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A TRAIN SERVICE PLANNING MODEL WITH DYNAMIC DEMAND FOR INTERCITY RAILWAY SYSTEMS

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Abstract: The journey time of intercity trains is usually more than hours. The train dispatched at its departure station may not be able to serve the passengers during the same hour at downstream stations. In addition, intercity trains usually consist of various stopping patterns. Consequently, typical method for determining train service plans based on maximum passenger load and train capacity is inapplicable for such systems. For these reasons, this paper develops a mathematical model for generating optimal train service plans for intercity railways, taking into account the dynamics of train movements and the transfer of unserved passengers. This model combines operator's revenue, operating cost, and users' travel-time cost as its objective. The operational requirements are formulated as the constraints. The Taiwan High Speed Rail is taken as an example to test the model. The case study shows that the model is very efficient and is very flexible for different operation strategies.

Key Words: *Intercity Railway System, Train Service Plan, Optimization*

1. INTRODUCTION

The operation planning of a railway system is a very complex decision-making process. From passengers' point of view, high service frequency, short travel time and comfortable seat arrangement are always desired. However, from an operator's standpoint, inadequately high quality service implies high operation cost. As a result, railway operators make great efforts in operation planning in order to provide sufficient train service with minimal operating and maintenance costs. Since traveler demand significantly differ in various time periods, efficiently utilizing infrastructures, vehicles, and crews to accommodate travel demand becomes an important issue for railway operators.

Due to the complexity of the problem, most railway operators apply a hierarchically structured planning process (Bussieck *et al.*, 1997). The planning process may have variations for different operators. But the underlying concept is similar. An example is introduced in Jong *et al.* (2006) and is shown in Figure 1. Operators usually estimate travel demand via marketing research, transportation planning model, and historical data. The demand is then inputted to a train service planning model to determine the service headway for each operation pattern in each hour. The resulting headways are then converted into a baseline schedule with train conflicts. The schedule is further refined to a published timetable through a train scheduling model to resolve the conflicts. Based on the resulting timetable, track occupancy, vehicle management, and crew management plans are then developed. The process is repeated regularly to better fit passenger demand and to efficiently utilize operators' resources.

From Figure 1 it can be seen that the train service planning process plays an important role in linking demand side and supply side. In fact, an inadequate service plan cannot meet demand, while over supply will increase operation costs and waste capacity. The activity of train service planning involves the determination of train service patterns, frequencies, and ridership allocation between different patterns. This paper focuses on the development of such a model for intercity railways. The paper would follow the previous research by Jong *et al* (2006) and amend the framework to formulate a more accurate mathematical model, especially dealing with the dynamics of train movements and the transfer of unserved passengers. Finally, the empirical study on the Taiwan High Speed Rail (HSR) is presented to show how the model works and to display all the resulting performance indices.

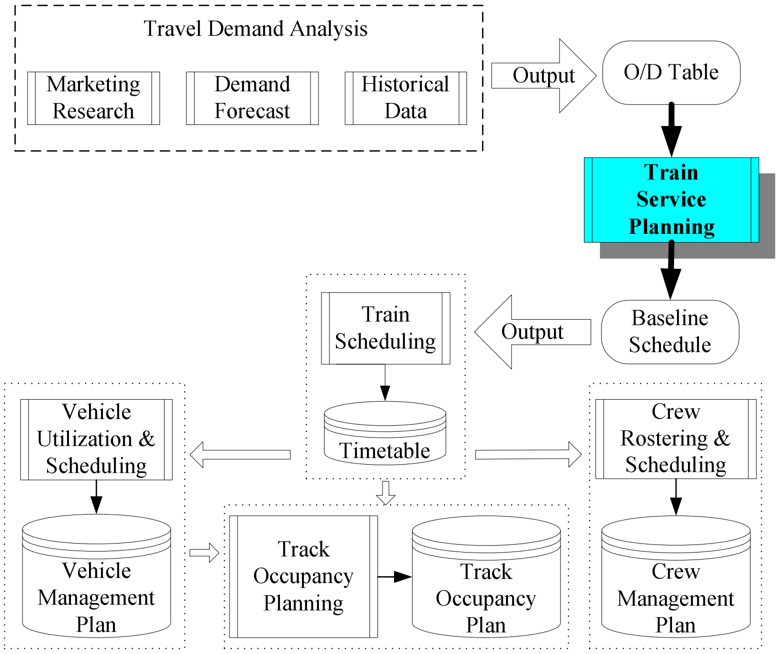


Figure 1 Planning process related to rail transportation

2. LITERATURE REVIEW

The nature of train service plan is inherently multi-objective. This is primarily due to the multiplicity of interests embodied by different stakeholders and social concerns. Current and Min (1986) points out that transportation planning objectives include cost, accessibility, environmental concerns, revenue, and regional equity among others. Some recent studies also show the advantages of using multi-objective functions for transportation planning, especially in transportation network problem (Current *et al.*, 1987), air services planning (Flynn and Ratick, 1988), train service planning (Shen, 1995, Chang *et al.*, 2000, Jong *et al.*, 2006), train scheduling (Ghoseiri *et al.*, 2004). For this reason, this research formulates the optimal train service planning problem as a multi-objective programming model from both the viewpoints of operator and passengers.

The train service planning problem could be solved by heuristic algorithms or mathematical programming method. Some recent studies have shown the pros and cons about both methods.

Heuristic algorithms could get an approximately good solution quickly no matter how enormous the problems are, but they have some risks of being trapped into the local optimum (Shen, 1995, Hung, 1998, Chang *et al.*, 2000). On the contrary, mathematical programming methods could get the exact optimal solution, but they usually take longer solution time, especially when dealing with complicated problems (Chen, 1998, Jong *et al.*, 2006). In order to achieve the exact optimal solution, the research decides to adopt mathematical programming approach.

