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The optimal ordering policy with trade credit under two different payment methods

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Abstract Suppliers' offering delay payment terms to retailers can be regarded as a type of price reduction. In today's ever competitive marketplace, offering delay payments has become a commonly adopted method to suppliers. Most of the inventory models with permissible delay in payments assumed that the entire lot size is delivered at the same time. However, in practice, goods ordered are usually arrived overtime in separate batches. In this study, we discuss an inventory problem with a finite replenishment rate under trade credit for two payment methods. We establish a theorem to find the optimal solution for each payment method. Numerical examples are also given to illustrate the solution procedure. Finally, to investigate the effect of changes of some main parameter values on the optimal solution, a sensitivity analysis is performed and some management interpretations are proposed.

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1 Introduction

In the real market, a supplier usually permits the retailer a delay of a fixed time period to settle the total amount owed to him. Usually, interest is not charged for the outstanding amount if it is paid within the permissible delay period. The permissible delay in payments produces benefits to the supplier. For example, it will attract some customers who consider it to be a type of price reduction and does not provoke competitors to reduce their prices and thus introduce lasting price reductions. Permissible delay in payments also provides advantages to the retailer due to the fact that the retailer can earn interest on the accumulated revenue received and delay the payment up to the last moment of the permissible period allowed by the supplier. However, if the payment is not paid within the permissible delay period, then interest is charged on the outstanding amount.

Goyal (1985) first developed an EOQ model under conditions of permissible delay in payments. Dave (1985) generalized Goyal's (1985) model by assuming that the selling price is necessarily higher than its purchase price. Aggarwal and Jaggi (1995) extended Goyal's (1985) model for deteriorating items. Jamal et al. (1997) further extended the model to allow shortages. Hwang and Shinn (1997) added the pricing strategy into the model and determined the optimal pricing and lot sizing for the retailer under the condition of permissible delay in payments. Sarker et al. (2000) presented an inventory model with deteriorating items for optimal cycle and payment times for a retailer when a supplier allows a specified credit period to the retailer for payment. Teng (2002) amended Goyal's (1985) model by considering the difference between the unit price and unit cost, and found that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. There are many interesting and relevant articles related to trade credit, such as Davis and Gaither (1985), Mandal and Phaujdar (1989), Shah (1993a, 1993b), Liao et al. (2000), Chang et al. (2003), Chang and Teng (2004), Chung and Liao (2004), Huang (2003), Ouyang et al. (2005, 2006), Teng et al. (2005), Goyal et al. (2007), and their references.

All of the above papers develop inventory models assuming that the entire order size is delivered at the same time (i.e., infinite replenishment rate). However, in practice, the order quantity is frequently received gradually over time and the inventory level is depleted at the same time it is being replenished (finite replenishment rate). The concept of "finite replenishment rate" can be observed in Stevenson (1996, p. 542) and Taylor (1999, p. 786). In extant literature of delay payment, few studies have explored conditions under finite replenishment. Chung and Huang (2003) extended Goyal's (1985) model to the case that the units are replenished at a finite rate. They calculated interest income based on the purchasing price. Huang (2004) established an inventory model with a finite replenishment rate under the supplier's trade credit policy. He assumed that at the end of the permissible period, the retailer only pay the supplier on goods already sold. The balance shall be paid through bank loans, and the retailer only payback bank loans at the end of the business cycle. In Chung and Huang's (2003) and Huang's (2004) models, they assumed that the retailer pays interest rates larger than the interest rate it can earn.

From the above we see that considering inventory problem with trade credit, extant scholars have fully discussed the problem with infinite replenishment rate. However,

few scholars have explored the problem with finite replenishment rate. Among the few researches that do pay attention to the finite replenishment rate, they only consider the situation that the retailer pays interest rates larger than the interest rate it can earn. However, in practice, retailers may invest what they have earned from sales revenue to more profitable channels and make profits on their investments at rates higher than the interest rates they pay. To more closely fit with real-world practice, this study develops the inventory model with trade credit and finite replenishment rate and relaxes the assumption that interest paid is always higher than interest earned. We calculate interest earned based on the sales revenue within permissible period. At the end of the trade period, we consider two different payment methods for the retailer to pay off the loan. One is that the retailer pays off all units sold and keeps the profits for other uses. The other payment method is that the retailer pays off the amount owed to the supplier whenever the supplier has money obtained from sales. We further develop separate theorems to find the optimal solution for the two payment methods and discuss some special cases while providing numerical examples to illustrate the theoretical results. To investigate the effect of changes in some main parameter values on the optimal solution, we also conduct a sensitivity analysis and discuss management implications.

2 Notation and assumptions

The mathematical model in this paper is developed on the basis of the following notation and assumptions.

