Degradation Algorithm of Compressive Sensing for Integer DCT with Application to H.264/AVC

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***Abstract*—Compressive sensing (CS) is an innovative approach for the acquisition of signals having a sparse or compressible representation in some basis. In this paper, we extend and modify the previous compression work that uses the degradation algorithm of CS for JPEG with 2-D DCT to the H.264/AVC with the integer 2-D DCT. Through simulation results we show that the complexity with the new proposed scheme can be dramatically reduced, and simultaneously to achieve better PSNR performance compared with the conventional CS approach using the 2-D DCT transform.**

Keywords- compressive sensing, degradation algorithm, sparsity, integer 2D-DCT, dimensionality reduction, PSNR.

I. INTRODUCTION

In conventional data acquisition process, we are first sampling the analog signals and then compressing them for reducing the quantity of sampled data that is indeed a kind of wasting [1-4]. It spends so much time to acquire data when we know actually that only a few of it will be preserved. Under such consideration, in recent years a new emerging technique, named as the compressive sensing (CS), has been developed for simultaneous measurement and compression [5]. It provides an innovative framework at data acquisition which is *S*-sparse in a specific domain, and is completely characterized by *M* measurements with $M << N$ (where *N* is traditional Nyquist based number of samples required) and $M > S$ [7]. That is, to reconstruct the original signal with CS approach, *M* depends on its sparsity rather than its bandwidth, which is much less than the Nyquist-based samples (*N*) [6]. Normally, it is also assumed that the location of sparse is unknown that brings much trouble and constraints in many practical applications. However, if the sparse basis is properly chosen, an accurate estimate of the original signal via an optimization process could be effectively achieved even in the presence of additive noise. Basically, there are three rough categories of signal recovery algorithms, viz., convex relaxation, signal algorithms and greedy pursuits.

Many natural signals have concise representations when expressed in the proper basis [8][9]. For instances, in image/video signal, with the 2-D discrete cosine transform (DCT), although nearly all pixels in the original image have nonzero values, the discrete cosine coefficients offer a concise summary. That is, most cosine coefficients are small, and the relatively few large coefficients are all that is required to reconstruct the original image pixels with high accuracy. Since in image/video signal compression the location of sparse values is knowable, the CS technique can be employed and degraded to a linear system. To do so, we first consider the dimensionality reduction aspect using the integer 2-D DCT transform, adopted in H.264/AVC, for compression. The concept is extended from the degradation algorithm of CS addressed in [10] for JPEG image with 2-D DCT transform, where the original signal (e.g., video/image) could be reconstructed through a linear process with the number of measurements to be the sparse values. We may expect that the proposed modified degradation algorithm of CS presented in this paper for integer 2-D DCT transform to be more effective, in terms of complexity and PSNR performance.

II. DEGRADATION ALGORITHM OF COMPRESSIVE SENSING FOR INTEGER 2-D DCT TRANSFORM

A. Conventional Compressive Sensing Method

Let us consider a *S*-sparse signal vector $\mathbf{x}$ in $\mathbf{\Psi}$, with size $N \times 1$, where $\mathbf{\Psi}$ is an orthogonal representation matrix with dimension $N \times N$. We can decompose $\mathbf{x}$ as:

$$\mathbf{x} = \mathbf{\Psi}\ \mathbf{f} \tag{1}$$

where vector $\mathbf{f}$ has at most S non-zero components. The CS theory proposes that for sparse-signals, we model the acquisition of signal vector $\mathbf{x}$ products the measurement matrix $\mathbf{\Phi}$ as:

$$\mathbf{y} = \mathbf{\Phi}\ \mathbf{x} \tag{2}$$

Here matrix $\mathbf{\Phi}$ with size $M \times N$ is useful information which can be captured through a small number of non-adaptive linear measurements. Usually, if the number of measurements is less than the number of unknowns, to recover the original signal with (2), we need to solve an undetermined system. But if the signal vector of interest $\mathbf{x}$ has a known basis, and $\mathbf{\Phi}$ can be chosen appropriately, then with CS technique $\mathbf{x}$ can be reconstructed with far fewer measurements than unknowns (that is, $M << N$), using existing optimization approaches [11-13]. One of the common approaches is to solve it by the convex optimization problem via ℓ_1 - norm minimization:

$$\min \|\hat{\mathbf{x}}\|_{\ell_1} \quad \text{subject to} \quad \mathbf{\Phi\Psi}\hat{\mathbf{x}} = \mathbf{y} \qquad (3)$$

for recovering S -sparse signals, for $M \geq S \log N$. The optimization process with (3) could be a trouble for hardware design.

