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JOINT PRICING AND ORDERING POLICIES FOR DETERIORATING ITEM WITH RETAIL PRICE-DEPENDENT DEMAND IN RESPONSE TO ANNOUNCED SUPPLY PRICE INCREASE

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Abstract. Recently, due to rapid economic development in emerging nations, the world's raw material prices have been rising. In today's unrestricted information environment, suppliers typically announce impending supply price increases at specific times. This allows retailers to replenish their stock at the present price, before the price increase takes effect. The supplier, however, will generally offer only limited quantities prior to the price increase, so as to avoid excessive orders. The retail price will usually reflect any supply price increases, as market demand is dependent on retail price. This paper considers deteriorating items and investigates (1) the possible effects of a supply price increase on retail pricing, and (2) ordering policies under the conditions that special order quantities are limited and demand is dependent on retail price. The purpose of this paper is to determine the optimal special order quantity and retail price to maximize profit. Our theoretical analysis examines the necessary and sufficient conditions for an optimal solution, and an algorithm is established to obtain the optimal solution. Furthermore, several numerical examples are given to illustrate the developed model and the solution procedure. Finally, a sensitivity analysis is conducted on the optimal solutions with respect to major parameters.

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1. **Introduction.** Recently, due to rapid economic development in emerging nations, the world's raw material prices have been rising, and the cost of many goods have been facing upwards pressure; a serious issue for enterprises. Moreover, in today's environment of unrestricted information, retailers can easily obtain supply price information through various methods. If a supplier attempts to hide information surrounding an impending supply price increase, he/she may lose customers in the long term. Hence, suppliers typically actively announce impending price increases to retailers. As retailers need to make inventory policy decisions, it is essential for them to consider price increases. In past studies, many authors have considered the issue of announcing price increases, and have proposed various analytical models to gain more insight into the inferences that can be made on inventory policy. Naddor [16], one of the first researchers in this area, proposed an infinite horizon economic order quantity (EOQ) model which can be applied when suppliers announce a price increase. Lev and Soyster [12] developed a finite horizon inventory model and determined optimal ordering policies based on knowledge of an ensuing price increase. Goyal [6] analyzed Lev and Soyster's [12] model and proposed an alternative method for determining the optimal policy. Taylor and Bradley [20] extended Naddor's [16] model, by developing optimal ordering strategies for situations where the time of the price increase does not coincide with the end of an EOQ cycle. Lev and Weiss [13] presented a structure of optimal policies and procedures for computing the optimal policy. Ghosh [5] presented an infinite-horizon deterministic inventory model that can account for inventory shortages under an announced price increase. Additionally, there have been several other interesting and relevant papers, such as Shah [18], Tersine and Grasso [21], Markowski [14], Jordan [11], Goyal and Bhatt [7], Yanasse [23] and Huang and Kulkarni [10].

Most of the above discussed research revealed that retailers will adopt a special order policy where order quantity is unlimited, when faced with an announced price increase. In practice, to avoid retailers ordering large quantities, suppliers can offer limited quantities prior to the price increase. Moreover, the above discussed literature does not account for price increases being passed on to consumers. Some retailers will pass on the cost of price increases to their customers, therefore it is reasonable that retailers will take limited order quantities and price responses into account when adopting a special order policy.

The inventory models discussed above account for the impact of price changes, and focus on determining the optimal special order quantity for the retailer. A weakness in most inventory models is that they neglect the deterioration of goods, a common phenomenon. It is well known that certain products, such as medicine, volatile liquids, fruits, and vegetables, will vaporize, spoil, or damage over time. For such products, losses due to deterioration cannot be ignored when determining the optimal ordering policy. Inventory problems relating to deteriorating items have been studied widely, for example Ghare and Schrader [4] were the first to establish an EOQ model for an exponentially-decaying item, for which there is constant demand. Later, Covert and Philip [1] extended this model and obtained an EOQ model for a variable deterioration rate, by assuming a two-parameter Weibull distribution. Philip [17] then developed an inventory model with a three-parameter Weibull distribution deterioration rate. Shah [19] extended Philip's model and considered the circumstances in which a shortage is allowed. Goyal and Giri [8] provided a detailed review of the deteriorating inventory literature since the early 1990s. Recently, Yu et al. [26] proposed a Three-Echelon deteriorating inventory model with two producers, a single distributor and two retailers. There is a vast amount of literature on deteriorating items, an outline of which can be found by reviewing Yao and Wang [25], Moon et al. [15], Deng et al. [2], He et al. [9] and others.

Consequently, the contribution of this paper, relative to previous studies, is that we explore inventory decisions in the context of the following three issues in regards to the traditional EOQ model: (1) when the retailer is informed by the supplier of a future price increase and decides whether to make a special order before the increase, and what their new retail price should be; (2) the quantities of the special order items are limited; and (3) the goods deteriorate at a constant rate. Furthermore, the special order date may or may not coincide with the replenishment date. Hence, the two situations developed in this study are: (a) when the special order date coincides with the retailer's replenishment date; and (b) when the special order date occurs during the retailer's sales period. The purpose of this study is to determine the retailer's optimal order policies and retail prices in response to a price increase, through the maximization of the total profit increase between special and regular orders, during the depletion time of the special order quantity. An algorithm is established to obtain the optimal solution. Several numerical examples are given to illustrate the theories in practical use, and a sensitivity analysis of the optimal solution is also conducted by examining the main parameters.

2. **Notation and assumptions.**

The following notation and assumptions are used in this study:

Notation

- ν unit purchasing price.
- k amount of the unit purchasing price increase.
- p retail price when the unit purchasing price is ν.
- p_r retail price when the unit purchasing price is $\nu + k$.
- p_s retail price for the special order quantity, a decision variable.
- $D(p)$ market demand rate, which is a decreasing function of the retail price.
- A ordering cost per order.
- h holding cost rate, as a fraction of the cost of the item carried in inventory per unit time, $0 < h < 1$.
- θ deterioration rate, where $0 \leq \theta < 1$ and is a constant.
- Q economic order quantity before the purchasing price increase.
- T the length of replenishment cycle time before the purchasing price increase.
- Q_r economic order quantity after the purchasing price increase.
- T_r the length of replenishment cycle time after the purchasing price increase.
- Q_s special order quantity before the purchasing price increase, a decision variable.
- T_s depletion time for the special order quantity Q_s, a decision variable.
- W limited special order quantity at the present purchasing price.
- T_W depletion time of the limited special order quantity W.
- q remnant of inventory level when special order is placed.
- t_q the length of time until special order is placed during the regular replenishment period.
- T_q depletion time for the inventory quantity $Q_s + q$.
- p^* optimal retail price when the unit purchasing price is ν.
- p_r^* optimal retail price when the unit purchasing price is $\nu + k$
- p_s^* optimal retail price for the special order quantity.