Notation:

D	The annual demand rate.
P	The annual replenishment rate, $P > D$.
ρ	$= 1 - D/P > 0$.
A	The ordering cost per order.
c	The unit purchase cost.
s	The selling price per unit, $s > c$.
h	The annual inventory holding cost per unit excluding interest charges.
I_c	The annual interest charged per \$ in stocks by the supplier or the bank.
I_e	The annual interest earned or return on investment per \$ for the retailer.
M	The permissible delay period in years.
T	The replenishment cycle time in years, which is a decision variable.
T^*	The optimal replenishment cycle time.
$TRC(T)$	The annual total relevant cost.
TRC^*	The minimum annual total relevant cost, i.e., $TRC^* = TRC(T^*)$.

Assumptions:

(1) Demand rate, D, is known and constant.
(2) Replenishment rate, P, is known and constant.
(3) Shortages are not allowed.
(4) Time horizon is infinite.

(5) The retailer would not consider paying the payment until receiving all items, i.e., $M > DT/P$.

(6) During the credit period, the retailer sells the items and uses the sales revenue to earn interest. At the end of this period, some retailers will keep their profits for emergency or other use rather than paying off the loan while some retailers will pay off the amount owed to the supplier whenever they have money obtained from sales. That is, the retailer has two possible methods to pay off the loan based on his/her need. One is that the retailer keeps his/her profits for other activities rather than paying off the loan. The other is that the retailer pays off the amount owed to the supplier whenever he/she has money obtained from sales. In this paper, we provide two possible payment methods. In different circumstances, the retailer can choose the payment methods he/she needs.

3 Mathematical model and solution procedure

In this paper, we consider two possibilities in the real market for the retailer to pay off the total amount owed to the supplier at the end of the trade credit period. One is that some retailers may keep their profits for emergency or other uses rather than paying off the loan. The other is that some retailers may pay off the amount owed to the supplier whenever they have money obtained from sales. In this section, we develop appropriate inventory models for both Payment Methods 1 and 2 respectively. We then present the solution procedure and establish a theorem to find the optimal solution for each payment method.

The total relevant cost per cycle consists of the following four elements:

(a) Cost of placing orders, which is A (1)
(b) Cost of carrying inventory, which is $DT^2 h\rho/2$ (2)
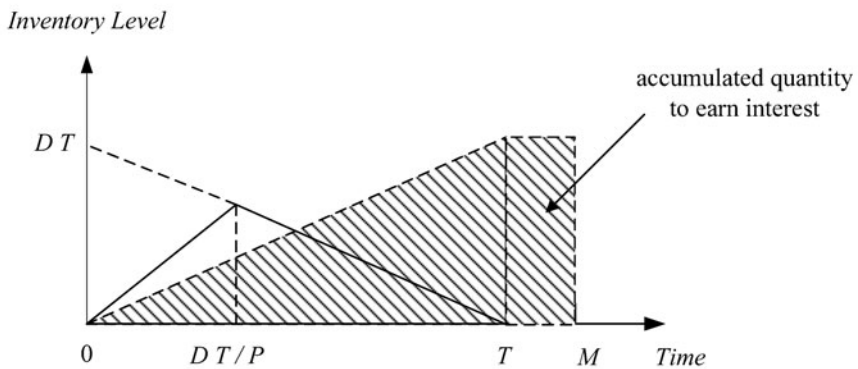(c) Interest payable, and
(d) Interest earned

Payment Method 1. The retailer keeps profits for other use rather than paying off the loan

In this payment method, at the end of the credit period, the retailer pays off all units sold, keeps the profits for other activities use, and starts paying interest charges on the unpaid balance. As the payment is made before or after the total depletion of inventory, considering the interest payable and interest earned, we have the following cases: (i) $T \leq M$ and (ii) $T \geq M$. These two cases are illustrated in Fig. 1.
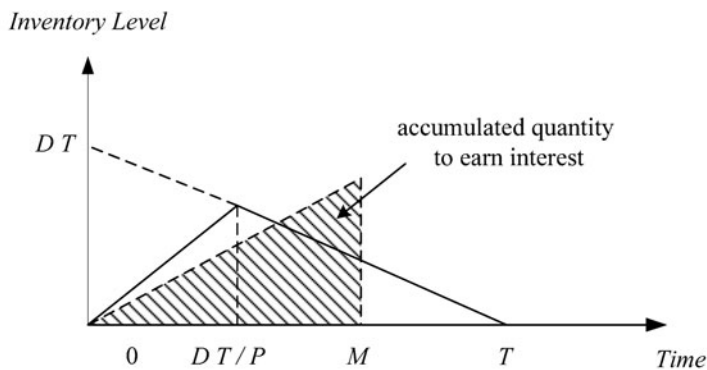
Case 1.1: $T \leq M$.

