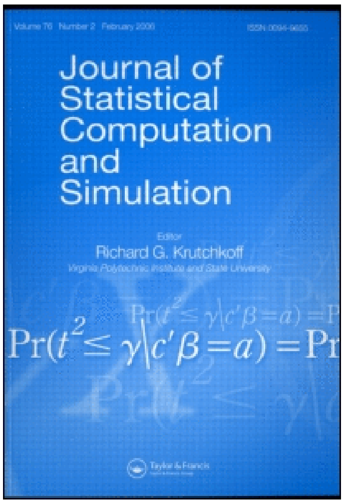

Journal of Statistical Computation and Simulation
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713650378

The simultaneous confidence intervals for all distances from the extreme populations for two-parameter exponential populations based on the multiply type II censored samples

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First published on: 09 March 2010

Journal of Statistical Computation and Simulation
Vol. 81, No. 2, February 2011, 137–165

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(*Received 8 October 2008; final version received 30 July 2009*)

Among k independent two-parameter exponential distributions which have the common scale parameter, the lower extreme population (LEP) is the one with the smallest location parameter and the upper extreme population (UEP) is the one with the largest location parameter. Given a multiply type II censored sample from each of these k independent two-parameter exponential distributions, 14 estimators for the unknown location parameters and the common unknown scale parameter are considered. Fourteen simultaneous confidence intervals (SCIs) for all distances from the extreme populations (UEP and LEP) and from the UEP from these k independent exponential distributions under the multiply type II censoring are proposed. The critical values are obtained by the Monte Carlo method. The optimal SCIs among 14 methods are identified based on the criteria of minimum confidence length for various censoring schemes. The subset selection procedures of extreme populations are also proposed and two numerical examples are given for illustration.

Keywords: extreme populations; multiply type II censoring; simultaneous confidence intervals; subset selection; two-parameter exponential distribution

1. Introduction

Experimenters sometimes want to identify the worst population (lower extreme population (LEP)) and the best population (upper extreme population (UEP)) from k independent populations at the same time for many biological or industrial experiments. There are several methods reported to determine the extreme populations, such as the simultaneous confidence interval (SCI) and subset selection procedure. Hsu [1] presented the SCIs for all distances from the UEP for the normal distributions and nonparametric models. Mishra and Dudewicz [2] studied the simultaneous selection of extreme populations under the normal distribution assumption for a complete sample, they formulated the subset selection procedure to select two non-empty subsets S_L and S_U, where S_L contains at least LEP and S_U contains at least UEP with a preset probability being at least $1 - \alpha$.

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ISSN 0094-9655 print/ISSN 1563-5163 online

DOI: 10.1080/00949650903222335
http://www.informaworld.com

In biological research or industrial areas, the normal probability model is applied universally. There are also many applications of exponential distribution in the analysis of reliability and life test experiments [3–6]. Suppose $\pi_1, \ldots, \pi_k$ represent k ($k \geq 2$) independent populations and the ith population π_i has a two-parameter exponential distribution denoted by $E(\mu_i, \theta)$, where μ_i is the unknown location parameter and θ is the common unknown scale parameter, $i = 1, 2, \ldots, k$. The location parameter of two-parameter exponential distribution is the so-called threshold value or 'guarantee time' parameter, and we can use it to identify the extreme populations. Misra and Dhariyal [7] proposed the SCIs for all distances from the worst and best populations for the two-parameter exponential distribution under complete samples.

Furthermore, in life-testing experiments, the experimenter may not always be in a position to observe the lifetimes of all the items put on test. This may be because of time limitations and/or other restrictions (such as money and material resources, etc.) on data collection. Censoring arises when some lifetimes of products are missing or for implementing some purposes of experimental designs. There are several types of censoring schemes and the type II censoring scheme is the most common. Many authors, Mann *et al*. [8], Lawless [9], and Meeker and Escobar [10] have studied the type II censoring with different failure time distributions. Let $Y_{i,1}, \ldots, Y_{i,n}$ be independent and identically distributed random variables from a population π_i and $Y_{i,(1)} \leq Y_{i,(2)} \leq \cdots \leq Y_{i,(n)}$ be the order statistics of $Y_{i,j}$, $j = 1, 2, \ldots, n$, for a given $i = 1, 2, \ldots, k$. Suppose that n items are put on life test, and an experimenter fails to observe the first r, the last s and the middle l observations, then only $n - r - l - s$ lifetimes are obtained and they are denoted by $(Y_{i,(r+1)}, \ldots, Y_{i,(r+u)}, Y_{i,(r+u+l+1)}, \ldots, Y_{i,(n-s)})$, $i = 1, 2, \ldots, k$. This type of censoring is known as the multiply type II censoring. Moreover, when $r = 0$, $l = 0$, $s > 0$, this is the so-called right type II-censored sample; when $r > 0$, $l = 0$, $s = 0$, this is the so-called left type II-censored sample; when $r > 0$, $l = 0$, $s > 0$, this is the so-called doubly type II-censored sample; when $r = 0$, $l > 0$, $s = 0$, this is the so-called middle type II-censored sample. For complete sample and right type II-censored sample, Chen and Vanichbuncha [11] proposed the SCIs for all distances from the UEP for the two-parameter exponential distributions.

However, if multiply type II censoring occurred, then the above methods may not be an appropriate choice. In this paper, for the common unknown scale parameter, 14 estimators including 12 weight moment estimators (WME) modified from Wu and Yang [12], approximate maximum likelihood estimator (AMLE) derived from Balakrishnan [13] and best linear unbiased estimator (BLUE) advanced by Balasubramanian and Balakrishnan [14] are considered. Then, we have 14 estimators for the location parameter of each exponential distribution. Based on these 14 estimators, we can propose the SCIs for all distances from the UEP and LEP for k independent exponential distributions under multiply type II censoring. The main theoretical results for the 14 SCIs and the Monte Carlo simulation procedure for obtaining the critical values are given in Section 2. The simulation comparison among 14 SCIs is described in Section 3. In Section 4, the subset selection procedure for extreme populations is provided. In Section 5, two numerical examples are given for illustration. Finally, the conclusions are summarized in Section 6.

2. SCIs for all distances from the LEP and UEP

For population π_i, let $Y_{i,(r+1)} \leq \cdots \leq Y_{i,(r+u)} \leq Y_{i,(r+u+l+1)} \leq \cdots \leq Y_{i,(n-s)}$ be a multiply type II-censored sample from a two-parameter exponential distribution denoted by $E(\mu_i, \theta)$, where μ_i is the unknown location parameter and θ is the common unknown scale parameter, $i = 1, 2, \ldots, k$. Let $\mu_{[1]} \leq \mu_{[2]} \leq \cdots \leq \mu_{[k]}$ be the ordered location parameters and let $\pi_{(i)}$ denote a population associated with $\mu_{[i]}$, $i = 1, 2, \dots, k$. The probability density function for the ith population is given by

$$f(y; \mu_i, \theta) = \begin{cases} \dfrac{1}{\theta} \exp\left\{-\dfrac{y-\mu_i}{\theta}\right\}, & \text{if } y > \mu_i, \mu_i > 0, \theta > 0 \\ 0, & \text{otherwise} \end{cases}, \tag{1}$$

$i = 1, 2, \dots, k$. Let $Y^*_{i,(j)} = (Y_{i,(j)} - \mu_i)/\theta$, then $(Y^*_{i,(r+1)}, \dots, Y^*_{i,(r+u)}, Y^*_{i,(r+u+l+1)}, \dots, Y^*_{i,(n-s)})$ is a multiply type II-censored sample from a standard exponential distribution, $i = 1, 2, \dots, k$.

In this paper, we consider 14 estimators to estimate the scale parameter θ. The first 12 estimators are the 12 WMEs modified from Wu and Yang [12], denoted by

$$\hat{\theta}^*_t = \frac{1}{k}\sum_{i=1}^{k} \hat{\theta}^*_{i,t}, \quad t = 1, 2, \dots, 12, \tag{2}$$

where

$$\hat{\theta}^*_{i,1} = w_{11}\sum_{j=r+2}^{r+u}(Y_{i,(j)} - Y_{i,(r+1)}) + w_{12}\sum_{j=r+u+l+1}^{n-s}(Y_{i,(j)} - Y_{i,(r+1)}) \equiv w_{11}C_{i,11} + w_{12}C_{i,12}, \tag{3}$$

$$\begin{aligned}\hat{\theta}^*_{i,2} &= w_{21}\left[r(Y_{i,(r+2)} - Y_{i,(r+1)}) + \sum_{j=r+2}^{r+u}(Y_{i,(j)} - Y_{i,(r+1)}) + l(Y_{i,(r+u)} - Y_{i,(r+1)})\right] \\ &\quad + w_{22}\left[\sum_{j=r+u+l+1}^{n-s}(Y_{i,(j)} - Y_{i,(r+1)}) + s(Y_{i,(n-s)} - Y_{i,(r+1)})\right] \\ &\equiv w_{21}C_{i,21} + w_{22}C_{i,22}\end{aligned} \tag{4}$$

$$\begin{aligned}\hat{\theta}^*_{i,3} &= w_{31}\left[r(Y_{i,(r+2)} - Y_{i,(r+1)}) + \sum_{j=r+2}^{r+u}(Y_{i,(j)} - Y_{i,(r+1)})\right] \\ &\quad + w_{32}\left[l(Y_{i,(r+u+l+1)} - Y_{i,(r+1)}) + \sum_{j=r+u+l+1}^{n-s}(Y_{i,(j)} - Y_{i,(r+1)}) + s(Y_{i,(n-s)} - Y_{i,(r+1)})\right] \\ &\equiv w_{31}C_{i,31} + w_{32}C_{i,32},\end{aligned} \tag{5}$$

$$\begin{aligned}\hat{\theta}^*_{i,4} &= w_{41}\left[\sum_{j=r+2}^{r+u}(Y_{i,(j)} - Y_{i,(r+1)}) + l(Y_{i,(r+u)} - Y_{i,(r+1)})\right] \\ &\quad + w_{42}\left[\sum_{j=r+u+l+1}^{n-s}(Y_{i,(j)} - Y_{i,(r+1)}) + s(Y_{i,(n-s)} - Y_{i,(r+1)})\right] \\ &\equiv w_{41}C_{i,41} + w_{42}C_{i,42},\end{aligned} \tag{6}$$

$$\hat{\theta}^*_{i,5} = w_{51} \sum_{j=r+2}^{r+u} (Y_{i,(j)} - Y_{i,(r+1)}) + w_{52} \left[l(Y_{i,(r+u+l+1)} - Y_{i,(r+1)}) + \sum_{j=r+u+l+1}^{n-s} (Y_{i,(j)} - Y_{i,(r+1)}) + s(Y_{i,(n-s)} - Y_{i,(r+1)}) \right] \equiv w_{51}C_{i,51} + w_{52}C_{i,52}, \tag{7}$$

$$\hat{\theta}^*_{i,6} = w_{61} \left[r(Y_{i,(r+2)} - Y_{i,(r+1)}) + \sum_{j=r+2}^{r+u} (Y_{i,(j)} - Y_{i,(r+1)}) + l(Y_{i,(r+u)} - Y_{i,(r+1)}) \right] + w_{62} \sum_{j=r+u+l+1}^{n-s} (Y_{i,(j)} - Y_{i,(r+1)}) \equiv w_{61}C_{i,61} + w_{62}C_{i,62}, \tag{8}$$

$$\hat{\theta}^*_{i,7} = w_{71} \left[r(Y_{i,(r+2)} - Y_{i,(r+1)}) + \sum_{j=r+2}^{r+u} (Y_{i,(j)} - Y_{i,(r+1)}) \right] + w_{72} \left[l(Y_{i,(r+u+l+1)} - Y_{i,(r+1)}) + \sum_{j=r+u+l+1}^{n-s} (Y_{i,(j)} - Y_{i,(r+1)}) \right] \equiv w_{71}C_{i,71} + w_{72}C_{i,72}, \tag{9}$$

$$\hat{\theta}^*_{i,8} = w_{81} \left[\sum_{j=r+2}^{r+u} (Y_{i,(j)} - Y_{i,(r+1)}) + l(Y_{i,(r+u)} - Y_{i,(r+1)}) \right] + w_{82} \sum_{j=r+u+l+1}^{n-s} (Y_{i,(j)} - Y_{i,(r+1)}) \equiv w_{81}C_{i,81} + w_{82}C_{i,82}, \tag{10}$$

$$\hat{\theta}^*_{i,9} = w_{91} \sum_{j=r+2}^{r+u} (Y_{i,(j)} - Y_{i,(r+1)}) + w_{92} \left[l(Y_{i,(r+u+l+1)} - Y_{i,(r+1)}) + \sum_{j=r+u+l+1}^{n-s} (Y_{i,(j)} - Y_{i,(r+1)}) \right] \equiv w_{91}C_{i,91} + w_{92}C_{i,92}, \tag{11}$$

$$\hat{\theta}^*_{i,10} = w_{101} \left[r(Y_{i,(r+2)} - Y_{i,(r+1)}) + \sum_{j=r+2}^{r+u} (Y_{i,(j)} - Y_{i,(r+1)}) \right] + w_{102} \left[\sum_{j=r+u+l+1}^{n-s} (Y_{i,(j)} - Y_{i,(r+1)}) + s(Y_{i,(n-s)} - Y_{i,(r+1)}) \right] \equiv w_{101}C_{i,101} + w_{102}C_{i,102}, \tag{12}$$

$$\hat{\theta}^*_{i,11} = w_{111} \sum_{j=r+2}^{r+u} (Y_{i,(j)} - Y_{i,(r+1)}) + w_{112} \left[\sum_{j=r+u+l+1}^{n-s} (Y_{i,(j)} - Y_{i,(r+1)}) + s(Y_{i,(n-s)} - Y_{i,(r+1)}) \right] \equiv w_{111}C_{i,111} + w_{112}C_{i,112}, \tag{13}$$

$$\hat{\theta}^*_{i,12} = w_{121}\left[r(Y_{i,(r+2)} - Y_{i,(r+1)}) + \sum_{j=r+2}^{r+u}(Y_{i,(j)} - Y_{i,(r+1)})\right] + w_{122}\sum_{j=r+u+l+1}^{n-s}(Y_{i,(j)} - Y_{i,(r+1)})$$
$$\equiv w_{121}C_{i,121} + w_{122}C_{i,122}, \tag{14}$$

$i = 1, 2, \ldots, k$. The weights w_{t1} and w_{t2} are determined by minimizing the mean squared error (MSE) given by

$$\text{MSE}(\hat{\theta}^*_t) = \theta^2\left[\frac{1}{k}(w_{t1}^2 b_{t1} + w_{t2}^2 b_{t2} + 2w_{t1}w_{t2}b_{t3}) + (w_{t1}a_{t1} + w_{t2}a_{t2} - 1)^2\right], \tag{15}$$

where $a_{tv} = E(C_{i,tv})/\theta$, $b_{tv} = \text{Var}(C_{i,tv})/\theta^2$, and $b_{t3} = \text{Cov}(C_{i,t1}, C_{i,t2})/\theta^2$, $t = 1, 2, \ldots, 12$, $v = 1, 2$, $i = 1, 2, \ldots, k$, are given in Chen [15]. Next, by differentiating Equation (15) with respect to w_{t1} and w_{t2}, and setting the derivatives equal to zero for each t, $t = 1, 2, \ldots, 12$. After some calculations, we get the weights for 12 WMEs as follows:

$$w_{t1} = \frac{(1/k)b_{t2}a_{t1} - (1/k)b_{t3}a_{t2}}{((1/k)b_{t1} + a_{t1}^2)((1/k)b_{t2} + a_{t2}^2) - ((1/k)b_{t3} + a_{t1}a_{t2})^2}, \tag{16}$$

and

$$w_{t2} = \frac{(1/k)b_{t1}a_{t2} - (1/k)b_{t3}a_{t1}}{((1/k)b_{t1} + a_{t1}^2)((1/k)b_{t2} + a_{t2}^2) - ((1/k)b_{t3} + a_{t1}a_{t2})^2}, \tag{17}$$

$t = 1, 2, \ldots, 12$. In addition, replacing weights in Equation (15) by Equations (16) and (17), we can obtain the minimum MSE given by:

$$\text{MSE}(\hat{\theta}^*_t) = \frac{((1/k^2)b_{t1}b_{t2} - (1/k^2)b_{t3}^2)\theta^2}{((1/k)b_{t1} + a_{t1}^2)((1/k)b_{t2} + a_{t2}^2) - ((1/k)b_{t3} + a_{t1}a_{t2})^2}, \quad t = 1, 2, \ldots, 12. \tag{18}$$

