A Fuzzy Inventory System with Deteriorating Items under Supplier Credits Linked to Ordering Quantity

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The inventory problem associated with trade credit is a popular topic in which interest income and interest payments are important issues. Most studies related to trade credit assume that the interest rate is both fixed and predetermined. However, in the real market, many factors such as financial policy, monetary policy and inflation, may affect the interest rate. Moreover, within the environment of merchandise storage, some distinctive factors arise which ultimately affect the quality of products such as temperature, humidity, and storage equipment. Thus, the rate of interest charges, the rate of interest earned, and the deterioration rate in a real inventory problem may be fuzzy. In this paper, we deal with these three imprecise parameters in inventory modeling by utilizing the fuzzy set theory. We develop the fuzzy inventory model based on Chang *et al*.'s [1] model by fuzzifying the rate of interest charges, the rate of interest earned, and the deterioration rate into the triangular fuzzy number. Subsequently, we discuss how to determine the optimal ordering policy so that the total relevant inventory cost, in the fuzzy sense, is minimal. Furthermore, we show that Chang *et al*.'s [1] model (the crisp model) is a special case of our model (the fuzzy model). Finally, numerical examples are provided to illustrate these results.

Keywords: inventory, deteriorating item, delay payments, fuzzy set, signed distance

1. INTRODUCTION

The fuzzy set theory is developed for solving the phenomenon of fuzziness prevalent in the real world. Up to this point, the fuzzy set theory has been widely applied in many fields, such as applied science, medicine and inventory management. The application of fuzzy set concepts in inventory models have been proposed by many researchers. Pertrovic and Sweeney [2] fuzzified the demand, lead time and inventory level into triangular fuzzy numbers in an inventory control model, and then determined the order quantity with the fuzzy propositions method. Yao *et al*. [3] investigated the Economic Lot Scheduling Problem (ELSP) with fuzzy demands. They used the 'Independent Solution' as well as the 'Common Cycle' approach to solve the fuzzy ELSP problem. Yao *et al*. [4] presented a fuzzy inventory system without the backorder model in which both the order quantity and the total demand were fuzzified as the triangular fuzzy numbers. Chang [5] discussed the Economic Order Quantity (EOQ) model with imperfect quality

items by applying the fuzzy sets theory, and proposed the model with both a fuzzy defective rate and a fuzzy annual demand. Chang *et al.* [6] considered the mixture inventory model involving variable lead time with backorders and lost sales. They fuzzified the random lead-time demand to be a fuzzy random variable and the total demand to be the triangular fuzzy number. Based on the centroid method of defuzzification, they derived an estimate of the total cost in the fuzzy sense. Chen *et al.* [7] introduced a fuzzy economic production quantity model with defective products in which they considered a fuzzy opportunity cost, trapezoidal fuzzy cost and quantities in the context of the traditional production inventory model. Maiti [8] developed a multi-item inventory model with stock-dependent demand and two-storage facilities in a fuzzy environment (where purchase cost, investment amount and storehouse capacity are imprecise) under inflation and incorporating the time value of money. Other related articles on this topic can be found in work by Chen and Wang [9], Vujosevic *et al.* [10], Gen *et al.* [11], Roy and Maiti [12], Ishii and Konno [13], Lee and Yao [14], Yao and Lee [15], Chang *et al.* [16], Chang *et al.* [17], Ouyang *et al.*[18], Yao *et al.* [19].

In today's business environment, trade credit plays an important role and the inventory problem associated with trade credit has become a popular topic in the inventory field. A supplier usually permits the retailer to delay in settling the total amount owed to them for a fixed period of time. Usually, interest does not begin accruing for the outstanding amount provided that it is paid within the permissible delay period. Therefore, the retailer can earn interest on the accumulated revenue received by deferring the payment until the last moment of this permissible period. Goyal [20] developed an EOQ model under conditions of permissible delay in payments, in which he calculated interest income based on the purchasing cost of goods sold within the permissible delay period. Beyond the permissible period, interest payments are calculated based on the purchasing cost of the goods not yet sold. Teng [21] amended Goyal's [20] model by calculating interest earned based on the selling price of goods sold. In Chang and Teng [22], the suppliers offer cash discounts or delay payment to retailers. Within the permissible delay period, retailers earn interest on sales revenue. Beyond the permissible period, interest is charged for the outstanding amount. Chang *et al.* [1] established an EOQ model for deteriorating items, in which the supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. The retailer can obtain interest income within the permissible delay period. Beyond the permissible period, interest payments accrue for the goods not yet sold. There are many interesting and relevant articles related to trade credit, such as Davis and Gaither [23], Ouyang *et al.* [24, 25] and Teng *et al.* [26].

When discussing trade credit, interest income and interest payments are important issues. The above mentioned studies on trade credit assumed that the rate of interest charges, the rate of interest earned, and the deterioration rate are fixed and predetermined. However, in the real market, many factors may cause fluctuations in interest rates. For instance, the growth rate of the economy may affect the interest rate. When the economy is developing at a faster rate, the demand for capital is strong, which in return pushes up the interest rate. Conversely, during economic downturns, demand for capital declines and interest rates fall. Another factor influencing interest rates is the supply of money in the economy. When the supply of money is high, interest rates fall and when supply is low, interest rates rise. The third factor affecting interest rates is the interest rates in other

countries. Since money flows between nations, the differing interest rates between countries mutually influence each other. Therefore, the rate of interest charges and the rate of interest earned are unlikely to remain constant due to various uncertainties. Moreover, within the environment of merchandise storage, some distinctive factors arise which ultimately affect the quality of products such as temperature, humidity, and storage equipment. Therefore, in the real world, the inventory deterioration rate is not known with certainty. Amongst extended studies on this topic, only Chen and Ouyang [27] have treated interest rates as variable, wherein the authors extended the model of Jamal *et al*. [28] by fuzzifying the carrying cost rate, interest paid rate and interest earned rate simultaneously. Summarizing the above, we observe that the inventory problem associated with trade credit has yet to be fully explored and understood especially when interest rates fluctuate.

In order to fill this gap, this study tries to recast Chang *et al*.'s [1] model by further fuzzifying the rate of interest charges, the rate of interest earned, and the deterioration rate into the triangular fuzzy number. We construct three different intervals to include the rate of interest charges, the rate of interest earned, and the deterioration rate, thus deriving the fuzzy total relevant inventory cost. By the signed distance method of defuzzification, we derive the estimate of the total relevant inventory cost in the fuzzy sense. Further, we discuss how to determine the optimal ordering policy such that the total relevant inventory cost in the fuzzy sense is minimal. Finally, numerical examples are given to illustrate the solution procedure. This article is organized as follows. In section 2, we provide the preliminaries of fuzzy mathematics. In section 3, we introduce the inventory problem. A brief review of Chang *et al*.'s [1] model is included in section 3.1 and the fuzzy inventory model is provided in section 3.2. We then use the signed distance method of defuzzification to derive the estimate of the total relevant inventory cost in the fuzzy sense. In section 4, we obtain the optimal replenishment time interval and the optimal order quantity by minimizing the estimate of the total relevant inventory cost in the fuzzy sense. Several numerical examples are given to illustrate the results in section 5. In section 6, we discuss two problems of the proposed model. Finally, section 7 draws conclusions and suggests potential directions for future research.

2. PRELIMINARIES

In this section, some concepts of the fuzzy set theory are reviewed. We introduce three definitions, decomposition theorem and one property which we will use throughout this article.

Definition 1   For $0 \leq \alpha \leq 1$ and $p < q$, the fuzzy set $[p, q; \alpha]$ on R is called *a level α fuzzy interval* if the membership function of $[p, q; \alpha]$ is given by

$$\mu_{[p,q;\alpha]}(x) = \begin{cases} \alpha, & p \leq x \leq q; \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Decomposition Theorem (see, Kaufmannan and Gupta [29])
Let $\tilde{D}$ be a fuzzy set on R and $\tilde{D} \in F_s$, $0 \leq \alpha \leq 1$. The α-cut of $\tilde{D}$ is $D(\alpha) = [D_L(\alpha),$

$D_U(\alpha)$]. Then, we have

$$\tilde{D} = \bigcup_{0 \le \alpha \le 1} \alpha D(\alpha), \tag{2}$$

or

$$\mu_{\tilde{D}}(x) = \bigvee_{0 \le \alpha \le 1} \alpha \cdot C_{D(\alpha)}(x), \tag{3}$$

where

(i) $\alpha D(\alpha)$ is a fuzzy set with membership function

$$\mu_{\alpha D(\alpha)}(x) = \begin{cases} \alpha, & x \in D(\alpha); \\ 0, & \text{otherwise.} \end{cases}$$

(ii) $C_{D(\alpha)}(x)$ is a characteristic function of $D(\alpha)$, that is,

$$C_{D(\alpha)}(x) = \begin{cases} 1, & x \in D(\alpha); \\ 0, & x \notin D(\alpha). \end{cases}$$

From the Decomposition Theorem and Eq. (2), we obtain

$$\tilde{D} = \bigcup_{0 \le \alpha \le 1} \alpha D(\alpha) = \bigcup_{0 \le \alpha \le 1} [D_L(\alpha), D_U(\alpha); \alpha], \tag{4}$$

or

$$\mu_{\tilde{D}}(x) = \bigvee_{0 \le \alpha \le 1} \alpha \cdot C_{D(\alpha)}(x) = \bigvee_{0 \le \alpha \le 1} \mu_{[D_L(\alpha), D_U(\alpha); \alpha]}(x). \tag{5}$$

For any $a, b, c, d, k \in R$, where $a < b$ and $c < d$, the following definitions of the interval operations can be found in [29].

