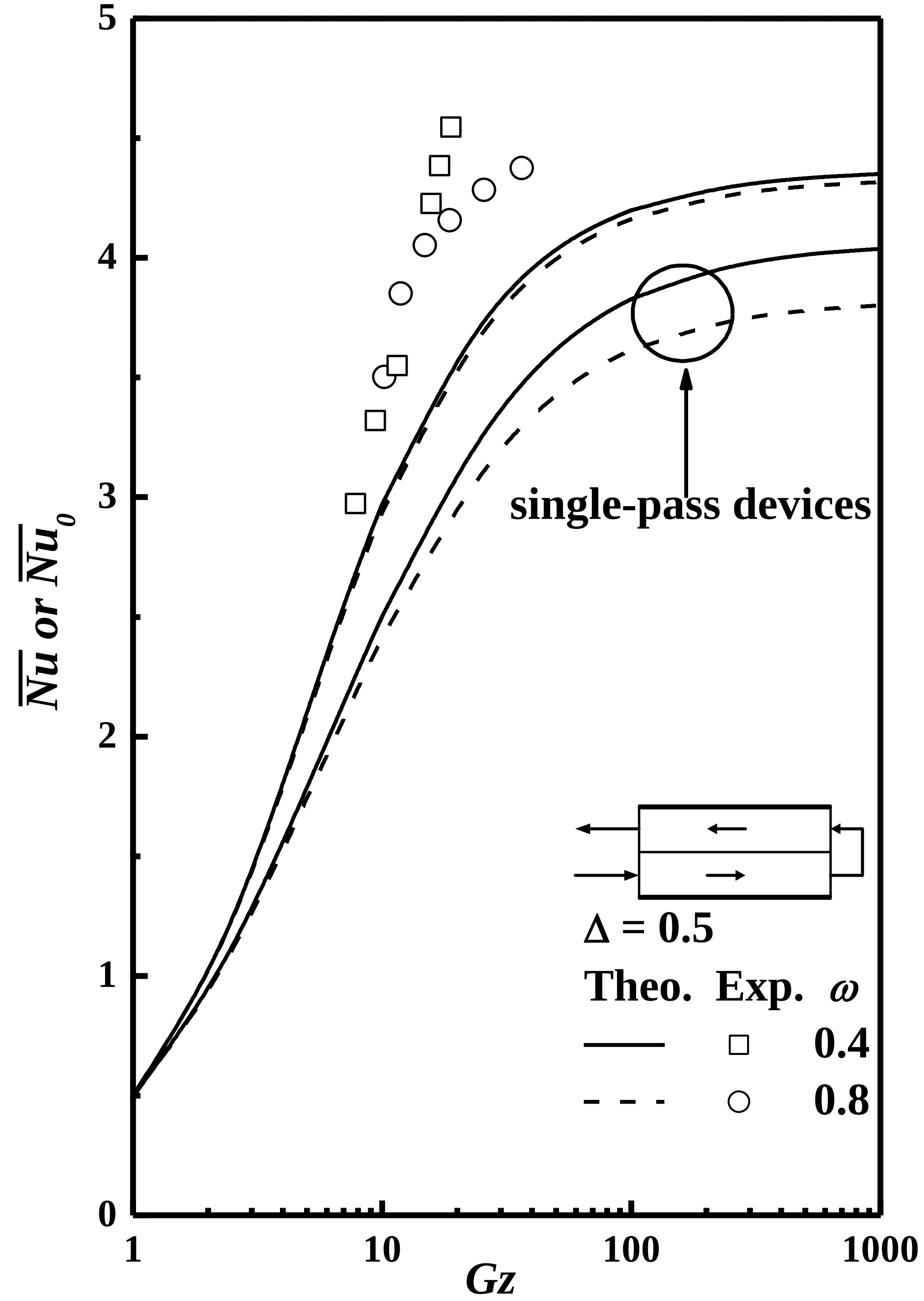

Figure 3. Average Nusselt number vs. *Gz* with ω as a parameter for the double-pass heat exchanger without internal recycling; Δ =0.5.

