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Reexamination of capital asset pricing model (CAPM): An application of quantile regression

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Capital asset pricing model (CAPM) plays a very important role in risky asset evaluation. This paper tries to explore the important aspect in CAPM, which is perfect linear relationship assumption between return and market portfolio risk and further discusses the application of CAPM. Empirical evidence shows that the model in ordinary least squares (OLS) supports the positive relationship between systematic risk and return. However, by quantile regression (QR) analysis, not all relationships between systematic risk and return are positive. For lower quantiles, the relationship is not significantly positive although the positive relationship is concluded for higher quantiles. To sum it up, it is not always sustainable for a positive relationship between systematic risk and return. Besides, non-parametric estimations show that the linear assumption between market portfolio risk and return in CAPM is suspicious. Therefore, we find that the two important associated assumptions, which are positive and linear relationships between market portfolio risk and return, do not necessarily exist.

Key words: CAPM, quantile regression, nonlinear.

INTRODUCTION

The heart of financial theory is to study how to achieve efficient resource allocation and investment decision-making behavior under the environment of uncertainty. However, under the influence of so many factors interact, we need a more precise mathematical tools and models to construct the basis of financial theory and then to provide practical application. In recent years, people want to see inside the compensation structure of securities and researchers and practitioners have developed a number

of different evaluation model. Due to the variety of different assumptions and limitations, it is necessary to discuss and analyze the accuracy of each model. Further, to put model into application may cause a bit of inconsistency, so it is important to explore the difference between the model and real world. In the past four decades, "financial theory" as a new science has gradually emerged. The new science tries to understand the mode of operation of financial markets, to explore how to make markets more efficient, and how to develop market specifications. At the same time, financial theory's accuracy and applicability for investors' decision-making behavior is great, so we can not overlook the subject how to provide investors a more accurate assessment of the model based on the theoretical basis. Undoubtedly, Markowitz (1952) is the foundation of modern investment theory. In this article, he establishes a strict mathematical model to illustrate how to select the least risky portfolio under the expected return. Markowitz (1952) theory is the basic theory of finance, and it also has been very widely applied in practice. Accordingly, Sharpe (1964) and Linter

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Abbreviations: CAPM, Capital asset pricing model; OLS, ordinary least squares; QR, quantile regression; SMB, small and medium businesses; ICAPM, intertemporal capital asset pricing model; C-CAPM, consumption capital asset pricing model; APT, arbitrage pricing theory; GMM, generalized moment method; NYSE, New York stock exchange; VaR, value at risk.

(1965) explore the equilibrium structure of asset prices, and establish so-called CAPM. After that, CAPM has become a very important basis for measuring investment performance in financial markets.

As CAPM in the financial markets is widely applied and plays an essential role in investment, there are some shortcomings. The most important one is that CAPM may produce contradictory situation to empirical results, so we often have to turn through appropriate adjustments to achieve more accurate result. In addition, CAPM is based on mean-variance efficiency and constructed under the condition which return is the mean. This model points out that under a perfect market assumption, when the market is in equilibrium, the relationship between the return of each individual security and the market systematic risk is linear and the systematic risk is the only one factor to explain the cross sectional expected return of each individual security. Under so strict assumptions, practical applications as well as empirical studies of many scholars generate a lot of inconsistencies in CAPM. In this paper, we try to practically analyze the inconsistencies through QR, furthermore, it explain and discuss the empirical results. In OLS, we assume that the population distribution of the sample is normal, so we may obtain the only regression coefficient estimation. However, Grauer and Janmaat (2009) point out that the population intercepts, slopes and R2 from cross-sectional regressions of expected returns on betas indicates that all three are unreliable indicators of whether the CAPM holds. It is relatively hard to observe the real form of the estimation. QR, however, is a statistical method applied to estimate, inference and process the Conditional Quantile Function. The concept is to extend the traditional regression method through minimizing residuals of the linear objective function and researching for the best regression coefficients. During the eighteenth century, Boscovich made the Median Regression concept and it is the first prototype of QR. Koenker and Bassett (1978) extend the concept to the calculation of quantiles beyond medians. QR provides flexibility.