The 13th estimator used in this paper is the AMLE derived by Balakrishnan [13] given as follows:

$$\hat{\theta}^*_{13} = \frac{\sum_{i=1}^{k}\hat{\theta}^*_{i,13}}{k}, \tag{19}$$

where

$$\hat{\theta}^*_{i,13} = \frac{\begin{array}{c}\sum_{j=r+1}^{r+u} Y_{i,(j)} + \sum_{j=r+u+l+1}^{n-s} Y_{i,(j)} + sY_{i,(n-s)} \\ +l\beta Y_{i,(r+u)} + l(1-\beta)Y_{i,(r+u+l+1)} - (n-r)Y_{i,(r+1)}\end{array}}{(n-r-s-l) - l\alpha}, \tag{20}$$

$$\alpha = \frac{q_{r+u+l+1}\ln q_{r+u+l+1} - q_{r+u}\ln q_{r+u}}{q_{r+u} - q_{r+u+l+1}} + \beta\ln q_{r+u} + (1-\beta)\ln q_{r+u+l+1},$$

$$\beta = \frac{q_{r+u}}{q_{r+u} - q_{r+u+l+1}} - \frac{q_{r+u}q_{r+u+l+1}}{(q_{r+u} - q_{r+u+l+1})^2}\ln\left(\frac{q_{r+u}}{q_{r+u+l+1}}\right),$$

and

$$q_j = 1 - \frac{j}{n}, \quad i = 1, 2, \ldots, k.$$

The last estimator is the BLUE derived by Balasubramanian and Balakrishnan [14], and it is as follows:

$$\hat{\theta}^*_{14} = \frac{\sum_{i=1}^{k}\hat{\theta}^*_{i,14}}{k}, \tag{21}$$

where

$$\hat{\theta}^*_{i,14} = \frac{1}{(n-s-r-l-2)+d_1}\left\{\sum_{j=r+1}^{r+u} Y_{i,(j)} - (n-r)Y_{i,(r+1)} + (n-r-u)Y_{i,(r+u)}\right.$$
$$\left. + \sum_{j=r+u+l+1}^{n-s} Y_{i,(j)} - (n-r-u-l)Y_{i,(r+u+l+1)} + sY_{i,(n-s)} + d_2(Y_{i,(r+u+l+1)} - Y_{i,(r+u)})\right\} \quad (22)$$

$$d_1 = \frac{\left(\sum_{j=r+u+1}^{r+u+l+1} 1/(n-j+1)\right)^2}{\sum_{j=r+u+1}^{r+u+l+1} 1/(n-j+1)^2}$$

and

$$d_2 = \frac{\sum_{j=r+u+1}^{r+u+l+1} 1/(n-j+1)}{\sum_{j=r+u+1}^{r+u+l+1} 1/(n-j+1)^2}, \quad i = 1, 2, \ldots, k.$$

Then the corresponding estimators of the location parameter of the ith population are

$$\hat{\mu}_{i,t} = Y_{i,(r+1)} - \hat{\theta}^*_t \sum_{j=1}^{r+1} \frac{1}{n-j+1}, \quad i = 1, 2, \ldots, k, \quad t = 1, 2, \ldots, 14. \quad (23)$$

We use those estimators to propose the following SCI for all distances from the LEP and UEP.

THEOREM 1 *When the scale parameter θ is unknown, let d_t be the percentile of the distribution of the random variable Z^*_t, where $Z^*_t = \max\{Z_{1,t} - Z_{i,t}, Z_{i,t} - Z_{k,t}, i \neq 1, k, Z_{1,t} - Z_{k,t}\}$ and $Z_{i,t} = n(Y_{i,(r+1)} - \mu_i)/\hat{\theta}^*_t$, $i = 1, 2, \ldots, k$, $t = 1, 2, \ldots, 14$. Then the $100(1-\alpha)\%$ SCI of $\mu_1 - \mu_{[1]}, \mu_2 - \mu_{[1]}, \ldots, \mu_k - \mu_{[1]}, \mu_{[k]} - \mu_1, \mu_{[k]} - \mu_2, \ldots, \mu_{[k]} - \mu_k$ are given by $[0, D_{1,t}]$, $[0, D_{2,t}], \ldots, [0, D_{k,t}]$, $[0, D^{1,t}]$, $[0, D^{2,t}], \ldots, [0, D^{k,t}]$, where*

$$D_{i,t} = \max\left\{\hat{\mu}_{i,t} - \min_{j\neq i} \hat{\mu}_{j,t} + d_t \frac{\hat{\theta}^*_t}{n}, 0\right\}$$
$$D^{i,t} = \max\left\{\max_{j\neq i} \hat{\mu}_{j,t} - \hat{\mu}_{i,t} + d_t \frac{\hat{\theta}^*_t}{n}, 0\right\}, \quad i = 1, 2, \ldots, k, \quad t = 1, 2, \ldots, 14. \quad (24)$$

Proof of Theorem 1 The proof is given in Appendix 1. ■

Since it is very hard to obtain the exact distribution of Z^*_t, the Monte Carlo method is used to obtain the approximate critical values d_t, $t = 1, 2, \ldots, 14$, and the procedure is described as follows:

(1) Generate a multiply type II-censored sample from a standard exponential distribution denoted by $(Y^*_{i,(r+1)}, \ldots, Y^*_{i,(r+u)}, Y^*_{i,(r+u+l+1)}, \ldots, Y^*_{i,(n-s)})$, $i = 1, 2, \ldots, k$.
(2) Compute $W^*_t = \hat{\theta}^*_t/\theta$, $t = 1, 2, \ldots, 14$, which are defined in Equations (3)–(14), (20), and (22) by replacing $Y_{i,(j)}$ by $Y^*_{i,(j)}$, $i = 1, 2, \ldots, k$, $j = r+1, \ldots, r+u, r+u+l+1, \ldots, n-s$.
(3) Compute $Z_{i,t} = nY^*_{i,(r+1)}/W^*_t$, $i = 1, 2, \ldots, k$, and then

$$Z^*_t = \max\{Z_{1,t} - Z_{2,t}, \ldots, Z_{1,t} - Z_{k-1,t}, Z_{1,t} - Z_{k,t}, \ldots, Z_{k-1,t} - Z_{k,t}\},$$
$$t = 1, 2, \ldots, 14.$$

(4) Repeat steps (1)–(3) 500,000 times.
(5) Sort Z_t^*s and the ordered Z_t^*s are denoted by $Z_{t,1}^* \leq Z_{t,2}^* \leq \ldots \leq Z_{t,500,000}^*$, $t = 1, 2, \ldots, 14$.
(6) Then the critical values of $d_t = Z_{t,[500,000\times(1-\alpha)]+1}^*$ can be obtained, where $[x]$ represents the greatest integer less than x, $t = 1, 2, \ldots, 14$.

The simulated critical values of d_t based on the multiply type II-censored sample are tabulated in Table 1 for $1 - \alpha = 0.95$. The results cover different censoring schemes assigning appropriate values to r, u, l and s for various combinations of $k = 2(2)\ 10$ and $n = 12, 24, 36$.

Remarks (1) Let $D_{i,t} = \infty$, $i = 1, 2, \ldots, k$, then the problem of SCI for all distances from the LEP and UEP is reduced to all distances from the UEP, and the critical values of d_t becomes the percentile of the distribution of the random variable Z_t^{**}, where $Z_t^{**} = \max_{i=1,\ldots,k-1}\{Z_{i,t} - Z_{k,t}\}$, $t = 1, 2, \ldots, 14$. The simulated critical values of d_t based on the multiply type II-censored sample are tabulated in Table 2 for $1 - \alpha = 0.95$.
(2) When $k = 2$, the problem of SCI for all distances from the LEP and UEP is identical to all distances from the UEP. Thus, the critical values in Table 1 are identical to the critical values in Table 2 when $k = 2$.
(3) Based on Tables 1 and 2, we summarize the findings as follows.
  (a) The direction of d_t based on 14 estimators is $d_{\text{AMLE}} > d_{\text{WME}} > d_{\text{BLUE}}$ for all situations. That is, the critical values of BLUE is always smaller than the critical values of WMEs (including WME1, WME2, ... , WME12), and the critical values of WMEs is always smaller than the critical values of AMLE.
  (b) For fixed n, d_t increases as k increases, and the speed of increase slows down when k increases.
  (c) For fixed k, d_t decreases as n increases, and the speed of decrease slows down when n increases.
  (d) For all situations, d_t increases as r increases for fixed k and n.
(4) For other combinations of n, k and $1 - \alpha$ under any type of censored sample, a software program to output the simulated critical values is written by using the IMSL Library of Absoft Fortran software package which is available upon request.

3. Simulation comparisons

Comparing the confidence length of SCIs given in Theorem 1 is equivalent to comparing the values of $d_t(\hat{\theta}_t^*/n)$. We use 500,000 simulation runs to obtain the values of $d_t(\hat{\theta}_t^*/n)$ for the 95% SCI, and summarize the results in Tables 3 and 4. The optimal SCI with minimum confidence length is marked by the symbol 'a'. We find that (1) for fixed n, the confidence length of SCIs increases as k increases, and the speed of increase slows down when k increases; (2) for fixed k, the confidence length of SCIs decreases as n increases, and the speed of decrease slows down when n increases; (3) for all situations, the confidence length of SCIs increases as r increases for fixed k and n; (4) the SCIs based on WME4, AMLE or BLUE have confidence length which is much shorter than the SCIs based on other estimators under different censored schemes for various combinations of k and n.

4. Subset selection of extreme populations

Mishra and Dudewicz [2] proposed the subset selection procedure which is to choose two nonempty subsets S_{L} and S_{U} simultaneously according to one decision rule such that S_{L} contains

Table 1. The critical values of d_t for the 95% SCI for all distances from the LEP and UEP.

k	n	r	u	l	s	WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
2	12	2	5	2	2	5.225	5.224	5.235	5.221	5.225	5.224	5.235	5.221	5.225	5.235	5.225	5.235	5.547	4.866
		3	5	2	1	6.342	6.340	6.368	6.336	6.342	6.340	6.368	6.336	6.342	6.368	6.342	6.368	6.706	5.898
		6	3	0	0	11.106	11.436	11.436	11.106	11.106	11.436	11.436	11.106	11.106	11.436	11.106	11.436	12.115	10.096
		3	3	0	3	6.840	6.763	6.763	6.735	6.735	7.030	7.030	6.840	6.840	6.763	6.735	7.030	7.348	6.123
		0	3	6	0	2.573	2.572	2.579	2.572	2.579	2.572	2.579	2.572	2.579	2.573	2.573	2.573	2.654	2.443
	24	2	5	2	2	4.462	4.458	4.458	4.457	4.458	4.462	4.465	4.462	4.465	4.458	4.457	4.461	4.571	4.342
		3	5	2	1	5.265	5.257	5.257	5.256	5.256	5.265	5.264	5.265	5.264	5.257	5.258	5.264	5.390	5.121
		6	3	0	0	7.620	7.621	7.621	7.620	7.620	7.621	7.621	7.620	7.620	7.621	7.620	7.621	7.838	7.402
		3	3	0	3	5.334	5.310	5.310	5.310	5.310	5.339	5.339	5.334	5.334	5.310	5.310	5.339	5.461	5.158
		0	3	6	0	2.408	2.408	2.410	2.408	2.410	2.408	2.410	2.408	2.410	2.408	2.408	2.408	2.460	2.358
	36	2	5	2	2	4.298	4.294	4.294	4.293	4.294	4.298	4.299	4.298	4.299	4.294	4.294	4.298	4.361	4.225
		3	5	2	1	5.014	5.012	5.011	5.012	5.011	5.013	5.014	5.014	5.014	5.012	5.013	5.014	5.090	4.931
		6	3	0	0	6.983	6.983	6.983	6.983	6.983	6.983	6.983	6.983	6.983	6.983	6.983	6.983	7.101	6.865
		3	3	0	3	5.038	5.033	5.033	5.033	5.033	5.039	5.039	5.038	5.038	5.033	5.033	5.039	5.119	4.948
		0	3	6	0	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.416	2.349
4	12	2	5	2	2	7.128	7.127	7.136	7.122	7.128	7.127	7.136	7.122	7.128	7.136	7.128	7.136	7.832	6.872
		3	5	2	1	8.610	8.609	8.624	8.605	8.610	8.609	8.624	8.605	8.610	8.624	8.610	8.624	9.434	8.297
		6	3	0	0	14.641	14.947	14.947	14.641	14.641	14.947	14.947	14.641	14.641	14.947	14.641	14.947	16.733	13.944
		3	3	0	3	9.086	9.003	9.003	8.983	8.983	9.245	9.245	9.086	9.086	9.003	8.983	9.245	10.266	8.555
		0	3	6	0	3.791	3.790	3.795	3.790	3.795	3.790	3.795	3.790	3.795	3.791	3.791	3.791	4.007	3.689
	24	2	5	2	2	6.335	6.330	6.328	6.330	6.328	6.334	6.339	6.335	6.339	6.331	6.331	6.335	6.576	6.248
		3	5	2	1	7.381	7.376	7.377	7.377	7.376	7.381	7.382	7.381	7.383	7.375	7.375	7.380	7.662	7.280
		6	3	0	0	10.509	10.514	10.514	10.509	10.509	10.514	10.514	10.509	10.509	10.514	10.509	10.514	10.965	10.356
		3	3	0	3	7.443	7.427	7.427	7.428	7.428	7.444	7.444	7.443	7.443	7.427	7.428	7.444	7.751	7.321
		0	3	6	0	3.652	3.653	3.653	3.653	3.653	3.653	3.653	3.653	3.653	3.652	3.652	3.652	3.770	3.614
	36	2	5	2	2	6.126	6.125	6.127	6.125	6.126	6.126	6.126	6.127	6.126	6.126	6.126	6.126	6.273	6.077
		3	5	2	1	7.109	7.106	7.106	7.106	7.106	7.109	7.108	7.109	7.108	7.106	7.107	7.108	7.276	7.049
		6	3	0	0	9.735	9.735	9.735	9.735	9.735	9.735	9.735	9.735	9.735	9.735	9.735	9.735	9.985	9.652
		3	3	0	3	7.129	7.123	7.123	7.123	7.123	7.128	7.128	7.129	7.129	7.123	7.123	7.128	7.306	7.062
		0	3	6	0	3.614	3.614	3.616	3.614	3.616	3.614	3.616	3.614	3.616	3.614	3.614	3.614	3.692	3.590