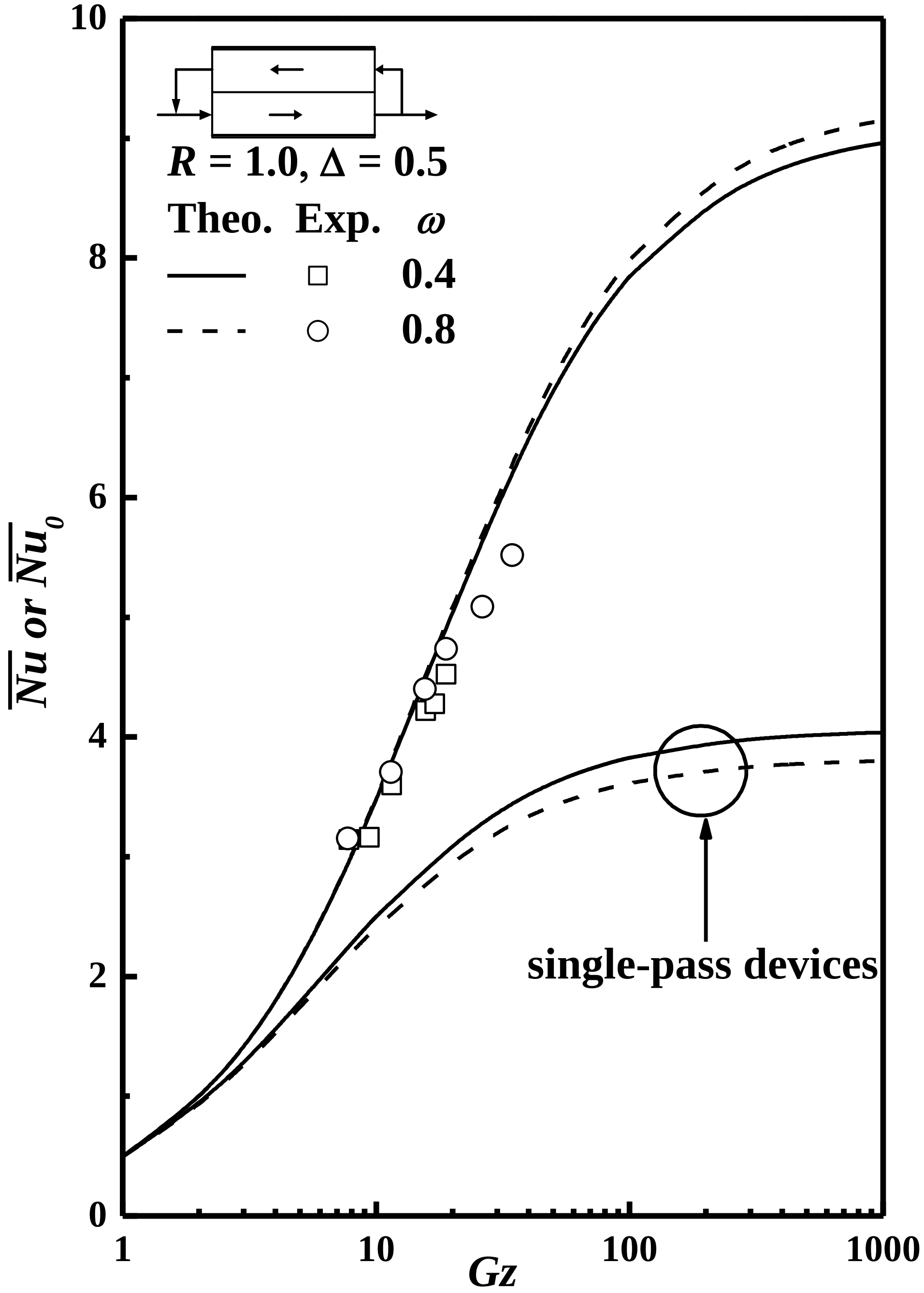

Figure 4. Average Nusselt number vs. *Gz* with ω as a parameter for the double-pass heat exchanger with internal recycling; *R*=1 and Δ=0.5.