Q^* optimal economic order quantity before the purchasing price increase.
T^* optimal length of replenishment cycle time before the purchasing price increase.
Q_r^* optimal economic order quantity after the purchasing price increase.
T_r^* optimal length of replenishment cycle time after the purchasing price increase.
Q_s^* optimal special order quantity before the purchasing price increase.
T_s^* optimal depletion time for the special order quantity Q_s^*.
$I(t)$ inventory level at time t before the purchasing price increase, $0 \le t \le T$.
$I_s(t)$ inventory level at time t when the special order policy is adopted, $0 \le t \le T_s$.
$I_q(t)$ inventory level at time t during the time interval $[0, T_q]$.
$TP(p,T)$ total profit per unit time during the replenishment period T.
$TP_r(p_r, T_r)$ total profit per unit time during the replenishment period T_r.
$g_i(p_s, T_s)$ total profit increase between the special order and regular order during the special cycle time for case i, $i = 1, 2$.
g_i^* maximum total profit increase between the special order and regular order during the special cycle time for case i, i.e., $g_i^*(p_s^*, T_s^*)$, $i = 1, 2$.

Assumptions

1. The demand rate $D(p)$ is a non-negative continuous function of the retail price p, and satisfies $D^{'}(p) < 0$ and $D^{''}(p) \le 0$.
2. The supplier announces that the unit price of an item will increased by a given amount, k, on a certain future date.
3. The retailer has only one opportunity to replenish with the present price before the purchasing price increases, and the special order quantity that the retailer can order at the present price is limited to W, i.e., $Q_s \le W$.
4. In general, the special order quantity at the present price, Q_s, is always greater than or equal to the optimal economic order quantity before the purchasing price increase, Q^*, i.e., $Q_s \ge Q^*$.
5. There is no replacement or repair of deteriorated units during the period of the consideration.
6. The replenishment is instantaneous and the lead time is zero.
7. Shortages are not allowed.

3. **Model formulation.** In the beginning, we explain the inventory level on hand change in inventory system: the depletion of the inventory occurs due to the combined effects of demand and physical deterioration. Hence, the change in inventory level before the purchasing price increase can be illustrated by the following differential equation:

$$\frac{dI(t)}{dt} = -\theta I(t) - D(p),\ 0 < t < T \tag{1}$$

Given the boundary condition $I(t) = 0$, the solution of (1) can be represented by

$$I(t) = \frac{D(p)}{\theta}\left[e^{\theta(T-t)} - 1\right],\ 0 \le t \le T \tag{2}$$

Thus, the order quantity is given by

$$Q = I(0) = \frac{D(p)}{\theta}\left(e^{\theta T} - 1\right) \tag{3}$$

Prior to the purchasing price increase, the retailer follows the regular economic order policy with unit purchasing cost, ν, and sell them with unit retail price, p.

In this situation, the total profit during the replenishment period T is the total revenue ($p\int_0^T D(p)dt$) minus the total relevant cost which is including the ordering cost (A), purchasing cost (νQ) and holding cost ($h\nu\int_0^T I(t)dt$). That is,

$$p\int_0^T D(p)dt - \left[A + \nu Q + h\nu\int_0^T I(t)dt\right]$$
$$= D(p)\left[(p-\nu)T - \frac{(\theta+h)\nu}{\theta^2}\left(e^{\theta T} - \theta T - 1\right)\right] - A. \tag{4}$$

Therefore, the total profit per unit time is

$$TP(p,T) = \frac{D(p)}{T}\left\{(p-\nu)T - \frac{(\theta+h)\nu}{\theta^2}\left(e^{\theta T} - \theta T - 1\right) - \frac{A}{D(p)}\right\}. \tag{5}$$

The objective of this problem is to determine the optimal pricing and ordering policies that correspond to maximizing the total profit per unit time. The optimal solutions can be obtained by using the following search procedure: We first prove that for any given retail price p, the optimal value of T not only exists but also is unique. And then for any given value of T, there exists a unique sell pricing p to maximize the objective function. The processes of proofs are similar to Dye [3], Wu et al. [22], Yang et al. [24], and hence are omitted here. Once the optimal retail price, p^*, and the length of replenishment cycle time, T^*, are obtained, the optimal order quantity, Q^*, is

$$Q^* = \frac{D(p^*)}{\theta}\left(e^{\theta T^*} - 1\right). \tag{6}$$

Next, when the unit purchasing cost increases from ν to $(\nu+k)$, the retailer will reflect the supply price increases on the retail price. Hence, the retail price increases from p to p_r. The total profit per unit time becomes

$$TP_r(p_r,T_r) = \frac{D(p_r)}{T_r}\left[(p_r-\nu-k)T_r - \frac{(\theta+h)(\nu+k)}{\theta^2}\left(e^{\theta T_r} - \theta T_r - 1\right)\right] - \frac{A}{T_r}. \tag{7}$$

Similarly, it can obtain the optimal solution (p_r^*, T_r^*) that maximizes $TP_r(p_r,T_r)$, and the corresponding optimal order quantity, is

$$Q_r^* = \frac{D(p_r^*)}{\theta}\left(e^{\theta T_r^*} - 1\right). \tag{8}$$

Subsequently, when a supplier announces a price increase that is effective starting on a particular future date, the retailer may place a special order to take advantage of the current lower purchasing price, ν, before the price increases. In order to response the marketing situation and increase the profit, the retailer will reflect supply price increases on retailer price. According to the demand-supply ruler, the more the price is, the less the demand will be. Alternatively, the retailer may ignore this notice and adopt a regular order policy. Anticipating the high likelihood of suppliers and retailers hoarding goods for later sale at a higher retail price, suppliers are only willing to offer limited quantities, W, prior to the price increase.

Our purpose is to determine the optimal special order quantity and the retail price by maximizing the total profit increase between special and regular orders during the depletion time of the special order quantity. As stated earlier, two specific situations are discussed in this study. Next, we will formulate the corresponding total relevant inventory cost saving function for these two cases.

Case 1: The special order date coincides with the retailer's replenishment date

In this case, if the retailer decides to adopt a special order policy and orders Q_s units, then the inventory level at time t will be

$$I_s(t) = \frac{D(p_s)}{\theta}\left[e^{\theta(T_s-t)} - 1\right],\ 0 \le t \le T_s. \tag{9}$$

The special order quantity with the original unit purchasing price, ν, is

$$Q_s = I_s(0) = \frac{D(p_s)}{\theta}\left(e^{\theta T_s} - 1\right). \tag{10}$$

From Assumptions 3 and 4, we know that the special order quantity Q_s is less than or equal to the limits quantity W and is always larger than or equal to the optimal regular order quantity Q^* (i.e., $Q^* \le Q_s \le W$). Substituting Q_s in (10) into the inequality $Q^* \le Q_s \le W$, we have

$$T_R \equiv \frac{1}{\theta}ln\left[\frac{\theta Q^* + D(p_s)}{D(p_s)}\right] \le T_s \le \frac{1}{\theta}ln\left[\frac{\theta W + D(p_s)}{D(p_s)}\right] \equiv T_w. \tag{11}$$