B. Model Description of Proposed Degradation Algorithm of Compressive Sensing

The fundamental consideration of degradation algorithm of compressive sensing (CS) presented in [10] is to construct the dimensionality reduction transform (DRT) matrix $\mathbf{T}$ as depicted in Figure 1. In degradation algorithm of CS it transfers the high dimensional signal into one dimensional signal with DRT matrix. However, due to the inherent formulation difference between the conventional 2-D DCT transform and integer 2-D DCT transform adopted in H.264/AVC, we could not directly apply the result of [10] to obtain the DRT matrix $\mathbf{T}$. To facilitate discussion, we consider a 2-D image with size 4×4 for compression using integer 2-D DCT transform. As in [10] we need to represent a 2-D image $\mathbf{I}$, with size $m \times n$ (e.g., 4×4), in an equivalent form as a column vector $\mathbf{f}$, with size $N \times 1$ $(e.g., N = 16)$, that is consistent with DRT matrix. Accordingly, if the integer 2-D DCT transform can be represented as DRT matrix $\mathbf{T}$ with size $N \times N$ (e.g. 16×16), then multiply it with a column vector $\mathbf{f}$ we obtain a $N \times 1$ column vector $\mathbf{x}$:

$$\mathbf{x} = \mathbf{T}\,\mathbf{f} \qquad (4)$$

Comparing (4) with (1), we know that the DRT matrix $\mathbf{T}$ is related to the signal sparse matrix $\mathbf{\Psi}$ which has many alternatives. For instances, the signal sparse matrix $\mathbf{\Psi}$ can be selected as the 2-D DCT transforms, wavelet transform, and singular value decompression etc. In this paper, the integer 2-D DCT transform can be viewed as the sparse matrix $\mathbf{\Psi}$ that is applied to video/image data during the compression processes. In what follows we would like to introduce the process of integer 2-D DCT in H.264/AVC for constructing the DRT matrix. First we consider the forward integer 2-D DCT transform for an $m \times n$ image $\mathbf{I}$ and to get

$$\mathbf{X} = (\mathbf{A}\,\mathbf{I}\,\mathbf{A}^T) \circ \mathbf{E} \qquad (5)$$

where $\circ$ denotes the *Hadamard product* [14] of two matrices, and $\mathbf{A}^T$ is the transpose matrix of $\mathbf{A}$. The entries of matrix $\mathbf{E}$ are defined as a=1/2 and $b = \sqrt{2/5}$, respectively. Both matrices $\mathbf{A}$ and $\mathbf{E}$ are defined as follows:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & -2 \\ 1 & -1 & -1 & 1 \\ 1 & -2 & 2 & -1 \end{bmatrix} \text{ and } \mathbf{E} = \begin{bmatrix} a^2 & \frac{ab}{2} & a^2 & \frac{ab}{2} \\ \frac{ab}{2} & \frac{b^2}{4} & \frac{ab}{2} & \frac{b^2}{4} \\ a^2 & \frac{ab}{2} & a^2 & \frac{ab}{2} \\ \frac{ab}{2} & \frac{b^2}{4} & \frac{ab}{2} & \frac{b^2}{4} \end{bmatrix}$$

For completing the dimensionality reduction shown in figure 1, we need to properly relate the DRT matrix $\mathbf{T}$ and the integer 2-D DCT transform. To do so, we introduce an intermediate sparse transform matrix $\mathbf{T}_s$ for reducing the dimensionality of an image/video data. Consequently the (s, t) element of the resulting matrix $\mathbf{X}$ after the sparse transformation of an image $\mathbf{I}$, can be represented as:

$$X(s,t) = \sum_{x=0}^{m-1}\sum_{y=0}^{m-1} T_s\,(s,x)\;I(x,y)\;T_s^T\,(y,t) \qquad (6)$$

where $I(x, y)$ is the (x, y) element of matrix $\mathbf{I}$ (or an image). On the other hand, by taking the inverse sparse transform (matrix $\mathbf{T}_s^{-1}$) to (6), we may recover the original image $\mathbf{I}$:

$$I(x,y) = \sum_{s=0}^{m-1}\sum_{t=0}^{m-1} T_s^{-1}(s,x)X(s,t)(T_s^{-1})^T\,(y,t) \qquad (7)$$