6	12	2	5	2	2	7.647	7.647	7.652	7.647	7.647	7.647	7.652	7.647	7.647	7.652	7.647	7.652	8.512	7.467
		3	5	2	1	9.230	9.226	9.244	9.225	9.230	9.226	9.244	9.225	9.230	9.244	9.230	9.244	10.237	9.003
		6	3	0	0	15.398	15.645	15.645	15.398	15.398	15.645	15.645	15.398	15.398	15.645	15.398	15.645	17.882	14.901
		3	3	0	3	9.591	9.527	9.527	9.507	9.507	9.718	9.718	9.591	9.591	9.527	9.507	9.718	11.040	9.200
		0	3	6	0	4.209	4.207	4.213	4.207	4.213	4.207	4.213	4.207	4.213	4.209	4.209	4.209	4.488	4.132
	24	2	5	2	2	6.931	6.926	6.925	6.926	6.924	6.932	6.935	6.932	6.935	6.927	6.926	6.931	7.225	6.864
		3	5	2	1	8.068	8.067	8.068	8.066	8.066	8.069	8.070	8.068	8.071	8.068	8.066	8.068	8.418	7.997
		6	3	0	0	11.381	11.383	11.383	11.381	11.381	11.383	11.383	11.381	11.381	11.383	11.381	11.383	11.933	11.270
		3	3	0	3	8.118	8.109	8.109	8.109	8.109	8.122	8.122	8.118	8.118	8.109	8.109	8.122	8.503	8.030
		0	3	6	0	4.099	4.099	4.098	4.099	4.098	4.099	4.098	4.099	4.098	4.099	4.099	4.099	4.245	4.069
	36	2	5	2	2	6.731	6.729	6.729	6.729	6.729	6.731	6.730	6.731	6.730	6.729	6.729	6.730	6.909	6.693
		3	5	2	1	7.791	7.790	7.789	7.790	7.790	7.791	7.790	7.791	7.790	7.790	7.790	7.792	7.998	7.748
		6	3	0	0	10.620	10.621	10.621	10.620	10.620	10.621	10.621	10.620	10.620	10.621	10.620	10.621	10.923	10.559
		3	3	0	3	7.797	7.788	7.788	7.788	7.788	7.796	7.796	7.797	7.797	7.788	7.788	7.796	8.010	7.743
		0	3	6	0	4.071	4.071	4.071	4.071	4.071	4.071	4.071	4.071	4.071	4.071	4.071	4.071	4.166	4.051
8	12	2	5	2	2	7.999	7.996	8.006	7.997	7.999	7.996	8.006	7.997	7.999	8.006	7.999	8.006	8.951	7.852
		3	5	2	1	9.618	9.624	9.630	9.619	9.618	9.624	9.630	9.619	9.618	9.630	9.618	9.630	10.738	9.444
		6	3	0	0	15.904	16.083	16.083	15.904	15.904	16.083	16.083	15.904	15.904	16.083	15.904	16.083	18.619	15.516
		3	3	0	3	9.905	9.852	9.852	9.844	9.844	10.009	10.009	9.905	9.905	9.852	9.844	10.009	11.525	9.604
		0	3	6	0	4.499	4.500	4.499	4.500	4.499	4.500	4.499	4.500	4.499	4.499	4.499	4.499	4.821	4.438
	24	2	5	2	2	7.297	7.292	7.292	7.291	7.291	7.298	7.299	7.298	7.298	7.291	7.292	7.297	7.625	7.244
		3	5	2	1	8.496	8.494	8.494	8.494	8.492	8.496	8.496	8.496	8.496	8.495	8.494	8.497	8.881	8.438
		6	3	0	0	11.933	11.936	11.936	11.933	11.933	11.936	11.936	11.933	11.933	11.936	11.933	11.936	12.542	11.846
		3	3	0	3	8.523	8.507	8.507	8.507	8.507	8.523	8.523	8.523	8.523	8.507	8.507	8.523	8.942	8.445
		0	3	6	0	4.397	4.397	4.396	4.397	4.396	4.397	4.396	4.397	4.396	4.397	4.397	4.397	4.561	4.373
	36	2	5	2	2	7.119	7.117	7.117	7.117	7.117	7.119	7.119	7.118	7.119	7.117	7.117	7.119	7.317	7.088
		3	5	2	1	8.217	8.218	8.217	8.217	8.217	8.217	8.218	8.217	8.218	8.218	8.217	8.217	8.447	8.184
		6	3	0	0	11.156	11.154	11.154	11.156	11.156	11.154	11.154	11.156	11.156	11.154	11.156	11.154	11.491	11.108
		3	3	0	3	8.215	8.213	8.213	8.213	8.213	8.215	8.215	8.215	8.215	8.213	8.213	8.215	8.459	8.177
		0	3	6	0	4.373	4.373	4.373	4.373	4.373	4.373	4.373	4.373	4.373	4.373	4.373	4.373	4.481	4.357

(*Continued*)

Table 1. Continued

k	n	r	u	l	s	WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
10	12	2	5	2	2	8.247	8.247	8.250	8.247	8.247	8.247	8.250	8.247	8.247	8.250	8.247	8.250	9.268	8.130
		3	5	2	1	9.910	9.906	9.920	9.903	9.910	9.906	9.920	9.903	9.910	9.920	9.910	9.920	11.097	9.760
		6	3	0	0	16.252	16.427	16.427	16.252	16.252	16.427	16.427	16.252	16.252	16.427	16.252	16.427	19.120	15.933
		3	3	0	3	10.163	10.124	10.124	10.115	10.115	10.247	10.247	10.163	10.163	10.124	10.115	10.247	11.900	9.916
		0	3	6	0	4.709	4.707	4.712	4.707	4.712	4.707	4.712	4.707	4.712	4.709	4.709	4.709	5.059	4.658
	24	2	5	2	2	7.580	7.574	7.574	7.574	7.574	7.580	7.584	7.581	7.585	7.575	7.575	7.580	7.931	7.535
		3	5	2	1	8.790	8.785	8.786	8.785	8.785	8.789	8.793	8.790	8.792	8.787	8.785	8.790	9.198	8.739
		6	3	0	0	12.280	12.283	12.283	12.280	12.280	12.283	12.283	12.280	12.280	12.283	12.280	12.283	12.926	12.208
		3	3	0	3	8.838	8.830	8.830	8.831	8.831	8.841	8.841	8.838	8.838	8.830	8.831	8.841	9.295	8.779
		0	3	6	0	4.614	4.614	4.613	4.614	4.613	4.614	4.613	4.614	4.613	4.614	4.614	4.614	4.792	4.594
	36	2	5	2	2	7.411	7.412	7.412	7.412	7.412	7.411	7.412	7.411	7.412	7.411	7.412	7.411	7.626	7.388
		3	5	2	1	8.527	8.527	8.526	8.527	8.526	8.527	8.528	8.527	8.528	8.527	8.527	8.527	8.773	8.499
		6	3	0	0	11.525	11.524	11.524	11.525	11.525	11.524	11.524	11.525	11.525	11.524	11.525	11.524	11.881	11.485
		3	3	0	3	8.537	8.535	8.535	8.535	8.535	8.538	8.538	8.537	8.537	8.535	8.535	8.538	8.799	8.506
		0	3	6	0	4.603	4.603	4.604	4.603	4.604	4.603	4.604	4.603	4.604	4.603	4.603	4.603	4.720	4.590

Table 2. The critical values of d_t for the 95% SCI for all distances from the UEP.

k	n	r	u	l	s	WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
2	12	2	5	2	2	5.225	5.224	5.235	5.221	5.225	5.224	5.235	5.221	5.225	5.235	5.225	5.235	5.547	4.866
		3	5	2	1	6.342	6.340	6.368	6.336	6.342	6.340	6.368	6.336	6.342	6.368	6.342	6.368	6.706	5.898
		6	3	0	0	11.106	11.436	11.436	11.106	11.106	11.436	11.436	11.106	11.106	11.436	11.106	11.436	12.115	10.096
		3	3	0	3	6.840	6.763	6.763	6.735	6.735	7.030	7.030	6.840	6.840	6.763	6.735	7.030	7.348	6.123
		0	3	6	0	2.573	2.572	2.579	2.572	2.579	2.572	2.579	2.572	2.579	2.573	2.573	2.573	2.654	2.443
	24	2	5	2	2	4.462	4.458	4.458	4.457	4.458	4.462	4.465	4.462	4.465	4.458	4.457	4.461	4.571	4.342
		3	5	2	1	5.265	5.257	5.257	5.256	5.256	5.265	5.264	5.265	5.264	5.257	5.258	5.264	5.390	5.121
		6	3	0	0	7.620	7.621	7.621	7.620	7.620	7.621	7.621	7.620	7.620	7.621	7.620	7.621	7.838	7.402
		3	3	0	3	5.334	5.310	5.310	5.310	5.310	5.339	5.339	5.334	5.334	5.310	5.310	5.339	5.461	5.158
		0	3	6	0	2.408	2.408	2.410	2.408	2.410	2.408	2.410	2.408	2.410	2.408	2.408	2.408	2.460	2.358
	36	2	5	2	2	4.298	4.294	4.294	4.293	4.294	4.298	4.299	4.298	4.299	4.294	4.294	4.298	4.361	4.225
		3	5	2	1	5.014	5.012	5.011	5.012	5.011	5.013	5.014	5.014	5.014	5.012	5.013	5.014	5.090	4.931
		6	3	0	0	6.983	6.983	6.983	6.983	6.983	6.983	6.983	6.983	6.983	6.983	6.983	6.983	7.101	6.865
		3	3	0	3	5.038	5.033	5.033	5.033	5.033	5.039	5.039	5.038	5.038	5.033	5.033	5.039	5.119	4.948
		0	3	6	0	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.384	2.416	2.349
4	12	2	5	2	2	6.678	6.678	6.687	6.675	6.678	6.678	6.687	6.675	6.678	6.687	6.678	6.687	7.343	6.441
		3	5	2	1	8.032	8.037	8.056	8.029	8.032	8.037	8.056	8.029	8.032	8.056	8.032	8.056	8.803	7.742
		6	3	0	0	13.573	13.860	13.860	13.573	13.573	13.860	13.860	13.573	13.573	13.860	13.573	13.860	15.512	12.927
		3	3	0	3	8.474	8.400	8.400	8.383	8.383	8.618	8.618	8.474	8.474	8.400	8.383	8.618	9.580	7.984
		0	3	6	0	3.608	3.609	3.613	3.609	3.613	3.609	3.613	3.609	3.613	3.608	3.608	3.608	3.817	3.514
	24	2	5	2	2	5.951	5.945	5.944	5.946	5.944	5.951	5.951	5.951	5.951	5.945	5.945	5.951	6.175	5.867
		3	5	2	1	6.930	6.925	6.925	6.925	6.925	6.932	6.931	6.931	6.931	6.925	6.925	6.931	7.194	6.835
		6	3	0	0	9.793	9.796	9.796	9.793	9.793	9.796	9.796	9.793	9.793	9.796	9.793	9.796	10.219	9.651
		3	3	0	3	6.990	6.973	6.973	6.972	6.972	6.991	6.991	6.990	6.990	6.973	6.972	6.991	7.275	6.871
		0	3	6	0	3.487	3.487	3.488	3.487	3.488	3.487	3.488	3.487	3.488	3.487	3.487	3.487	3.598	3.450
	36	2	5	2	2	5.769	5.765	5.765	5.765	5.765	5.769	5.768	5.769	5.768	5.765	5.765	5.769	5.903	5.719
		3	5	2	1	6.670	6.668	6.668	6.668	6.668	6.670	6.668	6.670	6.669	6.668	6.668	6.669	6.827	6.614
		6	3	0	0	9.089	9.088	9.088	9.089	9.089	9.088	9.088	9.089	9.089	9.088	9.089	9.088	9.322	9.011
		3	3	0	3	6.695	6.692	6.692	6.692	6.692	6.698	6.698	6.695	6.695	6.692	6.692	6.698	6.863	6.635
		0	3	6	0	3.446	3.446	3.447	3.446	3.447	3.446	3.447	3.446	3.447	3.446	3.446	3.446	3.520	3.422

(*Continued*)

Table 2. Continued

k	*n*	*r*	*u*	*l*	*s*	WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
6	12	2	5	2	2	7.259	7.256	7.261	7.257	7.259	7.256	7.261	7.257	7.259	7.261	7.259	7.261	8.075	7.084
		3	5	2	1	8.722	8.725	8.739	8.720	8.722	8.725	8.739	8.720	8.722	8.739	8.722	8.739	9.675	8.509
		6	3	0	0	14.416	14.631	14.631	14.416	14.416	14.631	14.631	14.416	14.416	14.631	14.416	14.631	16.741	13.951
		3	3	0	3	9.042	8.985	8.985	8.971	8.971	9.156	9.156	9.042	9.042	8.985	8.971	9.156	10.417	8.681
		0	3	6	0	4.067	4.064	4.066	4.064	4.066	4.064	4.066	4.064	4.066	4.067	4.067	4.067	4.338	3.994
	24	2	5	2	2	6.599	6.592	6.592	6.591	6.591	6.598	6.601	6.599	6.601	6.591	6.592	6.600	6.877	6.534
		3	5	2	1	7.651	7.648	7.646	7.647	7.647	7.650	7.649	7.650	7.649	7.648	7.648	7.651	7.979	7.581
		6	3	0	0	10.716	10.718	10.718	10.716	10.716	10.718	10.718	10.716	10.716	10.718	10.716	10.718	11.236	10.612
		3	3	0	3	7.696	7.684	7.684	7.684	7.684	7.701	7.701	7.696	7.696	7.684	7.684	7.701	8.057	7.610
		0	3	6	0	3.970	3.969	3.970	3.969	3.970	3.969	3.970	3.969	3.970	3.970	3.970	3.970	4.111	3.941
	36	2	5	2	2	6.408	6.407	6.407	6.406	6.406	6.407	6.409	6.407	6.409	6.407	6.406	6.408	6.578	6.373
		3	5	2	1	7.388	7.388	7.387	7.387	7.388	7.388	7.389	7.388	7.389	7.387	7.388	7.389	7.584	7.347
		6	3	0	0	10.009	10.009	10.009	10.009	10.009	10.009	10.009	10.009	10.009	10.009	10.009	10.009	10.295	9.952
		3	3	0	3	7.398	7.389	7.389	7.389	7.389	7.397	7.397	7.398	7.398	7.389	7.389	7.397	7.600	7.347
		0	3	6	0	3.936	3.936	3.938	3.936	3.938	3.936	3.938	3.936	3.938	3.936	3.936	3.936	4.029	3.918
8	12	2	5	2	2	7.672	7.671	7.676	7.669	7.672	7.671	7.676	7.669	7.672	7.676	7.672	7.676	8.586	7.532
		3	5	2	1	9.183	9.180	9.194	9.179	9.183	9.180	9.194	9.179	9.183	9.194	9.183	9.194	10.246	9.012
		6	3	0	0	15.067	15.238	15.238	15.067	15.067	15.238	15.238	15.067	15.067	15.238	15.067	15.238	17.639	14.700
		3	3	0	3	9.435	9.391	9.391	9.374	9.374	9.533	9.533	9.435	9.435	9.391	9.374	9.533	10.975	9.146
		0	3	6	0	4.389	4.387	4.387	4.387	4.387	4.387	4.387	4.387	4.387	4.389	4.389	4.389	4.699	4.326
	24	2	5	2	2	7.009	7.005	7.004	7.004	7.003	7.009	7.012	7.009	7.011	7.005	7.005	7.009	7.324	6.958
		3	5	2	1	8.115	8.111	8.111	8.111	8.112	8.114	8.118	8.114	8.117	8.111	8.110	8.114	8.482	8.059
		6	3	0	0	11.316	11.318	11.318	11.316	11.316	11.318	11.318	11.316	11.316	11.318	11.316	11.318	11.895	11.234
		3	3	0	3	8.147	8.134	8.134	8.134	8.134	8.148	8.148	8.147	8.147	8.134	8.134	8.148	8.549	8.074
		0	3	6	0	4.286	4.286	4.288	4.286	4.288	4.286	4.288	4.286	4.288	4.286	4.286	4.286	4.448	4.264

	36	2	5	2	2	6.832	6.831	6.830	6.831	6.831	6.832	6.832	6.832	6.832	6.830	6.831	6.831	7.023	6.803
		3	5	2	1	7.859	7.855	7.855	7.855	7.856	7.859	7.859	7.858	7.859	7.855	7.855	7.858	8.075	7.823
		6	3	0	0	10.584	10.581	10.581	10.584	10.584	10.581	10.581	10.584	10.584	10.581	10.584	10.581	10.902	10.539
		3	3	0	3	7.856	7.852	7.852	7.851	7.851	7.856	7.856	7.856	7.856	7.852	7.851	7.856	8.087	7.817
		0	3	6	0	4.269	4.269	4.267	4.269	4.267	4.269	4.267	4.269	4.267	4.269	4.269	4.269	4.375	4.253
10	12	2	5	2	2	7.963	7.963	7.966	7.961	7.963	7.963	7.966	7.961	7.963	7.966	7.963	7.966	8.945	7.847
		3	5	2	1	9.528	9.523	9.534	9.524	9.528	9.523	9.534	9.524	9.528	9.534	9.528	9.534	10.669	9.384
		6	3	0	0	15.501	15.638	15.638	15.501	15.501	15.638	15.638	15.501	15.501	15.638	15.501	15.638	18.237	15.197
		3	3	0	3	9.753	9.715	9.715	9.701	9.701	9.836	9.836	9.753	9.753	9.715	9.701	9.836	11.413	9.511
		0	3	6	0	4.613	4.612	4.619	4.612	4.619	4.612	4.619	4.612	4.619	4.613	4.613	4.613	4.957	4.564
	24	2	5	2	2	7.329	7.322	7.322	7.322	7.323	7.328	7.330	7.329	7.330	7.322	7.323	7.329	7.667	7.285
		3	5	2	1	8.456	8.453	8.454	8.453	8.454	8.456	8.455	8.455	8.456	8.453	8.454	8.456	8.851	8.409
		6	3	0	0	11.726	11.728	11.728	11.726	11.726	11.728	11.728	11.726	11.726	11.728	11.726	11.728	12.343	11.657
		3	3	0	3	8.505	8.499	8.499	8.498	8.498	8.503	8.503	8.505	8.505	8.499	8.498	8.503	8.945	8.448
		0	3	6	0	4.524	4.524	4.524	4.524	4.524	4.524	4.524	4.524	4.524	4.524	4.524	4.524	4.698	4.504
	36	2	5	2	2	7.161	7.161	7.160	7.160	7.160	7.161	7.162	7.161	7.162	7.161	7.160	7.161	7.367	7.137
		3	5	2	1	8.212	8.210	8.209	8.210	8.209	8.211	8.212	8.212	8.212	8.209	8.210	8.211	8.447	8.183
		6	3	0	0	11.005	11.005	11.005	11.005	11.005	11.005	11.005	11.005	11.005	11.005	11.005	11.005	11.345	10.967
		3	3	0	3	8.216	8.212	8.212	8.212	8.212	8.215	8.215	8.216	8.216	8.212	8.212	8.215	8.466	8.184
		0	3	6	0	4.513	4.513	4.513	4.513	4.513	4.513	4.513	4.513	4.513	4.513	4.513	4.513	4.628	4.500