The total profit of the special order during the time interval $[0, T_s]$ (denoted by $TPS_1(p_s, T_s)$) is equal to total revenue minus the total relevant cost which consists of the ordering cost, purchasing cost and holding cost, and can be expressed by

$$\begin{aligned}
&TPS_1(p_s, T_s)\\
=&p_s D(p_s)T_s - \left[A + \frac{\nu D(p_s)}{\theta}\left(e^{\theta T_s} - 1\right) + \frac{h\nu D(p_s)}{\theta^2}\left(e^{\theta T_s} - \theta T_s - 1\right)\right]\\
=&D(p_s)\left[(p_s - \nu)T_s - \frac{(\theta + h)\nu}{\theta^2}\left(e^{\theta T_s} - \theta T_s - 1\right)\right] - A.
\end{aligned} \tag{12}$$

If the retailer adopts its regular order policy, then the total cost of a regular order during the time interval $[0, T_s]$ will be divided into two periods (see Figure 1). In the first period, the retailer orders Q^* units at the unit purchasing price ν and retail price p. The corresponding total profit is similar to (4), and is represented by

$$D(p^*)\left[(p^* - \nu)T^* - \frac{(\theta + h)\nu}{\theta^2}\left(e^{\theta T^*} - \theta T^* - 1\right)\right] - A. \tag{13}$$

As to the rest period, the retailer follows regular EOQ policy with the unit purchasing price $\nu + k$ and retail price p_r. Thus, the total profit during the rest period is

$$\frac{T_s - T^*}{T_r^*}\left\{D(P_r^*)\left[(p_r^* - \nu - k)T_r^* - \frac{(\theta + h)(\nu + k)}{\theta^2}\left(e^{\theta T_r^*} - \theta T_r^* - 1\right)\right] - A\right\}. \tag{14}$$

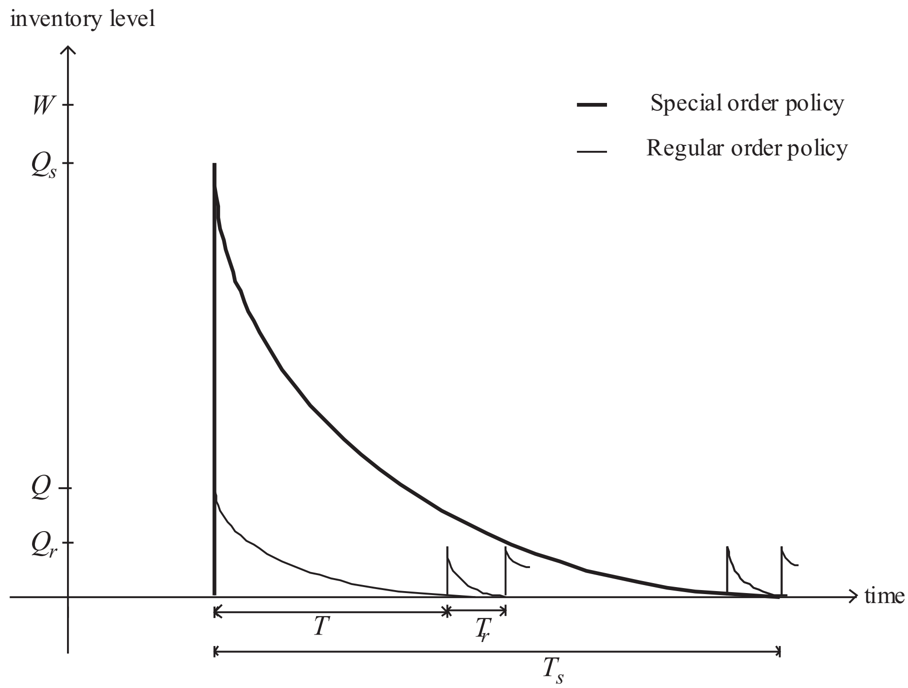

Figure 1. Special vs. regular order policies when the special order date coincides with the retailer's replenishment date.

Consequently, the total profit of a regular order during the time interval $[0, T_s]$ (denoted by $TPN_1(p_s, T_s)$) is

$$\begin{aligned} &TPN_1(p_s, T_s)\\ =&D(p^*)\left[(p^*-\nu)T^*-\frac{(\theta+h)\nu}{\theta^2}\left(e^{\theta T^*}-\theta T^*-1\right)\right]-A+\frac{T_s-T^*}{T_r^*}\\ \times&\left\{D(p_r^*)\left[(p_r^*-\nu-k)T_r^*-\frac{(\theta+h)(\nu+k)}{\theta^2}\left(e^{\theta T_r^*}-\theta T_r^*-1\right)\right]-A\right\}. \end{aligned} \tag{15}$$

Comparing (12) with (15), the total profit increase when the special order date coincides with the retailer's replenishment date (i.e., Case 1) can be given by

$$\begin{aligned} g_1(p_s, T_s) =&TPS_1(p_s, T_s)-TPN_1(p_s, T_s)\\ =&D(p_s)\left[(p_s-\nu)T_s-\frac{(\theta+h)\nu}{\theta^2}\left(e^{\theta T_s}-\theta T_s-1\right)\right]-D(p^*)\left[(p^*-\nu)T^*\right.\\ &\left.-\frac{(\theta+h)\nu}{\theta^2}\left(e^{\theta T^*}-\theta T^*-1\right)\right]-\frac{T_s-T^*}{T_r^*}\left\{D(p_r^*)\left[(p_r^*-\nu-k)T_r^*\right.\right.\\ &\left.\left.-\frac{(\theta+h)(\nu+k)}{\theta^2}\left(e^{\theta T_r^*}-\theta T_r^*-1\right)\right]-A\right\}. \end{aligned} \tag{16}$$

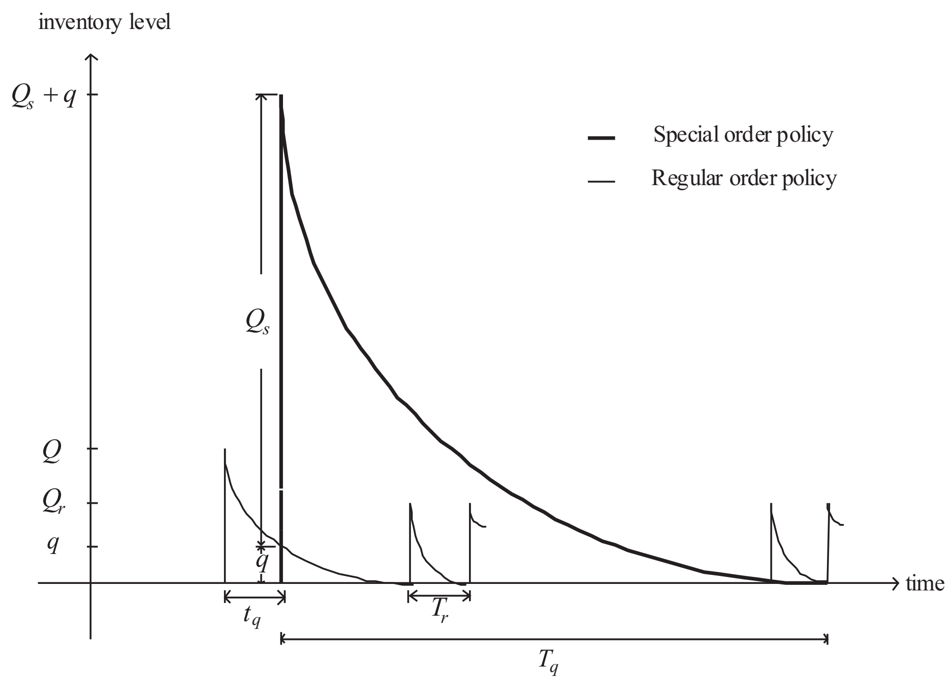

Figure 2. Special vs. regular order policies when the special order date occurs during the retailer's sales period.