Furthermore, due to the complexity of the problem, there have different mathematical programming methods to formulate the problem, such as multi-commodity flow (MCF) and integer programming (IP). A comprehensive survey on the literature dealing with the multi-commodity flow problems can be found in Assad (1978). A recent review is also given in Ghoseiri *et al.* (2004). The study compares multi-commodity flow and integer programming formulations in an edge capacity problem, and it concludes that multi-commodity flow requires more number of variables and restrictions than integer programming. In fact, we also conduct a test on a small scale problem for train service planning problem, and we get the same results. Since solution efficiency is related to the amounts of variables and constrains, integer programming is employed in this research to formulate the problem.

The forecast passenger demand for railway system is usually expressed in O/D matrices on an hourly basis, where the demand is regarded as a uniform distribution within each hour. The demand could be directly applied to MRT system for determining train service frequency because the journey time is usually less than one hour and the stopping patterns are the same for all trains. However, for intercity railway systems, the train journey time is usually more than one hour. As a result, the train departing from its origin station would not be able to serve the passengers during the same hour at far downstream stations. Therefore, the static demand must be converted into dynamic demand by taking the time-space characteristics of the train into account before inputting into train service planning models (Chen, 1998). Furthermore, since not all passengers can be served in the peak hour, some of them may take the trains in the next hour (Jong *et al*, 2006). For the above reasons, this research will consider both the dynamic demand and the transfer of unserved passengers in developing the train service planning model.

3. FUNCTIONAL REQUIREMENTS AND ASSUMPTIONS

3.1 Functional Requirements

The functional requirements for a train service planning model depend on the operation strategies and the resource availability of the railway company. The requirements may not be the same for different operators, but they are quite similar. In doing this research, we have discussed with the staffs in Taiwan High Speed Rail Corporation (THSRC) to identify the requirements. Below are some practical and physical conditions should be considered:

- More than one service pattern (combination of origin-destination and stopping mode).
- Considering the dynamic demand instead of static demand in the model, which makes the model more appropriate to the practical experience.
- Line capacity limitation along each section and each direction.
- Maximum number of trains could be dispatched in each hour, which is influenced by the layout of track configuration and the availability of crews.

- The fleet size limitation.
- Minimum service frequency for each service pattern, which would ensure the service quality.
- Train capacity limitation, which is restricted by the train's seats and standing space.
- Minimum service ratio of each original-destination (OD) pair, which would ensure the percentage of passengers to be served for each OD pair.
- The limitation on maximum loading factor, which would guarantee the level of service in trains.
- Optimizing the train service planning problem for the whole operation duration instead of dealing with it by each hour.

3.2 Assumptions

Prior to formulating the train service plan problem, several assumptions are made. These assumptions are:

- Travel demand is given and uniformly distributed in each hour. The research would adopt time-space adjustment in O/D matrix later.
- Different seat classes are treated the same.
- While traveling in the same itinerary, change trains are not allowed.
- Train equilibrium between southbound and northbound is not considered. The operator will dispatch returning trains if necessary.
- The origin and destination stations of each service pattern must be fixed due to the station layouts and depot locations. But the intermediate stopping stations can be modified.
- Transfer ratio is taken into consideration for unserved passengers.

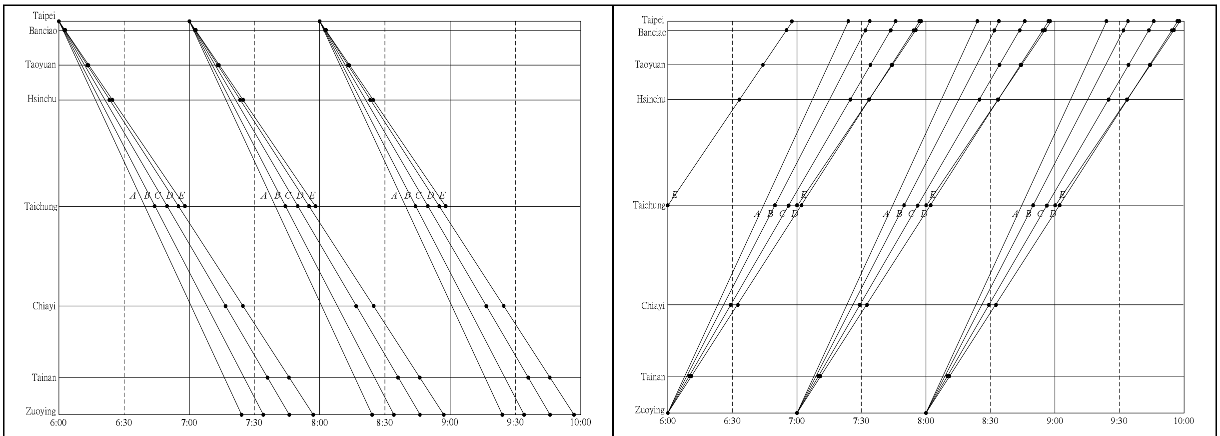
4. THE CALCULATION OF DYNAMIC DEMAND

Figure 2 shows that there are eight stations and five service patterns in the original operation plan for Taiwan HSR line. The departure station for southbound trains is Taipei and the destination station could be Taichung or Zuoying. On the other hand, the origin station for northbound trains could be Zuoying or Taichung and the destination station is Taipei.