That is, without assumptions of original distributions, we may obtain QR through adjusting quantile parameters. Due to the robustness for no assumptions of distribution, it is possible to estimate models accurately without the errors on model setting. Thus, we may observe the influence of independent variables on the dependable variable for given quantiles. Since we are able to make a more complete analysis compared with OLS, this paper contributes to literatures in econometrical methodology. Accordingly, this paper applies the model setting of Fama and MacBeth (1973) to discuss the relationship between return and the risk of market portfolio by QR. In addition, we concern about the accuracy of the assumption for the perfect linear model. In order to circumvent the possible errors due to the set of linearity, in this study we make no assumption for the non-linear part and then test the assumption of the perfect linearity hypothesis. We weight

the data for the different ways to observe whether it makes significant influence and further, to discuss the relationship between return and market risk with different quantiles. Besides, we compare it with the conventional OLS. By doing so, we construct the complete QR CAPM analytical framework. This paper aims to use the QR to estimate the performance of different quantiles for the CAPM empirical research and analysis. Taking Fama and MacBeth (1973) as basis and supplied by QR, we discuss the relationship between market portfolio risk and return and the accuracy of assumption of perfect linearity.

LITERATURE REVIEW

CAPM is a risky asset pricing model based on the mean-variance framework, indicating that the mean return on a risky asset is the function of the covariance of the asset and the return of market portfolio. The origin of CAPM is in the Markowitz (1952), which demonstrates how an investor selects his optimal portfolio for the criteria of its mean return and standard deviation. Tobin (1958) extends it and followed by Sharpe (1964), Lintner (1965), Mossin (1966) and Black et al. (1972) the capital market line based on 'mean variance' is formulated and it is regarded as the optimal portfolio selection criteria. Furthermore, security market line is derived and finally CAPM is formed. However, many studies show that this asset pricing model evaluation has errors and it produces a number of related follow-up studies. Markowitz (1952) first quantifies the concept of return and risk on portfolio selection theory. He quantifies risk as the standard deviation of expected return. Thus, under the expected return investors will select a portfolio with least standard deviation. On the other hand, if the risk is known and fixed, an investor will choose the portfolio with the greatest expected return. Tobin (1958) studies the optimal portfolio decisions of investors in risky assets other than by adding risk-free assets. He discusses how much investors will decide to invest in risky assets and risk-free assets. Sharpe (1964), Linter (1965), Mossin (1966) and Black et al. (1972) refer to Markowitz (1952) and develop CAPM. CAPM can be applied to assess the return of a specific portfolio or an individual security and the linear relationship of systematic risk. Thus, if we can estimate the systematic risk of each individual security, we may obtain the theoretical expected return of the security. That is,

$$E(R_i) = R_f + (R_M - R_f) * \beta_i + \varepsilon_i \quad (1)$$

where $E(R_i)$ is the expected return of security i , R_f is the risk-free interest rate, R_M is the expected return of market portfolio, β_i is the beta coefficient of

security i , and ε_i is the residual. However, the original CAPM builds on a number of stringent assumptions in order to over-simplify the complex real world. Many scholars challenge the perfect market assumption and the linear correlation between return and market portfolio risk. For example, Fama and French (1992,1995) derive the Fama-French three-factor model, which includes the traditional CAPM market portfolio return, return of portfolio of small companies less return of portfolio of large companies small and medium businesses (SMB), and return of high book value/market value ratio portfolio less return of low book value/market value ratio portfolio (HML). Besides, Kraus and Litzenberger (1976), Friend and Westerfield (1980), Lee et al. (1996), Fang and Lai (1997), Harvey and Siddique (2000) and Christie-David and Chaudhry (2001) all point out that CAPM do produce bias between theoretical model and empirical results.

