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STATISTICAL PROCESS CONTROL FOR MONITORING A DIFFUSION PROCESS

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Abstract. This study presents a new statistical process control (SPC) procedure for a process together with degradation and diffusion effects. One of such examples is the initial cool-down process of high-pressure hose production. The air temperature readings during the initial cool-down process often exhibit a non-increasing trend with a diffusion effect in that profiles generated from cycle to cycle deviates from each other more over time. A new charting procedure using the Wiener diffusion model is developed in this article. A real data set, generated from the cool-down process of high-pressure hose production, is used to demonstrate the application of proposed method.

Keywords: Control chart, normal distribution, profile monitoring, statistical process control, Wiener diffusion model.

1. Introduction.

The quality characteristics in some industrial manufacturing processes are often characterized as a profile rather than a number. For example, in a curing production where products are heated in a sealed, heated chamber, called an autoclave or vulcanizer, multiple thermocouples are equipped inside the chambers and/or their parts. One piece of the major curing information gathered from the thermocouples is the air temperature in the chamber. When the curing of products is finished, a cool-down process is needed to make the quality of final products satisfactory. The cool-down process is crucial to the product in that improper cool-down cycle may cause cosmetic blemishes such as blistering on the product surface. Because of the physical nature of this cool-down process, the air temperature profiles follow a downward trend whose degradation rate is not necessarily a constant. Although this study is developed for an autoclave cooling process, the core idea can be applied to any process together with the degradation phenomenon.

The cool-down stage of an autoclave curing process is hard to control because the shapes of the temperature profiles due to various venting and spraying cycles depend on how many reels of products are loaded inside the chamber. However, this information is not fed back into the control loop. In other words, the only control mechanism during the cooling stage is based on a pre-specified time slots of venting and water spraying. In the initial cool-down process, the air temperature degrades over time with a significant diffusion effect, then the diffusion effect declines in the following processes. It is important to develop proper SPC methods to monitor the initial cool-down process based on air temperature profiles and ensure that all temperature profiles are under the statistical control. When a profile deviates from the "normal" profiles, quality engineers should inspect this batch of reels of products for possible surface blemishes. In addition, they should also conduct an investigation for checking possible process malfunctions. For example, when the water tank runs out of water, the spraying cycle would fail as reflected in the temperature cycle.

The challenges of an SPC implementation are explained here. An illustrative example is regarding an air temperature profile during the cool-down stage of high-density hose products. In a typical curing cycle of high-pressure hose products manufacturing, the cool-down stage often starts with a vent-opening action followed by a series of water spraying and rest. A short rest breaks two water-spraying actions. The air temperature is expected to decline. However, the temperature-decline rates vary from one profile to another especially during the water-spraying stage. Note that such a diffusion effect is significant in the initial cool-down stage.

In this study, 153 sequences of air temperature readings are collected from a hose production factory. The present recipe of cool-down process calls for four sequences, denoted by S1 (vent-open stage), S2 (water spray 1 stage), S3 (resting stage) and S4 (water spray 2 stage), respectively. In S1, a valve is opened to vent off the pressure for a fixed period. The air temperature in S1 declines quickly and decreases over time. In S2 and S4, cool water is sprayed for a fixed duration to accelerate the air temperature degradation speed, and the recipe calls for a short rest of S3 between the stages of S2 and S4. After the second water spray is done, the chamber door is opened to unload the cured products. Since the timing and duration for each sequence is controlled by a programmable logic controller, process engineers would glance at the "general pattern" of a temperature profile for quality assurance. The current practice is very subjective. It is a great opportunity to introduce SPC methods for objective process monitoring and continuous quality improvement.

Literature works for profile monitoring when a target data profile is well defined can be found in [11], [12], [22], [8], [16], [2], [17]. Otherwise, a complicated profile can be modeled by nonlinear functions of which SPC literature can be found in [10], [7], [21], [15], [3], [19], [9] and [1]. However, these charting methods cannot handle the diffusion effect in the SPC process and could cause a higher false alarm rate.

The Wiener diffusion model has been abundantly used in reliability and financial studies, such as [4], [6], [18], [20], [13], [5] and [14]. Although there are no application of Wiener diffusion process for control charting studies is found according to our best knowledge, Wiener diffusion process model is capable of dealing with processes, which contain degradation and diffusion phenomenons.

The rest of this paper is organized as follows: statistical models are defined in Section 2. In addition, parameter estimation method is provided based on the phase I data. Moreover, an operable charting procedure using chi-square charts is suggested for SPC monitoring. In Section 3, a real data set of high-pressure hose products is used to demonstrate the application of the proposed method. Conclusions and future research are given in Section 4.