Pattern	Taipei	Banciao	Taoyuan	Hsinchu	Taichung	Chiayi	Tainan	Zuoying
A	●							●
B	●	●			●			●
C	●	●			●	●	●	●
D	●	●	●	●	●	●	●	●
E	●	●	●	●	●			

Figure 2 Expected service patterns for Taiwan HSR in the initial stage

Here we take southbound trains for instance to illustrate the time-space adjustment for the static demand. Figure 3(a) shows that if a train of pattern D departs from Taipei at 06:00, the train will just pass through Taichung around 07:00 and arrive at the Zuoying near 07:57. This means that passengers between Chiayi to Zuoying would not be served by the train although there has demand within 6:00 ~ 7:00. Therefore, the static hourly demand must be adjusted to correspond to the movement of the train. While proceeding with the adjustment, it should calculate train speed and estimate the accumulated operation time. For the reason that train speed depends on the service patterns, which is a variable in the model, the average speed is employed in this paper to estimate the movements of the trains. The numbers shown in the third and fourth columns in Table 1 are just an illustration based on an average speed of 200 km/hr.



(a) Southbound (b) Northbound

Figure 3 Time-Space diagram of each service pattern for southbound and northbound trains

Table 1 Example of the accumulated trip time

Station	Station Number	Mileage (km)	Accumulate operation time for southbound trains (Min)	Accumulate operation time for northbound trains (Min)
Taipei	1	5	0.00	102.00
Banciao	2	13	2.40	99.60
Taoyuan	3	42	11.10	90.90
Hsinchu	4	72	20.10	81.90
Taichung	5	165	48.00	54.00
Chiayi	6	251	73.80	28.20
Tainan	7	314	92.40	9.60
Zuoying	8	345	102.00	0.00

4.1 Time-Space Adjustment for Southbound Trains

Figure 4 is the service time zone for southbound trains in the morning. Assume that passenger demand is uniformly distributed in each hour. The dynamic demands for the 1st to 16th service hours can be derived as below:

$$\text{If } 0 \le t_{1i} \le 60, \text{ then } \overline{D}_{ij}^{t} = \frac{(60 - t_{1i})}{60} D_{ij}^{t} + \frac{t_{1i}}{60} D_{ij}^{t+1}, \ \forall j > i \tag{1}$$

$$\text{If } 60 \le t_{1i} \le 120, \text{ then } \overline{D}_{ij}^{t} = \frac{(120 - t_{1i})}{60} D_{ij}^{t+1} + \frac{(t_{1i} - 60)}{60} D_{ij}^{t+2}, \ \forall j > i \tag{2}$$

where t_{1i} = accumulated travel time for southbound trains from Taipei to station i (min)

$\overline{D}_{ij}^{t}$ = the average trip demand from station i to j during the service time zone of time period t (prs/hr)

D_{ij}^{t} = the original trip demand from station i to j during time period t (prs/hr)