Table 3. The values of $d_t(\hat{\theta}_t^*/n)$ for the 95% SCI for all distances from the LEP and UEP.

k	n	r	u	l	s	WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
2	12	2	5	2	2	0.4056	0.4056	0.4061	0.4054	0.4056	0.4056	0.4061	0.4054	0.4056	0.4061	0.4056	0.4061	0.4054	0.4053[a]
		3	5	2	1	0.4923	0.4922	0.4935	0.4920	0.4923	0.4922	0.4935	0.4920	0.4923	0.4935	0.4923	0.4935	0.4919[a]	0.4919[a]
		6	3	0	0	0.8417[a]	0.8550	0.8550	0.8417[a]	0.8417[a]	0.8550	0.8550	0.8417[a]	0.8417[a]	0.8550	0.8417[a]	0.8550	0.8417[a]	0.8417[a]
		3	3	0	3	0.5147	0.5117	0.5117	0.5105[a]	0.5105[a]	0.5226	0.5226	0.5147	0.5147	0.5117	0.5105[a]	0.5226	0.5105[a]	0.5105[a]
		0	3	6	0	0.2040	0.2040	0.2044	0.2040	0.2044	0.2040	0.2044	0.2040	0.2044	0.2040	0.2040	0.2040	0.2037[a]	0.2037[a]
	24	2	5	2	2	0.1810[a]	0.1810[a]	0.1810[a]	0.1810[a]	0.1810[a]	0.1810[a]	0.1811	0.1810[a]	0.1811	0.1810[a]	0.1810[a]	0.1810[a]	0.1810[a]	0.1810[a]
		3	5	2	1	0.2136	0.2134	0.2134	0.2133[a]	0.2134	0.2136	0.2136	0.2136	0.2136	0.2134	0.2134	0.2136	0.2133[a]	0.2133[a]
		6	3	0	0	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]
		3	3	0	3	0.2156	0.2150[a]	0.2150[a]	0.2150[a]	0.2150[a]	0.2158	0.2158	0.2156	0.2156	0.2150[a]	0.2150[a]	0.2158	0.2150[a]	0.2150[a]
		0	3	6	0	0.0982[a]	0.0982[a]	0.0983	0.0982[a]	0.0983	0.0982[a]	0.0983	0.0982[a]	0.0983	0.0982[a]	0.0982[a]	0.0982[a]	0.0983	0.0983
	36	2	5	2	2	0.1175	0.1174[a]	0.1174[a]	0.1174[a]	0.1174[a]	0.1174[a]	0.1175	0.1174[a]	0.1175	0.1174[a]	0.1174[a]	0.1174[a]	0.1174[a]	0.1174[a]
		3	5	2	1	0.1370[a]	0.1370[a]	0.1370[a]	0.1370[a]	0.1370[a]	0.1370[a]	0.1370[a]	0.1370[a]	0.1371	0.1370[a]	0.1370[a]	0.1371	0.1370[a]	0.1370[a]
		6	3	0	0	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]
		3	3	0	3	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]
		0	3	6	0	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0652[a]	0.0652[a]
4	12	2	5	2	2	0.5731	0.5730	0.5736	0.5727	0.5731	0.5730	0.5736	0.5727	0.5731	0.5736	0.5731	0.5736	0.5726[a]	0.5727
		3	5	2	1	0.6920	0.6919	0.6926	0.6917[a]	0.6920	0.6919	0.6926	0.6917[a]	0.6920	0.6926	0.6920	0.6926	0.6917[a]	0.6917[a]
		6	3	0	0	1.1622[a]	1.1781	1.1781	1.1622[a]	1.1622[a]	1.1781	1.1781	1.1622[a]	1.1622[a]	1.1781	1.1622[a]	1.1781	1.1622[a]	1.1622[a]
		3	3	0	3	0.7180	0.7134	0.7134	0.7125[a]	0.7125[a]	0.7260	0.7260	0.7180	0.7180	0.7134	0.7125[a]	0.7260	0.7125[a]	0.7125[a]
		0	3	6	0	0.3081	0.3080	0.3083	0.3080	0.3083	0.3080	0.3083	0.3080	0.3083	0.3081	0.3081	0.3081	0.3075[a]	0.3075[a]
	24	2	5	2	2	0.2604	0.2603	0.2602[a]	0.2603	0.2602[a]	0.2604	0.2605	0.2604	0.2605	0.2603	0.2603	0.2604	0.2603	0.2603
		3	5	2	1	0.3035	0.3034	0.3034	0.3034	0.3034	0.3035	0.3036	0.3035	0.3036	0.3033[a]	0.3033[a]	0.3035	0.3033[a]	0.3033[a]
		6	3	0	0	0.4316[a]	0.4317	0.4317	0.4316[a]	0.4316[a]	0.4317	0.4317	0.4316[a]	0.4316[a]	0.4317	0.4316[a]	0.4317	0.4316[a]	0.4316[a]
		3	3	0	3	0.3054	0.3050[a]	0.3050[a]	0.3050[a]	0.3050[a]	0.3054	0.3054	0.3054	0.3054	0.3050[a]	0.3050[a]	0.3054	0.3050[a]	0.3050[a]
		0	3	6	0	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]	0.1505[a]
	36	2	5	2	2	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]	0.1688[a]
		3	5	2	1	0.1959	0.1959	0.1958[a]	0.1958[a]	0.1958[a]	0.1959	0.1959	0.1959	0.1959	0.1958[a]	0.1959	0.1959	0.1958[a]	0.1958[a]
		6	3	0	0	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]	0.2681[a]
		3	3	0	3	0.1963	0.1962[a]	0.1962[a]	0.1962[a]	0.1962[a]	0.1962[a]	0.1962[a]	0.1963	0.1963	0.1962[a]	0.1962[a]	0.1962[a]	0.1962[a]	0.1962[a]
		0	3	6	0	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]	0.0997[a]

6	12	2	5	2	2	0.6222[a]	0.6222[a]	0.6224	0.6222[a]	0.6222[a]	0.6222[a]	0.6224	0.6222[a]	0.6222[a]	0.6224	0.6222[a]	0.6224	0.6223	0.6223
		3	5	2	1	0.7506	0.7502[a]	0.7513	0.7502[a]	0.7506	0.7502[a]	0.7513	0.7502[a]	0.7506	0.7513	0.7506	0.7513	0.7503	0.7502[a]
		6	3	0	0	1.2419[a]	1.2557	1.2557	1.2419[a]	1.2419[a]	1.2557	1.2557	1.2419[a]	1.2419[a]	1.2557	1.2419[a]	1.2557	1.2419[a]	1.2419[a]
		3	3	0	3	0.7713	0.7678	0.7678	0.7666[a]	0.7666[a]	0.7782	0.7782	0.7713	0.7713	0.7678	0.7666[a]	0.7782	0.7666[a]	0.7666[a]
		0	3	6	0	0.3449	0.3448	0.3452	0.3448	0.3452	0.3448	0.3452	0.3448	0.3452	0.3449	0.3449	0.3449	0.3443[a]	0.3444
	24	2	5	2	2	0.2862	0.2861	0.2861	0.2861	0.2860[a]	0.2863	0.2864	0.2863	0.2864	0.2861	0.2861	0.2862	0.2860[a]	0.2860[a]
		3	5	2	1	0.3332	0.3332	0.3332	0.3331[a]	0.3331[a]	0.3332	0.3332	0.3332	0.3333	0.3332	0.3331[a]	0.3332	0.3332	0.3332
		6	3	0	0	0.4696[a]	0.4697	0.4697	0.4696[a]	0.4696[a]	0.4697	0.4697	0.4696[a]	0.4696[a]	0.4697	0.4696[a]	0.4697	0.4696[a]	0.4696[a]
		3	3	0	3	0.3348	0.3346[a]	0.3346[a]	0.3346[a]	0.3346[a]	0.3349	0.3349	0.3348	0.3348	0.3346[a]	0.3346[a]	0.3349	0.3346[a]	0.3346[a]
		0	3	6	0	0.1696	0.1696	0.1695[a]	0.1696	0.1695[a]	0.1696	0.1695[a]	0.1696	0.1695[a]	0.1696	0.1696	0.1696	0.1696	0.1696
	36	2	5	2	2	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]	0.1859[a]
		3	5	2	1	0.2152[a]	0.2152[a]	0.2152[a]	0.2152[a]	0.2152[a]	0.2152[a]	0.2152[a]	0.2152[a]	0.2152[a]	0.2152[a]	0.2152[a]	0.2153	0.2152[a]	0.2152[a]
		6	3	0	0	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]	0.2933[a]
		3	3	0	3	0.2153	0.2151[a]	0.2151[a]	0.2151[a]	0.2151[a]	0.2153	0.2153	0.2153	0.2153	0.2151[a]	0.2151[a]	0.2153	0.2151[a]	0.2151[a]
		0	3	6	0	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]	0.1125[a]
8	12	2	5	2	2	0.6546	0.6544	0.6550	0.6544	0.6546	0.6544	0.6550	0.6544	0.6546	0.6550	0.6546	0.6550	0.6543[a]	0.6543[a]
		3	5	2	1	0.7865[a]	0.7870	0.7871	0.7867	0.7865[a]	0.7870	0.7871	0.7867	0.7865[a]	0.7871	0.7865[a]	0.7871	0.7866	0.7866
		6	3	0	0	1.2934[a]	1.3032	1.3032	1.2934[a]	1.2934[a]	1.3032	1.3032	1.2934[a]	1.2934[a]	1.3032	1.2934[a]	1.3032	1.2934[a]	1.2934[a]
		3	3	0	3	0.8039	0.8007	0.8007	0.8005[a]	0.8005[a]	0.8097	0.8097	0.8039	0.8039	0.8007	0.8005[a]	0.8097	0.8005[a]	0.8005[a]
		0	3	6	0	0.3701	0.3702	0.3700	0.3702	0.3700	0.3702	0.3700	0.3702	0.3700	0.3701	0.3701	0.3701	0.3698[a]	0.3698[a]
	24	2	5	2	2	0.3020	0.3018[a]	0.3018[a]	0.3018[a]	0.3018[a]	0.3020	0.3020	0.3020	0.3020	0.3018[a]	0.3018[a]	0.3020	0.3018[a]	0.3018[a]
		3	5	2	1	0.3517	0.3516[a]	0.3516[a]	0.3516[a]	0.3516[a]	0.3517	0.3517	0.3517	0.3516[a]	0.3517	0.3516[a]	0.3517	0.3516[a]	0.3516[a]
		6	3	0	0	0.4935[a]	0.4937	0.4937	0.4935[a]	0.4935[a]	0.4937	0.4937	0.4935[a]	0.4935[a]	0.4937	0.4935[a]	0.4937	0.4935[a]	0.4935[a]
		3	3	0	3	0.3524	0.3519[a]	0.3519[a]	0.3519[a]	0.3519[a]	0.3524	0.3524	0.3524	0.3524	0.3519[a]	0.3519[a]	0.3524	0.3519[a]	0.3519[a]
		0	3	6	0	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]	0.1822[a]
	36	2	5	2	2	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]	0.1969[a]
		3	5	2	1	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]
		6	3	0	0	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]	0.3085[a]
		3	3	0	3	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]	0.2272[a]
		0	3	6	0	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]	0.1210[a]

(*Continued*)

Table 3. Continued

k	n	r	u	l	s	WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
10	12	2	5	2	2	0.6776	0.6775[a]	0.6777	0.6776	0.6776	0.6775[a]	0.6777	0.6776	0.6776	0.6777	0.6776	0.6777	0.6777	0.6777
		3	5	2	1	0.8139	0.8135	0.8144	0.8134[a]	0.8139	0.8135	0.8144	0.8134[a]	0.8139	0.8144	0.8139	0.8144	0.8134[a]	0.8134[a]
		6	3	0	0	1.3278[a]	1.3381	1.3381	1.3278[a]	1.3278[a]	1.3381	1.3381	1.3278[a]	1.3278[a]	1.3381	1.3278[a]	1.3381	1.3278[a]	1.3278[a]
		3	3	0	3	0.8288	0.8265	0.8265	0.8260[a]	0.8260[a]	0.8334	0.8334	0.8288	0.8288	0.8265	0.8260[a]	0.8334	0.8260[a]	0.8260[a]
		0	3	6	0	0.3885	0.3883	0.3887	0.3883	0.3887	0.3883	0.3887	0.3883	0.3887	0.3885	0.3885	0.3885	0.3882[a]	0.3882[a]
	24	2	5	2	2	0.3141	0.3140	0.3139[a]	0.3139[a]	0.3139[a]	0.3142	0.3143	0.3142	0.3144	0.3140	0.3140	0.3141	0.3140	0.3140
		3	5	2	1	0.3643	0.3641[a]	0.3642	0.3641[a]	0.3641[a]	0.3643	0.3644	0.3643	0.3644	0.3642	0.3641[a]	0.3643	0.3641[a]	0.3641[a]
		6	3	0	0	0.5086[a]	0.5088	0.5088	0.5086[a]	0.5086[a]	0.5088	0.5088	0.5086[a]	0.5086[a]	0.5088	0.5086[a]	0.5088	0.5086[a]	0.5086[a]
		3	3	0	3	0.3659	0.3657[a]	0.3657[a]	0.3657[a]	0.3657[a]	0.3661	0.3661	0.3659	0.3659	0.3657[a]	0.3657[a]	0.3661	0.3657[a]	0.3657[a]
		0	3	6	0	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]	0.1914[a]
	36	2	5	2	2	0.2052[a]	0.2052[a]	0.2053	0.2052[a]	0.2052[a]	0.2052[a]	0.2052[a]	0.2052[a]	0.2052[a]	0.2052[a]	0.2052[a]	0.2052[a]	0.2052[a]	0.2052[a]
		3	5	2	1	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]	0.2361[a]
		6	3	0	0	0.3191	0.3190[a]	0.3190[a]	0.3191	0.3191	0.3190[a]	0.3190[a]	0.3191	0.3191	0.3190[a]	0.3191	0.3190[a]	0.3191	0.3191
		3	3	0	3	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]	0.2363[a]
		0	3	6	0	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]	0.1275[a]

Note: [a]SCI with minimum confidence length.