In empirical works, Roll (1977) points out that the only testable hypothesis of CAPM is the market portfolio with means different efficiency. The market portfolio efficiency concludes that the relationship between the expected return and systematic risk is linear. However, Roll (1977) challenge that this process can not be independently verified, because the market portfolio efficiency and the linear relationship questionable. Regardless of market portfolio generation process, from the sampling of observed value of security returns, there will be many ex-post efficiency portfolios. However, each portfolio will just be linear to individual expected return, but that is indifferent with the real world efficiency. Lintner (1965) provides an assessment of the ex-post performance. In addition, he also uses a two-pass regression. In the first pass, he regresses the time series of security returns against the single index to estimate systematic risk factor of each security. Black et al. (1972) propose a modified CAPM model, which includes the estimating period and calculating period. In addition, they also propose a zero-beta capital asset pricing model, which is a two-factor model, replacing the original risk-free interest rate assumption in CAPM. Similarly, Fama and MacBeth (1973) apply the technique of two stage regression and obtain conclusion analogous to Black et al. (1972). Further, adding two more explanatory variables proves that the only significant variable to explain risk is systematic risk. After that, Merton (1973) extends the single-period CAPM to intertemporal model. By maximizing the lifetime consumption utility and taking variable investment opportunity set into consideration makes CAPM more general, and establishes the Intertemporal capital asset pricing model (ICAPM). Breeden (1979) extends Merton's research by inducing Ito's Lemma, and transfers multi- β to single intertemporal β , which is so-called Consumption capital asset pricing model (C-CAPM). Ross (1976) proposes the Arbitrage Pricing Theory (APT) to reinforce what systematic risk in CAPM cannot explain. Furthermore, De Giorgi and Post (2009) show that the reward-risk CAPM captures the cross section of United State

stock returns better than the mean-variance CAPM does.

Most of the CAPM's empirical researches are mainly based on time series analysis and cross-sectional regression analysis (Fama and MacBeth, 1973). In addition, Longstaff (1989) applies Generalized Moment Method (GMM) to test stocks on the New York stock exchange (NYSE) and obtain supports for CAPM. However, recently there are a number of studies indicating that systematic risk and stock returns have the relationship other than linear (Banz, 1981; Basu, 1983; Fama and French, 1992; Fant and Peterson, 1995). Friend et al. (1976) consider the impact of inflation on CAPM and advocate replacing the real rate of return for nominal rate of return. Gonedes (1976) modify the homogeneous expectation assumption to heterogeneous expectation and concludes that "invest in people with the state of expectations" hypothesis amended as unusual state expected by the market portfolio is not necessarily efficient, so CAPM cannot be verified. Mayers (1973) extend the market portfolio for traded assets to non-traded assets, indicating the influence of portfolio for uncovered assets on traditional CAPM. Solnik (1984) extends CAPM from a single country to multinational countries and taking into account the exchange rate risk, inflation risk and other factors, to form a more complex multi-factor CAPM. Kraus and Litzenberger (1976) derive the third-order moment CAPM; they find that the skew factors on the measurement of prices of the securities do have a significant impact. Moreover, Ang and Chen (2007) point out that little evidence that the conditional alpha for a book-to-market trading strategy is different from zero.

METHODOLOGY

Regression coefficients of regression analysis are to measure the marginal effects of independent variables. However, the interpretation of the regression coefficients for OLS and QR has different meanings. The OLS estimation is to deal with independent variables on the dependent variables of the "average" of the marginal effects, so such estimation method emphasize more on the allocation of central tendency. On the other hand, the QR estimation is an order statistics-based estimation. It refers to the marginal effects for the independent variables on the dependent variables under a "specified percentile". Since evidence suggests that heavy-tailed errors do exist for many economic data, the estimated β in the traditional OLS method are very sensitive to these extreme values.

Using QR will not be affected by these outlier effects. Moreover, in a number of empirical studies, they are concerned not only average performance and even more concerned about the distribution of tail situation.