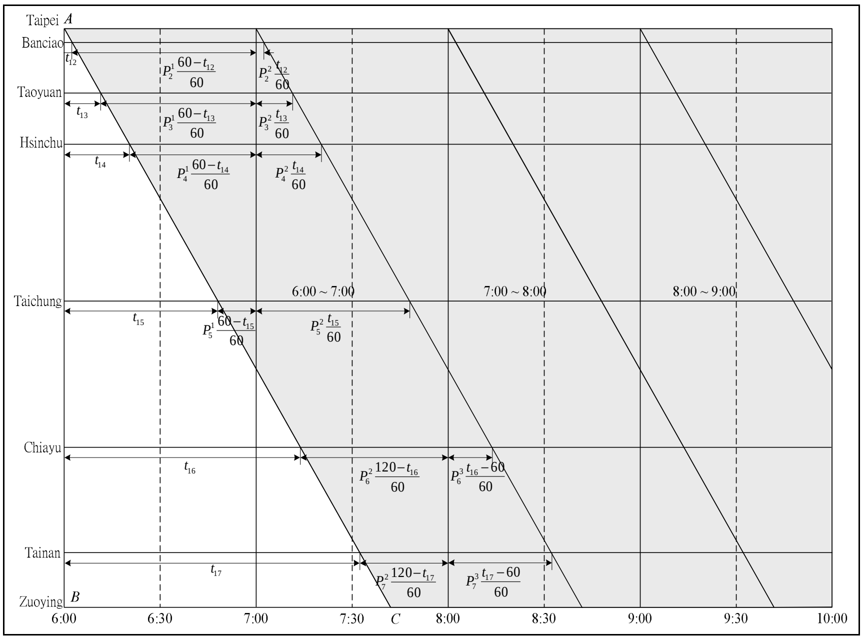


Figure 4 Service time zone for southbound trains in the morning

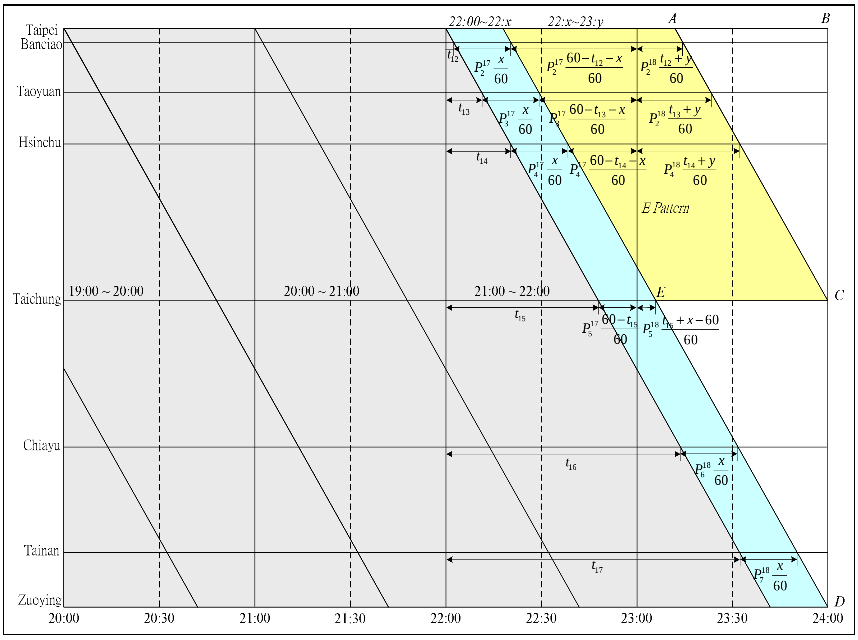


Figure 5 Service time zone for southbound trains in the evening

The operation time of Taiwan HSR is 06:00~24:00, which includes 18 time periods, and all of the trains must arrive at terminal stations before 24:00. Therefore, the last service time zone is not 23:00~24:00. From Figure 5 we could find the last time zone from Taipei to Zuoying is 22:00~22:x for all service patterns, and the last one from Taipei to Taichung is 22:x~23:y for pattern E only, where x and y are dependent upon the average speed of the service patterns. Thus, the dynamic demands for the 17th and 18th service hour can be derived as follows:

$$\text{If } 0 \le t_{1i} + x < 60 \text{, then } \overline{D}_{ij}^{17} = \frac{x}{60} D_{ij}^{17} \text{, } \forall j > i \qquad (3)$$

$$\text{If } t_{1i} < 60 \le t_{1i} + x \text{, then } \overline{D}_{ij}^{17} = \frac{(60 - t_{1i})}{60} D_{ij}^{17} + \frac{(t_{1i} + x - 60)}{60} D_{ij}^{18}, \ \forall j > i \tag{4}$$

$$\text{If } 60 \le t_{1i} < 120 \text{, then } \overline{D}_{ij}^{17} = \frac{x}{60} D_{ij}^{18}, \ \forall j > i \tag{5}$$

$$\overline{D}_{ij}^{18} = \frac{(60 - t_{1i} - x)}{60} D_{ij}^{17} + \frac{(t_{1i} + y)}{60} D_{ij}^{18}, \ \forall 5 \ge j > i \tag{6}$$

It is noted that passengers in the triangle ΔABC and ΔCDE of Figure 5 are unable to be served by the trains.

4.2 Time-Space Adjustment for Northbound Trains

Since the departure station for northbound trains could be Zuoying or Taichung, their service time zones will have some overlapping area, as shown in Figure 6. For the sake of producing train service plans on the same base, the corresponding time period for the trains starting from Taichung will have time lag behind those from Zuoying. Assume that the overlapping duration for pattern E is p minutes, then the first $60 - p$ minutes is regards as an individual time period during which the passengers can only be served by the trains of pattern E. Afterwards, the demand in each time zone could be served by all type of trains.

It is now clear that the dynamic demand for pattern E during the 1st service hour can be derived as below:

$$\overline{D}_{ij}^{1} = \frac{(60 - t_{5i})}{60} D_{ij}^{1} + \frac{(t_{5i} - p)}{60} D_{ij}^{2}, \ \forall j < i \le 5 \tag{7}$$

where t_{5i} = the accumulated travel time from Taichung to station i (min)

With regard to the 2nd to 17th time period, the dynamic demands could be calculated by:

$$\text{If } 0 \le t_{8i} < 60 \text{, then } \overline{D}_{ij}^{t} = \frac{(60 - t_{8i})}{60} D_{ij}^{t-1} + \frac{t_{8i}}{60} D_{ij}^{t}, \ \forall j < i \tag{8}$$

$$\text{If } 60 \le t_{8i} < 120 \text{, then } \overline{D}_{ij}^{t} = \frac{(120 - t_{8i})}{60} D_{ij}^{t} + \frac{(t_{8i} - 60)}{60} D_{ij}^{t+1}, \ \forall j < i \tag{9}$$

where t_{8i} = the accumulated travel time from Zuoying to station i (min)

Since all trains must arrive at terminal station before 24:00, Figure 7 shows that the last service time zone is from 22:00~22:q, where q is dependent upon the average speed of the supplied service patterns. Therefore, the static demand can be adjusted by the following equations:

$$\text{If } 0 \le t_{8i} + q < 60 \text{, then } \overline{D}_{ij}^{18} = \frac{q}{60} D_{ij}^{17}, \ \forall j < i \tag{10}$$

$$\text{If } t_{8i} < 60 \le t_{8i} + q \text{, then } \overline{D}_{ij}^{18} = \frac{(60 - t_{8i})}{60} D_{ij}^{17} + \frac{(t_{8i} + q - 60)}{60} D_{ij}^{18}, \ \forall j < i \tag{11}$$

$$\text{If } 60 \le t_{8i} < 120 \text{, then } \overline{D}_{ij}^{18} = \frac{q}{60} D_{ij}^{18},\ \forall j < i \tag{12}$$