(i) $[a, b](+)[c, d] = [a + c, b + d]$.
(ii) $[a, b](-)[c, d] = [a - d, b - c]$.
(iii) $k(\cdot)[a, b] = \begin{cases} [ka, kb], & k > 0; \\ [kb, ka], & k < 0. \end{cases}$ $\tag{6}$

If $0 \le a < b$ and $0 \le c < d$, then

(iv) $[a, b](\cdot)[c, d] = [ac, bd]$.

If $0 \le a < b$ and $0 < c < d$, then

(v) $[a, b](\div)[c, d] = [a/d, b/c]$.

Next, similar as Yao and Wu [30], we introduce the concept of the signed distance which will be needed later. We first consider the signed distance on R.

Definition 2 For any a and $0 \in R$, define the *signed distance* of a to 0 as $d_0(a, 0) = a$.

If $a > 0$, implies that a is on the right-hand side of origin 0 with distance $a = d_0(a, 0)$; and if $a < 0$, implies that a is on the left-hand side of origin 0 with distance $-a = -d_0(a, 0)$. So, we called $d_0(a, 0) = a$ is the signed distance of a to 0.

If $\tilde{D} \in F_s$, from Eq. (4), we have

$$\tilde{D} = \bigcup_{0 \leq \alpha \leq 1} [D_L(\alpha), D_U(\alpha); \alpha]. \tag{7}$$

And for every $\alpha \in [0, 1]$, there is an one-to-one mapping between the level α fuzzy interval $[D_L(\alpha), D_U(\alpha); \alpha]$ and real interval $[D_L(\alpha), D_U(\alpha)]$, that is, the following correspondence is one-to-one mapping:

$$[D_L(\alpha), D_U(\alpha)] \leftrightarrow [D_L(\alpha), D_U(\alpha); \alpha]. \tag{8}$$

We shall use this relation later.

From Definition 2, the signed distance of the left end point $D_L(\alpha)$ of the α-cut $[D_L(\alpha), D_U(\alpha)]$ of $\tilde{D}$ to the origin 0 is $D_L(\alpha)$, and the signed distance of the right end point $D_U(\alpha)$ to the origin 0 is $D_U(\alpha)$. Their average, $1/2[D_L(\alpha) + D_U(\alpha)]$, is defined as the signed distance of α-cut $[D_L(\alpha), D_U(\alpha)]$ to 0, that is, we define the signed distance of the interval $[D_L(\alpha), D_U(\alpha)]$ to 0 as:

$$d_0([D_L(\alpha), D_U(\alpha)], 0) = \frac{1}{2}[d_0(D_L(\alpha), 0) + d_0(D_U(\alpha), 0)] = \frac{1}{2}[D_L(\alpha) + D_U(\alpha)]. \tag{9}$$

Further, from Eqs. (8) and (9), the signed distance of level α fuzzy interval $[D_L(\alpha), D_U(\alpha); \alpha]$ to the fuzzy point $\tilde{0}$ can be defined as :

$$d([D_L(\alpha), D_U(\alpha); \alpha], \tilde{0}) = d_0([D_L(\alpha), D_U(\alpha)], 0) = \frac{1}{2}[D_L(\alpha) + D_U(\alpha)]. \tag{10}$$

Thus, from Eqs. (7) and (10), we can define the signed distance of a fuzzy set $\tilde{D} \in F_s$ to $\tilde{0}$ as follows.

Definition 3 For $\tilde{D} \in F_s$, define the *signed distance* of $\tilde{D}$ to $\tilde{0}$ as

$$d(\tilde{D}, \tilde{0}) = \int_0^1 d([D_L(\alpha), D_U(\alpha); \alpha], \tilde{0}) d\alpha = \frac{1}{2} \int_0^1 [D_L(\alpha) + D_U(\alpha)] d\alpha.$$

Let $\tilde{D}, \tilde{E} \in F_s$, from Eq. (7), we have $\tilde{D} = \bigcup_{0 \leq \alpha \leq 1} [D_L(\alpha), D_U(\alpha); \alpha]$, $\tilde{E} = \bigcup_{0 \leq \alpha \leq 1} [E_L(\alpha), E_U(\alpha); \alpha]$, and from Eq. (8), we have the following one-to-one mapping:

For every $\alpha \in [0, 1]$,

$$[D_L(\alpha), D_U(\alpha)] \leftrightarrow [D_L(\alpha), D_U(\alpha); \alpha],$$
$$[E_L(\alpha), E_U(\alpha)] \leftrightarrow [E_L(\alpha), E_U(\alpha); \alpha].$$

Then, from Eq. (6), we have

$$[D_L(\alpha), D_U(\alpha)](+)[E_L(\alpha), E_U(\alpha)]$$
$$= [D_L(\alpha) + E_L(\alpha), D_U(\alpha) + E_U(\alpha)] \leftrightarrow [D_L(\alpha) + E_L(\alpha), D_U(\alpha) + E_U(\alpha); \alpha].$$

Therefore, from the Decomposition Theorem, we can get

$$\tilde{D}(+)\tilde{E} = \bigcup_{0 \le \alpha \le 1} [D_L(\alpha) + E_L(\alpha), D_U(\alpha) + E_U(\alpha); \alpha]. \tag{11}$$

Similarly, we have

$$\tilde{k}(\cdot)\tilde{D} = \begin{cases} \bigcup_{0 \le \alpha \le 1} [kD_L(\alpha), kD_U(\alpha); \alpha], & k > 0; \\ \bigcup_{0 \le \alpha \le 1} [kD_U(\alpha), kD_L(\alpha); \alpha], & k < 0. \end{cases} \tag{12}$$

From the above discussion, we obtain the following property.

Property For $\tilde{D}, \tilde{E} \in F_s$, and $k \in R$,
(i) $d(\tilde{D}(+)\tilde{E}, \tilde{0}) = d(\tilde{D}, \tilde{0}) + d(\tilde{E}, \tilde{0})$.
(ii) $d(\tilde{k}(\cdot)\tilde{D}, \tilde{0}) = kd(\tilde{D}, \tilde{0})$. $\tag{13}$

Proof: The proof can be easily obtained from Eq. (11) and Definition 3.

3. THE INVENTORY PROBLEM

To develop the proposed model, we adopt the following notation and assumptions used in Chang *et al.* [1].

Notation:
D: the demand per year
h: the unit holding cost per year excluding interest charges
p: the selling price per unit
c: the unit purchasing cost, with $c < p$
I_c: the interest charges per \$ in stocks per year by the supplier
I_d: the interest earned per \$ per year
S: the ordering cost per order
M: the permissible delay in settling account (*i.e.*, the trade credit period)
Q: the order quantity
Q_d: the minimum order quantity at which the delay in payments is permitted
T_d: the time interval that Q_d units are depleted to zero due to both demand and deterioration
θ: the constant deterioration rate, where $0 \le \theta < 1$
$I(t)$: the level of inventory at time t, $0 \le t \le T$
T: the replenishment time interval
$Z(T)$: the total relevant inventory cost per year

Assumptions:
(1) The demand for the item is constant with time.
(2) Shortages are not allowed.
(3) Replenishment is instantaneous.
(4) If the order quantity is less than Q_d, then the payment for the items received must be made immediately.
(5) If the order quantity is greater than or equal to Q_d, then the delay in payments up to M is permitted. During the trade credit period the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of the permissible delay, the customer pays off all units ordered, and starts paying for the interest charges on the items in stocks.
(6) Time horizon is infinite.

The total relevant inventory cost consists of (a) cost of placing orders, (b) cost of deteriorated units, (c) cost of carrying inventory (excluding interest charges), (d) cost of interest charges for unsold items at the initial time or after the permissible delay M, and (e) interest earned from sales revenue during the permissible period.