2. Statistical Models and Parameter Estimation.

Assume that engineers emphasize on the monitoring of the initial cool-down process, the vent-open stage, mentioned in Section 1. The initial degradation sequences are well defined as y_0. All degradation readings are modeled, respectively, according to the following Weiner diffusion processes,

$$y_i = \beta t_i + \sigma w_i, \ i = 1, 2, \cdots , \tag{1}$$

where the reading value y_i is measured at the time t_i, β and σ are the drift and diffusion parameter, respectively for the degradation process, and $w_i = w(t_i)$ is random error. In this paper, all random errors w_i's are characterized by a standard Brownian motion with the following facts:

1. $w_0 = 0$ for the starting time t_0.
2. w_i is almost surely continuous.

3. w_i has independent and normally distributed increments of mean 0 and variance $\triangle t_i = t_i - t_{i-1}$. That is, $\triangle y_i = y_i - y_{i-1}$, $i = 1, 2, \cdots$ are independent normally distributed with mean $\beta(\triangle t_i)$ and variance $\sigma^2(\triangle t_i)$, labeled by $\triangle y_i \sim N(\beta(\triangle t_i), \sigma^2(\triangle t_i))$.

Let n degradation measurements of a profile be collected at times $t_1, t_2, \cdots, t_n$ with $\triangle t_i = \triangle t$, $i = 1, 2, \cdots, n$; That is, the spacing between times is equally. Therefore, $\triangle y_i \sim N(\beta(\triangle t), \sigma^2(\triangle t))$, $i = 1, 2, \cdots, m$. Let $z_i = \triangle y_i / \triangle t$, $i = 1, 2, \cdots, n$, $\bar{z} = \sum_{i=1}^{n} z_i / n$ and $S_z^2 = \sum_{i=1}^{n} (z_i - \bar{z})^2 / (n-1)$, then the unbiased estimators of β and σ^2 can be obtained, respectively, by

$$\hat{\beta} = \bar{z}, \tag{2}$$

and

$$\hat{\sigma}^2 = (\triangle t) S_z^2. \tag{3}$$

It can been shown that $\hat{\beta} \sim N(\beta, \sigma^2/(n\triangle t))$, and $(n-1)(\triangle t)S_z^2/\sigma^2$ has a chi-square distribution with $(n-1)$ degrees of freedom.

Assume that a phase I sample size m cool-down profiles is prepared for parameter estimation. Based on the m vent-open stage profiles, m estimates of (β, σ^2) can be obtained and denoted by $(\hat{\beta}_i, \hat{\sigma}_i^2)$, $i = 1, 2, \cdots, m$. Let $\bar{\beta} = \sum_{i=1}^{m} \hat{\beta}_i / m$, $\bar{\sigma}_z^2 = \sum_{i=1}^{m} \hat{\sigma}_i^2 / m = (\triangle t)\bar{S}_z^2$, where $\bar{S}_z^2 = \sum_{i=1}^{m} S_{z,i}^2 / m$. If m is large, it can been shown that the sequence of C_i, defined by

$$C_i = \frac{(\hat{\beta}_i - \bar{\beta})^2}{\bar{\sigma}^2 / n\triangle t} = \frac{n(\hat{\beta}_i - \bar{\beta})^2}{\bar{S}_z^2}, \quad i = 1, 2, \cdots, n, \tag{4}$$

have an asymptotic chi-square distribution with 1 degree of freedom, and the sequence of D_i, defined by

$$D_i = \frac{(n-1)(\triangle t)S_{z,i}^2}{\bar{\sigma}^2} = \frac{(n-1)S_{z,i}^2}{\bar{S}_z^2}, \quad i = 1, 2, \cdots, n, \tag{5}$$

have an asymptotic chi-square distribution with $(n-1)$ degrees of freedom. Two control charts at false alarm rate α are proposed as follows:

The BETA-chart: The control chart for monitoring parameter β, named BETA-chart. Plot statistics C_i, $i = 1, 2, \cdots, n$, on the BETA-chart with the upper control limit (UCL) and lower control limit (LCL), which are defined, respectively by

$$\text{UCL} = \chi^2_{\alpha/2,1}, \tag{6}$$

and

$$\text{LCL} = \chi^2_{1-\alpha/2,1}, \tag{7}$$

where $\chi^2_{\alpha/2,1}$ and $\chi^2_{1-\alpha/2,1}$ denote the upper and lower $\alpha/2$ percentile points of the chi-square distribution with 1 degree of freedom.