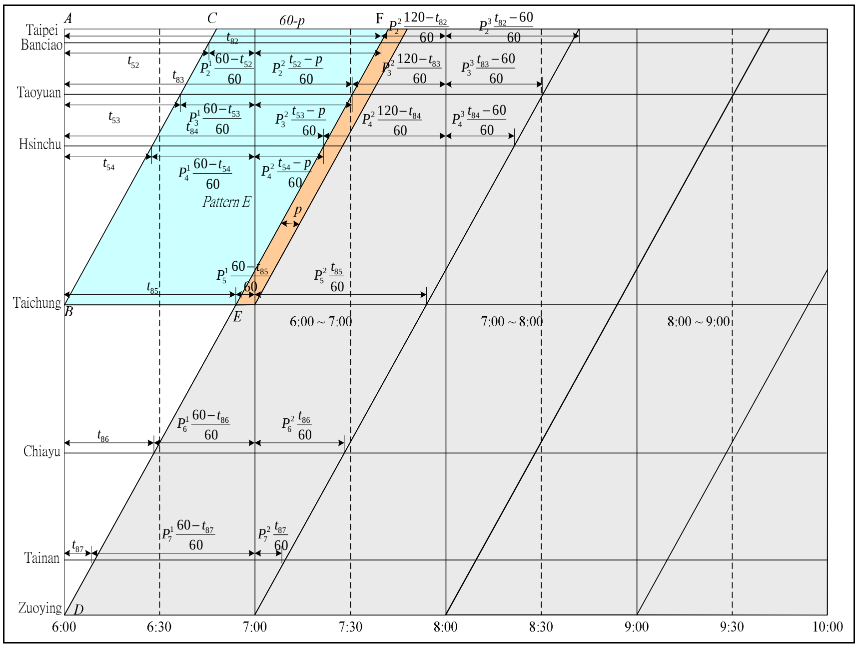


Figure 6 Service time zone for northbound trains in the morning

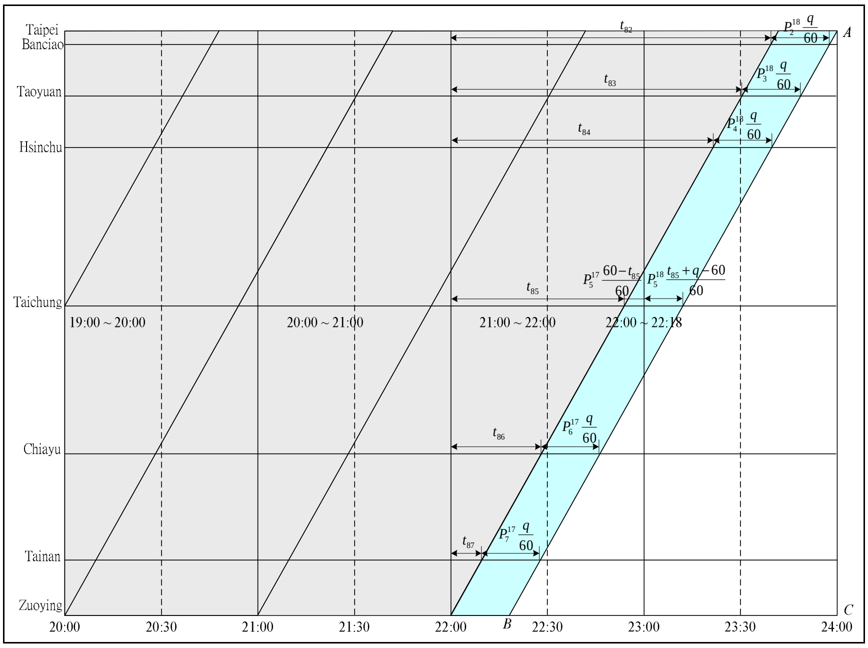


Figure 7 Service time zone for northbound trains in the evening

5. THE TRANSFER OF UNSERVED DEMAND

In real world, if the supplied service capacity is less than the demand, then some of the potential customers may take other transportation modes. But the others may consider taking the trains in the next time periods. In this section, we will illustrate how to combine the transfer of unserved passengers into the dynamic demand calculated from the previous section.

5.1 Transfer of Unserved Passengers for Southbound Trains

We would follow Figure 4 and Figure 5 to illustrate the transfer of unserved demand for southbound trains in each service time zone.

1. The 1st service time zone

The demand occurred within the triangle ΔABC in Figure 4 is regarded as the unserved passengers. Let β be the ratio of these passengers transferring to the first time period. Then the demand for the first service time zone can be computed as follows:

$$\text{If } 0 \le t_{1i} < 60\text{, then } T_{ij}^1 = \overline{D}_{ij}^1 + \beta(\frac{t_{1i}}{60} D_{ij}^1),\ \forall i < j \tag{13}$$

$$\text{If } 60 \le t_{1i} < 120\text{, then } T_{ij}^1 = \overline{D}_{ij}^1 + \beta(D_{ij}^1 + \frac{(t_{1i} - 60)}{60} D_{ij}^2),\ \forall i < j \tag{14}$$

where T_{ij}^1 = the trips from station i to j during the first time period, including dynamic demand and the transfer demand (prs /hr)

2. The 2nd to 16th service time zones

For the 2nd to 16th service time zones, the final demand can be computed by adding the original dynamic demand and those transferred from the previous time zone:

$$T_{ij}^{t+1} = \overline{D}_{ij}^{t+1} + \beta(T_{ij}^t - S_{ij}^t),\ \forall t = 2,....,16,,\ \forall i < j \tag{15}$$

where T_{ij}^{t+1} = the trip demand from station i to j during time period $t+1$ (prs /hr)

S_{ij}^t = the passengers who have been served from station i to j during time period t (prs /hr)

3. The 17th service time zone

This could be separated into two different conditions, with or without service pattern E.

(1) If service pattern E exists, there are two further situations to be discussed:

- While the southbound destination is at Taichung or to the north of Taichung (i.e., $j \le 5$), then there still have trains that can serve these trips in the next time period. In such a case, the transfer ratio must take x minutes duration into consideration, and the final demand is given by

$$T_{ij}^{17} = \overline{D}_{ij}^{17} + \beta \cdot (\frac{x}{60}) \cdot (T_{ij}^{16} - S_{ij}^{16}),\ \forall i < j \le 5 \tag{16}$$

- While the southbound destination is to the south of Taichung (i.e., $j > 5$), then the 17th service time zone is the last chance for these trips and the final demand is:

$$T_{ij}^{17} = \overline{D}_{ij}^{17} + \beta \cdot (T_{ij}^{16} - S_{ij}^{16}),\ \forall i < j \text{ and } j > 5 \tag{17}$$

(2) If service pattern E does not exist, the formulation is as below:

$$T_{ij}^{17} = \overline{D}_{ij}^{17} + \beta \cdot (T_{ij}^{16} - S_{ij}^{16}), \ \forall i < j \tag{18}$$