By using QR, it is possible to model the relationship between returns and beta for firms that over-perform and under-perform relative to the mean, or for firms that receive bad versus good news. In sum, QR provide some solutions of statistical problems about CAPM studies such as omitted variables bias, sensitivity to outliers and non-normal error distributions. QR has been widely applied in various disciplines. Buchinsky (1998) applies to analysis of wage structure and age, education, the relationship between demographic variables and Yu and Stander(2002) find that QR has

Table 1. Descriptive statistics for 100 portfolios.

Variable	Average value weighted returns		Average equal weighted returns	
	Rate of return (%)	Beta	Rate of return (%)	Beta
Mean	0.9279	1.2304	0.9790	1.2536
Median	0.9465	1.2067	0.9666	1.2294
Max	1.6527	1.7025	2.1841	1.7116
Min	0.2952	0.8969	0.2879	0.9406
Standard deviation	0.2807	0.1738	0.3366	0.1728
Observations	100	100	100	100

has been used in the pharmaceutical, survival analysis and other scientific fields. Lee and Saltoglu (2001) present that the main advantage of QR is to obtain a better statistical inference through the empirical quantile. Koenker and Bassett (1982) demonstrate that QR has another advantage for robustness because QR makes no assumption for population distribution (Abrevaya, 2001).. Taylor (1999) estimates the distribution of daily Value at Risk (VaR) on German mark, British pound sterling and the Japanese yen exchange, and find the good performance on QR. Based on the above description and discussion of the literature, we will explore the applicability of CAPM similar to Fama and MacBeth (1973) through QR proposed by Koenker and Bassett (1978). By doing so, we may observe the relationship between systematic risk return. According to Koenker and Bassett (1978), a linear model can be represented as:

$$y_i = x_i' \beta_\theta + u_{\theta i} \quad (2)$$

Where β_θ is an unknown $k \times 1$ vector of regression parameters associated with the θ th percentile, x_i is a $k \times 1$ vector of independent variables and y_i is the dependent variable. The θ th conditional quantile of y_i given x_i is

$$Quant_\theta(y_i | x_i) = x_i' \beta_\theta \quad (3)$$

The only necessary assumption concerning error term is

$$Quant_\theta(u_{\theta i} | x_i) = 0 \quad (4)$$

That is, the conditional θ th quantile of the error term is equal to zero. Therefore, the QR method can measure the marginal effects at different points in the conditional distribution by using several various values of θ , $\theta \in (0,1)$. It is in this way that QR allows for parameter heterogeneity across various types of assets. The QR estimator can be found by solve the following minimization function:

$$\hat{\beta}_\theta = \arg \min [\theta \int_{y \geq x_i \beta} |y - x_i \beta| + (1 - \theta) \int_{y < x_i \beta} |y - x_i \beta|] \quad (5)$$

When $\theta = 0.5$, the QR becomes the well-known median regression.

EMPIRICAL RESULTS

We derive data from the website of Kenneth R. French, who is well known for the research of empirical CAPM studies and the three-factor model. We adopt monthly based data. 100 Portfolios formed on size and book-to-market ratios, dated from July 1926 to September 2009, are used in this study. Rate of return of portfolios are constructed by two distinct ways: average value weighted returns and average equal weighted returns. Rate of return of market is weighted average return of NYSE, AMEX and NASDAQ index returns. 30 days T-Bill rate is utilized as risk free rate. Finally, we follow classic two pass method of Fama and MacBeth (1973) to estimate CAPM. Based on CAPM, we estimate betas using OLS and QR separately and discuss them. Tables 2 and 3 represent estimates results under both average value weighted returns and average equal weighted returns. Both tables show that CAPM is valid under OLS estimating method, however, QR provide some interesting discoveries. At lowest 5% quantile ($\theta=0.05$), Table 2 shows significant abnormal return and negative relationship between systematic risk and rate of return which violate the implications of CAPM. Table 3 shows non-significant negative relationship between systematic risk and rate of return but positive abnormal return is still significant. When quantile becomes higher, the positive relationship between systematic risk and rate of return reveals as well as negative abnormal return. The results also show that the higher quantile, the steeper relationship between systematic risk and portfolio return. Overall findings show that CAPM may not be appropriate at lower quantile which reveals the heavy tail effects. Figures1 and 2 provide viable representation of our results from Table 2 and 3. Estimated results of CAPM under 5, 25, 50, 75 and 95% quantiles are represented in the following figures. Based on different distinct quantiles, Figures 3 and 4 show the 95% confidence intervals of systematic risk (beta). We can see that when quantile is lower than about 25%, the beta is significant negative or insignificant positive (not reject the null under 5% significant level), which is obviously inconsistent with the implications of CAPM, that is, positive relationship between systematic risk and portfolio return. When