3.1 Review of Chang *et al.*'s Model

Under the above notation and assumptions, Chang *et al.* [1] considered the four cases: (1) $0 < T < T_d$, (2) $T_d \leq T < M$, (3) $T_d \leq M \leq T$ and (4) $M \leq T_d \leq T$ and obtained the total relevant inventory cost per year as follows:

$$Z(T) = \begin{cases} Z_1(T), & \text{if } 0 < T < T_d; \\ Z_2(T), & \text{if } T_d \leq T < M; \\ Z_3(T), & \text{if } T_d \leq M \leq T; \\ Z_4(T), & \text{if } M \leq T_d \leq T. \end{cases}$$

where

$$Z_1(T) = \frac{S}{T} - cD + \frac{D(h + c\theta + cI_c)}{\theta^2 T}(e^{\theta T} - 1) - \frac{D(h + cI_c)}{\theta}, \tag{14}$$

$$Z_2(T) = \frac{S}{T} - cD + \frac{D(h + c\theta)}{\theta^2 T}(e^{\theta T} - 1) - \frac{hD}{\theta} - pI_d D(M - T/2), \tag{15}$$

$$Z_3(T) = \frac{S}{T} - cD + \frac{D(h + c\theta)}{\theta^2 T}(e^{\theta T} - 1) - \frac{hD}{\theta} + \frac{cI_c D}{\theta^2 T}[e^{\theta(T-M)} - 1] \\ - \frac{cI_c D}{\theta T}(T - M) - \frac{pI_d D}{2T} M^2, \tag{16}$$

and $Z_4(T)$ is the same as $Z_3(T)$, *i.e.*,

$$Z_4(T) = \frac{S}{T} - cD + \frac{D(h + c\theta)}{\theta^2 T}(e^{\theta T} - 1) - \frac{hD}{\theta} + \frac{cI_c D}{\theta^2 T}[e^{\theta(T-M)} - 1] \\ - \frac{cI_c D}{\theta T}(T - M) - \frac{pI_d D}{2T} M^2. \tag{17}$$

3.2 The Fuzzy Inventory Model

In Chang *et al.*'s [1] model, the deterioration rate, the rate of interest charges, and the rate of interest earned are assumed to be constant. In a real-life inventory problem, however, it is not always easy to determine their values exactly. In most cases, it is likely they will have a little disturbance due to various uncertainties during the inventory period. First, we consider the deterioration rate near θ and fuzzify it as a triangular fuzzy number. From Yao *et al.* [19], we have the following concept: let $\theta - \Delta_1 < \theta < \theta + \Delta_2$ where θ is an any fixed point in $[\theta - \Delta_1, \theta + \Delta_2]$. Corresponding to the interval $[\theta - \Delta_1, \theta + \Delta_2]$, θ can be considered for fuzzification as the triangular fuzzy number $\tilde{\theta}$ in the following: a decision maker takes a point from the interval $[\theta - \Delta_1, \theta + \Delta_2]$, if the point is θ, the error between the point and fixed point θ is zero. Based on the confidence level concept; if the error is zero, then the confidence level is the maximum value and set to 1. If the point is taken from the interval $[\theta - \Delta_1, \theta)$, when the point moves away from θ, then the error between the point and θ becomes larger, *i.e.*, the confidence level becomes smaller. Moreover, if the point is equal to $\theta - \Delta_1$, the confidence level attains the minimum value and is thus set to 0. Similarly, if the point is taken from the interval $(\theta, \theta + \Delta_2]$, when the point moves away from θ, the confidence level becomes smaller. Moreover, if the point is equal to $\theta + \Delta_2$, the confidence level reaches 0. Therefore, corresponding to the interval $[\theta - \Delta_1, \theta + \Delta_2]$, the following triangular fuzzy number $\tilde{\theta}$ is set.

$$\tilde{\theta} = (\theta - \Delta_1, \theta, \theta + \Delta_2), \tag{18}$$

where $0 < \Delta_1 < \theta$ and $0 < \Delta_2$. Therefore, we obtain the membership function of $\tilde{\theta}$

$$\mu_{\tilde{\theta}}(x) = \begin{cases} \dfrac{x - \theta + \Delta_1}{\Delta_1}, & \theta - \Delta_1 \le x \le \theta; \\ \dfrac{\theta + \Delta_2 - x}{\Delta_2}, & \theta \le x \le \theta + \Delta_2; \\ 0, & \text{otherwise}, \end{cases} \tag{19}$$

and the left and right end points of the α-cut of $\tilde{\theta}$, $0 \le \alpha \le 1$, are

$$\theta_L(\alpha) = \theta - (1 - \alpha)\Delta_1 > 0, \text{ and } \theta_U(\alpha) = \theta + (1 - \alpha)\Delta_2 > 0, \tag{20}$$

respectively.

Let $y = g(x) = e^{xT}$, and by extension principle in fuzzy set (see, Zimmermann [31]), we get the membership function of fuzzy set $g(\tilde{\theta}) = e^{\tilde{\theta}T}$ as follows:

$$\mu_{g(\tilde{\theta})}(y) = \sup_{y = e^{xT}} \mu_{\tilde{\theta}}(x) = \mu_{\tilde{\theta}}(\ln y / T) = \begin{cases} \dfrac{\ln y - T\theta + T\Delta_1}{T\Delta_1}, & e^{T(\theta - \Delta_1)} \le y \le e^{T\theta}; \\ \dfrac{T\theta + T\Delta_2 - \ln y}{T\Delta_2}, & e^{T\theta} \le y \le e^{T(\theta + \Delta_2)}; \\ 0, & \text{otherwise}, \end{cases} \tag{21}$$

and the left and right end points of the a-cut of $e^{\tilde{\theta}T}$, $0 \le \alpha \le 1$, are

$$(e^{\theta T})_L(\alpha) = e^{[\theta-(1-\alpha)\Delta_1]T}, \text{ and } (e^{\theta T})_U(\alpha) = e^{[\theta+(1-\alpha)\Delta_2]T}, \tag{22}$$

respectively.

We also fuzzify the rate of interest charges I_c as the following triangular fuzzy number

$$\tilde{I}_c = (I_c - \Delta_3, I_c, I_c + \Delta_4), \tag{23}$$

where $0 < \Delta_3 < I_c$ and $0 < \Delta_4$. The left and right end points of the α-cut of $\tilde{I}_c$, $0 \le \alpha \le 1$, are

$$(I_c)_L(\alpha) = I_c - (1 - \alpha)\Delta_3 > 0, \text{ and } (I_c)_U(\alpha) = I_c + (1 - \alpha)\Delta_4 > 0, \tag{24}$$

respectively.

Further, we fuzzify the rate of interest earned I_d as the following triangular fuzzy number

$$\tilde{I}_d = (I_d - \Delta_5, I_d, I_d + \Delta_6), \tag{25}$$

where $0 < \Delta_5 < I_d$ and $0 < \Delta_6$. The left and right end points of the α-cut of $\tilde{I}_d$, $0 \le \alpha \le 1$, are

$$(I_d)_L(\alpha) = I_d - (1 - \alpha)\Delta_5 > 0, \text{ and } (I_d)_U(\alpha) = I_d + (1 - \alpha)\Delta_6 > 0, \tag{26}$$

respectively.

For convenience, we let

$$a_1 = \frac{S}{T} - cD, \ a_2 = \frac{Dh}{T}, \ a_3 = \frac{Dc}{T}, \ a_4 = Dh + \frac{Dc}{T}, \ a_5 = Dc, \ a_6 = pD\left(M - \frac{T}{2}\right),$$

$$a_7 = \frac{c(T-M)D}{T}, \ a_8 = \frac{pDM^2}{2T}, \ \tilde{P}_1 = e^{\tilde{\theta}T}(\div)\tilde{\theta}^2, \ \tilde{P}_2 = e^{\tilde{\theta}T}(\div)\tilde{\theta}, \ \tilde{P}_3 = \tilde{I}_c(\cdot)e^{\tilde{\theta}T}(\div)\tilde{\theta}^2,$$

$$\tilde{P}_4 = \tilde{1}(\div)\tilde{\theta}^2, \ \tilde{P}_5 = \tilde{1}(\div)\tilde{\theta}, \ \tilde{P}_6 = \tilde{I}_c(\div)\tilde{\theta}^2, \ \tilde{P}_7 = \tilde{I}_c(\div)\tilde{\theta}, \ \tilde{P}_8 = \tilde{I}_d \text{ and } \tilde{P}_9 = \tilde{I}_c(\cdot)e^{\tilde{\theta}(T-M)}$$

$$(\div)\tilde{\theta}^2 \text{ and let } \tilde{a}_j \text{ be a fuzzy point at real number } a_j, j = 1, 2, \ldots, 8. \tag{27}$$

Then, contrast to Eqs. (14)-(17), we have the total relevant inventory costs per year in the crisp case and the corresponding fuzzy case as follows:

Case 1: $0 < T < T_d$
From Eq. (14), we let

$$
\begin{aligned}
g_1(I_c, \theta\,; T) &\equiv Z_1(T) \\
&= \left(\frac{S}{T} - cD\right) + \frac{Dh}{T} \cdot \frac{e^{\theta T}}{\theta^2} + \frac{Dc}{T} \cdot \frac{e^{\theta T}}{\theta} + \frac{Dc}{T} \cdot \frac{I_c e^{\theta T}}{\theta^2} - \frac{Dh}{T} \cdot \frac{1}{\theta^2} - \left(Dh + \frac{Dc}{T}\right) \cdot \frac{1}{\theta} \\
&\quad - \frac{Dc}{T} \cdot \frac{I_c}{\theta^2} - Dc \cdot \frac{I_c}{\theta} \\
&= a_1 + a_2 \cdot \frac{e^{\theta T}}{\theta^2} + a_3 \cdot \frac{e^{\theta T}}{\theta} + a_3 \cdot \frac{I_c e^{\theta T}}{\theta^2} - a_2 \cdot \frac{1}{\theta^2} - a_4 \cdot \frac{1}{\theta} - a_3 \cdot \frac{I_c}{\theta^2} - a_5 \cdot \frac{I_c}{\theta}.
\end{aligned}
\tag{28}
$$

Thus, the fuzzy total relevant inventory cost per year is

$$g_1(\tilde{I}_c, \tilde{\theta}; T) = \tilde{a}_1(+)(\tilde{a}_2(\cdot)\tilde{P}_1)(+)(\tilde{a}_3(\cdot)\tilde{P}_2)(+)(\tilde{a}_3(\cdot)\tilde{P}_3)(-)(\tilde{a}_2(\cdot)\tilde{P}_4)(-)(\tilde{a}_4(\cdot)\tilde{P}_5)$$
$$(-)(\tilde{a}_3(\cdot)\tilde{P}_6)(-)(\tilde{a}_5(\cdot)\tilde{P}_7). \tag{29}$$

Case 2: $T_d \le T < M$

From Eq. (15), we let

$$g_2(I_d, \theta; T) \equiv Z_2(T)$$
$$= \left(\frac{S}{T} - cD\right) + \frac{Dh}{T} \cdot \frac{e^{\theta T}}{\theta^2} + \frac{Dc}{T} \cdot \frac{e^{\theta T}}{\theta} - \frac{Dh}{T} \cdot \frac{1}{\theta^2} - \left(\frac{Dc}{T} + Dh\right) \cdot \frac{1}{\theta} - \left(pD(M - \frac{T}{2})\right)I_d$$
$$= a_1 + a_2 \cdot \frac{e^{\theta T}}{\theta^2} + a_3 \cdot \frac{e^{\theta T}}{\theta} - a_2 \cdot \frac{1}{\theta^2} - a_4 \cdot \frac{1}{\theta} - a_6 I_d. \tag{30}$$

Then the fuzzy total relevant inventory cost per year is

$$g_2(\tilde{I}_d, \tilde{\theta}; T) = \tilde{a}_1(+)(\tilde{a}_2(\cdot)\tilde{P}_1)(+)(\tilde{a}_3(\cdot)\tilde{P}_2)(-)(\tilde{a}_2(\cdot)\tilde{P}_4)(-)(\tilde{a}_4(\cdot)\tilde{P}_5)(-)(\tilde{a}_6(\cdot)\tilde{P}_8). \tag{31}$$

Case 3: $T_d \le M \le T$

From Eq. (16), we let

$$g_3(I_c, I_d, \theta; T) \equiv Z_3(T)$$
$$= \left(\frac{S}{T} - cD\right) + \frac{Dh}{T} \cdot \frac{e^{\theta T}}{\theta^2} + \frac{Dc}{T} \cdot \frac{e^{\theta T}}{\theta} - \frac{Dh}{T} \cdot \frac{1}{\theta^2} - \left(\frac{Dc}{T} + Dh\right) \cdot \frac{1}{\theta} + \frac{Dc}{T} \cdot \frac{I_c e^{\theta(T-M)}}{\theta^2}$$
$$- \frac{Dc}{T} \cdot \frac{I_c}{\theta^2} - \frac{c(T-M)D}{T} \cdot \frac{I_c}{\theta} - \left(\frac{pDM^2}{2T}\right) \cdot I_d$$
$$= a_1 + a_2 \cdot \frac{e^{\theta T}}{\theta^2} + a_3 \cdot \frac{e^{\theta T}}{\theta} - a_2 \cdot \frac{1}{\theta^2} - a_4 \cdot \frac{1}{\theta} + a_3 \cdot \frac{I_c e^{\theta(T-M)}}{\theta^2} - a_3 \cdot \frac{I_c}{\theta^2} - a_7 \cdot \frac{I_c}{\theta} - a_8 \cdot I_d. \tag{32}$$

Then the fuzzy total relevant inventory cost per year is

$$g_3(\tilde{I}_c, \tilde{I}_d, \tilde{\theta}; T) = \tilde{a}_1(+)(\tilde{a}_2(\cdot)\tilde{P}_1)(+)(\tilde{a}_3(\cdot)\tilde{P}_2)(-)(\tilde{a}_2(\cdot)\tilde{P}_4)(-)(\tilde{a}_4(\cdot)\tilde{P}_5)(+)(\tilde{a}_3(\cdot)\tilde{P}_9)$$
$$(-)(\tilde{a}_3(\cdot)\tilde{P}_6)(-)(\tilde{a}_7(\cdot)\tilde{P}_7)(-)(\tilde{a}_8(\cdot)\tilde{P}_8). \tag{33}$$

Case 4: $M \le T_d \le T$

Case 4 is the similar to case 3. Therefore, the fuzzy total relevant inventory cost per year is

$$g_4(\tilde{I}_c, \tilde{I}_d, \tilde{\theta}; T) = \tilde{a}_1(+)(\tilde{a}_2(\cdot)\tilde{P}_1)(+)(\tilde{a}_3(\cdot)\tilde{P}_2)(-)(\tilde{a}_2(\cdot)\tilde{P}_4)(-)(\tilde{a}_4(\cdot)\tilde{P}_5)(+)(\tilde{a}_3(\cdot)\tilde{P}_9)$$
$$(-)(\tilde{a}_3(\cdot)\tilde{P}_6)(-)(\tilde{a}_7(\cdot)\tilde{P}_7)(-)(\tilde{a}_8(\cdot)\tilde{P}_8). \tag{34}$$

From Eqs. (20), (22), (24), (26) and (6), we obtain the left and right end points of the α-cut ($0 \le \alpha \le 1$) of fuzzy sets $\tilde{P}_j$ ($j = 1, 2, \dots, 9$) in Eq. (27) respectively as follows:

$$(P_1)_L(\alpha) = \frac{(e^{\theta T})_L(\alpha)}{[\theta_U(\alpha)]^2} = \frac{e^{[\theta - (1-\alpha)\Delta_1]T}}{[\theta + (1-\alpha)\Delta_2]^2} , \quad (P_1)_U(\alpha) = \frac{(e^{\theta T})_U(\alpha)}{[\theta_L(\alpha)]^2} = \frac{e^{[\theta + (1-\alpha)\Delta_2]T}}{[\theta - (1-\alpha)\Delta_1]^2} ,$$

(35)

$$(P_2)_L(\alpha) = \frac{(e^{\theta T})_L(\alpha)}{\theta_U(\alpha)} = \frac{e^{[\theta - (1-\alpha)\Delta_1]T}}{\theta + (1-\alpha)\Delta_2} , \quad (P_2)_U(\alpha) = \frac{(e^{\theta T})_U(\alpha)}{\theta_L(\alpha)} = \frac{e^{[\theta + (1-\alpha)\Delta_2]T}}{\theta - (1-\alpha)\Delta_1} , \quad (36)$$

$$(P_3)_L(\alpha) = \frac{(I_c)_L(\alpha)(e^{\theta T})_L(\alpha)}{[\theta_U(\alpha)]^2} = \frac{(I_c - (1-\alpha)\Delta_3)e^{[\theta - (1-\alpha)\Delta_1]T}}{[\theta + (1-\alpha)\Delta_2]^2} ,$$

$$(P_3)_U(\alpha) = \frac{(I_c)_U(\alpha)(e^{\theta T})_U(\alpha)}{[\theta_L(\alpha)]^2} = \frac{[I_c + (1-\alpha)\Delta_4]e^{[\theta + (1-\alpha)\Delta_2]T}}{[\theta - (1-\alpha)\Delta_1]^2} , \quad (37)$$

$$(P_4)_L(\alpha) = \frac{1}{[\theta_U(\alpha)]^2} = \frac{1}{[\theta + (1-\alpha)\Delta_2]^2} , \quad (P_4)_U(\alpha) = \frac{1}{[\theta_L(\alpha)]^2} = \frac{1}{[\theta - (1-\alpha)\Delta_1]^2} ,$$