4. The 18th service time zone

This time period is reserved for pattern E only and the demand is calculated as below:

$$T_{ij}^{18} = \overline{D}_{ij}^{18} + \beta \cdot \left[(\frac{60-x}{60}) \cdot (T_{ij}^{16} - S_{ij}^{16}) + (T_{ij}^{17} - S_{ij}^{17}) \right], \ \forall i < j \le 5 \tag{19}$$

5.2 Transfer of Unserved Passengers for Northbound Trains

The transfer of unserved passengers for northbound trains is illustrated in Figure 6 and Figure 7, and is discussed as below:

1. The 1st service time zone

First we should judge whether pattern E is in service or not. If it does not exist, then we could go to next time period. Otherwise, we should transfer the unserved trips within the triangle ΔABC in Figure 6 and combine them into the next service time zone:

$$T_{ij}^{1} = \overline{D}_{ij}^{1} + \beta \cdot (\frac{60-p}{60}) \cdot (\frac{t_{5i}}{60} D_{ij}^{1}), \ \forall j < i \le 5 \tag{20}$$

where t_{5i} = the accumulated travel time from Taichung to station i (min)

2. The 2nd service time zone

This could be separated into two different cases, with or without service pattern E.

(1) If service pattern E exists, there still have two further situations to be discussed:

- While the departure station is at Taichung or to the north of Taichung (i.e., $i \le 5$), then the demand should include part of unserved passengers within the triangle ΔABC in Figure 6, i.e.,

$$T_{ij}^{2} = \overline{D}_{ij}^{2} + \beta \cdot \left[(T_{ij}^{1} - S_{ij}^{1}) + (\frac{p}{60}) \cdot (\frac{t_{5i}}{60} D_{ij}^{1}) \right], \ \forall j < i \le 5 \tag{21}$$

- While the departure station is to the south of Taichung (i.e., $i > 5$), then the demand should consider part of unserved trips within the triangle ΔBDE in Figure 6, i.e.,

$$\text{If } 0 \le t_{8i} < 60, \text{ then } T_{ij}^{2} = \overline{D}_{ij}^{2} + \beta(\frac{t_{8i}}{60} D_{ij}^{1}), \ \forall i \ge 5 \text{ and } i > j \tag{22}$$

$$\text{If } 60 \le t_{8i} < 120\text{, then } T_{ij}^{2} = \overline{D}_{ij}^{2} + \beta(D_{ij}^{1} + \frac{(t_{8i} - 60)}{60} D_{ij}^{2}),\ \forall i \ge 5 \text{ and } i > j \tag{23}$$

where t_{8i} = the accumulated travel time from Zuoying to station i (min)

(2) If service pattern E does not exist, the demand should include part of unserved passengers within the triangle ΔADF in Figure 6, i.e.,

$$\text{If } 0 \le t_{8i} < 60\text{, then } T_{ij}^{2} = \overline{D}_{ij}^{2} + \beta(\frac{t_{8i}}{60} D_{ij}^{1}),\ \forall i > j \tag{24}$$

$$\text{If } 60 \le t_{8i} < 120\text{, then } T_{ij}^{2} = \overline{D}_{ij}^{2} + \beta(D_{ij}^{1} + \frac{(t_{8i} - 60)}{60} D_{ij}^{2}),\ \forall i > j \tag{25}$$

3. The 3rd to 18th service time zones

$$T_{ij}^{t+1} = \overline{D}_{ij}^{t+1} + \beta(T_{ij}^{t} - S_{ij}^{t}),\ \forall t = 3,....,18,\ \forall i > j \tag{26}$$

6. MODEL FORMULATION AND SOLUTION PROCESS

6.1 The model Formulation

The symbols for endogenous, assembly, and decision variables that are used in the model are listed in Table 2 to Table 4, followed by the formulation. A brief explanation of the model is given in this section. For more detailed discussions, please refer to Jong *et al*. (2006).

Table 2 Decision variables of the model

Symbol	Meaning	Unit	Remark
f_k^t	Service frequency of pattern k in time zone t	train/hr	unknown
S_k^t	Served trips of pattern k in time zone t	prs /hr	unknown
S_{ij}^t	Served trips from station i to j in time zone t	prs /hr	unknown
S_{ij}	Served trips from station i to j	prs /hr	unknown
S_{ijk}^t	Served trips of pattern k from station i to j in time zone t	prs /hr	unknown
L_{kmn}^t	Served trips of pattern k in the section (m,n) and in time zone t	prs /hr	unknown
E_{kmn}^t	Vacant seats of pattern k in the section (m,n) and in time zone t	seat/hr	unknown
T_{ij}^t	Travel demand from station i to j in time zone t	prs /hr	unknown
T_{ij}	Travel demand from station i to j	prs /hr	unknown

Table 3 Assembly of the model

Symbol	Meaning	Unit	Remark
A	Track section result from adjacent station	—	known
K	Assembly of train service mode	—	known
N	Assembly of station	—	known
T	Assembly of time period	—	known