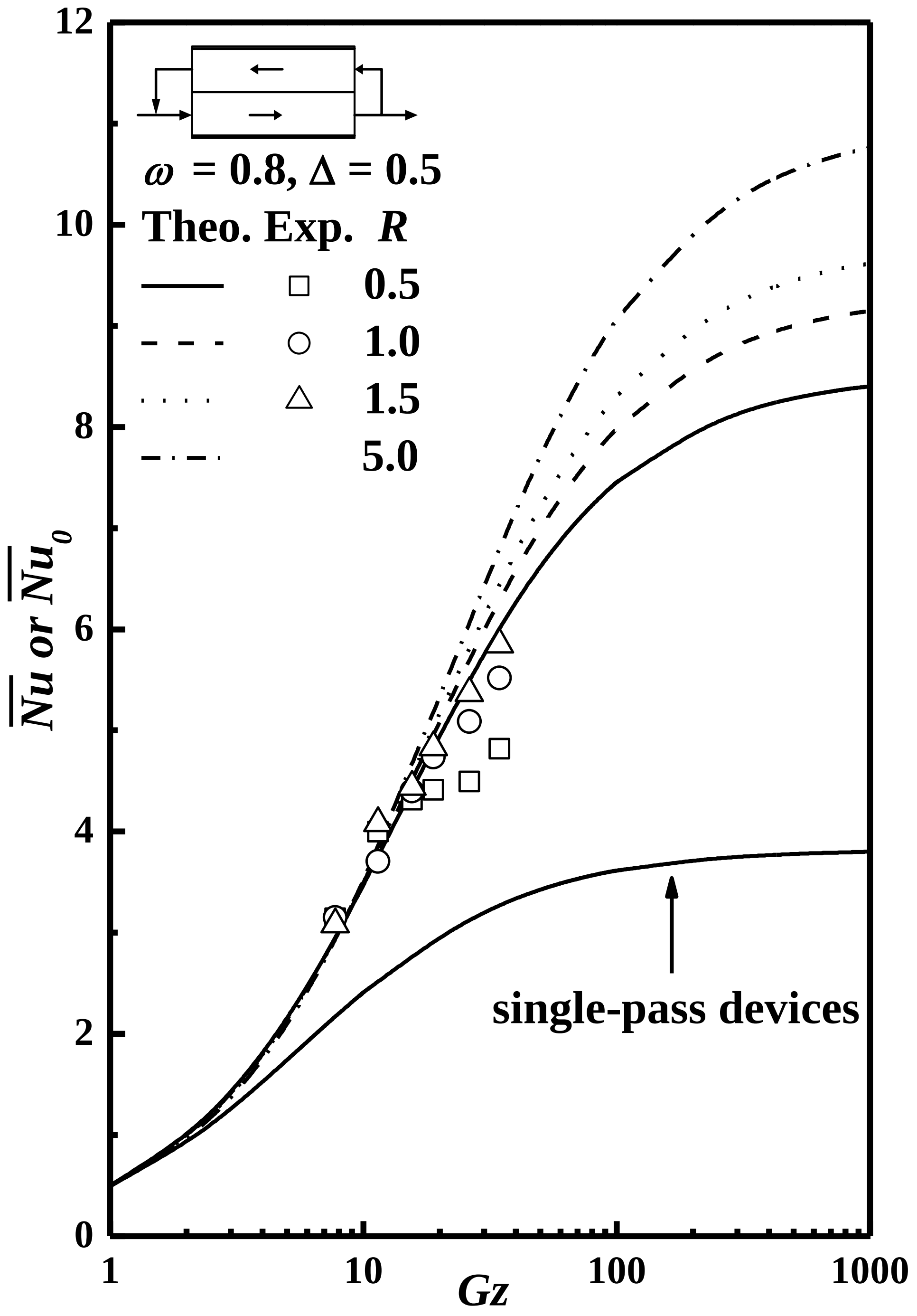

Figure 5. Average Nusselt number vs. Gz with R as a parameter for the double-pass heat exchanger with internal recycling; ω=0.8 and $\Delta = 0.5$.