In this case, the permissible payment time expires on or after the inventory is completely depleted. The retailer pays no interest charges for the purchased items. Moreover, during the credit period, the retailer sells the products and uses the sales revenue to earn interest at a rate of I_e. Thus, the interest earned per cycle is $s I_e [DT^2/2 + DT(M - T)] = s I_e DT(M - T/2)$.

Case 1. $T \leq M$

Case 2. $T \geq M$

Fig. 1 The retailer's inventory level and cumulative quantity to earn interest

Case 1.2: $T \geq M$.

In this case, the permissible payment time expires on or before the inventory is completely depleted. The interest payable per cycle is $cI_c D(T - M)^2/2$. In addition, by selling the items and investing at a rate I_e, until the end of the credit period M, the retailer can receive the interest earned per cycle as $sI_e DM^2/2$.

Therefore, the annual total relevant cost for the Payment Method 1 is as follows:

$$TRC(T) = (\text{ordering cost} + \text{carrying cost} + \text{interest charges} - \text{interest earned})/T$$

$$= \begin{cases} TRC_{1-1}(T) & \text{if } T \leq M, \\ TRC_{1-2}(T) & \text{if } T \geq M, \end{cases}$$

where

$$TRC_{1-1}(T) = \frac{A}{T} + \frac{DTh\rho}{2} - sI_e D\left(M - \frac{T}{2}\right) \quad \text{and} \tag{3}$$

$$TRC_{1-2}(T) = \frac{A}{T} + \frac{DTh\rho}{2} + \frac{cI_c D(T-M)^2}{2T} - \frac{sI_e DM^2}{2T}. \tag{4}$$

Since $TRC_{1-1}(M) = TRC_{1-2}(M)$, $TRC(T)$ is continuous in $T \in (0, \infty)$.

Our problem is to determine the retailer's optimal replenishment cycle time T^* which minimizes the annual total relevant cost $TRC(T)$.

Now, taking the first-order and second-order derivatives of $TRC_{1-1}(T)$ in (3) with respect to T, we obtain

$$\frac{dTRC_{1-1}(T)}{dT} = \frac{-A}{T^2} + \frac{Dh\rho}{2} + \frac{DsI_e}{2} \tag{5}$$

and

$$\frac{d^2 TRC_{1-1}(T)}{dT^2} = \frac{2A}{T^3}. \tag{6}$$

Due to $\frac{d^2 TRC_{1-1}(T)}{dT^2} > 0$, the optimal value of T (denoted by T_{1-1}) which minimizes $TRC_{1-1}(T)$ can be found by solving the equation $\frac{dTRC_{1-1}(T)}{dT} = 0$. We have

$$T_{1-1} = \sqrt{\frac{2A}{D(h\rho + sI_e)}}. \tag{7}$$

To ensure $T_{1-1} \leq M$ (i.e., Case 1.1), we substitute (7) into this inequality and get

$$\text{if } \Delta_1 \equiv DM^2(h\rho + sI_e) - 2A \geq 0, \quad \text{then } T_{1-1} \leq M. \tag{8}$$

The corresponding minimum annual total relevant cost can be obtained as

$$TRC_{1-1}(T_{1-1}) = \sqrt{2AD(h\rho + sI_e)} - sI_e DM. \tag{9}$$

Next, taking the first-order and second-order derivatives of $TRC_{1-2}(T)$ in (4) with respect to T, we obtain

$$\frac{dTRC_{1-2}(T)}{dT} = \frac{-A}{T^2} + \frac{Dh\rho}{2} + \frac{DcI_c}{2} - \frac{DM^2(cI_c - sI_e)}{2T^2} \tag{10}$$

and

$$\frac{d^2 TRC_{1-2}(T)}{dT^2} = \frac{2A}{T^3} + \frac{DM^2(cI_c - sI_e)}{T^3}. \tag{11}$$

Letting $\frac{dTRC_{1-2}(T)}{dT} = 0$ and solving this equation, we find the value of T (denoted by T_{1-2})

$$T_{1-2} = \sqrt{\frac{2A + DM^2(cI_c - sI_e)}{D(h\rho + cI_c)}}. \tag{12}$$

To ensure that the condition $T_{1-2} \geq M$ holds (i.e., Case 1.2), we substitute (12) into this inequality and obtain that

$$\text{if } \Delta_1 \leq 0, \quad \text{then } T_{1-2} \geq M, \quad \text{where } \Delta_1 \text{ is defined as above.} \tag{13}$$

Note that when $\Delta_1 \leq 0$, it can be shown that $2A + DM^2(cI_c - sI_e) \geq DM^2(h\rho + cI_c) > 0$. Hence, T_{1-2} in (12) is well defined, and $\frac{d^2TRC_{1-2}(T)}{dT^2} > 0$. Therefore, T_{1-2} is a unique optimal solution which minimizes $TRC_{1-2}(T)$, and the corresponding minimum annual total relevant cost is

$$TRC_{1-2}(T_{1-2}) = \sqrt{D(h\rho + cI_c)\left[2A + DM^2(cI_c - sI_e)\right]} - cI_c DM. \tag{14}$$

Summarizing the above arguments, we obtain the following theorem to get the explicitly optimal policy for Payment Method 1. We can find that Theorem 1 is a general form of the corresponding Theorem 3 in Chung and Huang (2003), in which it requires $I_c \geq I_e$ and $s = c$.