(38)

$$(P_5)_L(\alpha) = \frac{1}{\theta_U(\alpha)} = \frac{1}{\theta + (1-\alpha)\Delta_2} , \quad (P_5)_U(\alpha) = \frac{1}{\theta_L(\alpha)} = \frac{1}{\theta - (1-\alpha)\Delta_1} , \quad (39)$$

$$(P_6)_L(\alpha) = \frac{(I_c)_L(\alpha)}{[\theta_U(\alpha)]^2} = \frac{I_c - (1-\alpha)\Delta_3}{[\theta + (1-\alpha)\Delta_2]^2} , \quad (P_6)_U(\alpha) = \frac{(I_c)_U(\alpha)}{[\theta_L(\alpha)]^2} = \frac{I_c + (1-\alpha)\Delta_4}{[\theta - (1-\alpha)\Delta_1]^2} ,$$

(40)

$$(P_7)_L(\alpha) = \frac{(I_c)_L(\alpha)}{\theta_U(\alpha)} = \frac{I_c - (1-\alpha)\Delta_3}{\theta + (1-\alpha)\Delta_2} , \quad (P_7)_U(\alpha) = \frac{(I_c)_U(\alpha)}{\theta_L(\alpha)} = \frac{I_c + (1-\alpha)\Delta_4}{\theta - (1-\alpha)\Delta_1} , \quad (41)$$

$$(P_8)_L(\alpha) = (I_d)_L(\alpha) = I_d - (1-\alpha)\Delta_5, \quad (P_8)_U(\alpha) = (I_d)_U(\alpha) = I_d + (1-\alpha)\Delta_6, \quad (42)$$

$$(P_9)_L(\alpha) = \frac{(I_c)_L(\alpha)[e^{\theta(T-M)}]_L(\alpha)}{[\theta_U(\alpha)]^2} = \frac{[I_c - (1-\alpha)\Delta_3]e^{[\theta - (1-\alpha)\Delta_1](T-M)}}{[\theta + (1-\alpha)\Delta_2]^2} ,$$

and

$$(P_9)_U(\alpha) = \frac{(I_c)_U(\alpha)[e^{\theta(T-M)}]_U(\alpha)}{[\theta_L(\alpha)]^2} = \frac{[I_c + (1-\alpha)\Delta_4]e^{[\theta + (1-\alpha)\Delta_2](T-M)}}{[\theta - (1-\alpha)\Delta_1]^2} . \quad (43)$$

In order to calculate the signed distance of the fuzzy set $\tilde{P}_j$ ($j = 1, 2, \ldots, 9$) in Eq. (27), we consider the following functions and let $Z = w + u(1 - \alpha)$, where $u \neq 0$,

$$K_1(w_1, u_1, w, u; n) = \int_0^1 \frac{(w_1 + u_1(1-\alpha))^n}{w + u(1-\alpha)} d\alpha = \frac{1}{u} \int_w^{w+u} (w_1 - \frac{w}{u}u_1 + \frac{u_1}{u}Z)^n / Z dZ$$

$$= \frac{1}{u}\left[(w_1 - \frac{w}{u}u_1)^n \ln\left|\frac{w+u}{w}\right| + \sum_{r=1}^{n}\binom{n}{r}(w_1 - \frac{w}{u}u_1)^{n-r}(\frac{u_1}{u})^r \frac{((w+u)^r - w^r)}{r} \right], \text{ for } n \geq 1,$$

(44)

where $\binom{n}{r} = \frac{n!}{r!(n-r)!}$;

$$K_2(w_1, u_1, w, u; n) = \int_0^1 \frac{(w_1 + u_1(1-\alpha))^n}{(w + u\,(1-\alpha))^2}\,d\alpha = \frac{1}{u}\int_w^{w+u}(w_1 - \frac{w}{u}u_1 + \frac{u_1}{u}Z)^n/Z^2 dZ$$

$$= \frac{1}{u}\left[(w_1 - \frac{w}{u}u_1)^n(\frac{1}{w} - \frac{1}{w+u}) + n\,(w_1 - \frac{w}{u}u_1)^{n-1}(\frac{u_1}{u})\ln\left|\frac{w+u}{w}\right|\right.$$

$$\left. + \sum_{r=2}^n \binom{n}{r}(w_1 - \frac{w}{u}u_1)^{n-r}(\frac{u_1}{u})^r \frac{((w+u)^{r-1} - w^{r-1})}{r-1}\right], \text{ for } n \geq 1, \tag{45}$$

$$K_3(w_1, u_1, w_2, u_2, w, u; n) = \int_0^1 \frac{(w_1 + u_1(1-\alpha))(w_2 + u_2(1-\alpha))^n}{(w + u\,(1-\alpha))^2}\,d\alpha$$

$$= \frac{1}{u}\int_w^{w+u}(w_1 - \frac{w}{u}u_1 + \frac{u_1}{u}Z)(w_2 - \frac{w}{u}u_2 + \frac{u_2}{u}Z)^n/Z^2 dZ$$

$$= (w_1 - \frac{w}{u}u_1)K_2(w_2, u_2, w, u; n) + \frac{u_1}{u}K_1(w_2, u_2, w, u; n), \text{ for } n \geq 1. \tag{46}$$

Further, by using Taylor's formula, we have

$$e^{ax} \approx \sum_{m=0}^N \frac{1}{m!}a^m x^m, \text{ as } |ax| \text{ is enough small,}$$

where N is the positive integer which is decided by the decision-maker appropriately.

Therefore, from Eqs. (35)-(46) and using Taylor's formula, we get the signed distance of the fuzzy sets $\tilde{P}_j$ $(j = 1, 2, \ldots, 9)$ (in Eq. (27)) as follows.

$$d(\tilde{P}_1, \tilde{0}) = \frac{1}{2}\int_0^1 [(P_1)_L(\alpha) + (P_1)_U(\alpha)]d\alpha$$

$$= \frac{1}{2}e^{\theta T}\int_0^1 \left[\frac{e^{-(1-\alpha)\Delta_1 T}}{(\theta + (1-\alpha)\Delta_2)^2} + \frac{e^{(1-\alpha)\Delta_2 T}}{(\theta - (1-\alpha)\Delta_1)^2}\right]d\alpha$$

$$\approx \frac{1}{2}\left[\sum_{m=0}^N \frac{(\theta T)^m}{m!}\right]\left[\sum_{m=0}^{N_{11}} \frac{T^m}{m!}\int_0^1 \frac{(-(1-\alpha)\Delta_1)^m}{(\theta + (1-\alpha)\Delta_2)^2}\,d\alpha + \sum_{m=0}^{N_{12}} \frac{T^m}{m!}\int_0^1 \frac{((1-\alpha)\Delta_2)^m}{(\theta - (1-\alpha)\Delta_1)^2}\,d\alpha\right]$$

$$= \frac{1}{2}\left[\sum_{m=0}^N \frac{(\theta T)^m}{m!}\right]\left[\sum_{m=0}^{N_{11}} \frac{T^m}{m!}K_2(0, -\Delta_1, \theta, \Delta_2; m) + \sum_{m=0}^{N_{12}} \frac{T^m}{m!}K_2(0, \Delta_2, \theta, -\Delta_1; m)\right], \tag{47}$$

$$d(\tilde{P}_2, \tilde{0}) = \frac{1}{2}\int_0^1 [(P_2)_L(\alpha) + (P_2)_U(\alpha)]d\alpha = \frac{1}{2}e^{\theta T}\int_0^1 \left[\frac{e^{-(1-\alpha)\Delta_1 T}}{\theta + (1-\alpha)\Delta_2} + \frac{e^{(1-\alpha)\Delta_2 T}}{\theta - (1-\alpha)\Delta_1}\right]d\alpha$$

$$\approx \frac{1}{2}\left[\sum_{m=0}^N \frac{(\theta T)^m}{m!}\right]\left[\sum_{m=0}^{N_{21}} \frac{T^m}{m!}\int_0^1 \frac{(-(1-\alpha)\Delta_1)^m}{\theta + (1-\alpha)\Delta_2}d\alpha + \sum_{m=0}^{N_{22}} \frac{T^m}{m!}\int_0^1 \frac{((1-\alpha)\Delta_2)^m}{\theta - (1-\alpha)\Delta_1}d\alpha\right]$$

$$= \frac{1}{2}\left[\sum_{m=0}^N \frac{(\theta T)^m}{m!}\right]\left[\sum_{m=0}^{N_{21}} \frac{T^m}{m!}K_1(0, -\Delta_1, \theta, \Delta_2; m) + \sum_{m=0}^{N_{22}} \frac{T^m}{m!}K_1(0, \Delta_2, \theta, -\Delta_1; m)\right], \tag{48}$$

$$d(\tilde{P}_3, \tilde{0}) = \frac{1}{2} \int_0^1 [(P_3)_L(\alpha) + (P_3)_U(\alpha)] d\alpha$$