Table 4 Endogenous variables of the model

Symbol	Meaning	Unit	Remark
a_{ijk}	Whether pattern k could serve passengers from station i to j (1) pattern k stops at i and $j \Rightarrow a_{ijk} = 1$ (2) pattern k dose not stop at i and $j \Rightarrow a_{ijk} = 0$	—	known
b_{ijmn}	Whether OD pair (i, j) pass through the section (m,n) (1) (i, j) pass through $(m,n) \Rightarrow b_{ijmn} = 1$ (2) (i, j) dose not pass through $(m,n) \Rightarrow b_{ijmn} = 0$	—	known
C_k	The operation and maintenance cost of Pattern k	NT$/train	known
C_{mn}	Capacity of section (m,n)	train/hr	known
C_o	Operation Cost	NT$/day	known
C_T	Train capacity of both standees and seats	prs /train	known
C_S	Train seat capacity	seats/train	known
C_u	Traveling time cost	NT$/day	known
c_{kmn}	Whether pattern k pass through section (m,n) (1) pattern k pass through (m,n) $\Rightarrow c_{kmn} = 1$ (2) pattern k doesn't pass through (m,n) $\Rightarrow c_{kmn} = 0$	—	known
d_{ij}	Travel distance from station i to j	km	known
d_k	Total travel distance of pattern k	km	known
f_{km}	Minimal train service rate of pattern k	train/hr	known
l_{ij}	Minimal passengers service ratio from station i to j, $l_{ij} < 1$	—	known
N_T	Maximal no. of trains in service per hour per direction	train/hr	known
P_{ij}	Ticket fare from station i to j	NT$/ prs	known
R	Revenue	NT$/day	known
t_{ijk}	Travel time from station i to j for pattern k	min	known
v	Time value	NT$/ prs /min	known
α	Maximal loading factor of train	—	known
ϖ	Weight of traveling time cost to operation cost	—	known
γ^t	A ratio of time period t to an hour	—	known

Objective function:

$$\max \sum_{i \in N} \sum_{j \in N} S_{ij} P_{ij} - \sum_{t \in T} \sum_{k \in K} f_k^t C_k - \varpi \sum_{t \in T} \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} S_{ijk}^t t_{ijk} v \tag{27}$$

Subject to:

$$\sum_{k \in K} c_{kmn} f_k^t \leq \gamma^t C_{mn}, \ \forall (m,n) \in A, t \in T \tag{28}$$

$$\sum_{k \in K} f_k^t \leq \gamma^t N_T, \ \forall t \in T \tag{29}$$

$$f_k^t \geq \gamma^t f_{km}, \ \forall k \in K, t \in T \tag{30}$$

$$S_{ijk}^{t} \le a_{ijk} f_k^t C_T, \ \forall i, j \in N, k \in K, t \in T \tag{31}$$

$$S_{ij}^{t} = \sum_{k \in K} S_{ijk}^{t}, \ \forall i, j \in N, t \in T \tag{32}$$

$$S_{ij} = \sum_{t \in T} S_{ij}^{t}, \ \forall i, j \in N \tag{33}$$

$$S_{k}^{t} = \sum_{i \in N} \sum_{j \in N} S_{ijk}^{t}, \ \forall k \in K, t \in T \tag{34}$$

$$T_{ij} = \sum_{t \in T} T_{ij}^{t}, \ \forall i, j \in N \tag{35}$$

$$l_{ij} T_{ij} \le S_{ij}, \ \forall i, j \in N \tag{36}$$

$$S_{ij}^{t} \le T_{ij}^{t}, \ \forall i, j \in N, t \in T \tag{37}$$

$$\sum_{i \in N} \sum_{j \in N} S_{ij} d_{ij} \le \alpha \cdot C_S \sum_{t \in T} \sum_{k \in K} f_k^t d_k \tag{38}$$

$$\sum_{i \in N} \sum_{j \in N} b_{ijmn} S_{ijk}^{t} = L_{kmn}^{t}, \ \forall k \in K, (m,n) \in A, t \in T \tag{39}$$

$$L_{kmn}^{t} + E_{kmn}^{t} = C_T f_k^t, \ \forall k \in K, (m,n) \in A, t \in T \tag{40}$$

$$S_{ijk}^{t}, S_{ij}^{t}, S_{k}^{t}, L_{kmn}^{t}, E_{kmn}^{t}, f_k^t, T_{ij}^{t}, S_{ij}, T_{ij} \ge 0 \ \text{and are integers} \tag{41}$$

The objective function (27) is to maximize the operator revenue (profit minus operating cost) and minimize passengers' total travel time cost with a weighting factor (ϖ).

Constrains (28) specify the line capacity; (29) define the max amount of trains could be used per direction; (30) describe the minimum train frequency per hour; (31) impose that passengers served by train must less than or equal to train capacity; (32), (33), (34) and (35) describe the conservation of passenger flow; (36) specify the minimum passengers should be served while (37) specify the maximum passengers could be served; (38) define the loading factor of the train; (39) describe the conservation of passenger in each section (m,n); (40) describe the train capacity in each section (m,n); (41) describe the decision variables are integer and non-negative. Besides, the model should also subject to the transfer ratio from equations (13) to (26).

6.2 Solution Process

For the reason that the above model is in a pure Integer Programming (IP) format, the efficiency would be very poor when solved by Branch-and-Bound method. Although the decision variables are all integers, those related to passengers, such as S_{ijk}^{t}, S_{ij}^{t}, S_{k}^{t}, L_{kmn}^{t}, E_{kmn}^{t}, T_{ij}^{t}, S_{ij}, T_{ij}, are usually more than hundreds or thousands and thus, could be relaxed as real numbers without too much impact on the solutions to the original problem. On the other hand, the variables f_k^t for service frequencies are usually very small and very sensitive to different numbers so that it is inappropriate to relax them as real numbers. Therefore, equation (41) is modified as (42) and the problem will become a Mixed Integer programming (MIP) model. By doing so, the number of integer variables is greatly reduced and the solution efficiency will be significantly enhanced.

$$S_{ijk}^{t}, S_{ij}^{t}, S_{k}^{t}, L_{kmn}^{t}, E_{kmn}^{t}, T_{ij}^{t}, S_{ij}, T_{ij} \ge 0, \ f_k^t \ge 0 \ \text{and integers} \tag{42}$$

Figure 8 shows the solution procedure, which integrates C++/MFC programming technique to pre-process the parameters and O/D matrices with CPLEX API to optimize the problem on both directions. Finally, it will output the optimal service frequencies and the ridership allocations for different patterns.