Case 2: The special order date occurs during the retailer's sales period

Sometimes, the date of the price increase occurs during the retailer's sales period. In this situation, if the retailer decides to place a special order of quantity Q_s at the present purchasing price ν, retail price, p_s, then the inventory level will increase instantaneously from q to Q_s+q when the special order quantities are delivered (see Figure 2). On the other hand, if the retailer ignores notice of the purchasing price increase, then the retailer will not place any orders until the next replenishment. We will formulate the total profit functions for the special and regular order policies and then compare the two to calculate the total profit increase in Case 2.

When special orders are placed, the total profit increase during the time interval $[0, T_q]$ is equal to the total revenue, $p_s D(p_s) T_q$ minus the total relevant cost which consists of the ordering cost A, the purchasing cost $\nu(Q_s + q)$, and the holding cost which presented as follows:

As the special order quantities arrive, the maximum inventory is given by:

$$Q_s + q = \frac{D(p_s)}{\theta}(e^{\theta T_s} - 1) + \frac{D(p^*)}{\theta}[e^{\theta(T^* - t_q)} - 1]. \tag{17}$$

The inventory level at time t during the time interval $[0, T_q]$ can be obtained by

$$I_q(t) = \frac{D(p_s)}{\theta}\left[e^{\theta(T_q - t)} - 1\right], \ 0 \le t \le T_q. \tag{18}$$

Because $I_q(0) = Q_s + q$ from (17) and (18), we have

$$\frac{D(p_s)}{\theta}(e^{\theta T_q} - 1) = \frac{D(p_s)}{\theta}(e^{\theta T_s} - 1) + \frac{D(p^*)}{\theta}[e^{\theta(T^* - t_q)} - 1]. \tag{19}$$

Thus,

$$T_q = \frac{1}{\theta} ln\left[e^{\theta T_s} + \frac{D(p^*)[e^{\theta(T* - t_q)} - 1]}{D(p_s)}\right]. \tag{20}$$

For simplify, we let $z \equiv D(p^*)[e^{\theta(T^* - t_q)} - 1] > 0$. Therefore, the total holding cost of the special order policy is

$$\begin{aligned} h\nu \int_0^{T_q} I_q(t)dt &= \frac{h\nu D(p_s)}{\theta^2}\Big(e^{\theta T_q} - \theta T_q - 1\Big) \\ &= \frac{h\nu D(p_s)}{\theta^2}\left\{e^{\theta T_s} + \frac{z}{D(p_s)} - ln\left[e^{\theta T_s} + \frac{z}{D(p_s)}\right] - 1\right\}. \end{aligned} \tag{21}$$

Consequent to this, the total profit of the special order during the time interval $[0, T_q]$ (denoted by $TPS_2(p_s, T_s)$) can be formulated as follows:

$$\begin{aligned} TPS_2(p_s, T_s) =& D(p_s)\left\{\frac{\theta p_s + h\nu}{\theta^2} ln\left[e^{\theta T_s} + \frac{z}{D(p_s)}\right] - \frac{(\theta + h)\nu}{\theta^2}(e^{\theta T_s} - 1)\right\} \\ & - \frac{(\theta + h)\nu D(p^*)}{\theta^2}[e^{\theta(T^* - t_q)} - 1] - A. \end{aligned} \tag{22}$$

On the other hand, if the retailer ignores notification of the price increase and follows its regular order policy, the total profit during the time interval $[0, T_q]$ will also be divided into two periods. In the first period, the retailer only has the profit during the depletion time of remnant q, $T^* - t_q$, and we use the average cost analysis approach. It gives as follows

$$\frac{T^* - t_q}{T^*}\left\{D(p^*)\left[(p^* - \nu)T^* - \frac{(\theta + h)\nu}{\theta^2}\Big(e^{\theta T^*} - \theta T^* - 1\Big)\right] - A\right\}.$$

Next, the retailer follows the regular order policy with the unit purchase cost $\nu + k$ and retail price p_r during the rest period. To obtain the total profit in this period, we also use the average analysis approach, which is given by

$$\begin{aligned} &\frac{T_q - (T^* - t_q)}{T_r^*}\left\{D(p_r^*)\left[(p_r^* - \nu - k)T_r^* - \frac{(\theta + h)(\nu + k)}{\theta^2}\Big(e^{\theta T_r^*} - \theta T_r^* - 1\Big)\right] - A\right\} \\ &\quad = \left\{\frac{1}{\theta T_r^*} ln\left[e^{\theta T_s} + \frac{z}{D(p_s)}\right] - \frac{T^* - t_q}{T_r^*}\right\}\left\{D(p_r^*)\left[(p_r^* - \nu - k)T_r^*\right.\right. \\ &\qquad \left.\left. - \frac{(\theta + h)(\nu + k)(e^{\theta T_r^*} - \theta T_r^* - 1)}{\theta^2}\right] - A\right\}. \end{aligned} \tag{23}$$

As a result, if the retailer ignores the notification and follows its regular order policy during the time interval $[0, T_q]$, the total profit (denoted by $TPN_2(p_s, T_s)$)

is represented by:

$$\begin{aligned}TPN_2(p_s,T_s)=&\frac{T^*-t_q}{T^*}\left\{D(p^*)\left[(p^*-\nu)T^*-\frac{(\theta+h)\nu}{\theta^2}\left(e^{\theta T^*}-\theta T^*-1\right)\right]-A\right\}\\&+\left\{\frac{1}{\theta T_r^*}ln\left[e^{\theta T_s}+\frac{z}{D(p_s)}\right]-\frac{T^*-t_q}{T_r^*}\right\}\left\{D(p_r^*)\left[(p_r^*-\nu-k)T_r^*\right.\right.\\&\left.\left.-\frac{(\theta+h)(\nu+k)}{\theta^2}\left(e^{\theta T_r^*}-\theta T_r^*-1\right)\right]-A\right\}. \qquad (24)\end{aligned}$$