Theorem 1

(a) *If $\Delta_1 \geq 0$, then $T^* = T_{1-1}$ and $TRC^* = TRC_{1-1}(T_{1-1})$.*
(b) *If $\Delta_1 \leq 0$, then $T^* = T_{1-2}$ and $TRC^* = TRC_{1-2}(T_{1-2})$.*

Proof It immediately follows from (8) and (13). □

Once the optimal replenishment cycle time is obtained, the optimal order quantity per cycle $Q^* = DT^*$ follows.

Payment Method 2. The retailer pays off loan whenever he/she has money

In this payment method, at the end of the credit period, if the amount of revenue and interest earned is more than or equal to the purchase cost, the retailer pays off the amount owed to the supplier. Otherwise, the retailer pays the supplier the amount of revenue and interest earned and finances the difference. Thereafter, the retailer gradually reduces the financed loan from constant sales and revenue received. Similar to the previous discussion, according to the values of T and M, we have the following two possible cases: (i) $T \leq M$ and (ii) $T \geq M$.

Case 2.1: $T \leq M$.

This case is the same as Case 1.1. The retailer pays no interest charges, while the interest earned per cycle is $sI_e DT(M - T/2)$.

Case 2.2: $T \geq M$.

During $[0, M]$ period, the retailer sells DM units and receives sDM dollars. In addition, during this period, the interest earned is $sI_e DM^2/2$. Hence, the retailer has $sDM + sI_e DM^2/2$ dollars at time M. Since the retailer buys DT units at time 0, the retailer owes the supplier cDT dollars at time M. From the difference between the purchase cost and the amount the retailer has at time M, we have the following

two sub-cases to calculate interest charges: (i) $sDM + sI_e DM^2/2 \geq cDT$, which means the retailer can pay off the total purchase cost to the supplier at time M, and (ii) $sDM + sI_e DM^2/2 \leq cDT$, which means the retailer may not pay off the unpaid balance at time M.

Sub-case 2.2.1: $sDM + sI_e DM^2/2 \geq cDT$ and $T \geq M$ (i.e., $M \leq T \leq sM/c + sI_e M^2/(2c)$).

If the money the retailer has $sDM + sI_e DM^2/2$ at time M is greater than or equal to the purchase cost cDT, then there is no interest payable.

Sub-case 2.2.2: $sDM + sI_e DM^2/2 \leq cDT$ and $T \geq M$ (i.e., $T \geq sM/c + sI_e M^2/(2c)$).

If the money $sDM + sI_e DM^2/2$ is less than the purchase cost cDT, then the retailer needs to finance the difference $L = cDT - (sDM + sI_e DM^2/2)$ at time M. Thereafter, the retailer gradually reduces the financed loan from constant sales and revenue received. Hence, the interest payable per cycle is $I_c L[L/(sD)]/2 = \frac{I_c}{2sD}(cDT - sDM - \frac{sI_e DM^2}{2})^2$.

Therefore, the annual total relevant cost for the Payment Method 2 is as follows:

$$TRC(T) = \begin{cases} TRC_2(T) & \text{if } T \leq M, \\ TRC_{2-1}(T) & \text{if } M \leq T \leq sM/c + sI_e M^2/(2c), \\ TRC_{2-2}(T) & \text{if } T \geq sM/c + sI_e M^2/(2c), \end{cases}$$

where

$$TRC_2(T) = \frac{A}{T} + \frac{DTh\rho}{2} - sI_e D\left(M - \frac{T}{2}\right), \tag{15}$$

$$TRC_{2-1}(T) = \frac{A}{T} + \frac{DTh\rho}{2} - \frac{sI_e DM^2}{2T}, \tag{16}$$

$$TRC_{2-2}(T) = \frac{A}{T} + \frac{DTh\rho}{2}$$
$$+ \frac{I_c}{2sDT}\left(cDT - sDM - \frac{sI_e DM^2}{2}\right)^2 - \frac{sI_e DM^2}{2T}. \tag{17}$$

Since $TRC_2(M) = TRC_{2-1}(M)$ and $TRC_{2-1}(sM/c + sI_e M^2/(2c)) = TRC_{2-2}(sM/c + sI_e M^2/(2c))$, $TRC(T)$ is continuous in $T \in (0, \infty)$.