As in [10] we can construct a 16×16 matrix $\mathbf{T}_A$ corresponding to transform matrix $\mathbf{A}$ and its transpose $\mathbf{A}^T$ defined in (5) to simplify the problem. Also, its (i, j) element can be denoted as:

$$T_A(i,j) = A(s,x)\;A^T(y,t) \qquad (8)$$

Similar to (8), we may arrange a scalar matrix $\mathbf{E}$ defined in (5) as a 16×16 matrix $\mathbf{T}_E$. Consequently we can represent DRT matrix $\mathbf{T}$ as the *Hadamard product* of matrices $\mathbf{T}_A$ and $\mathbf{T}_E$:

$$\mathbf{T} = \mathbf{T}_A \circ \mathbf{T}_E \qquad (9)$$

This completes the construction of DRT matrix $\mathbf{T}$, where the above three matrices in (9) are all with same dimension (16×16). Now, as in [10] we may multiply the DRT matrix $\mathbf{T}$ with image $\mathbf{I}$ to obtain the resulting sparse signal vector $\mathbf{x}$ as depicted in Fig. 1. Next, to recover the original image, we simply take the backward procedures. To do so, we define the inverse DRT matrix of $\mathbf{T}$ is equivalent to the transpose of $\mathbf{T}$:

$$\mathbf{T}^{-1} = \mathbf{T}^T \qquad (10)$$

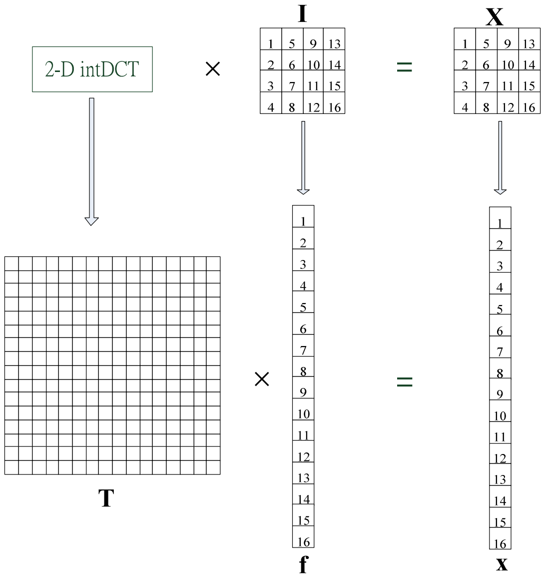


Figure 1. Configuration of dimensionality reduction.

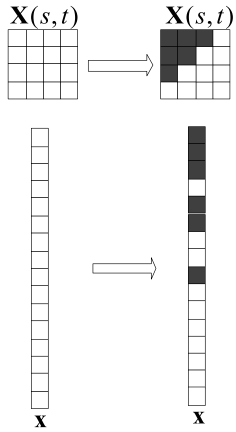


Figure 2. Mask control matrix

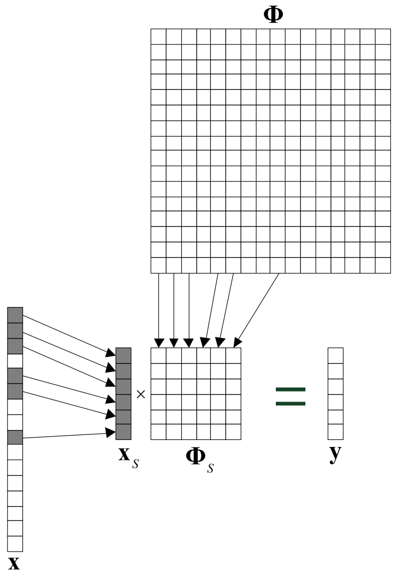


Figure 3. Signal acquisition schematic diagram

Next, we discuss the sparsity of the proposed scheme. Since in video/image compression, the mask matrix is employed during the compression processing; in which only the more significant information is of interest. Specifically, with integer 2-D DCT transform the related significant sparse values are concentrated in the low frequency domain of each image block as depicted in Figure 2. The sparse signal matrix $\mathbf{X}(s, t)$ with the mask matrix, called the mask of control matrix, is illustrated in the upper right hand side of Fig. 2. It can be used to set and control the corresponding desired sparse values.

As discussed earlier, unlike the conventional CS scheme, in which the number of measurements (M) is selected to be greater than or equal to $S \log N$, in our proposed degradation algorithm of CS, the number of measurements can be reduced, and depends highly on the selected mask matrix. As in (2) since $M << N$ we need to solve the undetermined system when the original signal (or image) is reconstructed. However, with our proposed degradation algorithm of CS, the number of measurements can be chosen to be the same as the sparse values ($M = S$). The original image (or signal vector $\mathbf{x}$) can be reconstructed, effectively, with the block diagram shown in Fig. 3. Due to the feature of the integer 2D-DCT transform, described earlier, we are able to identify and control the sparse values; which are concentrated in the low frequency parts. From the mask matrix depicted in figure 2, since six sparse values are chosen, the equivalent signal vector $\mathbf{x}$ with size 16×1 can be reduced to the signal vector $\mathbf{x}_s$, with size 6×1, and its entries are related to sparse values only. Using similar approach, we are able to reduce the dimensionality of the measurement matrix $\mathbf{\Phi}$ (see (2)) to the one (i.e. $\mathbf{\Phi}_s$) with size 6×6, depicted in Fig. 3.