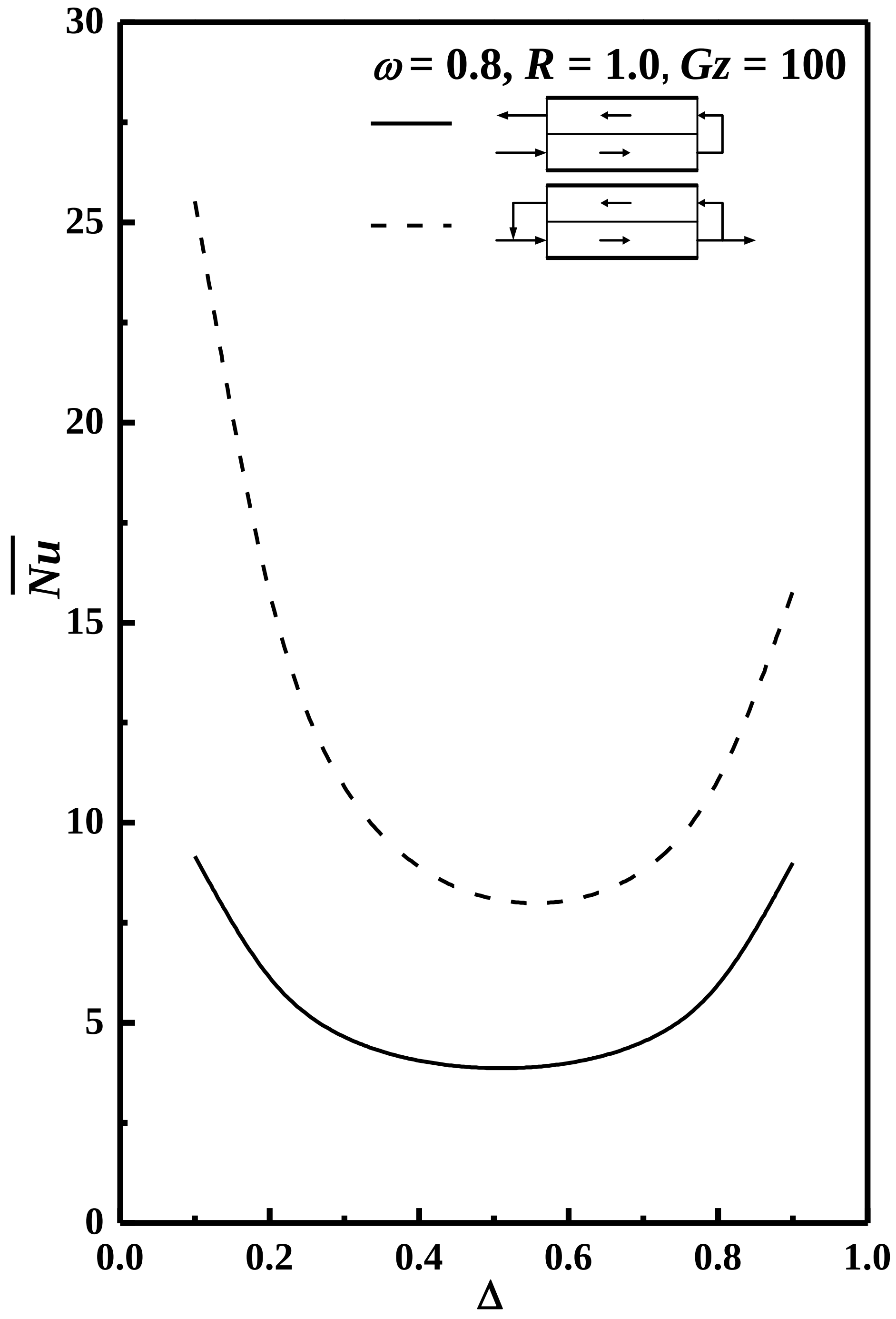

Figure 6. Average Nusselt number vs. Δ for the double-pass heat exchanger; ω=0.8, R=1 and Gz =100.

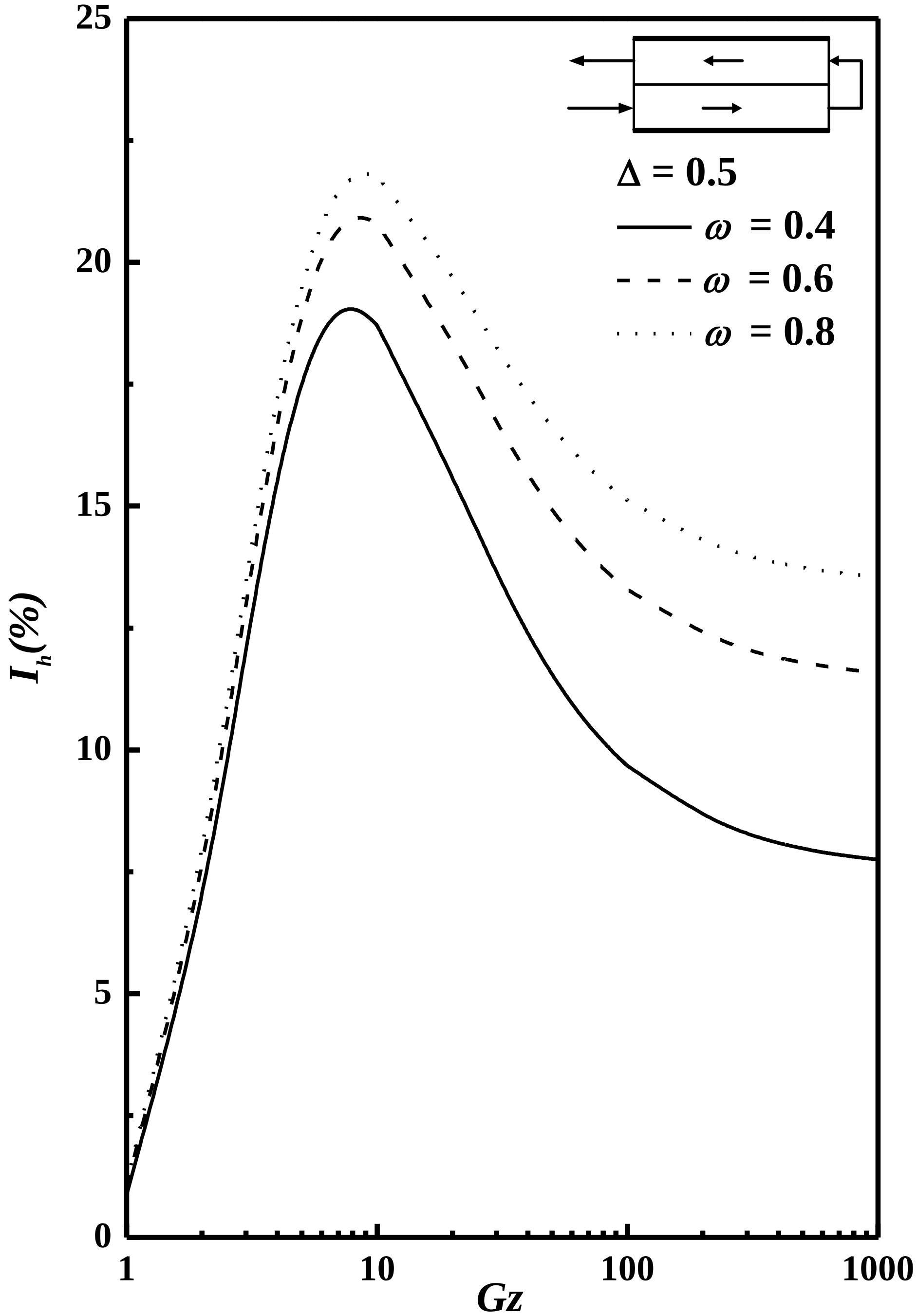

Figure 7. Percentage improvement in heat-transfer vs. *Gz* with *ω* as a parameter for the double-pass heat exchanger without internal recycling.