Our problem is to find the retailer's optimal replenishment cycle time T^* which minimizes the annual total relevant cost $TRC(T)$. As the annual total relevant cost $TRC_2(T)$ in (15) is the same as $TRC_{1-1}(T)$ in (3), the optimal value of T (denoted by T_2) which minimizes $TRC_2(T)$ can be obtained by the same approach as illustrated above and is given by

$$T_2 = \sqrt{\frac{2A}{D(h\rho + sI_e)}}. \tag{18}$$

Furthermore, we have the following result:

$$\text{if } \Delta_1 \geq 0, \quad \text{then } T_2 \leq M, \quad \text{where } \Delta_1 \text{ is defined as above.} \tag{19}$$

The corresponding minimum annual total relevant cost can be obtained as

$$TRC_2(T_2) = \sqrt{2AD(h\rho + sI_e)} - sI_e DM. \tag{20}$$

Next, taking the first-order and second-order derivatives of $TRC_{2-1}(T)$ in (16) and $TRC_{2-2}(T)$ in (17) with respect to T, we have

$$\frac{dTRC_{2-1}(T)}{dT} = \frac{-A}{T^2} + \frac{Dh\rho}{2} + \frac{DM^2 sI_e}{2T^2}, \tag{21}$$

$$\frac{dTRC_{2-2}(T)}{dT} = \frac{-A}{T^2} + \frac{Dh\rho}{2} + \frac{DI_c c^2}{2s} - \frac{DM^2 s}{2T^2}\left[I_c\left(1 + \frac{I_e M}{2}\right)^2 - I_e\right], \tag{22}$$

$$\frac{d^2 TRC_{2-1}(T)}{dT^2} = \frac{2A}{T^3} - \frac{DM^2 sI_e}{T^3}, \tag{23}$$

and

$$\frac{d^2 TRC_{2-2}(T)}{dT^2} = \frac{2A}{T^3} + \frac{DM^2 s}{T^3}\left[I_c\left(1 + \frac{I_e M}{2}\right)^2 - I_e\right]. \tag{24}$$

Letting $\frac{dTRC_{2-1}(T)}{dT} = 0$ and solving this equation, we find the value of T (denoted by T_{2-1}) as

$$T_{2-1} = \sqrt{\frac{2A - DM^2 sI_e}{Dh\rho}}. \tag{25}$$

To ensure $M \leq T_{2-1} \leq sM/c + sI_e M^2/(2c)$ (i.e., Sub-case 2.2.1), we substitute (25) into this inequality and get

$$\text{if } \Delta_1 \leq 0 \leq \Delta_2, \quad \text{then } M \leq T_{2-1} \leq sM/c + sI_e M^2/(2c),$$

$$\text{where } \Delta_1 \text{ is defined as above and } \Delta_2 \equiv DM^2 s\left[\frac{hs\rho}{c^2}\left(1 + \frac{I_e M}{2}\right)^2 + I_e\right] - 2A.$$

$$\tag{26}$$

Note that when $\Delta_1 \leq 0$, it can be shown that $2A - DM^2 sI_e \geq DM^2 h\rho > 0$, which implies T_{2-1} in (25) is well defined and $\frac{d^2 TRC_{2-1}(T)}{dT^2} > 0$. Therefore, T_{2-1} is a unique optimal solution which minimizes $TRC_{2-1}(T)$. In this situation, the minimum annual total relevant cost is

$$TRC_{2-1}(T_{2-1}) = \sqrt{Dh\rho\left(2A - sI_e DM^2\right)}. \tag{27}$$

Likewise, letting $\frac{dTRC_{2-2}(T)}{dT} = 0$ and solving this equation, we find the value of T (denoted by T_{2-2}) as

$$T_{2-2} = \sqrt{\frac{s\{2A + DM^2s[I_c(1 + \frac{I_eM}{2})^2 - I_e]\}}{DI_cc^2 + Dhs\rho}}. \tag{28}$$

To ensure $T_{2-2} \geq sM/c + sI_eM^2/(2c)$ (i.e., Sub-case 2.2.2), we substitute (28) into this inequality and get

if $\Delta_2 \leq 0$,   then $T_{2-2} \geq sM/c + sI_eM^2/(2c)$,

where Δ_2 is defined as above. (29)

It is noted that when $\Delta_2 \leq 0$, it can be shown that $2A + DM^2s[I_c(1 + \frac{I_eM}{2})^2 - I_e] > 0$ (for the proof, see Appendix), which implies T_{2-2} in (28) is well defined and $\frac{d^2TRC_{2-2}(T)}{dT^2} > 0$. Therefore, T_{2-2} is a unique optimal solution which minimizes $TRC_{2-2}(T)$. In this situation, the minimum annual total relevant cost is

$$TRC_{2-2}(T_{2-2}) = \sqrt{D\left(h\rho + \frac{c^2I_c}{s}\right)\left\{2A + DM^2s\left[I_c\left(1 + \frac{I_eM}{2}\right)^2 - I_e\right]\right\}}$$
$$- cI_cDM\left(1 + \frac{I_eM}{2}\right). \tag{30}$$

From the above arguments we can obtain the following result to get the explicitly optimal policy for Payment Method 2.