$$= \frac{1}{2} e^{\theta T} \int_0^1 \left[\frac{(I_c - (1-\alpha)\Delta_3)e^{-(1-\alpha)\Delta_1 T}}{(\theta + (1-\alpha)\Delta_2)^2} + \frac{(I_c + (1-\alpha)\Delta_4)e^{(1-\alpha)\Delta_2 T}}{(\theta - (1-\alpha)\Delta_1)^2} \right] d\alpha$$

$$\approx \frac{1}{2} \left[\sum_{m=0}^{N} \frac{(\theta T)^m}{m!} \right] \left[\sum_{m=0}^{N_{31}} \frac{T^m}{m!} \int_0^1 \frac{(I_c - (1-\alpha)\Delta_3)(-(1-\alpha)\Delta_1)^m}{(\theta + (1-\alpha)\Delta_2)^2} d\alpha \right.$$

$$+ \sum_{m=0}^{N_{32}} \frac{T^m}{m!} \int_0^1 \frac{(I_c + (1-\alpha)\Delta_4)((1-\alpha)\Delta_2)^m}{(\theta - (1-\alpha)\Delta_1)^2} d\alpha$$

$$= \frac{1}{2} \left[\sum_{m=0}^{N} \frac{(\theta T)^m}{m!} \right] \left[\sum_{m=0}^{N_{31}} \frac{T^m}{m!} K_3(I_c, -\Delta_3, 0, -\Delta_1, \theta, \Delta_2; m) \right.$$

$$\left. + \sum_{m=0}^{N_{32}} \frac{T^m}{m!} K_3(I_c, \Delta_4, 0, \Delta_2, \theta, -\Delta_1; m) \right], \tag{49}$$

$$d(\tilde{P}_4, \tilde{0}) = \frac{1}{2} \int_0^1 [(P_4)_L(\alpha) + (P_4)_U(\alpha)] d\alpha$$

$$= \frac{1}{2} \int_0^1 \frac{1}{(\theta + (1-\alpha)\Delta_2)^2} d\alpha + \frac{1}{2} \int_0^1 \frac{1}{(\theta - (1-\alpha)\Delta_1)^2} d\alpha$$

$$= \frac{1}{2} K_2(1, 0, \theta, \Delta_2; 0) + \frac{1}{2} K_2(1, 0, \theta, -\Delta_1; 0), \tag{50}$$

$$d(\tilde{P}_5, \tilde{0}) = \frac{1}{2} \int_0^1 [(P_5)_L(\alpha) + (P_5)_U(\alpha)] d\alpha$$

$$= \frac{1}{2} \int_0^1 \frac{1}{\theta + (1-\alpha)\Delta_2} d\alpha + \frac{1}{2} \int_0^1 \frac{1}{\theta - (1-\alpha)\Delta_1} d\alpha$$

$$= \frac{1}{2} K_1(1, 0, \theta, \Delta_2; 0) + \frac{1}{2} K_1(1, 0, \theta, -\Delta_1; 0), \tag{51}$$

$$d(\tilde{P}_6, \tilde{0}) = \frac{1}{2} \int_0^1 [(P_6)_L(\alpha) + (P_6)_U(\alpha)] d\alpha$$

$$= \frac{1}{2} \int_0^1 \frac{I_c - (1-\alpha)\Delta_3}{(\theta + (1-\alpha)\Delta_2)^2} d\alpha + \frac{1}{2} \int_0^1 \frac{I_c + (1-\alpha)\Delta_4}{(\theta - (1-\alpha)\Delta_1)^2} d\alpha$$

$$= \frac{1}{2} K_2(I_c, -\Delta_3, \theta, \Delta_2; 1) + \frac{1}{2} K_2(I_c, \Delta_4, \theta, -\Delta_1; 1), \tag{52}$$

$$d(\tilde{P}_7, \tilde{0}) = \frac{1}{2} \int_0^1 [(P_7)_L(\alpha) + (P_7)_U(\alpha)] d\alpha$$

$$= \frac{1}{2} \int_0^1 \frac{I_c - (1-\alpha)\Delta_3}{\theta + (1-\alpha)\Delta_2} d\alpha + \frac{1}{2} \int_0^1 \frac{I_c + (1-\alpha)\Delta_4}{\theta - (1-\alpha)\Delta_1} d\alpha$$

$$= \frac{1}{2} K_1(I_c, -\Delta_3, \theta, \Delta_2; 1) + \frac{1}{2} K_1(I_c, \Delta_4, \theta, -\Delta_1; 1), \tag{53}$$

$$d(\tilde{P}_8, \tilde{0}) = \frac{1}{2}\int_0^1 [(P_8)_L(\alpha) + (P_8)_U(\alpha)]d\alpha$$

$$= \frac{1}{2}\int_0^1 I_d - (1-\alpha)\Delta_5 d\alpha + \frac{1}{2}\int_0^1 I_d + (1-\alpha)\Delta_6 d\alpha = I_d + \frac{1}{4}(\Delta_6 - \Delta_5), \qquad (54)$$

and

$$d(\tilde{P}_9, \tilde{0}) = \frac{1}{2}\int_0^1 [(P_9)_L(\alpha) + (P_9)_U(\alpha)]d\alpha$$

$$= \frac{1}{2}e^{\theta(T-M)}\int_0^1 \left[\frac{(I_c - (1-\alpha)\Delta_3)e^{-(1-\alpha)\Delta_1(T-M)}}{(\theta + (1-\alpha)\Delta_2)^2} + \frac{(I_c + (1-\alpha)\Delta_4)e^{(1-\alpha)\Delta_2(T-M)}}{(\theta - (1-\alpha)\Delta_1)^2} \right] d\alpha$$

$$\approx \frac{1}{2}\left[\sum_{m=0}^{N} \frac{[\theta(T-M)]^m}{m!} \right]\left[\sum_{m=0}^{N_{41}} \frac{(T-M)^m}{m!} \int_0^1 \frac{(I_c - (1-\alpha)\Delta_3)(-(1-\alpha)\Delta_1)^m}{(\theta + (1-\alpha)\Delta_2)^2} d\alpha \right.$$

$$\left. + \sum_{m=0}^{N_{42}} \frac{(T-M)^m}{m!} \int_0^1 \frac{(I_c + (1-\alpha)\Delta_4)((1-\alpha)\Delta_2)^m}{(\theta - (1-\alpha)\Delta_1)^2} d\alpha \right]$$

$$= \frac{1}{2}\left[\sum_{m=0}^{N} \frac{[\theta(T-M)]^m}{m!} \right]\left[\sum_{m=0}^{N_{41}} \frac{(T-M)^m}{m!} K_3(I_c, -\Delta_3, 0, -\Delta_1, \theta, \Delta_2; m) \right.$$

$$\left. + \sum_{m=0}^{N_{42}} \frac{(T-M)^m}{m!} K_3(I_c, \Delta_4, 0, \Delta_2, \theta, -\Delta_1; m) \right]. \qquad (55)$$

In Eqs. (47)-(49) and (55), N and N_{ij} ($i = 1, 2, 3, 4$ and $j = 1, 2$) are decided by the decision-maker appropriately.

Now, using Eqs. (47)-(55) and the property in section 2, we can defuzzify $g_1(\tilde{I}_c, \tilde{\theta}; T)$, $g_2(\tilde{I}_d, \tilde{\theta}; T)$, $g_3(\tilde{I}_c, \tilde{I}_d, \tilde{\theta}; T)$ and $g_4(\tilde{I}_c, \tilde{I}_d, \tilde{\theta}; T)$ in Eqs. (29), (31), (33) and (34), respectively and get the estimates of the total relevant inventory cost per year in the fuzzy sense as follows:

Case 1: $0 < T < T_d$

$$Z_1^*(T) \equiv d(g_1(\tilde{I}_c, \tilde{\theta}; T), \tilde{0})$$
$$= a_1 + a_2 d(\tilde{P}_1, \tilde{0}) + a_3 d(\tilde{P}_2, \tilde{0}) + a_3 d(\tilde{P}_3, \tilde{0}) - a_2 d(\tilde{P}_4, \tilde{0}) - a_4 d(\tilde{P}_5, \tilde{0}) \qquad (56)$$
$$- a_3 d(\tilde{P}_6, \tilde{0}) - a_5 d(\tilde{P}_7, \tilde{0}).$$

Case 2: $T_d \leq T < M$

$$Z_2^*(T) \equiv d(g_2(\tilde{I}_d, \tilde{\theta}; T), \tilde{0})$$
$$= a_1 + a_2 d(\tilde{P}_1, \tilde{0}) + a_3 d(\tilde{P}_2, \tilde{0}) - a_2 d(\tilde{P}_4, \tilde{0}) - a_4 d(\tilde{P}_5, \tilde{0}) + a_6 d(\tilde{P}_8, \tilde{0}). \qquad (57)$$

Case 3: $T_d \leq M \leq T$

$$Z_3^*(T) \equiv d(g_3(\tilde{I}_c, \tilde{I}_d, \tilde{\theta}; T), \tilde{0})$$
$$= a_1 + a_2 d(\tilde{P}_1, \tilde{0}) + a_3 d(\tilde{P}_2, \tilde{0}) - a_2 d(\tilde{P}_4, \tilde{0}) - a_4 d(\tilde{P}_5, \tilde{0}) + a_3 d(\tilde{P}_9, \tilde{0}) \qquad (58)$$
$$- a_3 d(\tilde{P}_6, \tilde{0}) - a_7 d(\tilde{P}_7, \tilde{0}) - a_8 d(\tilde{P}_8, \tilde{0}).$$

Case 4: $M \leq T_d \leq T$

$$Z_4^*(T) \equiv d(g_4(\tilde{I}_c, \tilde{I}_d, \tilde{\theta}; T), \tilde{0}) = Z_3^*(T). \tag{59}$$

4. THE OPTIMAL SOLUTION

In this section our objective is to determine the optimal values of T which minimize the total relevant inventory cost per year $Z_1^*(T)$, $Z_2^*(T)$, $Z_3^*(T)$ and $Z_4^*(T)$ for cases 1-4, respectively.