Table 2. Estimates of CAPM: average value weighted returns.

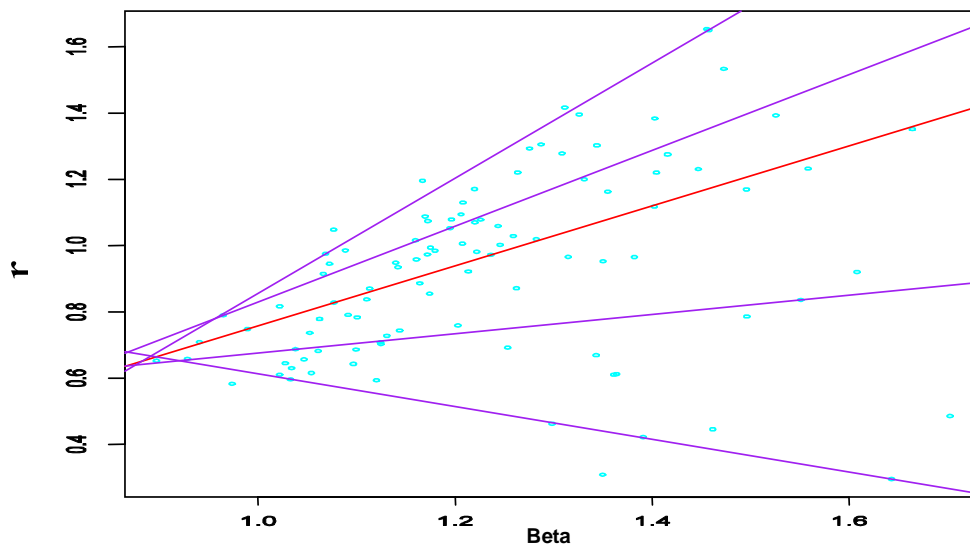
	OLS	QR	QR	QR	QR	QR
		$\theta=0.05$	$\theta=0.25$	$\theta=0.5$	$\theta=0.75$	$\theta=0.95$
α_0	0.2454 (0.1904)	1.1053*** (0.2260)	0.3867 (0.2762)	-0.1464 (0.1844)	-0.3119** (0.1351)	-0.8779*** (0.1377)
$E(r_M) - r_f$	0.5547*** (0.1532)	-0.4930** (0.2130)	0.2895 (0.2675)	0.9046*** (0.1776)	1.1419*** (0.1314)	1.7342*** (0.1090)

*, **, *** denotes for 10, 5 and 1% significance level respectively. Number in parentheses denotes for standard error.

Table 3. Estimates of CAPM: Average equal weighted returns.

	OLS	QR	QR	QR	QR	QR
		$\theta=0.05$	$\theta=0.25$	$\theta=0.5$	$\theta=0.75$	$\theta=0.95$
α_0	-0.0294 (0.2268)	0.9686** (0.3882)	0.3027 (0.3154)	-0.4417* (0.2226)	-0.7787*** (0.1611)	-1.7158*** (0.2693)
$E(r_M) - r_f$	0.8043*** (0.1792)	-0.3411 (0.3324)	0.3651 (0.2919)	1.1582*** (0.2095)	1.5527*** (0.1461)	2.5127*** (0.2455)

*, **, *** denotes for 10, 5 and 1% significance level. Number in parentheses denotes for standard error.