Therefore, the total profit increase when the special order date occurs during the retailer's sales period (i.e., Case 2) can be formulated as follows:

$$\begin{aligned}g_2(p_s,T_s)=&TPS_2(p_s,T_s)-TPN_2(p_s,T_s)\\=&D(p_s)\left\{\frac{\theta p_s+h\nu}{\theta^2}ln\left[e^{\theta T_s}+\frac{z}{D(p_s)}\right]-\frac{(\theta+h)\nu}{\theta^2}(e^{\theta T_s}-1)\right\}\\&-\frac{(\theta+h)\nu D(p^*)}{\theta^2}[e^{\theta(T^*-t_q)}-1]-A-\frac{T^*-t_q}{T^*}\left\{D(p^*\left[(p^*-\nu)T^*\right.\right.\\&\left.\left.-\frac{(\theta+h)\nu}{\theta^2}\left(e^{\theta T^*}-\theta T^*-1\right)-A\right]\right\}\\&-\left\{\frac{1}{\theta T_r^*}ln\left[e^{\theta T_s}+\frac{z}{D(p_s)}\right]-\frac{T^*-t_q}{T_r^*}\right\}\\&\times\left\{D(p_r^*)\left[(p_r^*-\nu-k)T_r^*-\frac{(\theta+h)(\nu+k)}{\theta^2}\left(e^{\theta T_r^*}-\theta T_r^*-1\right)\right]-A\right\}. \qquad (25)\end{aligned}$$

Remark 1. When $q=0$ (i.e., $t_q=T^*$), the two total profit increase functions $g_1(p_s,T_s)$ and $g_2(p_s,T_s)$ have the following relationship from (16) and (25):

$$\begin{aligned}&g_2(p_s,T_s)-g_1(p_s,T_s)\\=&D(p^*)\left[(p^*-\nu)T^*-\frac{(\theta+h)\nu}{\theta^2}\left(e^{\theta T^*}-\theta T^*-1\right)\right]-A\\&-\frac{T^*}{T_r^*}\left\{D(p_r^*)\left[(p_r^*-\nu-k)T_r^*-\frac{(\theta+h)(\nu+k)}{\theta^2}\left(e^{\theta T_r^*}-\theta T_r^*-1\right)\right]-A\right\}.\end{aligned}$$

It is noted that the right-hand side of the above equation is a constant as the values of parameters are given. Therefore, the total profit increase function for Case 1 is a little different from Case 2 with a constant item.

Remark 2. Note that it is worth placing a special order when the total profit increase is positive for the above two cases. Otherwise, the special order policy will be ignored by the retailer.

4. **Theoretical results.** In this section, the optimal solution of (p_s,T_s) that maximizes the total profit increase function is determined. From Remark 1, we can see that the total profit increase function for Case 1 is a little different from Case 2 with a constant item when $q=0$ ($t_q=T^*$). Here we only discuss how to find

the optimal solution of (p_s, T_s) that maximizes the total profit increase function for Case 2. The detail solution procedure is shown as follows.

The necessary conditions for the increase function in (25) to be maximized are $\partial g_2(p_s, T_s)/\partial T_s = 0$ and $\partial g_2(p_s, T_s)/\partial p_s = 0$, simultaneously. That is,

$$D(p_s)\left\{\frac{(\theta p_s + h\nu)e^{\theta T_s}}{\theta[e^{\theta T_s} + \frac{z}{D(p_s)}]} - \frac{(\theta + h)\nu e^{\theta T_s}}{\theta}\right\} - \frac{e^{\theta T_s} y}{e^{\theta T_s} + \frac{z}{D(p_s)}} = 0, \tag{26}$$

and

$$\frac{D^{'}(p_s)}{\theta^2}\left\{(\theta p_s + h\nu)ln\left[e^{\theta T_s} + \frac{z}{D(p_s)}\right] - (\theta + h)\nu(e^{\theta T_s} - 1) - \frac{[(\theta p_s + h\nu)D(p_s) - \theta y]z/D(p_s)}{[e^{\theta T_s} + z/D(p_s)]D(p_s)}\right\} + \frac{D(p_s)}{\theta}ln\left[e^{\theta T_s} + \frac{z}{D(p_s)}\right] = 0, \tag{27}$$

where $y \equiv \frac{1}{T_r^*}\left\{D(p_r^*)\left[(p_r^* - \nu - k)T_r^* - \frac{(\theta+h)(\nu+k)}{\theta^2}\left(e^{\theta T_r^*} - \theta T_r^* - 1\right)\right] - A\right\} > 0,$ and $D^{'}(p_s)$ is the derivative of $D(p_s)$ with respect to p_s.

It is not easy to find the closed-form solution of (p_s, T_s) from (26) and (27). Besides, due to the high-power expression of the exponential function, the concavity property of the total profit increase function in (25) cannot be proved by using the Hessian matrix. Instead, we solve the problem by the following search procedure.

For any given p_s, the necessary condition for the total profit increase in (25) to be maximized is $dg_2(p_s, Ts)/dT_s = 0$, leads to

$$\frac{(\theta p_s + h\nu)D(p_s) - \theta y}{e^{\theta T_s} + z/D(p_s)} = (\theta + h)\nu D(p_s), \tag{28}$$

and the second-order sufficient condition must satisfy $d^2 g_2(T_s)/dT_s^2 < 0$. Because

$$\frac{d^2 g_2(T_s)}{dT_s^2} = e^{\theta T_s}\left\{\frac{[(\theta p_s + h\nu)D(p_s) - \theta y]z/D(p_s)}{[e^{\theta T_s} + z/D(p_s)]^2} - (\theta + h)\nu D(p_s)\right\}$$
$$= -e^{2\theta T_s}\left\{\frac{(\theta p_s + h\nu)D(p_s) - \theta y}{[e^{\theta T_s} + z/D(p_s)]^2}\right\} \text{ (by(28))}.$$

Moreover, due to the RHS in (28) is positive which implies $(\theta P_s + h\nu)D(p_s) - \theta y > 0$, and hence $d^2 g_2(T_s)/dT_s^2 < 0$. Thus the optimal value of T_s (say T_{s_2}) can be obtained by solving (28), and is

$$T_{s_2} = \frac{1}{\theta}ln\left\{\frac{(\theta p_s + h\nu)D(p_s) - \theta y - (\theta + h)\nu z}{(\theta + h)\nu D(p_s)}\right\}. \tag{29}$$

Furthermore, to ensure $Q^* \le Q_s \le W$ (i.e., $T_R \le T_s \le T_W$), we substitute (29) into this inequality, and obtain that

$$\text{if } \Delta_1(p_s) \le 0 \le \Delta_2(p_s), \text{ then } T_R \le T_{s_2} \le T_W, \tag{30}$$

where

$$\Delta_1(p_s) \equiv (\theta + h)\nu(\theta Q^* + z) - \theta[(p_s - \nu)D(p_s) - y], \tag{31}$$

and

$$\Delta_2(p_s) \equiv (\theta + h)\nu(\theta W + z) - \theta[(p_s - \nu)D(p_s) - y], \tag{32}$$

T_R and T_W are defined as in (11).