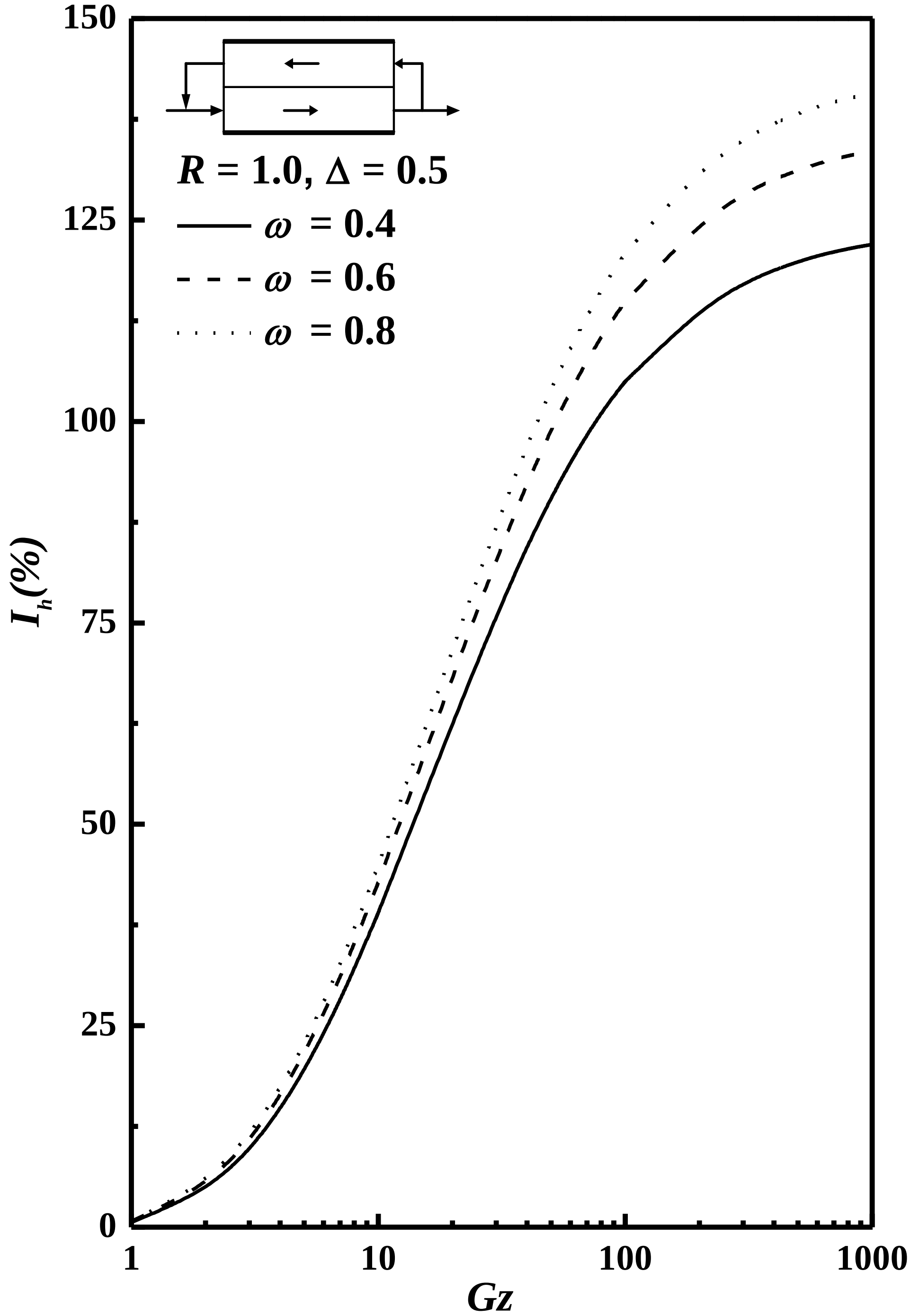

Figure 8. Percentage improvement in heat-transfer vs. *Gz* with ω as a parameter for the double-pass heat exchanger with internal recycling; R=1 and $\Delta = 0.5$.