Theorem 2

(a) *If $\Delta_1 \geq 0$, then $T^* = T_2$ and $TRC^* = TRC_2(T_2)$.*
(b) *If $\Delta_1 \leq 0 \leq \Delta_2$, then $T^* = T_{2-1}$ and $TRC^* = TRC_{2-1}(T_{2-1})$.*
(c) *If $\Delta_2 \leq 0$, then $T^* = T_{2-2}$ and $TRC^* = TRC_{2-2}(T_{2-2})$.*

Proof It immediately follows from (19), (26), and (29).   □

Once the optimal replenishment cycle time T^* is obtained, the optimal order quantity per cycle $Q^* = DT^*$ follows.

Remark For $T \leq M$, from (3) and (15) we have $TRC_{1-1}(T) = TRC_2(T)$. On the other hand, for $T \geq M$, from (4), (16), and (17) we have

$$TRC_{1-2}(T) - TRC_{2-1}(T) = \frac{cI_cD(T - M)^2}{2T} > 0$$

and

$$TRC_{1-2}(T) - TRC_{2-2}(T)$$

$$= \frac{cI_c D(T-M)^2}{2T} - \frac{I_c}{2sDT}\left(cDT - sDM - \frac{sI_e DM^2}{2}\right)^2$$

$$= \frac{cI_c D(T-M)^2}{2T} - \frac{c^2 I_c D}{2sT}\left(T - \frac{sM}{c} - \frac{sI_e M^2}{2c}\right)^2$$

$$> \frac{cI_c D(T-M)^2}{2T} - \frac{cI_c D}{2T}\left(T - M - \frac{sI_e M^2}{2c}\right)^2 \quad \text{(because } s > c\text{)}$$

$$= \frac{cI_c D}{2T}\left[(T-M)^2\right.$$

$$\left. - \left(T - M - \frac{sI_e M^2}{2c}\right)^2\right] > 0 \quad \left(\text{because } T \geq M + \frac{sI_e M^2}{2c}\right).$$

Therefore, the annual total relevant cost in Payment Method 2 is always lower than or equal to that in Payment Method 1. However, if the retailer needs to keep some money for emergency or other use, he/she will choose Payment Method 1. For example, some retailers will keep some money for fragmentary purchase and other fragmentary disbursement.

4 Special cases

In this section, we present three special cases: (I) $P \to \infty$ (i.e., EOQ model with permissible delay in payments), (II) $M = 0$ (i.e., EPQ model whereby the supplier must be paid for the items at the starting point of each cycle), and (III) $P \to \infty$ and $M = 0$ (i.e., EOQ model and the supplier must be paid for the items as soon as the retailer receives them).

For simplicity, we will discuss only the case in which the retailer keeps profits for other use rather than paying off the loan (i.e., Payment Method 1). The reader can obtain similar results for the other case (i.e., Payment Method 2).

Case I. EOQ model with permissible delay in payments.

If $P \to \infty$, then (3) and (4) are reduced as follows:

$$TRC_{1-1}(T) = \frac{A}{T} + \frac{DTh}{2} - sI_e D\left(M - \frac{T}{2}\right), \quad T \leq M, \quad (31)$$

$$TRC_{1-2}(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_c D(T-M)^2}{2T} - \frac{sI_e DM^2}{2T}, \quad T \geq M. \quad (32)$$

The corresponding optimal replenishment cycle time and the minimum annual total relevant cost are as follows:

$$T_{1-1} = \sqrt{\frac{2A}{D(h + sI_e)}}, \quad (33)$$

$$TRC_{1-1}(T_{1-1}) = \sqrt{2AD(h + sI_e)} - sI_eDM, \tag{34}$$

$$T_{1-2} = \sqrt{\frac{2A + DM^2(cI_c - sI_e)}{D(h + cI_c)}}, \tag{35}$$

and

$$TRC_{1-2}(T_{1-2}) = \sqrt{D(h + cI_c)[2A + DM^2(cI_c - sI_e)]} - cI_cDM. \tag{36}$$

Note that, if $s = c$, then (31) and (32) are consistent with (4) and (1) in Goyal (1985), respectively. That is, Goyal's (1985) model is a special case of our model.

Case II. EPQ model whereby the supplier must be paid for the items at the starting point of each cycle.