Conversely, as $\Delta_2(p_s) < 0$, we have

$$y < (p_s - \nu)D(p_s) - (\theta + h)\nu(\theta W + z)/\theta,$$

which implies

$$\begin{aligned}\frac{dg_2(p_s, T_s)}{dT_s} &= e^{\theta T_s}\left\{\frac{(\theta p_s + h\nu)D(p_s) - \theta y}{\theta[e^{\theta T_s} + z/D(p_s)]} - \frac{(\theta + h)\nu D(p_s)}{\theta}\right\} \\ &> e^{\theta T_s}\left\{\frac{(\theta + h)\nu[\theta W - D(p_s)(e^{\theta T_s} - 1)]}{\theta[e^{\theta T_s} + z/D(p_s)]}\right\} \\ &= e^{\theta T_s}\left[\frac{(\theta + h)\nu(W - Q_s)}{e^{\theta T_s} + z/D(p_s)}\right] \geq 0 \text{ (because } Q_s \leq W).\end{aligned}$$

Hence, for any given p_s, $g_2(p_s, T_s)$ is a strictly increasing function of $T_s \in [T_R, T_W]$, and has a maximum value at the upper boundary point $T_s = T_W$.

On the other hand, if $\Delta_1(p_s) > 0$, $y > (p_s - \nu)D(p_s) - (\theta + h)\nu(\theta Q^* + z)/\theta$, which implies

$$\begin{aligned}\frac{dg_2(p_s, T_s)}{dT_s} &= e^{\theta T_s}\left\{\frac{(\theta p_s + h\nu)D(p_s) - \theta y}{\theta[e^{\theta T_s} + z/D(p_s)]} - \frac{(\theta + h)\nu D(p_s)}{\theta}\right\} \\ &< e^{\theta T_s}\left\{\frac{(\theta + h)\nu[\theta Q^* - D(p_s)(e^{\theta T_s} - 1)]}{\theta[e^{\theta T_s} + z/D(p_s)]}\right\} \\ &= e^{\theta T_s}\left[\frac{(\theta + h)\nu(Q^* - Q_s)}{e^{\theta T_s} + z/D(p_s)}\right] \leq 0 \text{ (because } Q_s \geq Q^*).\end{aligned}$$

Thus, for any given p_s, $g_2(p_s, T_s)$ is a strictly decreasing function of $T_s \in [T_R, T_W]$, and hence has a maximum value at the lower boundary point $T_s = T_R$.

From the above arguments, for any given p_s, we can obtain the optimal value of T_s (denoted by Y_s^*) as follows:

$$T_s^* = \begin{cases} T_R, & if\ \Delta_1(p_s) > 0, \\ T_{s_2}, & if\ \Delta_1(p_s) \leq 0 \leq \Delta_2(p_s), \\ T_W, & if\ \Delta_2(p_s) < 0. \end{cases}$$

Remark 3. Note that when the optimal length of replenishment cycle time $T_s^* = T_R$ or T_{s_2}, from (11) and (29), it can be found that the value of T_s^* is independent of the limited special order quantity by supplier supplied W, and hence the corresponding optimal special order quantity Q_s^* is also independent of W.

Next, the problem remaining is to find the optimal value of p_s which maximizes $g_2(p_s, T_s^*)$. Similarly, the necessary condition for the total profit increase in (25) to be maximized is $dg_2(p_s,T_s^*)/dp_s=0$, gives

$$\frac{D^{'}(p_s)}{\theta^2}\left\{(\theta p_s+h\nu)\left\{ln\left[e^{\theta T_s^*}+\frac{z}{D(p_s)}\right]-\frac{z/D(p_s)}{e^{\theta T_s^*}+z/D(p_s)}\right\}-(\theta+h)\nu(e^{\theta T_s^*}-1)\right.$$
$$\left.+\frac{\theta yz/D(p_s)}{[e^{\theta T_s^*}+z/D(p_s)]D(p_s)}\right\}+\frac{D(p_s)}{\theta}ln\left[e^{\theta T_s^*}+\frac{z}{D(p_s)}\right]=0, \tag{33}$$

and the sufficient condition must satisfy $d^2g_2(p_s,T_s^*)/dp_s^2<0$. Because

$$\begin{aligned}\frac{d^2g_2(p_s,T_s^*)}{dp_s^2}=&\frac{D^{''}(p_s)}{\theta^2}\left\{(\theta p_s+h\nu)\left\{ln\left[e^{\theta T_s^*}+\frac{z}{D(p_s)}\right]-\frac{z/D(p_s)}{e^{\theta T_s^*}+z/D(p_s)}\right\}\right.\\&\left.-(\theta+h)\nu(e^{\theta T_s^*}-1)+\frac{\theta yz/D(p_s)}{[e^{\theta T_s^*}+z/D(p_s)]D(p_s)}\right\}\\&-\frac{(\theta p_s+h\nu)[D^{'}(p_s)]^2}{\theta^2D(p_s)}\times\left[\frac{z/D(p_s)}{e^{\theta T_s^*}+z/D(p_s)}\right]^2\\&-\frac{y[D^{'}(p_s)]^2}{\theta[D(p_s)]^2}\left\{\frac{2[z/D(p_s)]e^{\theta T_s^*}+[z/D(p_s)]^2}{[e^{\theta T_s^*}+z/D(p_s)]^2}\right\}\\&+\frac{2D^{'}(p_s)}{\theta}\left\{ln\left[e^{\theta T_s^*}+\frac{z}{D(p_s)}\right]-\left[\frac{z/D(ps)}{e^{\theta T_s^*}+z/D(p_s)}\right]\right\},\end{aligned} \tag{34}$$

where $D^{'}(p_s)$ and $D^{''}(p_s)$ are the first and second-order derivatives of $D(p_s)$ with respect to p_s, respectively. By the assumptions $D^{'}(p_s)<0$ and $D^{''}(p_s)\le 0$, and from (33), it is known that the first brace term of RHS in (34) is positive, i.e.,

$$(\theta p_s+h\nu)\left\{ln[e^{\theta T_s^*}+z/D(p_s)]-\frac{z/D(p_s)}{e^{\theta T_s^*}+z/D(p_s)}\right\}-(\theta+h)\nu(e^{\theta T_s^*}-1)$$
$$+\frac{\theta yz/D(p_s)}{[e^{\theta T_s^*}+z/D(p_s)]D(p_s)}>0,$$

and the last brace term $ln[e^{\theta T_s^*}+z/D(p_s)]-[z/D(p_s)]/[e^{\theta T_s^*}+z/D(p_s)]$ is also positive (the proof sees in the Appendix). Therefore, it can be obtained that $d^2g_2(p_s^*,T_s^*)/dp_s^2<0$. Consequently, for a given T_s^*, $g_2(p_s,T_s^*)$ is a concave function of p_s, and hence there exists a unique value of p_s (say p_{s_2}) which maximizes $g_2(p_s,T_s^*)$. It is obvious p_{s_2} can be found by solving $dg_2(p_s,T_s^*)/dp_s=0$, that is, p_{s_2} can be determined by solving (33).