**Figure 1.** Estimates of CAPM: average value weighted returns.

quantile is higher than about 25%, positive relationship between systematic risk and portfolio return is valid and becomes steeper when quantile continues to grow. Such results show that CAPM is sound only when the quantile is not lower.

Conclusion

CAPM plays a very important role in risky asset evaluation, but also plays an important role. This paper tries to explore the important aspect in CAPM, which is perfect linear relationship assumption between return and

market portfolio risk, and further discuss the application of CAPM. Since literatures (Grauer and Janmaat, 2009) point out that the population intercepts, slopes and R2 from cross-sectional regressions of expected returns on betas indicates that all three are unreliable indicators of whether the CAPM holds, we apply QR to reexamine CAPM. Empirical evidence shows that for the model in OLS, it supports the positive relationship between systematic risk and return. However, by QR analysis, not all relationships between systematic risk and return are positive. For lower quantiles, the relationship is not significantly positive although the positive relationship is concluded for higher quantiles. To sum it up, it is not

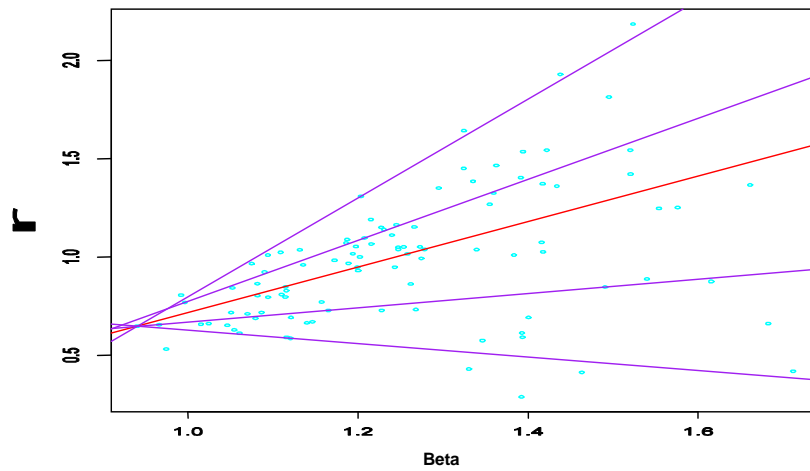


Figure 2. Estimates of CAPM: average equal weighted returns.