Case 1: $0 < T < T_d$

Solving $\frac{\partial}{\partial T} Z_1^*(T) = 0$, we get T, and then obtain $Q = \frac{D}{\theta}(e^{\theta T} - 1)$. Furthermore, we need to check whether $0 < T < T_d$ holds. If it does, then (T, Q) is indeed the optimal solution to case 1. We let $(T^*, Q^*) = (T, Q)$, and hence obtain Z_1^* from Eq. (56). Otherwise, there is no feasible solution for case 1.

Case 2: $T_d \leq T < M$

Solving $\frac{\partial}{\partial T} Z_2^*(T) = 0$, we get T, and then obtain $Q = \frac{D}{\theta}(e^{\theta T} - 1)$. Furthermore, we need to check whether $T_d \leq T < M$ holds. If it does, then (T, Q) is indeed the optimal solution to case 2. We let $(T^*, Q^*) = (T, Q)$, and hence obtain Z_2^* from Eq. (57). Otherwise, there is no feasible solution for case 2.

Case 3: $T_d \leq M \leq T$

Solving $\frac{\partial}{\partial T} Z_3^*(T) = 0$, we get T, and then obtain $Q = \frac{D}{\theta}(e^{\theta T} - 1)$. Furthermore, we need to check whether $T_d \leq M \leq T$ holds. If it does, then (T, Q) is indeed the optimal solution to case 3. We let $(T^*, Q^*) = (T, Q)$, and hence obtain Z_3^* from Eq. (58). Otherwise, there is no feasible solution for case 3.

Case 4: $M \leq T_d \leq T$

Solving $\frac{\partial}{\partial T} Z_4^*(T) = 0$, we get T, and then obtain $Q = \frac{D}{\theta}(e^{\theta T} - 1)$. Furthermore, we need to check whether $M \leq T_d \leq T$ holds. If it does, then (T, Q) is indeed the optimal solution to case 4. We let $(T^*, Q^*) = (T, Q)$, and hence obtain Z_4^* from Eq. (59). Otherwise, there is no feasible solution for case 4.

5. NUMERICAL EXAMPLES

Several numerical examples are given to illustrate the above solution procedure. In order to compare the results with those obtained from the crisp case, we consider the data used in Chang *et al.* [1].

Example 1: Given $D = 1000$ units/year, $h = \$4$/unit/year, $I_c = 0.09/\$$/year, $I_d = 0.06/\$$/year, $c = \$20$ per unit, $p = \$30$ per unit, $\theta = 0.03$, $M = 30$ days (0.082192 years) and $Q_d = 70$ units. In addition, we let $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 0.0005$. We also consider $S = 10, 20$ and 30 per order and then get the computational results as shown in Table 1.

Table 1. The optimal solutions of example 1.

Ordering Cost S	Replenishment Cycle $T*$	Economic Order Quantity $Q(T*)$	Total Relevant Cost per Year $Z(T*)$	Increment $\delta(\%)$
10	0.055850	55.8966	$Z_1^*(T) = 437.042$	22.1237
20	0.078992	79.0861	$Z_2^*(T) = 411.392$	14.8610
30	0.093233	93.3630	$Z_3^*(T) = 528.699$	12.0313

Note: $\delta = \dfrac{\text{fuzzy annual total cost} - \text{crisp annual total cost}}{\text{crisp annual total cost}} \times 100\%$

Table 2. The optimal solutions of example 2.

Min Order Quantity Q_d	Replenishment Cycle $T*$	Economic Order Quantity $Q(T*)$	Total Relevant Cost per Year $Z(T*)$	Increment $\delta(\%)$
80	0.085490	85.5997	$Z_3^*(T) = 522.429$	12.3938
90	0.088741	88.8589	$Z_1^*(T) = 769.686$	–
100	0.088741	88.8589	$Z_1^*(T) = 769.686$	13.9258

Note: $\delta = \dfrac{\text{fuzzy annual total cost} - \text{crisp annual total cost}}{\text{crisp annual total cost}} \times 100\%$

Table 3. The optimal solutions of example 3.

Credit Period M	Replenishment Cycle $T*$	Economic Order Quantity $Q(T*)$	Total Relevant Cost per Year $Z(T*)$	Increment $\delta(\%)$
20	0.084933	85.0415	$Z_4^*(T) = 524.821$	13.5686
30	0.082572	82.6746	$Z_3^*(T) = 460.290$	13.2814
40	0.086315	86.4270	$Z_2^*(T) = 402.068$	15.2706

Note: $\delta = \dfrac{\text{fuzzy annual total cost} - \text{crisp annual total cost}}{\text{crisp annual total cost}} \times 100\%$

Example 2: Given $D = 1000$ units/year, $h = \$4$/unit/year, $I_c = 0.09/\$$/year, $I_d = 0.06/\$$/year, $c = \$30$ per unit, $p = \$40$ per unit, $\theta = 0.03$, $M = 30$ days (0.082192 years) and $S = 30$ per order. In addition, we let $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 0.0005$. If $Q_d = 80$, 90 or 100 units, then $T_d = 0.079904$, 0.089879 or 0.099850 year. We have the computational results as shown in Table 2.

Example 3: Given $D = 1000$ units/year, $h = \$4$/unit/year, $I_c = 0.09/\$$/year, $I_d = 0.06/\$$/year, $c = \$20$ per unit, $p = \$35$ per unit, $\theta = 0.03$, $S = 25$ per order and $Q_d = 80$ units. In addition, we let $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 0.0005$. Consequently, we obtain $T_d = 0.079904$ year. If $M = 20$, 30 or 40 days, then we can easily obtain the optimal solutions as shown in Table 3.

Example 4: In this example, we let $\Delta^* = \Delta_1 = \Delta_3 = \Delta_5$, $\Delta^{**} = \Delta_2 = \Delta_4 = \Delta_6$ and $S = 20$ and consider several values of different (Δ^*, Δ^{**}). The remaining are the same as the values in Example 1 and the optimal solutions are summarized in Table 4.

Table 4. The optimal solutions of example 4 for different values of (Δ^*, Δ^{**}).

Δ^*	Δ^{**}	Replenishment Cycle T^*	Economic Order Quantity $Q(T^*)$	Total Relevant Cost per Year $Z(T^*)$	Increment $\delta(\%)$
0.0002	0.0003	0.079000	79.0931	$Z_2^*(T) = 371.406$	3.69690
	0.0002	0.079009	79.1026	$Z_2^*(T) = 366.677$	2.37656
	0.0001	0.079018	79.1118	$Z_2^*(T) = 362.981$	1.34463
0.0001	0.00015	0.079007	79.1007	$Z_2^*(T) = 361.476$	0.92443
	0.0001	0.079011	79.1050	$Z_2^*(T) = 360.291$	0.59358
	0.00005	0.079016	79.1092	$Z_2^*(T) = 359.368$	0.33588
0.00005	0.000075	0.079010	79.1036	$Z_2^*(T) = 358.990$	0.23034
	0.00005	0.079012	79.1056	$Z_2^*(T) = 358.695$	0.14798
	0.000025	0.079014	79.1076	$Z_2^*(T) = 358.465$	0.08376
0.000025	0.000025	0.079012	79.1058	$Z_2^*(T) = 358.296$	0.03658
0.000015	0.000015	0.079012	79.1058	$Z_2^*(T) = 358.210$	0.01256
0.000005	0.000005	0.079012	79.1058	$Z_2^*(T) = 358.168$	0.000838
0.000001	0.000001	0.079012	79.1058	$Z_2^*(T) = 358.163$	$-$ 0.000560
0.0000005	0.0000005	0.079012	79.1058	$Z_2^*(T) = 358.163$	$-$ 0.000560

Note: $\delta = \dfrac{\text{fuzzy annual total cost} - \text{crisp annual total cost}}{\text{crisp annual total cost}} \times 100\%$

The computational results in Table 1 reveal that a higher value of ordering cost implies a lower value for the difference between the total relevant inventory cost per year in the crisp and fuzzy model, but higher values of order quantity and replenishment cycle. Table 2 shows that a higher value of the minimum order quantity at which the delay in payments is permitted causes higher values of the difference between the total relevant inventory cost per year in the crisp and fuzzy model and the total relevant inventory cost, order quantity and replenishment cycle. Table 3 reveals that a higher value of the trade credit period implies a lower value of the total relevant inventory cost. From Table 4, we find that when $\Delta^* = \Delta^{**} \to 0$, the total relevant inventory cost in the fuzzy sense is getting closer to the crisp total relevant inventory cost in Chang *et al*. [1]. This phenomenon is discussed in section 6.2.