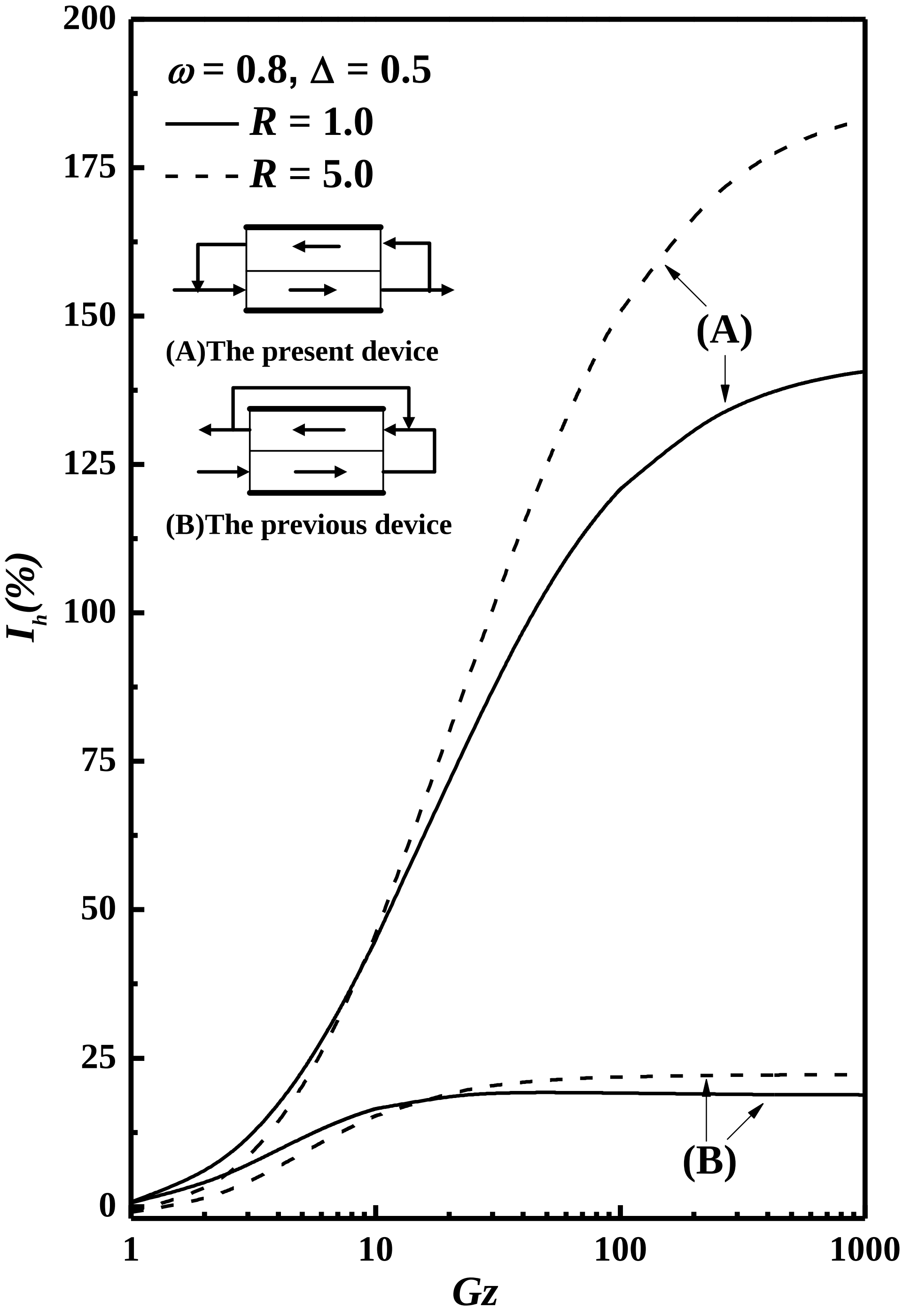

Figure 9. Percentage improvement in heat-transfer vs. *Gz* with *R* as a parameter; (A) the present device with internal recycling, (B) the previous device with external recycling [18].