Summarize the above results, for $q>0$ (i.e., Case 2), we can develop an algorithm to obtain the optimal solution (T_s^*,p_s^*). As to the case $q=0$ (i.e., Case 1), let $t_q=T^*$ and by using the similar algorithm, we also can find the optimal solution.

Algorithm:

Step 1. Determine T^*, Q^*, p^*, T_r^*, p_r^*, z and y, respectively.
Step 2. Start with $j=0$ and the initial value is $p_{s,j}=\nu$.
Step 3. Put $p_{s,j}$ into (31) and (32) to obtain
$\Delta_1(p_{s,j})\equiv(\theta+h)\nu[\theta Q^*+z]-\theta[(p_{s,j}-\nu)D(p_{s,j})-y]$ and
$\Delta_2(p_{s,j})\equiv(\theta+h)\nu[\theta W+z]-\theta[(p_{s,j}-\nu)D(p_{s,j})-y]$.

(i) If $\Delta_1(p_{s,j}) \leq 0 \leq \Delta_2(p_{s,j})$, find the value $T_{s_2,j}$ from (29). Then put $T_{s_2,j}$ into (33) and solve this equation to obtain the corresponding value $p_{s_2,j+1}$. Let $p_{s,j+1} = p_{s_2,j+1}$ and $T_{s,j} = T_{s_2,j}$.

(ii) If $\Delta_1(p_{s,j}) > o$, find the value $T_{R,j}$ from (11). Then put $T_{R,j}$ into (33) and solve this equation to obtain the corresponding value $p_{s_2,j+1}$. Let $p_{s,j+1} = p_{s_2,j+1}$ and $T_{s,j} = T_{R,j}$.

(iii) If $\Delta_2(p_{s,j}) < 0$, find the value $T_{W,j}$ from (11). Then put $T_{W,j}$ into (33) and solve this equation to obtain the corresponding value $p_{s_2,j+1}$. Let $p_{s,j+1} = p_{s_2,j+1}$ and $T_{s,j} = T_{W,j}$.

Step 4. If the difference between $p_{s,j}$ and $p_{s,j+1}$ is enough small, (i.e., $|p_{s,j} - p_{s,j+1}| \leq 10^{-5}$), then set $p_s^* = p_{s,j}$ and $T_s^* = T_{s,j}$. Thus (p_s^*, T_s^*) is the optimal solution. Otherwise, set $j = j + 1$ and go back to Step 3.

Once the optimal solution (p_s^*, T_s^*) is obtained, we can determine the optimal special order quantity $Q_s^* = D(p_s^*)(e^{\theta T_s^*} - 1)/\theta$, and the corresponding maximum profit increasing $g_i^* = g_i(p_s^*, T_s^*)$, $i = 1, 2$.

5. **Numerical examples.** To illustrate the optimal ordering policy, the following example is presented:

Example 1. Given an inventory system with the following parameters:

$D(p) = 1000 - 8p$ units/year, where $p < 125$, $\nu = \$30$/unit, $k = \$5$/unit, $A = \$250$/order, $\theta = 0.05$, $h = 0.3$, and $q = 50$. Before unit purchasing cost increase, the optimal retail price, p^*, the length of replenishment cycle time, T^* and the optimal order quantity, Q^*, can be obtained as $p^* = \$78.4386$/unit, $T^* = 0.3554$ years and $Q^* = 133.58$ units. After unit purchasing cost increase, the optimal retail price, p_r^*, the length of replenishment cycle time, T_r^* and the optimal order quantity, Q_r^*, can be obtained as $p_r^* = \$81.0434$/unit, $T_r^* = 0.3388$ years and $Q_r^* = 120.14$ units. It is shown that the increase rate of purchasing cost is $(k/\nu) \times 100\% = 16.67\%$ and the retailer will reflect supply price increases on retail price with the rate $[(p_r^* - p^*)/p^*] \times 100\% = 3.32\%$. When a supplier announces a price increase that is effective starting on a particular future date, it can be found that the optimal ordering policies depend on the limited special order quantity W. The computational results are shown in Table 1.

From Table 1, some observations can be made as follows. First, for the cases with lower limited special order quantity (for example, the cases with $W \leq 250$ units in Table 1), the optimal policy for the retailer is to adopt the special order policy and order an upper boundary quantity by supplier supplied, that is $Q_s^* = W$.

Table 1. Optimal solutions of Example 1 under the different value of W

W	p_s^*	T_s^*	Q_{s^*}	g_2^*	r^*
150	79.337	$T_W = 0.406458$	150	712.748	1.1454%
200	79.4738	$T_W = 0.541730$	200	838.907	1.3198%
250	79.6899	$T_{s_2} = 0.678066$	250	894.172	1.5953%
300	79.7562	$T_{s_2} = 0.714557$	263.31	896.747	1.6798%
350	79.7562	$T_{s_2} = 0.714557$	263.31	896.747	1.6798%
400	79.7562	$T_{s_2} = 0.714557$	263.31	896.747	1.6798%

Note: r^* denotes the rate of retail price increase and is defined by

$$r^* = \frac{p_s^* - p^*}{p^*} \times 100\%$$

In such situation, the higher the limited special order quantity is, the higher the special order quantity will be. Contrarily, for higher limited special order quantity (for example, the cases with $W \geq 300$ units in Table 1), the optimal policy for the retailer is to adopt the special order policy which is unaffected by the limited special order quantity. This result can be shown in Remark 3.

In addition, when a supplier announces a price increase that is effective starting on a particular future date, the optimal retail price adopting special order policy will be between regular order policy before price increase and that after price increase due to price-dependent demand rate. That is, the optimal rate of retail price increase (denoted by $r^* = [(p_s^* - p^*)/p^*] \times 100\%$) will be less than or equal to the rate $[(p_r^* - p^*)/p^*] \times 100\% = 3.32\%$. It implies when the retailer places a special order to take advantage of current lower price before purchasing price increases, it will reflect the cost saving on retail price which is related to market demand to increase the profit.

Example 2. The data used is the same as those in Example 1 except we consider the case with quadratic demand function, $D(p) = 1000 + 8p - 0.5p^2$ units/year, where $8 < p < 53.4312$, Similarly, $p^* = \$43.3341$, $T^* = 0.3398$, $Q^* = 139.74$, $p_r^* = \$45.7033$, $T_r^* = 0.3544$ and $Q_r^* = 114.85$. It is shown that the increase rate of purchasing cost is 16.67% and the retailer will reflect supply price increases on retail price with the rate 5.47%. When a supplier announces a price increase that is effective starting on a particular future date, it can be found that the optimal ordering policies depend on the limited special order quantity W. The computational results are shown in Table 2. The management insights are similar to Example 1.