Table 4. The values of $d_t(\hat{\theta}_t^*/n)$ for the 95% SCI for all distances from the UEP.

k	n	r	u	l	s	WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
2	12	2	5	2	2	0.4056	0.4056	0.4061	0.4054	0.4056	0.4056	0.4061	0.4054	0.4056	0.4061	0.4056	0.4061	0.4054	0.4053[a]
		3	5	2	1	0.4923	0.4922	0.4935	0.4920	0.4923	0.4922	0.4935	0.4920	0.4923	0.4935	0.4923	0.4935	0.4919[a]	0.4919[a]
		6	3	0	0	0.8417[a]	0.8550	0.8550	0.8417[a]	0.8417[a]	0.8550	0.8550	0.8417[a]	0.8417[a]	0.8550	0.8417[a]	0.8550	0.8417[a]	0.8417[a]
		3	3	0	3	0.5147	0.5117	0.5117	0.5105[a]	0.5105[a]	0.5226	0.5226	0.5147	0.5147	0.5117	0.5105[a]	0.5226	0.5105[a]	0.5105[a]
		0	3	6	0	0.2040	0.2040	0.2044	0.2040	0.2044	0.2040	0.2044	0.2040	0.2044	0.2040	0.2040	0.2040	0.2037[a]	0.2037[a]
	24	2	5	2	2	0.1810[a]	0.1810[a]	0.1810[a]	0.1810[a]	0.1810[a]	0.1810[a]	0.1811	0.1810[a]	0.1811	0.1810[a]	0.1810[a]	0.1810[a]	0.1810[a]	0.1810[a]
		3	5	2	1	0.2136	0.2134	0.2134	0.2133[a]	0.2134	0.2136	0.2136	0.2136	0.2136	0.2134	0.2134	0.2136	0.2133[a]	0.2133[a]
		6	3	0	0	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]	0.3083[a]
		3	3	0	3	0.2156	0.2150[a]	0.2150[a]	0.2150[a]	0.2150[a]	0.2158	0.2158	0.2156	0.2156	0.2150[a]	0.2150[a]	0.2158	0.2150[a]	0.2150[a]
		0	3	6	0	0.0982[a]	0.0982[a]	0.0983	0.0982[a]	0.0983	0.0982[a]	0.0983	0.0982[a]	0.0983	0.0982[a]	0.0982[a]	0.0982[a]	0.0983	0.0983
	36	2	5	2	2	0.1175	0.1174[a]	0.1174[a]	0.1174[a]	0.1174[a]	0.1174[a]	0.1175	0.1174[a]	0.1175	0.1174[a]	0.1174[a]	0.1174[a]	0.1174[a]	0.1174[a]
		3	5	2	1	0.1370[a]	0.1370[a]	0.1370[a]	0.1370[a]	0.1370[a]	0.1370[a]	0.1370[a]	0.1370[a]	0.1371	0.1370[a]	0.1370[a]	0.1371	0.1370[a]	0.1370[a]
		6	3	0	0	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]	0.1907[a]
		3	3	0	3	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]	0.1375[a]
		0	3	6	0	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0653	0.0652[a]	0.0652[a]
4	12	2	5	2	2	0.5369	0.5369	0.5375	0.5367[a]	0.5369	0.5369	0.5375	0.5367[a]	0.5369	0.5375	0.5369	0.5375	0.5369	0.5368
		3	5	2	1	0.6456	0.6460	0.6469	0.6455	0.6456	0.6460	0.6469	0.6455	0.6456	0.6469	0.6456	0.6469	0.6454[a]	0.6454[a]
		6	3	0	0	1.0774[a]	1.0925	1.0925	1.0774[a]	1.0774[a]	1.0925	1.0925	1.0774[a]	1.0774[a]	1.0925	1.0774[a]	1.0925	1.0774[a]	1.0774[a]
		3	3	0	3	0.6697	0.6657	0.6657	0.6649[a]	0.6649[a]	0.6767	0.6767	0.6697	0.6697	0.6657	0.6649[a]	0.6767	0.6649[a]	0.6649[a]
		0	3	6	0	0.2932	0.2933	0.2935	0.2933	0.2935	0.2933	0.2935	0.2933	0.2935	0.2932	0.2932	0.2932	0.2929[a]	0.2929[a]
	24	2	5	2	2	0.2446	0.2445	0.2444[a]	0.2445	0.2444[a]	0.2446	0.2446	0.2446	0.2446	0.2445	0.2445	0.2446	0.2444[a]	0.2444[a]
		3	5	2	1	0.2850	0.2848[a]	0.2848[a]	0.2848[a]	0.2848[a]	0.2850	0.2850	0.2850	0.2850	0.2848[a]	0.2848[a]	0.2850	0.2848[a]	0.2848[a]
		6	3	0	0	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]	0.4022[a]
		3	3	0	3	0.2868	0.2863[a]	0.2863[a]	0.2863[a]	0.2863[a]	0.2868	0.2868	0.2868	0.2868	0.2863[a]	0.2863[a]	0.2868	0.2863[a]	0.2863[a]
		0	3	6	0	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]	0.1437[a]
	36	2	5	2	2	0.1589	0.1589	0.1588[a]	0.1588[a]	0.1589	0.1589	0.1589	0.1589	0.1589	0.1588[a]	0.1588[a]	0.1589	0.1588[a]	0.1588[a]
		3	5	2	1	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]	0.1838[a]
		6	3	0	0	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]	0.2503[a]
		3	3	0	3	0.1843[a]	0.1843[a]	0.1843[a]	0.1843[a]	0.1843[a]	0.1844	0.1844	0.1843[a]	0.1843[a]	0.1843[a]	0.1843[a]	0.1844	0.1843[a]	0.1843[a]
		0	3	6	0	0.0950[a]	0.0950[a]	0.0951	0.0950[a]	0.0951	0.0950[a]	0.0951	0.0950[a]	0.0951	0.0950[a]	0.0950[a]	0.0950[a]	0.0951	0.0951

(*Continued*)

Table 4. Continued

k	*n*	*r*	*u*	*l*	*s*	WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
6	12	2	5	2	2	0.5906	0.5904[a]	0.5906	0.5904[a]	0.5906	0.5904[a]	0.5906	0.5904[a]	0.5906	0.5906	0.5906	0.5906	0.5904[a]	0.5904[a]
		3	5	2	1	0.7092	0.7095	0.7102	0.7092	0.7092	0.7095	0.7102	0.7092	0.7092	0.7102	0.7092	0.7102	0.7091	0.7090[a]
		6	3	0	0	1.1627[a]	1.1744	1.1744	1.1627[a]	1.1627[a]	1.1744	1.1744	1.1627[a]	1.1627[a]	1.1744	1.1627[a]	1.1744	1.1627[a]	1.1627[a]
		3	3	0	3	0.7271	0.7240	0.7240	0.7234[a]	0.7234[a]	0.7332	0.7332	0.7271	0.7271	0.7240	0.7234[a]	0.7332	0.7234[a]	0.7234[a]
		0	3	6	0	0.3332	0.3330	0.3331	0.3330	0.3331	0.3330	0.3331	0.3330	0.3331	0.3332	0.3332	0.3332	0.3329[a]	0.3329[a]
	24	2	5	2	2	0.2725	0.2723	0.2723	0.2723	0.2722[a]	0.2725	0.2726	0.2725	0.2726	0.2723	0.2723	0.2725	0.2723	0.2723
		3	5	2	1	0.3159	0.3159	0.3158[a]	0.3158[a]	0.3158[a]	0.3159	0.3159	0.3159	0.3158[a]	0.3159	0.3159	0.3159	0.3158[a]	0.3158[a]
		6	3	0	0	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]	0.4422[a]
		3	3	0	3	0.3174	0.3171[a]	0.3171[a]	0.3171[a]	0.3171[a]	0.3176	0.3176	0.3174	0.3174	0.3171[a]	0.3171[a]	0.3176	0.3171[a]	0.3171[a]
		0	3	6	0	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]	0.1642[a]
	36	2	5	2	2	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]	0.1770[a]
		3	5	2	1	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]
		6	3	0	0	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]	0.2765[a]
		3	3	0	3	0.2043	0.2041[a]	0.2041[a]	0.2041[a]	0.2041[a]	0.2042	0.2042	0.2043	0.2043	0.2041[a]	0.2041[a]	0.2042	0.2041[a]	0.2041[a]
		0	3	6	0	0.1088[a]	0.1088[a]	0.1089	0.1088[a]	0.1089	0.1088[a]	0.1089	0.1088[a]	0.1089	0.1088[a]	0.1088[a]	0.1088[a]	0.1088[a]	0.1088[a]
8	12	2	5	2	2	0.6278	0.6278	0.6280	0.6276[a]	0.6278	0.6278	0.6280	0.6276[a]	0.6278	0.6280	0.6278	0.6280	0.6277	0.6276[a]
		3	5	2	1	0.7509	0.7507	0.7515	0.7506[a]	0.7509	0.7507	0.7515	0.7506[a]	0.7509	0.7515	0.7509	0.7515	0.7506[a]	0.7506[a]
		6	3	0	0	1.2253[a]	1.2347	1.2347	1.2253[a]	1.2253[a]	1.2347	1.2347	1.2253[a]	1.2253[a]	1.2347	1.2253[a]	1.2347	1.2253[a]	1.2253[a]
		3	3	0	3	0.7658	0.7633	0.7633	0.7623[a]	0.7623[a]	0.7712	0.7712	0.7658	0.7658	0.7633	0.7623[a]	0.7712	0.7623[a]	0.7623[a]
		0	3	6	0	0.3611	0.3609	0.3608	0.3609	0.3608	0.3609	0.3608	0.3609	0.3608	0.3611	0.3611	0.3611	0.3604[a]	0.3604[a]
	24	2	5	2	2	0.2901	0.2899[a]	0.2899[a]	0.2899[a]	0.2899[a]	0.2901	0.2901	0.2901	0.2901	0.2900	0.2899[a]	0.2901	0.2899[a]	0.2899[a]
		3	5	2	1	0.3359	0.3358	0.3358	0.3358	0.3358	0.3359	0.3360	0.3359	0.3360	0.3358	0.3357[a]	0.3359	0.3358	0.3358
		6	3	0	0	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]	0.4681[a]
		3	3	0	3	0.3368	0.3365	0.3365	0.3364[a]	0.3364[a]	0.3369	0.3369	0.3368	0.3368	0.3365	0.3364[a]	0.3369	0.3364[a]	0.3364[a]
		0	3	6	0	0.1776[a]	0.1776[a]	0.1777	0.1776[a]	0.1777	0.1776[a]	0.1777	0.1776[a]	0.1777	0.1776[a]	0.1776[a]	0.1776[a]	0.1776[a]	0.1776[a]

	36	2	5	2	2	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]	0.1890[a]
		3	5	2	1	0.2174	0.2173[a]	0.2173[a]	0.2173[a]	0.2173[a]	0.2174	0.2174	0.2174	0.2174	0.2173[a]	0.2173[a]	0.2174	0.2173[a]	0.2173[a]
		6	3	0	0	0.2927	0.2926[a]	0.2926[a]	0.2927	0.2927	0.2926[a]	0.2926[a]	0.2927	0.2927	0.2926[a]	0.2927	0.2926[a]	0.2927	0.2927
		3	3	0	3	0.2173	0.2172[a]	0.2172[a]	0.2172[a]	0.2172[a]	0.2173	0.2173	0.2173	0.2173	0.2172[a]	0.2172[a]	0.2173	0.2172[a]	0.2172[a]
		0	3	6	0	0.1182	0.1182	0.1181[a]	0.1182	0.1181[a]	0.1182	0.1181[a]	0.1182	0.1181[a]	0.1182	0.1182	0.1182	0.1181[a]	0.1181[a]
10	12	2	5	2	2	0.6542	0.6542	0.6544	0.6541[a]	0.6542	0.6542	0.6544	0.6541[a]	0.6542	0.6544	0.6542	0.6544	0.6541[a]	0.6541[a]
		3	5	2	1	0.7825	0.7821[a]	0.7827	0.7822	0.7825	0.7821[a]	0.7827	0.7822	0.7825	0.7827	0.7825	0.7827	0.7821[a]	0.7821[a]
		6	3	0	0	1.2664[a]	1.2739	1.2739	1.2664[a]	1.2664[a]	1.2739	1.2739	1.2664[a]	1.2664[a]	1.2739	1.2664[a]	1.2739	1.2664[a]	1.2664[a]
		3	3	0	3	0.7954	0.7931	0.7931	0.7923[a]	0.7923[a]	0.8000	0.8000	0.7954	0.7954	0.7931	0.7923[a]	0.8000	0.7923[a]	0.7923[a]
		0	3	6	0	0.3805	0.3804	0.3810	0.3804	0.3810	0.3804	0.3810	0.3804	0.3810	0.3805	0.3805	0.3805	0.3803[a]	0.3803[a]
	24	2	5	2	2	0.3037	0.3035[a]	0.3035[a]	0.3035[a]	0.3036	0.3037	0.3038	0.3037	0.3038	0.3035[a]	0.3035[a]	0.3037	0.3035[a]	0.3035[a]
		3	5	2	1	0.3505	0.3503[a]	0.3504	0.3504	0.3504	0.3505	0.3504	0.3504	0.3505	0.3504	0.3504	0.3505	0.3504	0.3504
		6	3	0	0	0.4857[a]	0.4858	0.4858	0.4857[a]	0.4857[a]	0.4858	0.4858	0.4857[a]	0.4857[a]	0.4858	0.4857[a]	0.4858	0.4857[a]	0.4857[a]
		3	3	0	3	0.3521	0.3520	0.3520	0.3519[a]	0.3519[a]	0.3521	0.3521	0.3521	0.3521	0.3520	0.3519[a]	0.3521	0.3519[a]	0.3519[a]
		0	3	6	0	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]	0.1877[a]
	36	2	5	2	2	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]	0.1983[a]
		3	5	2	1	0.2274	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2274	0.2273[a]	0.2274	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]
		6	3	0	0	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]	0.3047[a]
		3	3	0	3	0.2274	0.2273[a]	0.2273[a]	0.2273[a]	0.2273[a]	0.2274	0.2274	0.2274	0.2274	0.2273[a]	0.2273[a]	0.2274	0.2273[a]	0.2273[a]
		0	3	6	0	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]	0.1250[a]

Note: [a]SCI with minimum confidence length.

at least LEP and S_{U} contains at least UEP with a preset probability not less than $1-\alpha$. To modify the procedure, the decision rule for the subset selection procedure for censored two-parameter exponential distribution is defined as:

$$R_{\mathrm{MD}}: \quad \pi_i \text{ is included in a nonempty subset of } S_{\mathrm{L}} \text{ if and only if } \hat{\mu}_{i,t} \le \hat{\mu}_{[1],t} + d_t(\hat{\theta}_t^*/n),$$

$$\pi_i \text{ is included in a nonempty subset of } S_{\mathrm{U}} \text{ if and only if } \hat{\mu}_{i,t} \ge \hat{\mu}_{[k],t} - d_t(\hat{\theta}_t^*/n),$$

where $\hat{\mu}_{i,t}$ and $\hat{\theta}_t^*$ are defined as in Section 2, d_t is determined in Theorem 1, $\hat{\mu}_{[1],t} = \min(\hat{\mu}_{1,t}, \ldots, \hat{\mu}_{k,t})$ and $\hat{\mu}_{[k],t} = \max(\hat{\mu}_{1,t}, \ldots, \hat{\mu}_{k,t})$, $i = 1, 2, \ldots, k$, $t = 1, 2, \ldots, 14$. A 'correct selection (CS)' is made when the LEP is contained in the subset of S_L, namely $\pi_{(1)} \in S_{\mathrm{L}}$, and the UEP is contained in the subset of S_{U}, namely $\pi_{(k)} \in S_{\mathrm{U}}$. Then according to decision rule R_{MD}, the probability of CS is at least $1-\alpha$, that is,

$$P(\mathrm{CS}|R_{\mathrm{MD}}) \ge 1 - \alpha, \tag{25}$$

and the proof is given in the section of Appendix 2. The SCI of $1-\alpha$ significance level can also be obtained for all distances from the LEP and UEP along with the proof.

5. Numerical example

5.1. *Example 1*

Suppose that we want to select the worst population and the best population at the same time. A random sample of size $n = 24$ following four independent distributions $E(\mu_i, \theta)$, $i = 1, \ldots, 4$, with $(\mu_1, \mu_2, \mu_3, \mu_4, \theta) = (2, 3, 6, 8, 2)$ is generated and the data is listed in Table 5. Considering the censoring schemes of $(r, u, l, s) = (2, 5, 2, 2)$, $(6, 3, 0, 0)$, $(3, 3, 0, 3)$ and $(0, 3, 6, 0)$, the required statistics and critical values of d_t for $1-\alpha = 0.95$ are listed in Table 6.

The SCIs of $\mu_1 - \mu_{[1]}$, $\mu_2 - \mu_{[1]}$, $\mu_3 - \mu_{[1]}$, $\mu_4 - \mu_{[1]}$, $\mu_{[4]} - \mu_1$, $\mu_{[4]} - \mu_2$, $\mu_{[4]} - \mu_3$ and $\mu_{[4]} - \mu_4$ are listed in Table 7. Based on the criterion of the minimum confidence length, population 1 and 4 are identified as the LEP and UEP, respectively, for all censoring schemes. As to the subset selection procedure, the results are listed in Table 8. It is observed that population 1 is included in S_L and population 4 is included in S_{U} for all censoring schemes.

5.2. *Example 2*

The time intervals of successive failures of the air conditioning system in Boeing 720 jet airplanes are shown in Table 9. Sun and Kim [16] assumed that the time between successive

Table 5. Simulated data for four populations.