If $M = 0$, then (4) is reduced to

$$TRC_{1-2}(T) = \frac{A}{T} + \frac{DT(h\rho + cI_c)}{2}, \quad T > 0. \tag{37}$$

Consequently, we obtain the corresponding optimal replenishment cycle time and the minimum annual total relevant cost as follows:

$$T_{1-2} = \sqrt{\frac{2A}{D(h\rho + cI_c)}} \tag{38}$$

and

$$TRC_{1-2}(T_{1-2}) = \sqrt{2AD(h\rho + cI_c)}. \tag{39}$$

From above we know that (37) is the classical EPQ model.

Case III. EOQ model whereby the supplier must be paid for the items as soon as the retailer receives them.

If $P \to \infty$ and $M = 0$, then (4) is reduced to

$$TRC_{1-2}(T) = \frac{A}{T} + \frac{DT(h + cI_c)}{2}, \quad T > 0. \tag{40}$$

Thus, we obtain the corresponding optimal replenishment cycle time and the minimum annual total relevant cost as follows:

$$T_{1-2} = \sqrt{\frac{2A}{D(h + cI_c)}} \tag{41}$$

and

$$TRC_{1-2}(T_{1-2}) = \sqrt{2AD(h + cI_c)}. \tag{42}$$

It is noted that (40) is the classical EOQ model.

Table 1 The sensitivity analysis of parameters A and h

Parameter	Tendency	Payment Methods 1 and 2	
		T^*	TRC^*
A	↗	↗	↗
h	↗	↘	↗

Note: ↗ denote increasing tendency; ↘ denote decreasing tendency

5 Numerical examples

In order to illustrate the solution procedure and investigate the effect of changes in some main parameter values on the optimal solution in our models, let us consider the following examples. We applied the data in Teng (2002). Given $s = \$1/\text{unit}$, $c = \$0.50/\text{unit}$, $D = 3{,}600$ units/year, $h = \$0.5/\text{unit/year}$. Besides, we let $A = \$20/\text{order}$, $M = 1/6$ year (60 days), $I_c = 0.04$, $I_e = 0.1$, and $P = 4{,}000$ units.

Example 1 For a retailer who will keep his/her profits for emergency or other use rather than paying off the loan, he/she will adopt Payment Method 1. Since $\Delta_1 = -24.994 < 0$, we know from Theorem 1(b) that the optimal solution is $T^* = T_{1-2}$ and $TRC^* = TRC_{1-2}(T_{1-2})$. Therefore, we obtain $T^* = 0.3563$ from (12), $TRC^* = 77.7929$ from (14), and $Q^* = DT^* = 1282.79$.

Example 2 For a retailer who will pay off the amount owed to the supplier whenever he/she has money obtained from sales, he/she will adopt Payment Method 2. Since $\Delta_2 = -9.6531 < 0$, we know from Theorem 2(c) that the optimal solution is $T^* = T_{2-2}$ and $TRC^* = TRC_{2-2}(T_{2-2})$. Therefore, we obtain $T^* = 0.3971$ from (28), $TRC^* = 73.6761$ from (30), and $Q^* = DT^* = 1429.64$.

Example 3 In this example, we first discuss the effects of changing the values of the parameters, the ordering cost A and the holding cost h, on the optimal replenishment cycle time T^* and the minimum annual total relevant cost TRC^*. By observing the solutions of the replenishment cycle time in (7), (12), (18), (25), and (28) and the annual total relevant costs in (9), (14), (20), (27), and (30), we can find that a higher value of the ordering cost A results in higher values for the replenishment cycle time T^* and the annual total relevant cost TRC^*. This means that the retailer needs to increase the replenishment cycle time (i.e., increase the order quantity) to reduce the number of orders if the ordering cost is more expensive. Also, the positive change in the holding cost h causes a negative change in the replenishment cycle time T^* and a positive change in the annual total relevant cost TRC^*, which implies that the retailers will reduce their replenishment cycle time (i.e., reduce their order quantity) due to the higher holding cost. We summarize the results and present the effects in Table 1.

Furthermore, we study the effects of the changes in the parameter values I_c and I_e on the optimal solutions for Payment Methods 1 and 2. The sensitivity analysis is performed by considering $I_c \in \{0.06, 0.11, 0.16, 0.18\}$ and $I_e \in \{0.05, 0.10, 0.15, 0.17\}$, taking one parameter at a time and keeping the remaining parameters unchanged. From Theorems 1 and 2, the computational results for the two different payment methods are shown in Table 2.