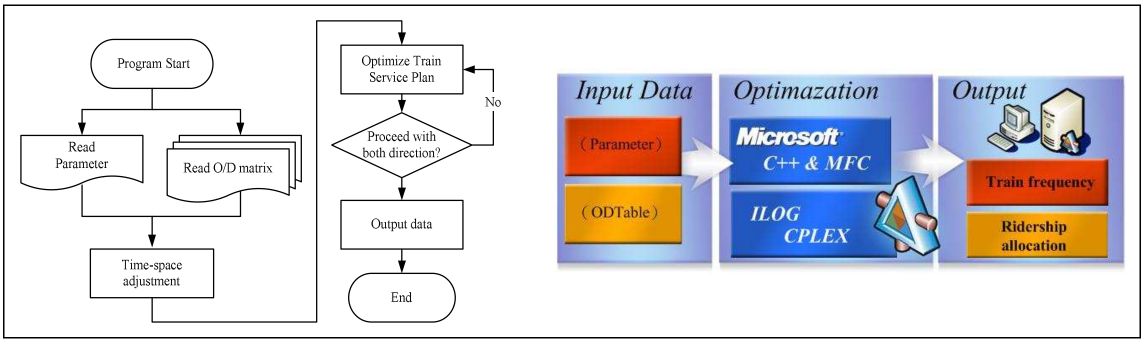


Figure 8 The solution procedure diagram

7. CASE STUDY

7.1 Taiwan High Speed Raid System

Taiwan HSR system is about 340 kilometer in length along the west corridor of Taiwan. It connects three major cities, Taipei, Taichung and Kaohsiung, with 8 intermediate stations. Table 5 summaries parameters to the model and Table 6 lists the assumed demand for each OD pair.

Table 5 Parameters to the model

Parameter	Value / Explanation
1. Ticket fare	Weighted fare of standing and seat fee
2. Minimum passengers service rate	The longer the trip the higher the ratio
3. Operating cost	Total amount of fixed and variable cost
4. Minimum service frequency	Set pattern B & pattern D 1 train per hour at least
5. Seats capacity	989 prs /train
6. Stand capacity	150 prs /train
7. Max loading factor	0.75
8. Transfer rate for non-serviced passengers	0.6
9. Time value	5 price/min/ prs
10. Weighting of ϖ	1
11. Peak hours	07:00~09:00, 17:00~19:00

Table 6 O/D matrix for peak and non-peak hour (Assumed by the research)

Peak hour

	1	2	3	4	5	6	7	8
1	-	0	251	498	1068	331	281	851
2	0	-	63	125	267	83	70	213
3	286	71	-	36	244	92	106	177
4	460	115	35	-	106	30	42	100
5	896	224	243	106	-	196	211	541
6	320	80	79	19	200	-	150	357
7	299	75	98	30	227	169	-	163
8	816	204	176	78	583	376	149	-

Non-peak hour

	1	2	3	4	5	6	7	8
1	-	0	130	298	783	212	198	674
2	0	-	32	74	196	53	49	168
3	142	35	-	17	193	73	92	165
4	301	75	18	-	64	25	36	80
5	794	199	200	67	-	134	168	421
6	254	63	67	17	133	-	111	281
7	237	59	87	23	171	116	-	101
8	698	174	177	62	441	289	91	-

7.2 Analysis of the Optimal Train Service Plan

The optimal train service plans found by the model are given in Table 7and Table 8, which show that the total amount of train services is 66 a day for southbound direction and 67 a day for northbound direction. The service plans satisfy all operation requirements and maximize the objective function. It is also found that only 2 trains of pattern E are in service for southbound direction in the evening and for northbound direction in the morning. This is probably due to the reason that pattern E has the same stopping stations as pattern D with shorter service scope and thus, could be substituted in most time zones.

Table 7 Optimal service plan for southbound trains

Pattern	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	Total
A	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	0	0	14
B	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	17
C	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	16
D	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	17
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2
Total	4	4	4	3	4	3	4	4	4	4	4	4	4	4	4	4	2	2	66

Table 8 Optimal service plan for northbound trains

Pattern	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	Total
A	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	15
B	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	17
C	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	16
D	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	17
E	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
Total	2	4	4	4	4	3	4	4	4	4	4	4	4	4	4	4	4	2	67

We can also expand the model's capability by fixing the total number of trains for each direction and let model optimize the best service frequencies and the ridership allocation. For example, if the operator decides to dispatch only 60 trains a day in southbound direction under the same conditions as the foregoing case, then the model will yield the optimal solution as shown in Table 9. This illustrates how the model works excellently under the research achievement.

Table 9 Optimal solution for downward patterns given total 60 trains

Pattern	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	Total
A	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	15
B	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	17
C	1	1	1	0	1	0	1	1	1	0	1	1	1	1	0	0	0	0	11
D	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	17
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	4	4	4	3	4	3	4	4	4	3	4	4	4	4	3	2	2	0	60

8. CONCLUSION

The research develops a model for optimizing train service plans for intercity trains with various service patterns and long journey time. The model tries to maximize operator's profit and minimize passenger's traveling time cost by combining these two indexes with a weighted factor into the objective, while taking into account the dynamic demand and the transfer of unserved passengers. The model is originally formulated as an IP format and then modified to a MIP form to enhance the solution efficiency. The model can generate the optimal train service plans, including service frequencies, ridership allocations, and other performance indices. The empirical study has shown the excellent work applied to Taiwan HSR. For a given input data, the model not only produces the details of each service pattern, but also could fix the total number of trains per direction and then trace back to the optimal service frequencies. In particular, the solution time and speed is very quickly and efficiency. In conclusion, the research could provide various and efficient information to policy makers under the limited resources and fit in with the requirement of both operator and passenger.

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