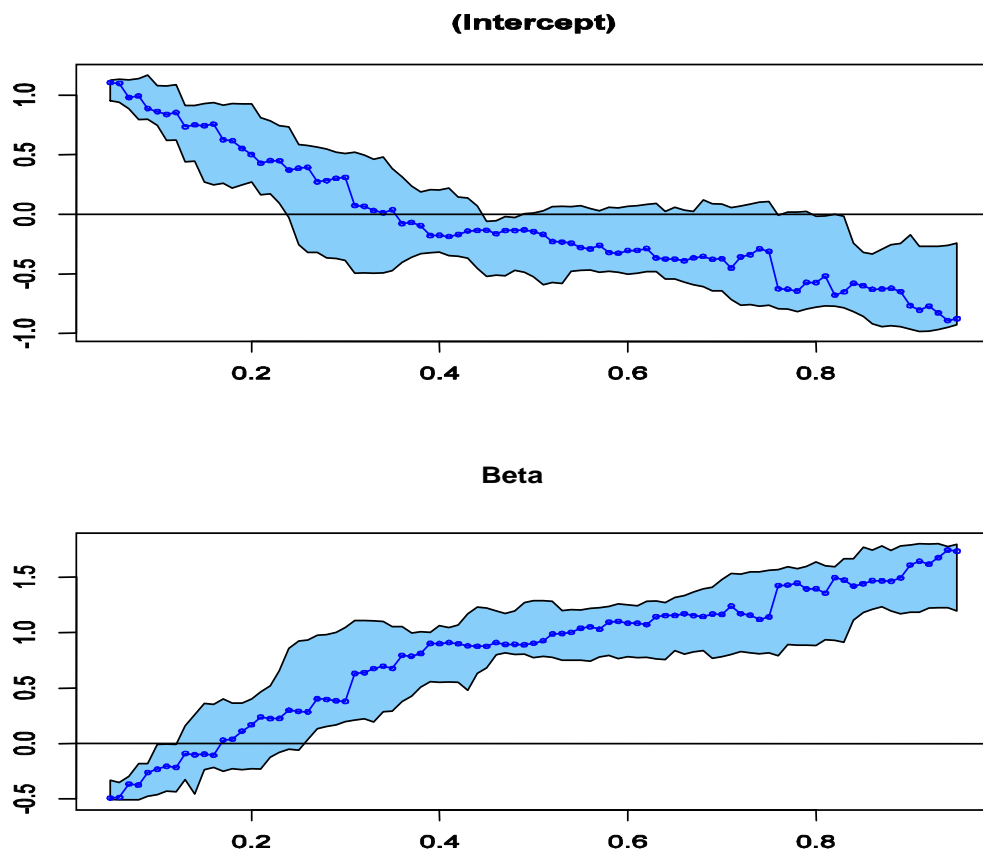


Figure 3. Estimates of coefficients for different quantiles: average value weighted returns.

always sustainable for a positive relationship between systematic risk and return. Besides, non-parametric estimations show that the linear assumption between market

portfolio risk and return in CAPM is suspicious. Therefore, we find that the two important associated assumptions, which are positive and linear relationships

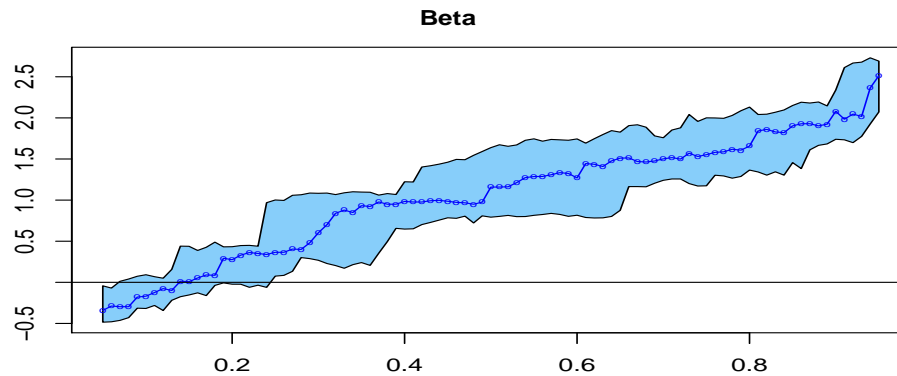


Figure 4. Estimates of coefficients for different quantiles: average equal weighted returns.

between market portfolio risk and return, do not necessarily exist.