Through the above mentioned two steps, original signals (or blocks) reconstruction process becomes a linear problem. That will be much easier to manipulate the desired signals for recovery. Also, it is more effective in reducing the number of sensors and improving efficiently the whole operation. Of course, we can improve the PSNR performance by setting the mask to increase the number of preservation.

III. *Experimental Results*

In this section, to verify the advantage of the proposed degradation CS algorithm for integer 2D-DCT transform, we compare it with the conventional compressive sensing approach which is with different number of measurements. The test image with size 64×64 shown in Figure 4 is adopted from the Lena picture of size 512×512. Also, in the conventional compressive sensing approach, the number of measurements is chosen to be 6 and 15 for investigating the quality of recovery images, as illustrated in Fig. 5. On the other hand the experimental result with proposed degradation algorithm of CS for integer 2-D DCT transform is given in Fig. 6. Next, it is of interest to investigate the PSNR performance of the proposed degradation algorithm of CS with integer 2-D DCT transform. We then compare it with the conventional CS with sparse matrix $\mathbf{\Psi}$ of (1) to be 2-D DCT transform; the results are shown in Fig. 7. From Fig. 5, we learn that with the conventional CS scheme, if the number of measurements (M) is increased from 6 to 15, the quality of recovering image will be closer to the original image as shown in figure 4, with the penalty of increasing the complexity. With our proposed scheme the number of measurements (related to the mask matrix) is fixed (M=6), as observed from Fig.6, the image quality is similar to that given in Fig.5 (b) (with M=15). This is consistent with PSNR performance demonstrated in Fig. 7, where the PSNR with our proposed *CS* scheme (with M=6) 28.75 dB, that is normally performed better than the conventional CS approach using (3) with 2-D DCT matrix.

IV. Joint Design of Quantization and Integer 2-D DCT Transform

In H.264/AVC codec, the integer 2-D DCT transform and quantization processes are jointly designed to provide an efficient encoding process of video data. With this approach we can reduce the ‘drift’, due to the mismatch between the decoded data at encoder and decoder, to achieve low complexity implementations [15-16]. In compression process

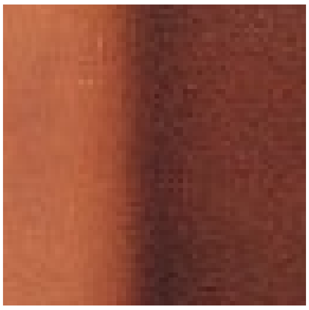
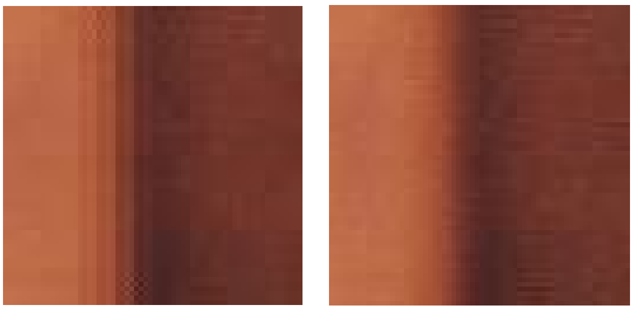


Figure 4. Original image



(a) M= 6 (b) M=15

Figure 5. Conventional compressive sensing method

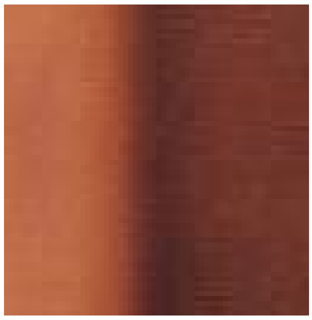


Figure 6. Resulting image with proposed degradation algorithm

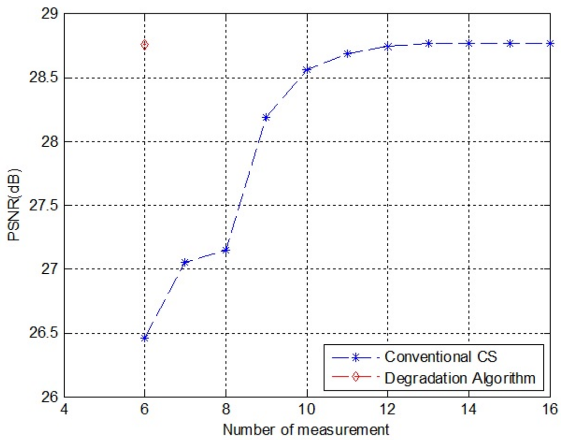


Figure 7. PSNR comparison of the proposed degradation algorithm with the conventional CS approach using 2-D DCT transform, for different number of measurements.