The SIGMA-chart: The control chart for monitoring parameter σ^2, named SIGMA-chart. Plot statistics D_i, $i = 1, 2, \cdots, n$, on the SIGMA-chart with only the UCL, which is defined by

$$\text{UCL} = \chi^2_{\alpha,n-1}. \tag{8}$$

3. **An Application.** The data set of air temperature readings in vent-open stage had been collected in 2011 from a cool-down process of high-pressure hose production factory. All air temperature readings of 153 profiles are used to illustrate the application of proposed method and given in Figure 1. Engineers believe that all 153 profiles are in statistical control. The BETA-chart and SIGMA-chart at false alarm rate $\alpha = 0.0027$ were constructed and given in Figure 2 and Figure 3, respectively.

The BETA-chart in Figure 2 shows that the drift parameter of the initial cool-down process is in-control because all statistics are plotted within the control limits. But profile 50 in Figure 3 is suspected as a potential abnormal profile due to this profile diffuses more than the others. The D estimate of profile 50 is close to the control limit and could be ignored if traditional control charting methods are used for process monitoring.

4. **Conclusion.** In this paper, a new charting method is proposed and developed with the Wiener diffusion model for monitoring a degradation process together with a diffusion effect. Since the existing nonlinear profile monitoring methods cannot handle the diffusion effect together with the profile drift effect, the proposed method is capable of monitoring a degradation process in manufacturing. A data set of air temperature readings from the vent-open stage in a cool-down process of high-pressure hose production is used for illustration. The proposed method can be used for monitoring other processes together with degradation and diffusion phenomenons.

The present study does not process the association relationship among profile. How to involve the existing association relationship among profiles in the proposed method is concerned and will be studied in the future. It is also a challenge to use the proposed charting method with a non-Wiener diffusion process.

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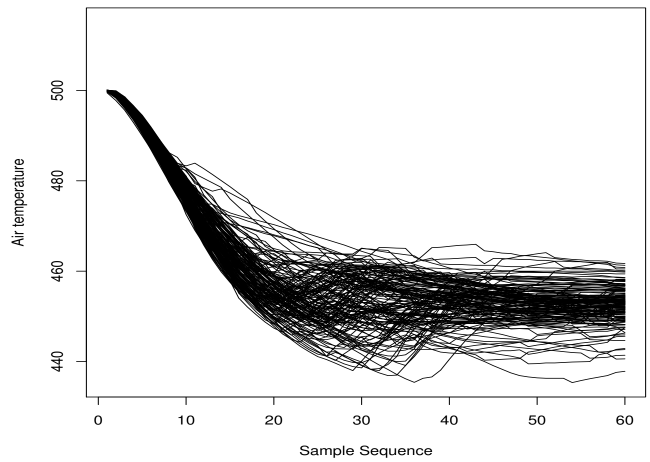

FIGURE 1. 153 in-control profiles in the vent-open stage.

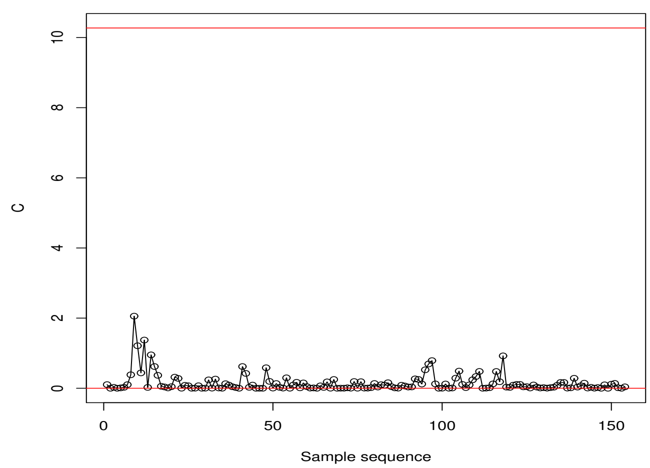

FIGURE 2. The BETA-chart for 153 vent-open profiles.

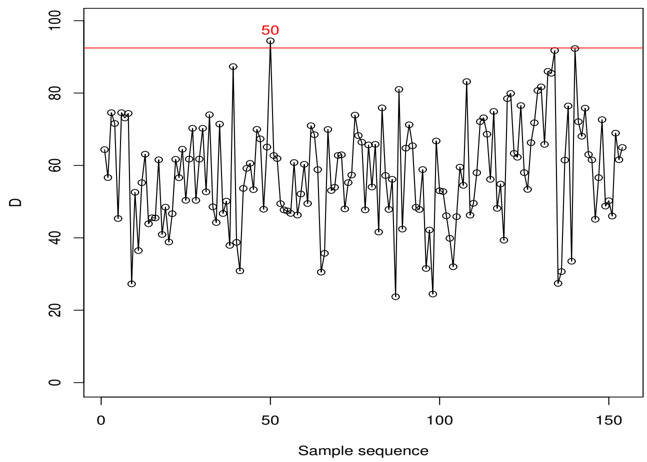

FIGURE 3. The SIGMA-chart for 153 vent-open profiles.