Technology Licensing in Vertically Related Markets: A Case of Vertically-Integrated Firm

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Abstract

In this paper, we consider the licensing behavior from an upstream firm to a vertically-integrated firm. We find that the optimal contract includes only a per-unit royalty rate when the innovation is small. Moreover, the royalty rate may be even larger than the innovation level. Under such a circumstance, the competing firms can achieve a collusive outcome through technology licensing; however, the social welfare improves. Furthermore, if the upstream firm determines the input price while dealing the licensing contract with the vertically-integrated firm, the upstream firm licenses to the vertically-integrated firm by a fixed-fee and determines a high input price to deter other downstream firms from entry. That may distort the social welfare.

Key Words: Technology Licensing, Vertically-related Markets

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1 Introduction

In the last two decades, technology licensing has been a common business behavior and grown substantially. Nadiri (1993) shows that the international payments for technology have an amazing growth. For Japan and U.K. the total transaction between 1970 to 1988 increased by about 400%, France and the U.S. experienced an increase of about 550% while West Germany had a spectacular increase of over 1000% between 1979 and 1988. As the importance of intellectual property right to be recognized nowadays, technology licensing itself sometimes even becomes a strategic tool. It can be used as a tool to deter the rival firm from entry (eg., Gallini (1984)) or to do the R&D (eg., Gallini and Winter (1985)). The discussion in the literature is also flourishing.

In the development of literature in technology licensing, the issue of optimal contract has been largely discussed. Rostocker (1984) finds that royalty alone is used 39% of the time, fixed fee alone is 13%, and both instruments together are 46%, and the percentage seems not changed largely. Numerous studies have been devoted to explain this result. Depends on whether the innovator is the inside competitor or outside independent firm, the discussion in the literature has two main branches, and the optimal contract form may change accordingly. Given the innovator is also an inside competitor, Wang (1998) shows that the optimal contract is the royalty licensing. Moreover, if there is an upstream input monopolist in the market and the technology licensing is among the downstream firms, Arya and Mittendorf (2006) show that the optimal contract is also the royalty licensing. On the other hand, Kamien and Tauman (1986) show that if the innovator is an outsider, the optimal contract is the fixed-fee licensing. However, when there is an input monopolist in the industry, the optimal contract of the outside innovator is the royalty licensing.

The intuition of the above papers goes as follows: The merit of fixed-fee licensing is it lowers the industry cost, and the innovator exploits the extra rents coming from the lowered industry cost. However, if there are too many firms licensed under fixed fee licensing, the profit of the innovator lowers owing to the too intense competition among the licensees. On the other hand, royalty licensing controls the level of competition among the licensees since the output of the downstream firms does not increase as much as that under fixed-fee licensing. Therefore, if the innovator is also the competitor in the market, he prefers the royalty licensing; otherwise, the fixed-fee licensing. If there is an input monopolist in the industry, whether the innovator is an insider or outsider, he prefers the royalty licensing. The intuition is that fixed-fee licensing lowers the industry cost and therefore increases the final outputs. That in turn results in a higher derived demand of input. Under such a circumstance, the input monopolist would charges the downstream firms a higher input prices and exploits part of the benefits coming from technology licensing.

Besides the above papers, there are still many articles discussing the optimal licensing contract. Given the patentee is an outside innovator, Kamien and Tauman (1984), Katz and Shapiro (1985, 1986) find the similar results as Kamien and Tauman (1986) that license by means of a fixed fee or auction is superior to royalty for both the innovator and the consumers. Moreover, Kamien et al. (1992) extend theses results to non-linear demand function. On the other hand, given the patentee is also an inside competitor, Wang (2002) extends the results in Wang (1998) to a differentiated Cournot model, he finds that royalty licensing prefers to fixed-fee licensing if and only if the products are not too differentiated; moreover, the innovator may even license a drastic innovation by means of royalty when the goods are imperfect substitutes. Faulí-Oller and Sandonís (2002) have the similar result as Wang (2002). Wang and Yang (1999) show that this finding also holds if the firms compete in Bertrand fashion as long as their products are not too differentiated. Kamien and

Tauman (2002) synthesize the two branches.

Sen (2005) starts from another point of view to explain why royalty licensing could be superior to both fixed fee and auction, that is, the number of licenses can take only integer values. Therefore, the innovator's profit under fixed fee licensing or auction is a step function, it is possible for royalty licensing to be a better licensing method. Bousquet et al. (1998) studies the design of the licensing contracts under demand or cost uncertainty, and find that the optimal contract consists of, in general, a mix of a fixed fee and royalties. Poddar and Shinha (2004) use a linear city model to investigate the optimal licensing strategy for an outsider patentee and an insider patentee. Under the assumption of Bertrand competition, they find that royalty licensing is the optimal licensing contract for an outside patentee. Moreover, when the innovator is also an inside competitor, royalty licensing is superior to fixed-fee licensing when the innovation is non-drastic; however, no licensing is the optimal strategy when the innovation is drastic. Poddar and Shinha (2010) consider the case in which technology licensing happens from a cost-inefficient firm to a cost-efficient firm. They show that the optimal licensing contract includes a positive royalty rate when the innovation is drastic or the cost difference between the firms is moderate.