the operation of quantization introduces signal loss. Its implementation is complicated by two requirements. First, we have to avoid the division operation and/or floating point arithmetic, and then incorporate the scalar matrix **E** defined in (5). In forward quantization operation, we quantize coefficient of the (s, t) element of matrix **X**, i.e. X_{st} of (5) during compression, i.e.

$$Z_{st} = round(X_{st} / Qstep) \tag{11}$$

In (11) parameter *Qstep* is a step-size of quantization and Z_{st} denotes the quantized coefficient of the (s,t) element of matrix **Z**. Since a wide range of step sizes makes it possible for an encoder to control the tradeoff accurately and flexibly between bit rate and quality. We can combine the scalar coefficient E_{st}, the (s,t) element of matrix **E,** into the operation of quantization. Since in forward quantization process of (11) the input image block **I** is transformed to give a resulting block of un-scaled coefficients $\mathbf{W}=\mathbf{AIA}^T$. Hence coefficient of the (s, t) element of **W**, denoted as W_{st}, could be quantized and scaled in a single operation:

$$Z_{st} = round(W_{st} \frac{E_{st}}{Qstep}) \tag{12}$$

A disadvantage of the quantization formula of (12) is that it involves the division operation at the encoder. In order to achieve calculation process that is avoiding the division operation, the scalar matrix is incorporated into the forward quantization process. However, in order to simplify the arithmetic, the factor (E_{st} /*Qstep*) is implemented in [17] as a multiplication by a factor matrix **H** and a right shift to avoid any divisions. We can rewrite (12) with the coefficient of the (s, t) element of **H** (H_{st}) as

$$Z_{st} = round(W_{st} \frac{H_{st}}{2^{qbits}}) \tag{13}$$

where $H_{st} /2^{qbits} = E_{st} /Qstep$ and $qbits=15 + floor\,(QP/6)$. A total of 52 values of *Qstep* are supported by the standard, indexed by a Quantization Parameter, QP [1]. For integer arithmetic, (13) can be implemented as

$$Z_{st} = round\left[(W_{st} H_{st}) \frac{1}{2^{qbits}} \right] \tag{14}$$

Finally, we consider the implementation of the DRT matrix **T**. Similarly to (8), we may arrange the factor matrix **H** as matrix $\mathbf{T}_H$, with size 16×16 . Consequently we represent DRT matrix **T** as the *Hadamard product* of matrices $\mathbf{T}_A$ and $\mathbf{T}_H$:

$$\mathbf{T} = \mathbf{T}_A \circ \mathbf{T}_H \tag{15}$$

This completes the final DRT matrix **T**. Accordingly, the jointly processing of transform and quantization is given as

$$\mathbf{x} = round(\mathbf{T}\,\mathbf{f} \frac{1}{2^{qbits}}) \tag{16}$$

Eq. (16) includes the resulting DRT matrix in degradation CS algorithm to implement integer 2-D DCT transform and the quantization process together. This new resulting transform is a scaling integer approximation to integer 2-D DCT transform that allows computation of the forward or inverse transforms with simply an addition and binary shift to avoid the use of multiplications. Also, the joint design of the integer 2-D DCT transform and quantization processes described above can avoid the divisions. Hence summing up the above features, we are able to achieve a minimal computational complexity.

V. CONCLUSIONS

In this paper, we proposed a new degradation algorithm of CS framework for integer 2-D DCT transform, which can be applied to H.264/AVC for data compression. Since the implementation of the integer 2-D DCT transform introduced a scaling matrix (**E**) in (5), we needed to implement it using the similar idea of dimensionality reduction transform (DRT) matrix addressed in [10], to obtain the reduction transform matrix denoted in (9), to complete the overall DRT matrix **T**. As demonstrated in the results of computer simulation, our proposed scheme outperformed the conventional CS with 1-D DCT and 2-D DCT transform approaches in terms of quality of image recovery and with much less number of measurements required for reconstruction the original signal. We concluded that with the degradation algorithm of CS proposed in this paper the overall processing became easier and effective, compared with the conventional CS approach.

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