REFERENCES

- Abrevaya J (2001). The effects of demographics and maternal behavior on the distribution of birth outcomes. *Empir. Econ.*, 26: 247-57.
- Ang A, Chen J (2007). CAPM over the long run: 1926–2001. *J. Empir. Finan.*, 14: 1-40.
- Banz RW (1981). The Relationship between return and market value of common stocks. *J. Financ. Econ.*, 9: 3-18.
- Basu S (1983). The relationship between earnings yield, market value, and return for NYSE common stocks: further evidence. *J. Financ. Econ.*, 12: 129-156.
- Black F, Jensen MC, Scholes M (1972). The capital asset pricing model: some empirical test, *Studies in the Theory of Capital Market*, Praeger, N.Y.
- Boudoukh J, Richardson M, Whitelaw R (1998). The best of both worlds. *J. Financ. Econ.*, 5: 64-67.
- Breeden DT (1979). An intertemporal asset pricing model with stochastic consumption and investment. *J. Financ. Econ.*, 7: 265-296.
- Christie DR, Chaudhry M (2001). Coskewness and cokurtosis in futures markets. *J. Empirical Finan.*, 8: 55-81.
- De Giorgi E, Post T (2009). Second-order stochastic dominance, reward-risk portfolio selection, and the CAPM. *J. Financ. Quant. Anal.*, 43: 525-546.
- Fama E, French KR (1992). The cross-section of expected stock returns. *J. Finan.*, 47: 427-465.
- Fama E, French KR (1993). Common risk factors in the returns on bonds and stocks. *J. Financ. Econ.*, 33: 3-56.
- Fama E, French KR (1995). Size and book-to-market factors in earnings and returns. *J. Finan.*, 50: 131-155.
- Fama E, Macbeth R (1973). Risk, return and equilibrium: empirical tests. *J. Politic. Econ.*, 71: 607-636.
- Fang E, Lai TY (1997). Co-kurtosis and capital asset pricing. *Financ. Rev.*, 32: 293-307.
- Fant LF, Peterson DR (1995). The effect of size, book-to-market equity, prior returns, and beta on stock returns: January versus the remainder of the year. *J. Financ. Res.*, 18: 129-142.
- Friend I, Yoram L, Etienne L (1976). The demand for risky assets under uncertain inflation. *J. Finance*, 31: 1287-1297.
- Friend I, Westerfield R (1976). Co-skewness and capital asset pricing. *J. Financ.*, 35: 897-913.
- Gonedes NJ (1976). Capital market equilibrium for a class of heterogeneous expectations in a two- parameter world. *J. Financ.*, 31: 1-15.
- Grauer RR, Janmaat JA (2009). On the power of cross-sectional and multivariate tests of the CAPM. *J. Bank. Financ.*, 33: 775-787.
- Harvey CR, Siddique A (2000). Conditional skewness in asset pricing tests. *J. Financ.*, 55: 1263-1295.
- Koenker R, Bassett GW (1978). Regression quantiles. *Econometrica*, 46: 33-50.
- Koenker R, Bassett GW (1982). Robust tests for heteroscedasticity based on regression quantiles. *Econometrica*, 46: 33-50.
- Koenker R, Hallock KF (2001). Quantile regression. *J. Econ.*, 15: 143-156.
- Kraus A, Litzenberger RH (1976). Skewness preference and the valuation of risk assets. *J. Financ.*, 31: 1085-1100.
- Lee T, Saltoglu B (2001). Evaluating predictive performance of Value-at-Risk models in emerging markets: a reality check, Working Paper.
- Lee A, Moy RL, Lee CF (1996). A multivariate test of the covariance-coskewness restriction for the three moment CAPM. *J. Econ. Bus.*, 48: 515-523.
- Lintner J (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Rev. Econ. Stat.*, 47: 13-37.
- Longstaff F (1989). Temporal aggregation and the continuous-time capital asset pricing model. *J. Financ.*, 44: 871-887.
- Markowitz HM (1952). Portfolio selection. *J. Finan.*, 7: 77-91.
- Mayers D (1973). Nonmarketable assets and capital market equilibrium under uncertainty, *Theory of Capital Market*, 2nd ed., Praeger Publishers, N. Y.
- Merton RC (1973). An intertemporal capital asset pricing model. *Econometrica*, 41: 867-887.
- Mossin J (1966). Equilibrium in a capital asset market. *Econometrica*, 34: 768-783.
- Roll RA (1977). Critique of the asset pricing theory's tests. Part 1: On past and potential testability of the theory. *J. Financ. Econ.*, 4: 129-176.
- Ross SA (1976). The arbitrage theory of capital asset pricing. *J. Econ. Theory*, 13: 341-360.
- Sharpe W (1964). Capital asset prices: a theory of market equilibrium under conditions of risk. *J. Financ.*, 19: 425-442.
- Solnik BH (1984). An equilibrium model of the international capital market. *Econ. Theory*, 8: 500-524.
- Taylor JW (1999). A quantile regression approach to estimating the conditional density of multiperiod returns. *J. Forecast.*, 19: 299-311.
- Tobin J (1958). Liquidity preference as behavior toward risk. *Rev. Econ. Stud.*, 7: 65-86.
- Yu K, Lu Z, Stander J (2002). Quantile regression: applications and current research areas, Working Paper.