Table 2 The optimal solutions for different interest rates I_c and I_e

I_c	I_e	Payment Method 1			Payment Method 2		
		T^*	Q^*	TRC^*	T^*	Q^*	TRC^*
0.06	0.05	0.3632	1307.66	86.6089	0.4188	1507.84	79.9307
	0.10	0.3385	1218.55	79.4807	0.3928	1413.97	73.7546
	0.15	0.3118	1122.40	71.7885	0.3648	1313.43	67.1440
	0.17	0.3004	1081.55	68.5203	0.3531	1270.98	64.3552
0.11	0.05	0.3273	1178.38	90.7232	0.4065	1463.27	80.2592
	0.10	0.3064	1103.21	82.8039	0.3842	1383.15	73.9121
	0.15	0.2840	1022.54	74.3599	0.3606	1298.09	67.1829
	0.17	0.2746	988.43	70.7781	0.3507	1262.47	64.3672
0.16	0.05	0.3031	1091.24	93.8515	0.3973	1430.22	80.5099
	0.10	0.2849	1025.81	85.3457	0.3779	1360.45	74.0304
	0.15	0.2655	955.91	76.2590	0.3575	1286.91	67.2117
	0.17	0.2574	926.48	72.4326	0.3490	1256.29	64.3759
0.18	0.05	0.2955	1063.71	94.9080	0.3942	1419.27	80.5944
	0.10	0.2782	1001.42	86.1883	0.3758	1352.96	74.0699
	0.15	0.2597	935.00	76.8891	0.3565	1283.23	67.2211
	0.17	0.2520	907.07	72.9789	0.3484	1254.26	64.3788

The computational results demonstrate that a higher value of interest charges I_c results in lower values for the optimal replenishment cycle time T^* and the optimal economic order quantity Q^*, but a higher value for the optimal annual total relevant cost TRC^*. Moreover, a higher value of interest earned I_e results in lower values for the optimal replenishment cycle time T^*, the optimal economic order quantity Q^*, and the optimal annual total relevant cost TRC^*. Consequently, a higher value of I_c implies a higher value of the cost, and a higher value of I_e implies a higher value of the benefit from the permissible delay. A simple management interpretation is that the retailer should order less quantity and take the benefits of the permissible delay more frequently.

6 Conclusions

Most of the inventory models with trade credit assumed that the entire lot size is delivered at the same time. However, in the real marketplace, it is common that goods ordered are arrived overtime in separate batches. Up to now, few scholars have explored the inventory problem with trade credit and finite replenishment rate. Among the few researches that do pay attention to the finite replenishment rate, they only consider the situation that the retailer pays interest rates larger than the interest rate it can earn. Nevertheless, in their day-to-day business, retailers may reinvest the funds they obtained from sales on other businesses and earn profits at rates higher than the interest rates payable. To more closely fit with real-world practice, in this paper, we incorporate the situation that the replenishment rate is finite and that interest earned

may be greater than interest paid. We develop an inventory model with trade credit and finite replenishment rate for two payment methods. Theorems 1 and 2 are established to provide the retailer with an explicit closed-form solution to determine the optimal replenishment cycle time and the minimum annual total relevant cost for each payment method, respectively. Our model is created in a general framework that includes Chung and Huang (2003), Goyal (1985), the classical EPQ model, and the classical EOQ model as special cases.

Moreover, numerical examples are given to illustrate our solution procedure. We then conducted a sensitivity analysis of some main parameter values where we find that the retailer needs to increase the replenishment cycle time so as to reduce the number of orders if the ordering cost is more expensive. On the other hand, retailers tend to reduce the replenishment cycle time due to the higher holding cost. In addition, a higher value of the interest rate charged by the supplier implies lower values for the optimal replenishment cycle time and the optimal economic order quantity, but a higher value for the optimal annual total relevant cost. A higher value of the retailer's return rate on investment implies lower values for the optimal replenishment cycle time, the optimal economic order quantity, and the optimal annual total relevant cost. Consequently, the retailer should order less quantity and take the benefits of the permissible delay more frequently.

In future research our model can be extended in several ways. For instance, we will extend the constant demand rate to a varying demand rate such as a function of time, selling price, or inventory level. Also, we may generalize the model to allow for shortages, quantity discounts, deteriorating items, and others.

Appendix

Proof that if $\Delta_2 \leq 0$, then $2A + DM^2 s[I_c(1 + \frac{I_e M}{2})^2 - I_e] > 0$.

Proof If $\Delta_2 \leq 0$, i.e., $2A \geq DM^2 s[\frac{hs\rho}{c^2}(1 + \frac{I_e M}{2})^2 + I_e]$, then we have

$$2A + DM^2 s\left[I_c\left(1 + \frac{I_e M}{2}\right)^2 - I_e\right]$$

$$\geq DM^2 s\left[\frac{hs\rho}{c^2}\left(1 + \frac{I_e M}{2}\right)^2 + I_e\right] + DM^2 s\left[I_c\left(1 + \frac{I_e M}{2}\right)^2 - I_e\right]$$

$$= DM^2 s\left(1 + \frac{I_e M}{2}\right)^2\left(\frac{hs\rho}{c^2} + I_c\right) > 0.$$

This completes the proof. □

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