Population1	2.062, 2.102, 2.504, 2.505, 2.519, 2.629, 2.685, 2.778, 2.924, 3.080, 3.099, 3.412, 3.847, 3.854, 3.984, 4.009, 4.067, 4.160, 4.389, 4.619, 6.421, 6.483, 6.995, 7.391
Population2	3.105, 3.111, 3.360, 3.480, 3.673, 3.770, 3.848, 3.893, 3.908, 3.987, 3.990, 4.356, 5.086, 5.203, 5.248, 5.563, 5.687, 6.198, 6.351, 6.897, 6.960, 7.147, 7.154, 8.041
Population3	3.105, 3.111, 3.360, 3.480, 3.673, 3.770, 3.848, 3.893, 3.908, 3.987, 3.990, 4.356, 5.086, 5.203, 5.248, 5.563, 5.687, 6.198, 6.351, 6.897, 6.960, 7.147, 7.154, 8.041
Population4	8.022, 8.180, 8.239, 8.408, 8.677, 8.704, 8.782, 9.070, 9.081, 9.481, 9.529, 9.771, 9.933, 9.987, 10.084, 10.442, 10.489, 10.681, 11.105, 11.464, 11.465, 12.106, 12.167, 14.853

Table 6. The required statistics for Example 1.

(r, u, l, s)		WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
(2, 5, 2, 2)	$\hat{\theta}_t^*$	1.902	1.876	1.885	1.877	1.886	1.901	1.912	1.901	1.912	1.875	1.878	1.901	1.811	1.906
	$\hat{\mu}_{1,t}$	2.256	2.259	2.258	2.259	2.258	2.256	2.255	2.256	2.255	2.259	2.259	2.256	2.268	2.255
	$\hat{\mu}_{2,t}$	3.112	3.115	3.114	3.115	3.114	3.112	3.110	3.112	3.110	3.115	3.115	3.112	3.123	3.111
	$\hat{\mu}_{3,t}$	6.194	6.198	6.197	6.198	6.196	6.194	6.193	6.194	6.193	6.198	6.197	6.194	6.206	6.194
	$\hat{\mu}_{4,t}$	7.990	7.994	7.993	7.994	7.992	7.991	7.989	7.990	7.989	7.994	7.994	7.991	8.002	7.990
	d_t	6.335	6.330	6.328	6.330	6.328	6.334	6.339	6.335	6.339	6.331	6.331	6.335	6.576	6.248
	$d_t \times (\hat{\theta}_t^*/n)$	0.502	0.495	0.497	0.495	0.497	0.502	0.505	0.502	0.505	0.495	0.495	0.502	0.496	0.496
(6, 3, 0, 0)	$\hat{\theta}_t^*$	1.753	1.754	1.754	1.753	1.753	1.754	1.754	1.753	1.753	1.754	1.753	1.754	1.680	1.778
	$\hat{\mu}_{1,t}$	2.095	2.095	2.095	2.095	2.095	2.095	2.095	2.095	2.095	2.095	2.095	2.095	2.119	2.086
	$\hat{\mu}_{2,t}$	3.259	3.258	3.258	3.259	3.259	3.258	3.258	3.259	3.259	3.258	3.259	3.258	3.283	3.250
	$\hat{\mu}_{3,t}$	6.276	6.275	6.275	6.276	6.276	6.275	6.275	6.276	6.276	6.275	6.276	6.275	6.300	6.267
	$\hat{\mu}_{4,t}$	8.192	8.192	8.192	8.192	8.192	8.192	8.192	8.192	8.192	8.192	8.192	8.192	8.217	8.184
	d_t	10.509	10.514	10.514	10.509	10.509	10.514	10.514	10.509	10.509	10.514	10.509	10.514	10.965	10.356
	$d_t \times (\hat{\theta}_t^*/n)$	0.767	0.768	0.768	0.767	0.767	0.768	0.768	0.767	0.767	0.768	0.767	0.768	0.767	0.767
(3, 3,0, 3)	$\hat{\theta}_t^*$	1.937	1.907	1.907	1.898	1.898	1.949	1.949	1.937	1.937	1.907	1.898	1.949	1.819	1.926
	$\hat{\mu}_{1,t}$	2.160	2.165	2.165	2.167	2.167	2.158	2.158	2.160	2.160	2.165	2.167	2.158	2.181	2.162
	$\hat{\mu}_{2,t}$	3.135	3.140	3.140	3.142	3.142	3.133	3.133	3.135	3.135	3.140	3.142	3.133	3.156	3.137
	$\hat{\mu}_{3,t}$	6.145	6.150	6.150	6.152	6.152	6.143	6.143	6.145	6.145	6.150	6.152	6.143	6.166	6.147
	$\hat{\mu}_{4,t}$	8.063	8.068	8.068	8.070	8.070	8.061	8.061	8.063	8.063	8.068	8.070	8.061	8.084	8.065
	d_t	7.443	7.427	7.427	7.428	7.428	7.444	7.444	7.443	7.443	7.427	7.428	7.444	7.751	7.321
	$d_t \times (\hat{\theta}_t^*/n)$	0.601	0.590	0.590	0.587	0.587	0.604	0.604	0.601	0.601	0.590	0.587	0.604	0.587	0.588
(0, 3, 6, 0)	$\hat{\theta}_t^*$	1.916	1.927	1.957	1.927	1.957	1.927	1.957	1.927	1.957	1.916	1.916	1.916	1.881	1.962
	$\hat{\mu}_{1,t}$	1.983	1.982	1.981	1.982	1.981	1.982	1.981	1.982	1.981	1.983	1.983	1.983	1.984	1.981
	$\hat{\mu}_{2,t}$	3.025	3.025	3.024	3.025	3.024	3.025	3.024	3.025	3.024	3.025	3.025	3.025	3.027	3.023
	$\hat{\mu}_{3,t}$	6.059	6.059	6.058	6.059	6.058	6.059	6.058	6.059	6.058	6.059	6.059	6.059	6.061	6.057
	$\hat{\mu}_{4,t}$	7.942	7.941	7.940	7.941	7.940	7.941	7.940	7.941	7.940	7.942	7.942	7.942	7.943	7.940
	d_t	3.652	3.653	3.653	3.653	3.653	3.653	3.653	3.653	3.653	3.652	3.652	3.652	3.770	3.614
	$d_t \times (\hat{\theta}_t^*/n)$	0.291	0.293	0.298	0.293	0.298	0.293	0.298	0.293	0.298	0.291	0.291	0.291	0.296	0.295

Table 7. The SCIs for Example 1.

(r, u, l, s)		WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
(2, 5, 2, 2)	$\mu_1 - \mu_{[1]}$	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]
	$\mu_2 - \mu_{[1]}$	[0, 1.358]	[0, 1.350]	[0, 1.353]	[0, 1.351]	[0, 1.353]	[0, 1.357]	[0, 1.361]	[0, 1.358]	[0, 1.361]	[0, 1.350]	[0, 1.351]	[0, 1.357]	[0, 1.352]	[0, 1.352]
	$\mu_3 - \mu_{[1]}$	[0, 4.441]	[0, 4.433]	[0, 4.435]	[0, 4.434]	[0, 4.436]	[0, 4.440]	[0, 4.443]	[0, 4.440]	[0, 4.444]	[0, 4.433]	[0, 4.434]	[0, 4.440]	[0, 4.435]	[0, 4.435]
	$\mu_4 - \mu_{[1]}$	[0, 6.237]	[0, 6.229]	[0, 6.231]	[0, 6.230]	[0, 6.232]	[0, 6.236]	[0, 6.239]	[0, 6.236]	[0, 6.240]	[0, 6.229]	[0, 6.230]	[0, 6.236]	[0, 6.231]	[0, 6.231]
	$\mu_{[4]} - \mu_1$	[0, 6.237]	[0, 6.229]	[0, 6.231]	[0, 6.230]	[0, 6.232]	[0, 6.236]	[0, 6.239]	[0, 6.236]	[0, 6.240]	[0, 6.229]	[0, 6.230]	[0, 6.236]	[0, 6.231]	[0, 6.231]
	$\mu_{[4]} - \mu_2$	[0, 5.381]	[0, 5.374]	[0, 5.376]	[0, 5.374]	[0, 5.376]	[0, 5.380]	[0, 5.384]	[0, 5.381]	[0, 5.384]	[0, 5.374]	[0, 5.374]	[0, 5.381]	[0, 5.375]	[0, 5.375]
	$\mu_{[4]} - \mu_3$	[0, 2.298]	[0, 2.291]	[0, 2.293]	[0, 2.291]	[0, 2.293]	[0, 2.298]	[0, 2.301]	[0, 2.298]	[0, 2.301]	[0, 2.291]	[0, 2.291]	[0, 2.298]	[0, 2.292]	[0, 2.292]
	$\mu_{[4]} - \mu_4$	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]
(6, 3, 0, 0)	$\mu_1 - \mu_{[1]}$	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]
	$\mu_2 - \mu_{[1]}$	[0, 1.931]	[0, 1.932]	[0, 1.932]	[0, 1.931]	[0, 1.931]	[0, 1.932]	[0, 1.932]	[0, 1.931]	[0, 1.931]	[0, 1.932]	[0, 1.931]	[0, 1.932]	[0, 1.931]	[0, 1.931]
	$\mu_3 - \mu_{[1]}$	[0, 4.948]	[0, 4.949]	[0, 4.949]	[0, 4.948]	[0, 4.948]	[0, 4.949]	[0, 4.949]	[0, 4.948]	[0, 4.948]	[0, 4.949]	[0, 4.948]	[0, 4.949]	[0, 4.948]	[0, 4.948]
	$\mu_4 - \mu_{[1]}$	[0, 6.865]	[0, 6.866]	[0, 6.866]	[0, 6.865]	[0, 6.865]	[0, 6.866]	[0, 6.866]	[0, 6.865]	[0, 6.865]	[0, 6.866]	[0, 6.865]	[0, 6.866]	[0, 6.865]	[0, 6.865]
	$\mu_{[4]} - \mu_1$	[0, 6.865]	[0, 6.866]	[0, 6.866]	[0, 6.865]	[0, 6.865]	[0, 6.866]	[0, 6.866]	[0, 6.865]	[0, 6.865]	[0, 6.866]	[0, 6.865]	[0, 6.866]	[0, 6.865]	[0, 6.865]
	$\mu_{[4]} - \mu_2$	[0, 5.701]	[0, 5.702]	[0, 5.702]	[0, 5.701]	[0, 5.701]	[0, 5.702]	[0, 5.702]	[0, 5.701]	[0, 5.701]	[0, 5.702]	[0, 5.701]	[0, 5.702]	[0, 5.701]	[0, 5.701]
	$\mu_{[4]} - \mu_3$	[0, 2.684]	[0, 2.685]	[0, 2.685]	[0, 2.684]	[0, 2.684]	[0, 2.685]	[0, 2.685]	[0, 2.684]	[0, 2.684]	[0, 2.685]	[0, 2.684]	[0, 2.685]	[0, 2.684]	[0, 2.684]
	$\mu_{[4]} - \mu_4$	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]
(3, 3, 0, 3)	$\mu_1 - \mu_{[1]}$	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]
	$\mu_2 - \mu_{[1]}$	[0, 1.576]	[0, 1.565]	[0, 1.565]	[0, 1.563]	[0, 1.563]	[0, 1.580]	[0, 1.580]	[0, 1.576]	[0, 1.576]	[0, 1.565]	[0, 1.563]	[0, 1.580]	[0, 1.563]	[0, 1.563]
	$\mu_3 - \mu_{[1]}$	[0, 4.586]	[0, 4.575]	[0, 4.575]	[0, 4.573]	[0, 4.573]	[0, 4.590]	[0, 4.590]	[0, 4.586]	[0, 4.586]	[0, 4.575]	[0, 4.573]	[0, 4.590]	[0, 4.573]	[0, 4.573]
	$\mu_4 - \mu_{[1]}$	[0, 6.504]	[0, 6.493]	[0, 6.493]	[0, 6.490]	[0, 6.490]	[0, 6.507]	[0, 6.507]	[0, 6.504]	[0, 6.504]	[0, 6.493]	[0, 6.490]	[0, 6.507]	[0, 6.490]	[0, 6.491]
	$\mu_{[4]} - \mu_1$	[0, 6.504]	[0, 6.493]	[0, 6.493]	[0, 6.490]	[0, 6.490]	[0, 6.507]	[0, 6.507]	[0, 6.504]	[0, 6.504]	[0, 6.493]	[0, 6.490]	[0, 6.507]	[0, 6.490]	[0, 6.491]
	$\mu_{[4]} - \mu_2$	[0, 5.528]	[0, 5.518]	[0, 5.518]	[0, 5.515]	[0, 5.515]	[0, 5.532]	[0, 5.532]	[0, 5.528]	[0, 5.528]	[0, 5.518]	[0, 5.515]	[0, 5.532]	[0, 5.515]	[0, 5.515]
	$\mu_{[4]} - \mu_3$	[0, 2.518]	[0, 2.508]	[0, 2.508]	[0, 2.505]	[0, 2.505]	[0, 2.522]	[0, 2.522]	[0, 2.518]	[0, 2.518]	[0, 2.508]	[0, 2.505]	[0, 2.522]	[0, 2.505]	[0, 2.505]
	$\mu_{[4]} - \mu_4$	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]
(0, 3, 6, 0)	$\mu_1 - \mu_{[1]}$	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]
	$\mu_2 - \mu_{[1]}$	[0, 1.334]	[0, 1.336]	[0, 1.341]	[0, 1.336]	[0, 1.341]	[0, 1.336]	[0, 1.341]	[0, 1.336]	[0, 1.341]	[0, 1.334]	[0, 1.334]	[0, 1.334]	[0, 1.338]	[0, 1.338]
	$\mu_3 - \mu_{[1]}$	[0, 4.368]	[0, 4.370]	[0, 4.375]	[0, 4.370]	[0, 4.375]	[0, 4.370]	[0, 4.375]	[0, 4.370]	[0, 4.375]	[0, 4.368]	[0, 4.368]	[0, 4.368]	[0, 4.372]	[0, 4.372]
	$\mu_4 - \mu_{[1]}$	[0, 6.251]	[0, 6.252]	[0, 6.257]	[0, 6.252]	[0, 6.257]	[0, 6.252]	[0, 6.257]	[0, 6.252]	[0, 6.257]	[0, 6.251]	[0, 6.251]	[0, 6.251]	[0, 6.255]	[0, 6.255]
	$\mu_{[4]} - \mu_1$	[0, 6.251]	[0, 6.252]	[0, 6.257]	[0, 6.252]	[0, 6.257]	[0, 6.252]	[0, 6.257]	[0, 6.252]	[0, 6.257]	[0, 6.251]	[0, 6.251]	[0, 6.251]	[0, 6.255]	[0, 6.255]
	$\mu_{[4]} - \mu_2$	[0, 5.208]	[0, 5.210]	[0, 5.214]	[0, 5.210]	[0, 5.214]	[0, 5.210]	[0, 5.214]	[0, 5.210]	[0, 5.214]	[0, 5.208]	[0, 5.208]	[0, 5.208]	[0, 5.212]	[0, 5.212]
	$\mu_{[4]} - \mu_3$	[0, 2.174]	[0, 2.176]	[0, 2.180]	[0, 2.176]	[0, 2.180]	[0, 2.176]	[0, 2.180]	[0, 2.176]	[0, 2.180]	[0, 2.174]	[0, 2.174]	[0, 2.174]	[0, 2.178]	[0, 2.178]
	$\mu_{[4]} - \mu_4$	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]	[0, 0.000]

Table 8. Results of subset selection for Example 1.