Example 3. Due to the insights of Examples 1 and 2 are similar, we only discuss the influences of changes in major parameters ν, k, A, θ, and h on p_s^*, T_s^*, Q_s^*, g_2^* and r^* of the Example 1. For convenience, the case with fixed $W = 350$ is taken into account. The sensitivity analysis is performed by changing each of the parameters by -20%, -10%, $+10\%$ and $+20\%$ taking one parameter at a time and keeping the remains unchanged. The computational results are shown in Table 3.

On the basis of the results in Table 3, the following observations can be made.

(a) The optimal retail price for special order quantity p_s^* increases while the optimal length of replenishment cycle T_s^*, the optimal special order quantity Q_s^*, the maximum total profit increase g_2^* and the optimal rate of retail price increase r^* decrease with increases in the values of h and A.

Table 2. Optimal solutions of Example 2 under the different value of W

W	p_s^*	T_s^*	Q_{s^*}	g_2^*	r^*
150	44.0927	$T_W = 0.390224$	150	699.473	1.7506%
200	44.2177	$T_W = 0.524771$	200	827.986	2.0390%
250	44.4246	$T_{s_2} = 0.666953$	250	886.184	2.5167%
300	44.4838	$T_{s_2} = 0.702568$	262.04	888.585	2.6533%
350	44.4838	$T_{s_2} = 0.702568$	262.04	888.585	2.6533%
400	44.4838	$T_{s_2} = 0.702568$	262.04	888.585	2.6533%

Note: r^* denotes the rate of retail price increase and is defined by
$$r^* = \frac{p_s^* - p^*}{p^*} \times 100\%$$

(b) All the optimal retail price for special order quantity p_s^*, the optimal length of replenishment cycle T_s^*, the optimal special order quantity Q_s^*, the maximum total profit increase g_2^* and the optimal rate r^* increase with the increases in the values of ν and k.

(c) If the value of θ increases, the optimal retail price for special order quantity p_s^*, and the optimal rate r^* will increase but the optimal length of replenishment cycle T_s^*, the optimal special order quantity Q_s^* and the maximum total profit increase g_2^* will decrease.

6. **Conclusions.** Recently, due to rapid economic development in emerging nations, the world's raw material prices have been rising, and the cost of many goods have been facing upwards pressure; a serious issue for enterprises. Moreover, in today's environment of unrestricted information, where retailers can easily obtain supply price information through various methods, suppliers typically announce impending price increases before they come into effect. In this situation, the retailer not only has the opportunity to replenish at the present price before the supply price increase, but also to pass the increased costs on to consumers. In addition, to avoid excessive retail orders in this situation, suppliers can enforce limited order quantities prior to the price increase. Therefore, in this paper, we investigated the possible effects of a supply price increase on the retail price and ordering policies of retailers under the conditions of a limited special order quantity and retail

Table 3. Effect of changes in major parameters of the Example 1

		% change in				
parameter	%change	p_s^*	T_s^*	Q_s^*	g_2^*	r^*
ν	-20	-0.33	-13.53	-13.24	-27.81	-19.80
	-10	-0.16	-6.75	-6.59	-14.37	-9.89
	10	0.16	6.72	6.54	15.28	9.87
	20	0.33	13.41	13.04	31.45	19.70
k	-20	-0.14	-5.63	-5.50	-8.63	-0.53
	-10	-0.07	-2.74	-2.67	-4.20	-0.26
	10	0.06	2.60	2.54	4.01	0.24
	20	0.12	5.10	4.96	7.86	0.49
A	-20	-0.02	3.00	2.70	2.87	0.11
	-10	-0.01	1.48	1.33	1.42	0.05
	10	0.01	-1.43	-1.30	-1.38	-0.05
	20	0.02	-2.82	-2.56	-2.72	-0.09
θ	-20	-0.12	18.37	19.01	17.32	-0.65
	-10	-0.06	8.40	8.68	7.95	-0.31
	10	0.06	-7.19	-7.40	-6.85	0.30
	20	0.11	-13.41	-13.78	-12.81	0.58
h	-20	-3.92	22.45	31.44	30.02	4.27
	-10	-1.96	10.08	14.09	13.53	2.08
	10	1.96	-8.41	-11.71	-11.34	-1.96
	20	3.92	-15.56	-21.61	-21.03	-3.83

price-dependent demand. This has not been previously studied in the relevant literature. The conditions of the optimal solutions were derived, and an algorithm was established to obtain the optimal solution. Finally, the theoretical and numerical results reveal that: (i) the total profit increase function for Case 1 is a little different from Case 2 with a constant item when $q = 0$. (ii) When the limited special order quantity is lower, the optimal policy for the retailer is to adopt the special order policy and order an upper boundary quantity allowed by the supplier. The higher the limited special order quantity is, the higher the special order quantity will be. (iii) As the retailer places a special order to take advantage of the current lower price, to increase profit they will pass the cost saving onto consumers. It is our belief that these observations will provide the basis for enterprises to make decisions in inventory management. The proposed model can be extended in several ways. For example, it is usually observed in the supermarket that display of the consumer goods in large quantities attracts more customers and generates higher demand. Hence, the proposed inventory model may deal with the demand rate as a function of the on-hand inventory. Furthermore, shortages are not allowed and inflation is not considered in this study. In the future, we hope the model can also be generalized to allow for shortages and take inflation into account.

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7. **Appendix.** To prove the last brace term of RHS in (34) is positive, i.e., $ln[e^{\theta T_s^*} + z/D(p_s)] - [z/D(p_s)]/[e^{\theta T_s^*} + z/D(p_s)] > 0$, we first let $x \equiv ln[e^{\theta T_s^*} + z/D(p_s)]$. It is obvious that $x \geq ln[e^{\theta T_s^*}] = \theta T_s^*$, and $e^x = e^{\theta T_s^*} + z/D(p_s)$. Hence,

$$ln\left[e^{\theta T_s^*} + \frac{z}{D(p_s)}\right] - \left[\frac{z/D(p_s)}{e^{\theta T_s^*} + z/D(p_s)}\right] = \frac{xe^x - e^x + e^{\theta T_s^*}}{e^x}. \quad \text{(A1)}$$

Next, we let $F(x) = xe^x - e^x + e^{\theta T_s^*}$ and then take the derivative of $F(x)$ with respect to $x \in (\theta T_s^*, \infty)$, it gets $dF(x)/dx = xe^x > 0$. Thus, $F(x)$ is a strictly increasing function of $x \in [\theta T_s^*, \infty)$. Moreover, we know that $F(\theta T_s^*) = \theta T_s^* e^{\theta T_s^*} > 0$. Therefore, $F(x) > 0$ for $x \in [\theta T_s^*, \infty)$, which implies $ln[e^{\theta T_s^*} + z/D(p_s)] - [z/D(p_s)]/[e^{\theta T_s^*} + z/D(p_s)] = F(x)/e^x > 0$. This completes the proof.

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