6. DISCUSSION

6.1 The Reasoning Behind the Choice to Defuzzify the Fuzzy Total Relevant Inventory Cost in Cases 1-4 by Using the Signed Distance Method Instead of the Centroid Method

If we use the centroid method to defuzzify the fuzzy set $\tilde{A}$, we obtain $C(\tilde{A}) = \int_{-\infty}^{\infty} x \, \mu_{\tilde{A}}(x)dx \Big/ \int_{-\infty}^{\infty} \mu_{\tilde{A}}(x)dx$ which implies we have to find the membership function of fuzzy set $\tilde{A}$ first. However, in this paper, since the fuzzy sets of the fuzzy total relevant inventory cost in Eqs. (29), (31), (33), and (34) are obtained through complicated computational operators including $(+)$, $(-)$, $(\cdot)$, $(\div)$, thus it is difficult to find their membership function by using the extension principle. Therefore we apply the signed distance method to defuzzify the fuzzy total relevant inventory cost.

6.2 The Reasoning Behind the Choice to Apply the Interval Operations Instead of the Standard Fuzzy Arithmetic Operations

Multiple-occurrence of same fuzzy parameters in a fuzzy function will cause a fuzzy number been overestimated or illegal when the standard fuzzy arithmetic is applied to solve the problem (see Chang [32]). In this paper, instead of the standard fuzzy arithmetic operations, we applied the interval operations to deal with our model. The interval operations are adopted by many researchers to handle complicated computation between fuzzy numbers (see Chang [5], Ouyang *et al*. [18] and Yao [19]).

6.3 The Relationship between the Fuzzy Case and the Crisp Case

From Eq. (44), we obtain

$$K_1(0, -b, \theta, b; 0) = \frac{1}{b} \ln(\frac{\theta + b}{\theta}) \to \frac{1}{\theta} \text{ as } b \to 0, \text{ and for } n \geq 1,$$

$$K_1(0, -b, \theta, b; n) = \theta^n \frac{1}{b} \ln \left| \frac{\theta + b}{\theta} \right| + \sum_{r=1}^{n} \binom{n}{r} \theta^{n-r} (-1)^r \frac{(\theta + b)^r - \theta^r}{br}.$$

Due to $\lim_{b \to 0} \frac{1}{b} \ln \left| \frac{\theta + b}{\theta} \right| = \frac{1}{\theta}$ and $\lim_{b \to 0} \frac{(\theta + b)^r - \theta^r}{b r} = \theta^{r-1}$, we get

$$\lim_{b \to 0} K_1(0, -b, \theta, b; n) = \theta^{n-1} \sum_{r=0}^{n} \binom{n}{r} (-1)^r = \theta^{n-1} (1 - 1)^n = 0. \tag{60}$$

Similarly, from Eqs. (45) and (46), we have

$$\lim_{b \to 0} K_2(0, -b, \theta, b; n) = \theta^{n-2} \sum_{r=0}^{n} \binom{n}{r} (-1)^r = 0, \tag{61}$$

and

$$K_3(I_c, -b, 0, -b, \theta, b; 0) = K_2(I_c, -b, \theta, b; 1)$$
$$= \frac{1}{b} \left[(I_c + \theta) (\frac{1}{\theta} - \frac{1}{\theta + b}) - \ln \left| \frac{\theta + b}{\theta} \right| \right] \to (I_c + \theta) \frac{1}{\theta^2} - \frac{1}{\theta} = \frac{I_c}{\theta^2} \text{ as } b \to 0,$$

respectively.

Therefore, by Eqs. (60) and (61), we have

$$\lim_{b \to 0} K_3(I_c, -b, 0, -b, \theta, b; n)$$
$$= (I_c + \theta) \lim_{b \to 0} K_2(0, -b, \theta, b; n) - \lim_{b \to 0} K_1(0, -b, \theta, b; n) = 0, \text{ for } n \geq 1. \tag{62}$$

Furthermore, when $\Delta_2 = \Delta_1 = b \to 0$, from Eqs. (47) and (61), (48) and (60), (50) and (61), (51) and (60), we obtain

$$\lim_{b \to 0} d(\tilde{P}_1, \tilde{0}) = \frac{e^{\theta T}}{\theta^2},$$ (63)

$$\lim_{b \to 0} d(\tilde{P}_2, \tilde{0}) = \frac{e^{\theta T}}{\theta},$$ (64)

$$\lim_{b \to 0} d(\tilde{P}_4, \tilde{0}) = \frac{1}{\theta^2},$$ (65)

and

$$\lim_{b \to 0} d(\tilde{P}_5, \tilde{0}) = \frac{1}{\theta}.$$ (66)

When $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = b$, from Eqs. (49), (55) and (62), we get

$$\lim_{b \to 0} d(\tilde{P}_3, \tilde{0}) = \frac{I_c e^{\theta T}}{\theta^2},$$ (67)

and

$$\lim_{b \to 0} d(\tilde{P}_9, \tilde{0}) = \frac{I_c e^{\theta(T-M)}}{\theta^2}.$$ (68)

From Eqs. (44) and (45), we have

$$K_1(I_c, -b, \theta, b; 1) = \frac{1}{b}\left[(I_c + \theta)\ln\left|\frac{\theta + b}{\theta}\right| - b\right] \to \frac{(I_c + \theta)}{\theta} - 1 = \frac{I_c}{\theta} \text{ as } b \to 0$$ (69)

and

$$K_2(I_c, -b, \theta, b; 1)$$
$$= \frac{1}{b}\left[(I_c + \theta)\left[\frac{1}{\theta} - \frac{1}{\theta + b}\right] - \ln\left|\frac{\theta + b}{\theta}\right|\right] \to \frac{(I_c + \theta)}{\theta^2} - \frac{1}{\theta} = \frac{I_c}{\theta^2} \text{ as } b \to 0.$$ (70)

When $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = b$, from Eqs. (52), (53) and (69), (70), we obtain

$$\lim_{b \to 0} d(\tilde{P}_6, \tilde{0}) = \frac{I_c}{\theta^2}$$ (71)

and

$$\lim_{b \to 0} d(\tilde{P}_7, \tilde{0}) = \frac{I_c}{\theta}.$$ (72)

When $\Delta_5 = \Delta_6 = 0$, from Eq. (54), we get

$$d(\tilde{P}_8, \tilde{0}) = I_d.$$ (73)

Combing the Eqs. (63)-(68), (71)-(73), (56)-(59), (28), (30) and (32), we get the following result.

When $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = b \to 0$ and $\Delta_5 = \Delta_6 = 0$, we have

$$\lim_{b \to 0} Z_j^*(T) = Z_j(T), \ j = 1, 2, 3, 4.$$ (74)

That is, when $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = b \to 0$ and $\Delta_5 = \Delta_6 = 0$, the fuzzy case is the same as the crisp case. Thus, a crisp case is the special situation of a fuzzy case.

7. CONCLUSION

Due to the various uncertainties that may occur in the context of the real-world inventory problem, the rate of interest charges, the rate of interest earned, and the deterioration rate may not remain constant. To deal with these uncertainties, we apply the fuzzy set theory based on Chang *et al.*'s [1] model. We construct three different intervals to include the rate of interest charges, the rate of interest earned, and deterioration rate and subsequently derive the fuzzy total relevant inventory cost. By utilizing the signed distance method of defuzzification, we derive the estimate of the total relevant inventory cost in the fuzzy sense. We also discuss how to determine the optimal ordering policy so that the total relevant inventory cost in the fuzzy sense is minimal. The optimal ordering policy is determined in a fuzzy environment which provides the decision maker with a deeper insight into the problem. Finally, some numerical examples are provided to illustrate the solution procedure. Table 4 shows that as the interval variation of the rate of interest charges, the rate of interest earned and the deterioration rate becomes sufficiently small, the fuzzy total relevant inventory cost per year becomes close to the crisp total relevant inventory cost per year. That is, the crisp model can be viewed as a special case of the fuzzy model. Future research on this problem may include additional sources of uncertainty in the fuzzy models, such as an uncertain demand rate.

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