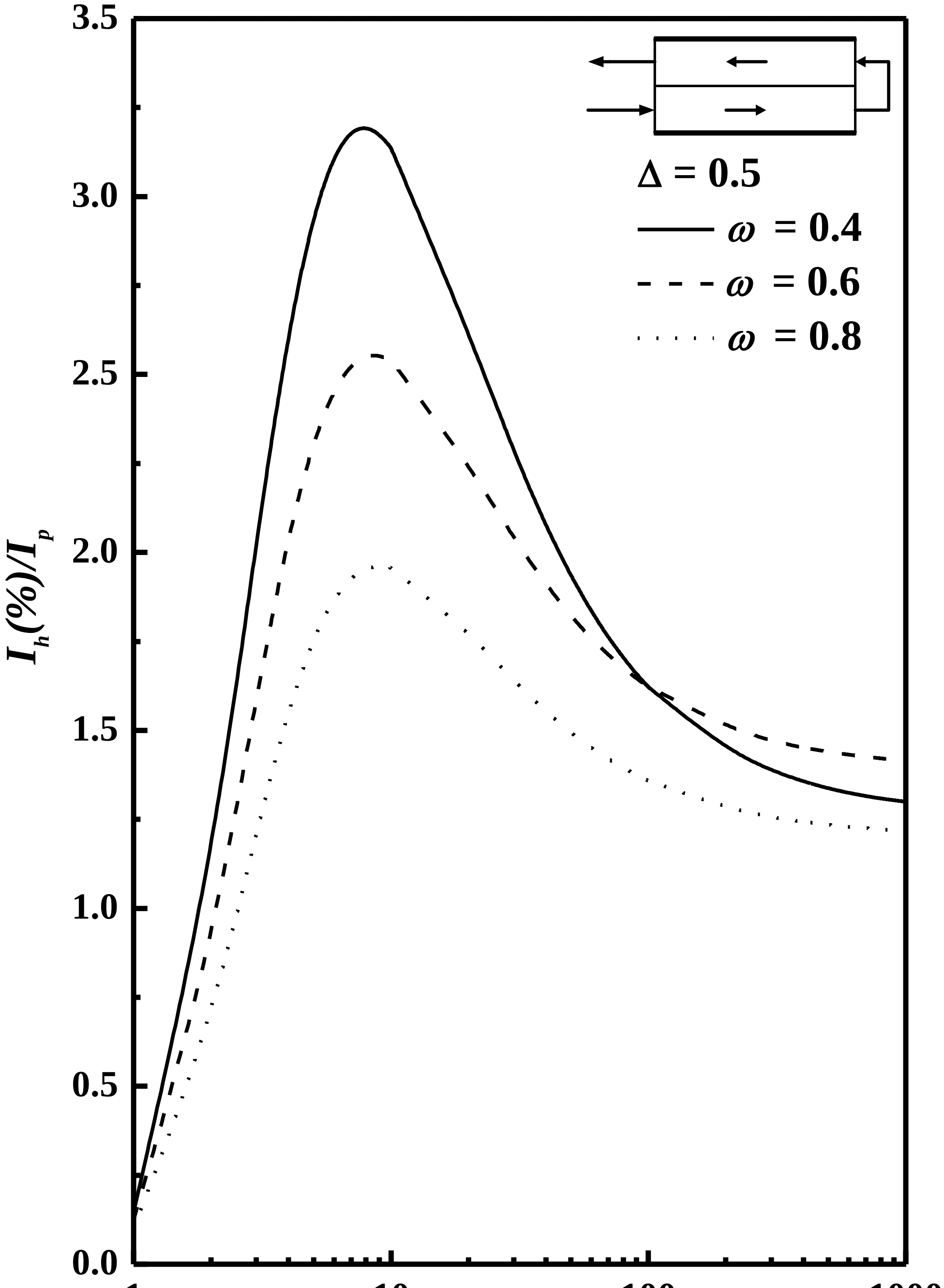

Figure 10. Ratio of I_h/I_p vs. Gz with ω as a parameter for the double-pass heat exchanger without internal recycling.