(r, u, l, s)		WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
(2, 5, 2, 2)	$\hat{\mu}_{[1],t} + (d_t \times (\hat{\theta}_t^*/n))$	2.758	2.754	2.755	2.754	2.755	2.758	2.760	2.758	2.760	2.754	2.754	2.758	2.764	2.752
	$\hat{\mu}_{[4],t} - (d_t \times (\hat{\theta}_t^*/n))$	7.488	7.499	7.496	7.499	7.495	7.489	7.484	7.489	7.484	7.499	7.498	7.489	7.506	7.494
	S_L	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	S_U	4	4	4	4	4	4	4	4	4	4	4	4	4	4
(6, 3, 0, 0)	$\hat{\mu}_{[1],t} + (d_t \times (\hat{\theta}_t^*/n))$	2.862	2.863	2.863	2.862	2.862	2.863	2.863	2.862	2.862	2.863	2.862	2.863	2.887	2.854
	$\hat{\mu}_{[4],t} - (d_t \times (\hat{\theta}_t^*/n))$	7.425	7.424	7.424	7.425	7.425	7.424	7.424	7.425	7.425	7.424	7.425	7.424	7.449	7.416
	S_L	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	S_U	4	4	4	4	4	4	4	4	4	4	4	4	4	4
(3, 3, 0, 3)	$\hat{\mu}_{[1],t} + (d_t \times (\hat{\theta}_t^*/n))$	2.760	2.755	2.755	2.754	2.754	2.762	2.762	2.760	2.760	2.755	2.754	2.762	2.768	2.749
	$\hat{\mu}_{[4],t} - (d_t \times (\hat{\theta}_t^*/n))$	7.462	7.478	7.478	7.482	7.482	7.456	7.456	7.462	7.462	7.478	7.482	7.456	7.496	7.477
	S_L	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	S_U	4	4	4	4	4	4	4	4	4	4	4	4	4	4
(0, 3, 6, 0)	$\hat{\mu}_{[1],t} + (d_t \times (\hat{\theta}_t^*/n))$	2.274	2.275	2.279	2.275	2.279	2.275	2.279	2.275	2.279	2.274	2.274	2.274	2.280	2.276
	$\hat{\mu}_{[4],t} - (d_t \times (\hat{\theta}_t^*/n))$	7.650	7.648	7.642	7.648	7.642	7.648	7.642	7.648	7.642	7.650	7.650	7.650	7.648	7.644
	S_L	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	S_U	4	4	4	4	4	4	4	4	4	4	4	4	4	4

Table 9. Time intervals between failures.

Plane1	3, 5, 5, 13, 14, 15, 22, 22, 23, 30, 36, 39, 44, 46, 50, 72, 79, 88, 97, 102, 139, 188, 197, 210
Plane2	10, 14, 20, 23, 24, 25, 26, 29, 44, 44, 49, 56, 59, 60, 61, 62, 70, 76, 79, 84, 90, 101, 118, 130
Plane3	1, 3, 5, 7, 11, 11, 11, 12, 14, 14, 14, 16, 16, 20, 21, 23, 42, 47, 52, 62, 71, 71, 87, 90
Plane4	1, 4, 11, 16, 18, 18, 18, 24, 31, 39, 46, 51, 54, 63, 68, 77, 80, 82, 97, 106, 111, 141, 142, 163

Table 10. The required statistics for Example 2.

(r, u, l, s)		WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
(2, 5, 2, 2)	$\hat{\theta}_t^*$	51.701	52.016	52.250	51.822	52.089	51.694	51.855	51.581	51.792	52.263	52.027	51.848	50.000	52.625
	$\hat{\mu}_{1,t}$	−1.752	−1.793	−1.824	−1.768	−1.803	−1.751	−1.772	−1.737	−1.764	−1.825	−1.795	−1.771	−1.530	−1.873
	$\hat{\mu}_{2,t}$	13.248	13.207	13.176	13.232	13.197	13.249	13.228	13.263	13.236	13.175	13.205	13.229	13.470	13.127
	$\hat{\mu}_{3,t}$	−1.752	−1.793	−1.824	−1.768	−1.803	−1.751	−1.772	−1.737	−1.764	−1.825	−1.795	−1.771	−1.530	−1.873
	$\hat{\mu}_{4,t}$	4.248	4.207	4.176	4.232	4.197	4.249	4.228	4.263	4.236	4.175	4.205	4.229	4.470	4.127
	d_t	6.335	6.330	6.328	6.330	6.328	6.334	6.339	6.335	6.339	6.331	6.331	6.335	6.576	6.248
	$d_t \times (\hat{\theta}_t^*/n)$	13.647	13.719	13.776	13.668	13.734	13.643	13.696	13.615	13.679	13.786	13.724	13.686	13.700	13.700
(6, 3, 0, 0)	$\hat{\theta}_t^*$	49.449	49.291	49.291	49.449	49.449	49.291	49.291	49.449	49.449	49.291	49.449	49.291	47.389	50.176
	$\hat{\mu}_{1,t}$	5.365	5.418	5.418	5.365	5.365	5.418	5.418	5.365	5.365	5.418	5.365	5.418	6.058	5.120
	$\hat{\mu}_{2,t}$	9.365	9.418	9.418	9.365	9.365	9.418	9.418	9.365	9.365	9.418	9.365	9.418	10.058	9.120
	$\hat{\mu}_{3,t}$	−5.635	−5.582	−5.582	−5.635	−5.635	−5.582	−5.582	−5.635	−5.635	−5.582	−5.635	−5.582	−4.942	−5.880
	$\hat{\mu}_{4,t}$	1.365	1.418	1.418	1.365	1.365	1.418	1.418	1.365	1.365	1.418	1.365	1.418	2.058	1.120
	d_t	10.509	10.514	10.514	10.509	10.509	10.514	10.514	10.509	10.509	10.514	10.509	10.514	10.965	10.356
	$d_t \times (\hat{\theta}_t^*/n)$	21.653	21.593	21.593	21.653	21.653	21.593	21.593	21.653	21.653	21.593	21.653	21.593	21.651	21.651
(3, 3, 0, 3)	$\hat{\theta}_t^*$	48.217	48.456	48.456	48.536	48.536	48.101	48.101	48.217	48.217	48.456	48.536	48.101	46.514	49.250
	$\hat{\mu}_{1,t}$	4.407	4.364	4.364	4.350	4.350	4.428	4.428	4.407	4.407	4.364	4.350	4.428	4.710	4.223
	$\hat{\mu}_{2,t}$	14.407	14.364	14.364	14.350	14.350	14.428	14.428	14.407	14.407	14.364	14.350	14.428	14.710	14.223
	$\hat{\mu}_{3,t}$	−1.593	−1.636	−1.636	−1.650	−1.650	−1.572	−1.572	−1.593	−1.593	−1.636	−1.650	−1.572	−1.290	−1.777
	$\hat{\mu}_{4,t}$	7.407	7.364	7.364	7.350	7.350	7.428	7.428	7.407	7.407	7.364	7.350	7.428	7.710	7.223
	d_t	7.443	7.427	7.427	7.428	7.428	7.444	7.444	7.443	7.443	7.427	7.428	7.444	7.751	7.321
	$d_t \times (\hat{\theta}_t^*/n)$	14.953	14.995	14.995	15.022	15.022	14.919	14.919	14.953	14.953	14.995	15.022	14.919	15.022	15.023
(0, 3, 6, 0)	$\hat{\theta}_t^*$	50.493	50.601	51.104	50.601	51.104	50.601	51.104	50.601	51.104	50.493	50.493	50.493	49.264	51.387
	$\hat{\mu}_{1,t}$	0.896	0.892	0.871	0.892	0.871	0.892	0.871	0.892	0.871	0.896	0.896	0.896	0.947	0.859
	$\hat{\mu}_{2,t}$	7.896	7.892	7.871	7.892	7.871	7.892	7.871	7.892	7.871	7.896	7.896	7.896	7.947	7.859
	$\hat{\mu}_{3,t}$	−1.104	−1.108	−1.129	−1.108	−1.129	−1.108	−1.129	−1.108	−1.129	−1.104	−1.104	−1.104	−1.053	−1.141
	$\hat{\mu}_{4,t}$	−1.104	−1.108	−1.129	−1.108	−1.129	−1.108	−1.129	−1.108	−1.129	−1.104	−1.104	−1.104	−1.053	−1.141
	d_t	3.652	3.653	3.653	3.653	3.653	3.653	3.653	3.653	3.653	3.652	3.652	3.652	3.770	3.614
	$d_t \times (\hat{\theta}_t^*/n)$	7.683	7.702	7.778	7.702	7.778	7.702	7.778	7.702	7.778	7.683	7.683	7.683	7.739	7.738

Table 11. The SCIs for Example 2.

(r, u, l, s)		WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
(2, 5, 2, 2)	$\mu_1 - \mu_{[1]}$	[0, 13.647]	[0, 13.719]	[0, 13.776]	[0, 13.668]	[0, 13.734]	[0, 13.643]	[0, 13.696]	[0, 13.615]	[0, 13.679]	[0, 13.786]	[0, 13.724]	[0, 13.686]	[0, 13.700]	[0, 13.700]
	$\mu_2 - \mu_{[1]}$	[0, 28.647]	[0, 28.719]	[0, 28.776]	[0, 28.668]	[0, 28.734]	[0, 28.643]	[0, 28.696]	[0, 28.615]	[0, 28.679]	[0, 28.786]	[0, 28.724]	[0, 28.686]	[0, 28.700]	[0, 28.700]
	$\mu_3 - \mu_{[1]}$	[0, 13.647]	[0, 13.719]	[0, 13.776]	[0, 13.668]	[0, 13.734]	[0, 13.643]	[0, 13.696]	[0, 13.615]	[0, 13.679]	[0, 13.786]	[0, 13.724]	[0, 13.686]	[0, 13.700]	[0, 13.700]
	$\mu_4 - \mu_{[1]}$	[0, 19.647]	[0, 19.719]	[0, 19.776]	[0, 19.668]	[0, 19.734]	[0, 19.643]	[0, 19.696]	[0, 19.615]	[0, 19.679]	[0, 19.786]	[0, 19.724]	[0, 19.686]	[0, 19.700]	[0, 19.700]
	$\mu_{[4]} - \mu_1$	[0, 28.647]	[0, 28.719]	[0, 28.776]	[0, 28.668]	[0, 28.734]	[0, 28.643]	[0, 28.696]	[0, 28.615]	[0, 28.679]	[0, 28.786]	[0, 28.724]	[0, 28.686]	[0, 28.700]	[0, 28.700]
	$\mu_{[4]} - \mu_2$	[0, 4.647]	[0, 4.719]	[0, 4.776]	[0, 4.668]	[0, 4.734]	[0, 4.643]	[0, 4.696]	[0, 4.615]	[0, 4.679]	[0, 4.786]	[0, 4.724]	[0, 4.686]	[0, 4.700]	[0, 4.700]
	$\mu_{[4]} - \mu_3$	[0, 28.647]	[0, 28.719]	[0, 28.776]	[0, 28.668]	[0, 28.734]	[0, 28.643]	[0, 28.696]	[0, 28.615]	[0, 28.679]	[0, 28.786]	[0, 28.724]	[0, 28.686]	[0, 28.700]	[0, 28.700]
	$\mu_{[4]} - \mu_4$	[0, 22.647]	[0, 22.719]	[0, 22.776]	[0, 22.668]	[0, 22.734]	[0, 22.643]	[0, 22.696]	[0, 22.615]	[0, 22.679]	[0, 22.786]	[0, 22.724]	[0, 22.686]	[0, 22.700]	[0, 22.700]
(6, 3, 0, 0)	$\mu_1 - \mu_{[1]}$	[0, 32.653]	[0, 32.593]	[0, 32.593]	[0, 32.653]	[0, 32.653]	[0, 32.593]	[0, 32.593]	[0, 32.653]	[0, 32.653]	[0, 32.593]	[0, 32.653]	[0, 32.593]	[0, 32.651]	[0, 32.651]
	$\mu_2 - \mu_{[1]}$	[0, 36.653]	[0, 36.593]	[0, 36.593]	[0, 36.653]	[0, 36.653]	[0, 36.593]	[0, 36.593]	[0, 36.653]	[0, 36.653]	[0, 36.593]	[0, 36.653]	[0, 36.593]	[0, 36.651]	[0, 36.651]
	$\mu_3 - \mu_{[1]}$	[0, 14.653]	[0, 14.593]	[0, 14.593]	[0, 14.653]	[0, 14.653]	[0, 14.593]	[0, 14.593]	[0, 14.653]	[0, 14.653]	[0, 14.593]	[0, 14.653]	[0, 14.593]	[0, 14.651]	[0, 14.651]
	$\mu_4 - \mu_{[1]}$	[0, 28.653]	[0, 28.593]	[0, 28.593]	[0, 28.653]	[0, 28.653]	[0, 28.593]	[0, 28.593]	[0, 28.653]	[0, 28.653]	[0, 28.593]	[0, 28.653]	[0, 28.593]	[0, 28.651]	[0, 28.651]
	$\mu_{[4]} - \mu_1$	[0, 25.653]	[0, 25.593]	[0, 25.593]	[0, 25.653]	[0, 25.653]	[0, 25.593]	[0, 25.593]	[0, 25.653]	[0, 25.653]	[0, 25.593]	[0, 25.653]	[0, 25.593]	[0, 25.651]	[0, 25.651]
	$\mu_{[4]} - \mu_2$	[0, 17.653]	[0, 17.593]	[0, 17.593]	[0, 17.653]	[0, 17.653]	[0, 17.593]	[0, 17.593]	[0, 17.653]	[0, 17.653]	[0, 17.593]	[0, 17.653]	[0, 17.593]	[0, 17.651]	[0, 17.651]
	$\mu_{[4]} - \mu_3$	[0, 36.653]	[0, 36.593]	[0, 36.593]	[0, 36.653]	[0, 36.653]	[0, 36.593]	[0, 36.593]	[0, 36.653]	[0, 36.653]	[0, 36.593]	[0, 36.653]	[0, 36.593]	[0, 36.651]	[0, 36.651]
	$\mu_{[4]} - \mu_4$	[0, 29.653]	[0, 29.593]	[0, 29.593]	[0, 29.653]	[0, 29.653]	[0, 29.593]	[0, 29.593]	[0, 29.653]	[0, 29.653]	[0, 29.593]	[0, 29.653]	[0, 29.593]	[0, 29.651]	[0, 29.651]
(3, 3, 0, 3)	$\mu_1 - \mu_{[1]}$	[0, 20.953]	[0, 20.995]	[0, 20.995]	[0, 21.022]	[0, 21.022]	[0, 20.919]	[0, 20.919]	[0, 20.953]	[0, 20.953]	[0, 20.995]	[0, 21.022]	[0, 20.919]	[0, 21.022]	[0, 21.023]
	$\mu_2 - \mu_{[1]}$	[0, 30.953]	[0, 30.995]	[0, 30.995]	[0, 31.022]	[0, 31.022]	[0, 30.919]	[0, 30.919]	[0, 30.953]	[0, 30.953]	[0, 30.995]	[0, 31.022]	[0, 30.919]	[0, 31.022]	[0, 31.023]
	$\mu_3 - \mu_{[1]}$	[0, 8.953]	[0, 8.995]	[0, 8.995]	[0, 9.022]	[0, 9.022]	[0, 8.919]	[0, 8.919]	[0, 8.953]	[0, 8.953]	[0, 8.995]	[0, 9.022]	[0, 8.919]	[0, 9.022]	[0, 9.023]
	$\mu_4 - \mu_{[1]}$	[0, 23.953]	[0, 23.995]	[0, 23.995]	[0, 24.022]	[0, 24.022]	[0, 23.919]	[0, 23.919]	[0, 23.953]	[0, 23.953]	[0, 23.995]	[0, 24.022]	[0, 23.919]	[0, 24.022]	[0, 24.023]
	$\mu_{[4]} - \mu_1$	[0, 24.953]	[0, 24.995]	[0, 24.995]	[0, 25.022]	[0, 25.022]	[0, 24.919]	[0, 24.919]	[0, 24.953]	[0, 24.953]	[0, 24.995]	[0, 25.022]	[0, 24.919]	[0, 25.022]	[0, 25.023]
	$\mu_{[4]} - \mu_2$	[0, 7.953]	[0, 7.995]	[0, 7.995]	[0, 8.022]	[0, 8.022]	[0, 7.919]	[0, 7.919]	[0, 7.953]	[0, 7.953]	[0, 7.995]	[0, 8.022]	[0, 7.919]	[0, 8.022]	[0, 8.023]
	$\mu_{[4]} - \mu_3$	[0, 30.953]	[0, 30.995]	[0, 30.995]	[0, 31.022]	[0, 31.022]	[0, 30.919]	[0, 30.919]	[0, 30.953]	[0, 30.953]	[0, 30.995]	[0, 31.022]	[0, 30.919]	[0, 31.022]	[0, 31.023]
	$\mu_{[4]} - \mu_4$	[0, 21.953]	[0, 21.995]	[0, 21.995]	[0, 22.022]	[0, 22.022]	[0, 21.919]	[0, 21.919]	[0, 21.953]	[0, 21.953]	[0, 21.995]	[0, 22.022]	[0, 21.919]	[0, 22.022]	[0, 22.023]
(0, 3, 6, 0)	$\mu_1 - \mu_{[1]}$	[0, 9.683]	[0, 9.702]	[0, 9.778]	[0, 9.702]	[0, 9.778]	[0, 9.702]	[0, 9.778]	[0, 9.702]	[0, 9.778]	[0, 9.683]	[0, 9.683]	[0, 9.683]	[0, 9.739]	[0, 9.738]
	$\mu_2 - \mu_{[1]}$	[0, 16.683]	[0, 16.702]	[0, 16.778]	[0, 16.702]	[0, 16.778]	[0, 16.702]	[0, 16.778]	[0, 16.702]	[0, 16.778]	[0, 16.683]	[0, 16.683]	[0, 16.683]	[0, 16.739]	[0, 16.738]
	$\mu_3 - \mu_{[1]}$	[0, 7.683]	[0, 7.702]	[0, 7.778]	[0, 7.702]	[0, 7.778]	[0, 7.702]	[0, 7.778]	[0, 7.702]	[0, 7.778]	[0, 7.683]	[0, 7.683]	[0, 7.683]	[0, 7.739]	[0, 7.738]
	$\mu_4 - \mu_{[1]}$	[0, 7.683]	[0, 7.702]	[0, 7.778]	[0, 7.702]	[0, 7.778]	[0, 7.702]	[0, 7.778]	[0, 7.702]	[0, 7.778]	[0, 7.683]	[0, 7.683]	[0, 7.683]	[0, 7.739]	[0, 7.738]
	$\mu_{[4]} - \mu_1$	[0, 14.683]	[0, 14.702]	[0, 14.778]	[0, 14.702]	[0, 14.778]	[0, 14.702]	[0, 14.778]	[0, 14.702]	[0, 14.778]	[0, 14.683]	[0, 14.683]	[0, 14.683]	[0, 14.739]	[0, 14.738]
	$\mu_{[4]} - \mu_2$	[0, 0.683]	[0, 0.702]	[0, 0.778]	[0, 0.702]	[0, 0.778]	[0, 0.702]	[0, 0.778]	[0, 0.702]	[0, 0.778]	[0, 0.683]	[0, 0.683]	[0, 0.683]	[0, 0.739]	[0, 0.738]
	$\mu_{[4]} - \mu_3$	[0, 16.683]	[0, 16.702]	[0, 16.778]	[0, 16.702]	[0, 16.778]	[0, 16.702]	[0, 16.778]	[0, 16.702]	[0, 16.778]	[0, 16.683]	[0, 16.683]	[0, 16.683]	[0, 16.739]	[0, 16.738]
	$\mu_{[4]} - \mu_4$	[0, 16.683]	[0, 16.702]	[0, 16.778]	[0, 16.702]	[0, 16.778]	[0, 16.702]	[0, 16.778]	[0, 16.702]	[0, 16.778]	[0, 16.683]	[0, 16.683]	[0, 16.683]	[0, 16.739]	[0, 16.738]

Table 12. Results of subset selection for Example 2.

(r, u, l, s)		WME1	WME2	WME3	WME4	WME5	WME6	WME7	WME8	WME9	WME10	WME11	WME12	AMLE	BLUE
(2, 5, 2, 2)	$\hat{\mu}_{[1],t} + (d_t \times (\hat{\theta}_t^*/n))$	11.895	11.926	11.953	11.900	11.931	11.892	11.924	11.879	11.916	11.961	11.930	11.914	12.170	11.827
	$\hat{\mu}_{[4],t} - (d_t \times (\hat{\theta}_t^*/n))$	−0.399	−0.512	−0.600	−0.436	−0.537	−0.394	−0.468	−0.352	−0.443	−0.612	−0.519	−0.457	−0.230	−0.573
	S_L	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4
	S_U	2, 4	2, 4	2, 4	2, 4	2, 4	2, 4	2, 4	2, 4	2, 4	2, 4	2, 4	2, 4	2, 4	2, 4
(6, 3, 0, 0)	$\hat{\mu}_{[1],t} + (d_t \times (\hat{\theta}_t^*/n))$	16.018	16.012	16.012	16.018	16.018	16.012	16.012	16.018	16.018	16.012	16.018	16.012	16.709	15.771
	$\hat{\mu}_{[4],t} - (d_t \times (\hat{\theta}_t^*/n))$	−12.288	−12.175	−12.175	−12.288	−12.288	−12.175	−12.175	−12.288	−12.288	−12.175	−12.288	−12.175	−11.593	−12.531
	S_L	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4
	S_U	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4	1, 2, 3, 4
(3, 3, 0, 3)	$\hat{\mu}_{[1],t} + (d_t \times (\hat{\theta}_t^*/n))$	13.360	13.359	13.359	13.372	13.372	13.347	13.347	13.360	13.360	13.359	13.372	13.347	13.732	13.246
	$\hat{\mu}_{[4],t} - (d_t \times (\hat{\theta}_t^*/n))$	−0.547	−0.631	−0.631	−0.672	−0.672	−0.492	−0.492	−0.547	−0.547	−0.631	−0.672	−0.492	−0.312	−0.801
	S_L	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4
	S_U	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4	1, 2, 4
(0, 3, 6, 0)	$\hat{\mu}_{[1],t} + (d_t \times (\hat{\theta}_t^*/n))$	6.579	6.594	6.649	6.594	6.649	6.594	6.649	6.594	6.649	6.579	6.579	6.579	6.686	6.597
	$\hat{\mu}_{[4],t} - (d_t \times (\hat{\theta}_t^*/n))$	0.213	0.190	0.092	0.190	0.092	0.190	0.092	0.190	0.092	0.213	0.213	0.213	0.209	0.121
	S_L	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4	1, 3, 4
	S_U	1, 2	1, 2	1, 2	1, 2	1, 2	1, 2	1, 2	1, 2	1, 2	1, 2	1, 2	1, 2	1, 2	1, 2

failures for each plane is independent and exponentially distributed. The likelihood ratio asymptotic χ^2-test [9] had shown a non-significant difference among the scale parameters of four two-parameter exponential distributions. Considering the censoring schemes of $(r, u, l, s) = (2, 5, 2, 2)$, $(6, 3, 0, 0)$, $(3, 3, 0, 3)$ and $(0, 3, 6, 0)$, the required statistics and critical values of d_t for $1 - \alpha = 0.95$ are listed in Table 10.

The SCIs of $\mu_1 - \mu_{[1]}$, $\mu_2 - \mu_{[1]}$, $\mu_3 - \mu_{[1]}$, $\mu_4 - \mu_{[1]}$, $\mu_{[4]} - \mu_1$, $\mu_{[4]} - \mu_2$, $\mu_{[4]} - \mu_3$ and $\mu_{[4]} - \mu_4$ are listed in Table 11. Based on the criterion of the minimum confidence length, plane 2 is identified as the UEP and plane 3 is identified as the LEP for all censoring schemes. But for the censoring scheme $(r, u, l, s) = (2, 5, 2, 2)$, plane 1 is also identified as the LEP. For censoring scheme $(r, u, l, s) = (0, 3, 6, 0)$, plane 4 is also identified as LEP. As to the subset selection procedure, the results are listed in Table 12. The size of selected subset S_{L} is greater than or equal to the size of selected subset S_{U} for all cases. The subset selection procedure does not perform well under the censoring scheme of $(r, u, l, s) = (6, 3, 0, 0)$, since the selected subset size is too large.

6. Conclusions

In this paper, 14 SCIs for all distances from the extreme populations and from the UEP from these k independent exponential distributions under the multiply type II censoring are proposed. From the simulation results, WME4, AMLE, and BLUE are recommended for use based on the criteria of minimum confidence length for various censoring schemes. Without decreasing the probability of CS, the subset selection procedures of extreme populations are also proposed. Finally, two numerical examples are provided to illustrate the proposed procedures.

References

[1] J.C. Hsu, *Simultaneous confidence intervals for all distances from the best*, Ann. Statist. 9 (1981), pp. 1026–1034.

[2] S.N. Mishra and E.J. Dudewicz, *Simultaneous selection of extreme populations: A subset selection approach*, Biom. J. 29(4) (1987), pp. 471–483.

[3] N.L. Johnson, S. Kotz, and N. Balakrishnan, *Continuous Univariate Distributions*, Wiley, New York, 1994.

[4] L.J. Bain and M. Engelhardt, *Statistical Analysis of Reliability and Life Testing Models*, Marcel Dekker, New York, 1991.

[5] J.F. Lawless and K. Singhal, *Analysis of data from life test experiments under an exponential model*, Nav. Res. Logist. Q. 27 (1980), pp. 323–334.

[6] M. Zelen, *Application of exponential models to problems in cancer research*, J. R. Stat. Soc. Ser. A (General) 129 (1966), pp. 368–398.

[7] N. Misra and I.D. Dhariyal, *Simultaneous confidence intervals for all distances from the worst and best populations*, Comm. Statist. Theory Methods 22(11) (1993), pp. 3097–3116.

[8] N.R. Mann, R.E. Schafer, and N.D. Singpurwalla, *Methods for Statistical Analysis of Reliability and Life Data*, Wiley, New York, 1974.

[9] J.F. Lawless, *Statistical Models and Methods for Lifetime Data*, Wiley, New York, 2003.

[10] W.Q. Meeker and L.A. Escobar, *Statistical Methods for Reliability Data*, Wiley, New York, 1998.

[11] H.J. Chen and K. Vanichbuncha, *Simultaneous upper confidence bounds for distances from the best two-parameter exponential distribution*, Comm. Statist. Theory Methods 18(8) (1989), pp. 3019–3031.

[12] J.W. Wu and C.C. Yang, *Weighted moments estimation of the scale parameter of the exponential distribution based on a multiply type II censored sample*, Qual. Reliab. Eng. Int. 18 (2002), pp. 149–154.

[13] N. Balakrishnan, *On the maximum likelihood estimation of the location and scale parameters of exponential distribution based on multiply type II censored samples*, J. Appl. Stat. 17 (1990), pp. 55–61.

[14] K. Balasubramanian and N. Balakrishnan, *Estimation for one- and two-parameter exponential distributions under multiple type II censoring*, Statist. Papers 33 (1992), pp. 203–216.

[15] S.C. Chen, *Prediction intervals of future observation and waiting time from exponential and Pareto distributions based on multiply type II censored samples*, Master diss., Tamkang University, Tamsui, Taipei, Taiwan, ROC, 2003.

[16] D. Sun and S.W. Kim, *Intrinsic priors for testing ordered exponential means*, Tech. Rep., University of Missouri, 1999.

Appendix 1. Proof of Theorem 1

Let $\hat{\mu}_{(1),t}$ be associated with the population $\pi_{(1)}$ with the smallest location parameter $\mu_{[1]}$ and $\hat{\mu}_{(k),t}$ be associated with the population $\pi_{(k)}$ with the largest location parameter $\mu_{[k]}$. Then

$$\begin{aligned}
P_{\vec{\mu}}(\mathrm{SCI}) &= P_{\vec{\mu}}(D_{i,t} \geq \mu_i - \mu_{[1]},\ D^{i,t} \geq \mu_{[k]} - \mu_i,\ i = 1, 2, \ldots, k) \\
&= P_{\vec{\mu}}\Bigg(\max\left\{Y_{i,(r+1)} - \min_{j\neq i} Y_{j,(r+1)} + d_t \frac{\hat{\theta}_t^*}{n}, 0\right\} \geq \mu_i - \mu_{[1]}, \\
&\qquad \max\left\{\max_{j\neq i} Y_{j,(r+1)} - Y_{i,(r+1)} + d_t \frac{\hat{\theta}_t^*}{n}, 0\right\} \geq \mu_{[k]} - \mu_i,\ i = 1, 2, \ldots, k\Bigg) \\
&\geq P_{\vec{\mu}}\Bigg(Y_{i,(r+1)} - \min_{j\neq i} Y_{j,(r+1)} + d_t \frac{\hat{\theta}_t^*}{n} \geq \mu_i - \mu_{[1]},\ i \neq (1), \\
&\qquad \max_{j\neq i} Y_{j,(r+1)} - Y_{i,(r+1)} + d_t \frac{\hat{\theta}_t^*}{n} \geq \mu_{[k]} - \mu_i,\ i \neq (k)\Bigg) \\
&\geq P_{\vec{\mu}}\Bigg(Y_{i,(r+1)} - Y_{(1),(r+1)} + d_t \frac{\hat{\theta}_t^*}{n} \geq \mu_i - \mu_{[1]},\ i \neq (1), \\
&\qquad Y_{(k),(r+1)} - Y_{i,(r+1)} + d_t \frac{\hat{\theta}_t^*}{n} \geq \mu_{[k]} - \mu_i,\ i \neq (k)\Bigg) \\
&= P_{\vec{\mu}}(Z_{i,t} \geq Z_{(1),t} - d_t,\ i \neq (1),\ Z_{i,t} \leq Z_{(k),t} + d_t,\ i \neq (k)) \\
&= P_{\vec{\mu}}(Z_{(1),t} - d_t \leq Z_{i,t} \leq Z_{(k),t} + d_t,\ i \neq (1)(k),\ Z_{(1),t} \leq Z_{(k),t} + d_t) \\
&= P_{\vec{\mu}}(Z_t^* \leq d_t) \\
&= 1 - \alpha,
\end{aligned}$$

where

$$\begin{aligned}
Z_t^* &= \max\{Z_{1,t} - Z_{i,t}, Z_{i,t} - Z_{k,t}, i \neq 1, k, Z_{1,t} - Z_{k,t}\}, \\
Z_{i,t} &= \frac{n(Y_{i,(r+1)} - \mu_i)}{\hat{\theta}_t^*} = \frac{nY_{i,(r+1)}^*}{W_t^*}, \\
Y_{i,(r+1)}^* &= \frac{Y_{i,(r+1)} - \mu_i}{\theta}
\end{aligned}$$

and

$$W_t^* = \frac{\hat{\theta}_t^*}{\theta}, \quad i = 1, 2, \ldots, k, \quad t = 1, 2, \ldots, 14.$$

Since $Y_{i,(r+1)}^*$ is the $(r+1)$th order statistic from a standard exponential distribution and the distribution of W_t^* is independent of parameters, then we can see that the distribution of Z_t^* is independent of parameters and only depends on $n, r, u, l,$ and s, $i = 1, 2, \ldots, k$, $t = 1, 2, \ldots, 14$. The proof of Theorem 1 is completed.

Appendix 2. Proof of Equation (25)

$$\begin{aligned}
1 - \alpha &= P_{\vec{\mu}}(Z_{(1),t} - d_t \leq Z_{i,t} \leq Z_{(k),t} + d_t,\ i \neq (1)(k),\ Z_{(1),t} \leq Z_{(k),t} + d_t) \\
&= P_{\vec{\mu}}(Z_{i,t} \geq Z_{(1),t} - d_t,\ i \neq (1),\ Z_{(k),t} \geq Z_{i,t} - d_t,\ i \neq (k)) \\
&= P_{\vec{\mu}}\Bigg(Y_{i,(r+1)} - Y_{(1),(r+1)} + d_t \frac{\hat{\theta}_t^*}{n} \geq \mu_i - \mu_{[1]},\ i \neq (1), \\
&\qquad Y_{(k),(r+1)} - Y_{i,(r+1)} + d_t \frac{\hat{\theta}_t^*}{n} \geq \mu_{[k]} - \mu_i,\ i \neq (k)\Bigg)
\end{aligned}$$

$$\leq P_{\vec{\mu}}\Bigg(Y_{[1],(r+1)} - Y_{(1),(r+1)} + d_t\frac{\hat{\theta}_t^*}{n} \geq 0,\ Y_{(k),(r+1)} - Y_{[k],(r+1)} + d_t\frac{\hat{\theta}_t^*}{n} \geq 0,$$

$$D_{i,t} \geq \mu_i - \mu_{[1]},\ D^{i,t} \geq \mu_{[k]} - \mu_i, \quad i = 1, 2, \ldots, k\Big)$$

$$= P_{\vec{\mu}}(\pi_{(1)} \in S_L,\ \pi_{(k)} \in S_U,\ D_{i,t} \geq \mu_i - \mu_{[1]},\ D^{i,t} \geq \mu_{[k]} - \mu_i, \quad i = 1, 2, \ldots, k),$$

$t = 1, 2, \ldots, 14$. Thus, we can assert Equation (25).