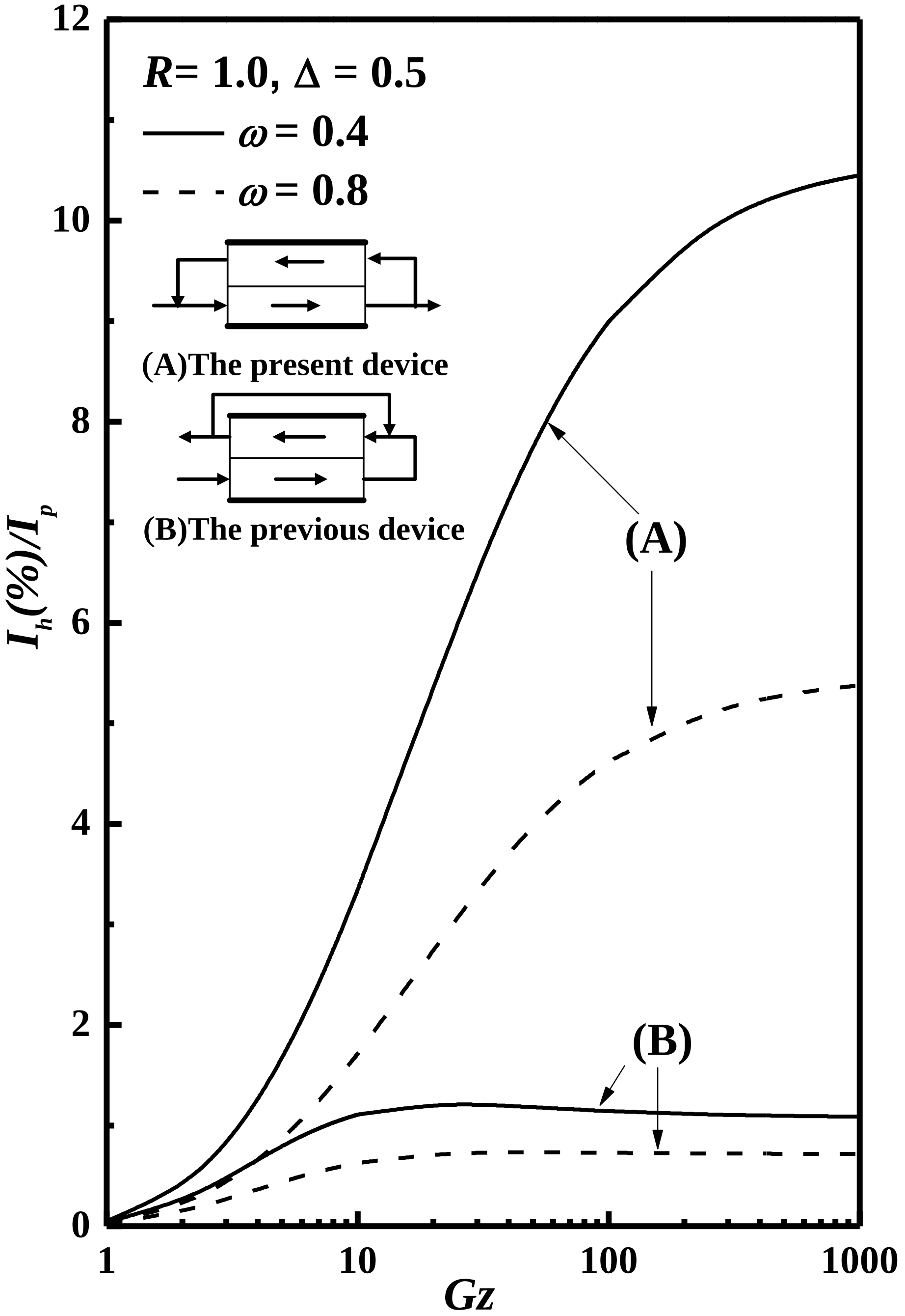

Figure 11. Ratio of I_h/I_p vs. Gz with ω as a parameter; (A) present device with internal recycling, (B) previous device with external recycling [18].

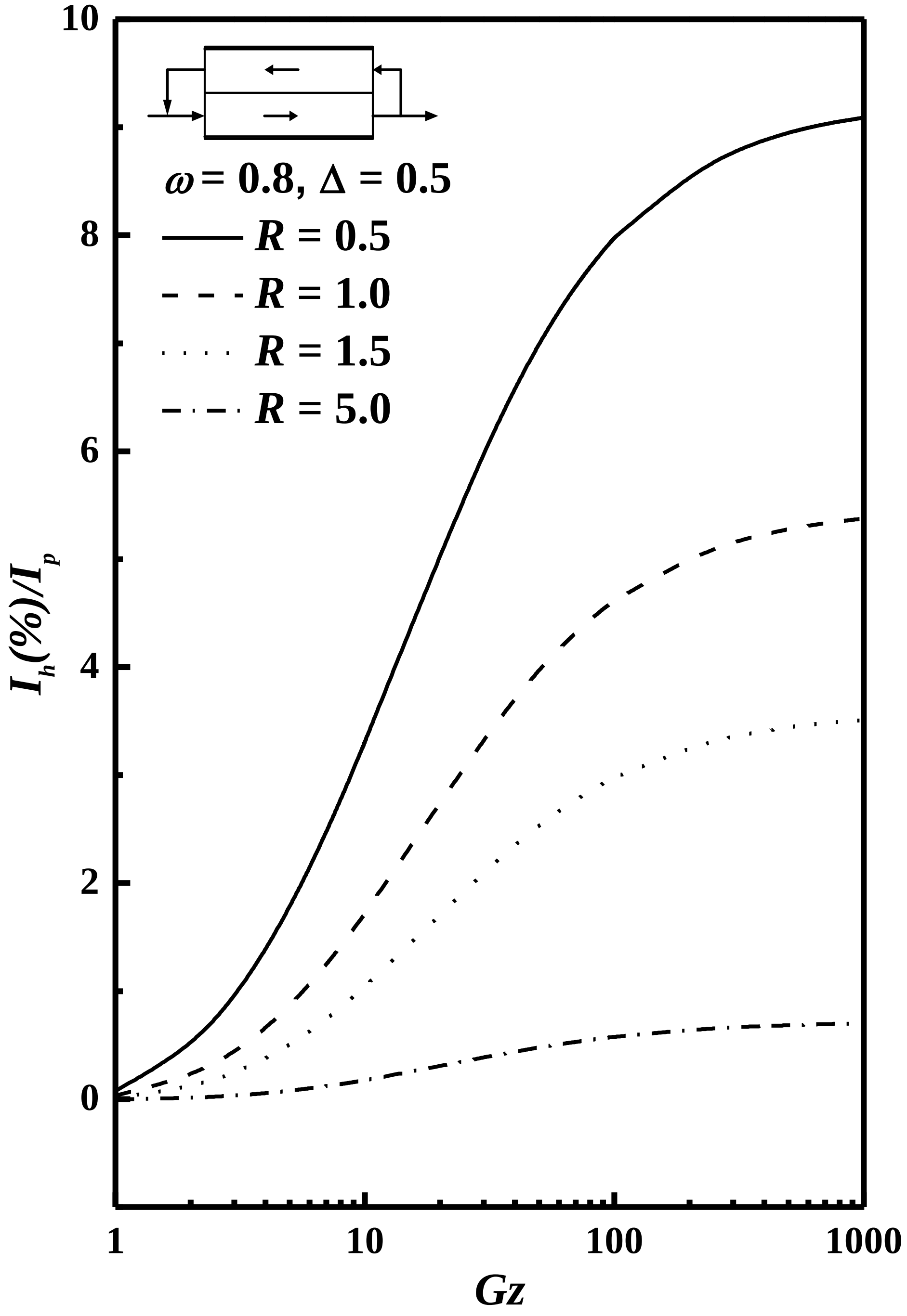

Figure 12. Ratio of I_h/I_p vs. Gz with R as a parameter for the double-pass heat exchanger with internal recycling; ω=0.8 and $\Delta = 0.5$