As the production chain now is more and more complex, the behavior of technology licensing is also more and more complicated. As we show above, the market structure plays an important role. If there are upstream firms in the market, the optimal licensing strategy may be different. However, in the literature of technology licensing, the licensing happens only among the pure downstream firms. In the real world, the market may not only consist of pure downstream firms or upstream firms. There are also vertically-integrated firms, such as Apple, Samsung, and so on. The possibility that technology licensing happens from a pure upstream (downstream) firm to vertically-integrated firm is neglected, and vice versa. For example, Silicon Image, the leading company in Serial ATA, provides its chip set to MSI, GIGABYTE, and so on, and reaches a licensing agreement in Serial ATA chip set with Samsung. In this case, the technology licensing happens from the upstream firm, Silicon Image, to the vertically-integrated firm, Samsung. It is hardly to define whether Silicon Image is the inside innovator or outside innovator. The intuition that we discussed above cannot be directly applied. Therefore, we would like to complete the blank in the literature of technology licensing.

Moreover, we would like to discuss an interesting issue which is also neglected in the literature of technology licensing. All of the literature assumes the technology licensing is a rather long-term contract than the decision making of input price and should be considered in prior of the determination of input price. However, given the technology licensing is from the upstream firm to the vertically-integrated firm, the upstream firm has the option to decide the licensing mode and the input price at the same time. I would like to examine whether the setup of game structure matters. We introduce our model setup in section 2, followed up the equilibrium if the upstream firm makes the decision of licensing mode and the input price sequentially. Then we derive the equilibrium that the upstream firm makes the decision of licensing mode and the input price simultaneously in section 3. Section 4 compares the equilibriums to see whether there are differences if the upstream firm can make his decision sequentially or simultaneously. Section 5 concludes this paper.

2 Model Setup

This paper aims to analysis the issue of technology licensing from an upstream firm to a vertically-integrated firm. We assume the upstream firm, firm U, sells the intermediate goods to the downstream firm, firm D, with input price w. Moreover, there is a vertically-integrated firm, firm I, which produces the intermediate good by itself and competes with firm D in the final good market. However, we assume firm U has the more advanced technology than firm I in producing the intermediate good. With this advanced technology, the vertically-integrated firm can lower the marginal production cost in intermediate good from c to $c-\varepsilon$. For simplicity, I assume one unit of input to produce one unit of output. The market demand of the final good is defined as $P(Q)=a-Q$, where Q is the output. Furthermore, I assume the upstream firm has all the bargaining power in the licensing game and the contract form is of two-part tariff, that is, a per-unit royalty rate and a fixed payment. As shown in Sen and Tauman (2007), if the two-part tariff contract includes only a royalty rate, then the royalty licensing necessarily generates more revenues for innovator than fixed-fee licensing. On the other hand, if the two-part tariff contract includes only a fixed-fee payment, then the fixed-fee licensing creates higher profits for the innovator than royalty licensing. However, we do not exclude the possibility that the optimal contract includes both royalty rate and fixed-fee.

2.1 Basic Model: No Technology Licensing

The game structure for the basic model is as follows: In the first stage, the upstream firm determines the input price. Then, in the second stage, firm D and firm I compete in Cournot fashion in the final good market. We look for the subgame perfect Nash equilibrium and solve it by backward induction.

The profit of the downstream firm and the vertically-integrated firm are as follows:

$$\pi_D=\left[P(Q)-w\right]q_D, \tag{1}$$

$$\pi_I=\left[P(Q)-c\right]q_I, \tag{2}$$

where the subscript "D" and "I" denote the variables associated with downstream firm and vertically-integrated firm, and $Q = q_D + q_I$. For simplicity, we assume the marginal cost in producing the final goods is zero. Differentiating (1) and (2) with respect to q_D and q_I respectively, we can have the first-order conditions. By solving the first-order conditions simultaneously, the equilibrium outputs in the final stage are as follows:

$$q_D = \frac{a-2w+c}{3} \text{ and } q_I = \frac{a+w-2c}{3}. \tag{3}$$

In the first stage, the upstream firm determines the optimal input price. The profit function of the upstream firm is as follows:

$$\pi_U = (w-c+\varepsilon)x, \tag{4}$$

where $x = q_D$ as we assume one unit of input to produce one unit of output, and the subscript "*U*" denotes the variables associated with the upstream firm. Differentiating (4) with respect to *w*, we have the first-order condition of the upstream firm. The equilibrium input price is therefore as follows:

$$w^N = \frac{a+3c-2\varepsilon}{4}, \tag{5}$$

the superscript "*N*" denotes the equilibrium that there is no technology licensing. Substituting (5) into (3), we can have the equilibrium final output as follows:

$$q_D^N = \frac{a-c+2\varepsilon}{6} \text{ and } q_I^N = \frac{5a-5c-2\varepsilon}{12}, \tag{6}$$

where $q_I^N > 0$ if $0 < \varepsilon < 5(a-c)/2$. Moreover, substituting (5) and (6) into (1), (2) and (4), we have the equilibrium profits of the competing firms as follows:

$$\pi_U^N = \frac{(a-c+2\varepsilon)^2}{24}, \quad \pi_D^N = \frac{(a-c+2\varepsilon)^2}{36}, \text{ and } \pi_I^N = \frac{(5a-5c-2\varepsilon)^2}{144}.$$

The social welfare which consists of all the producer surpluses and the consumer surplus is as follows:

$$SW^N = \frac{1}{288}\left[119(a-c)^2 + 68(a-c)\varepsilon + 92\varepsilon^2\right].$$

2.2 The Equilibriums under Technology Licensing

In this sub-section, we would like to discuss the issue of technology licensing from an upstream firm to a vertically-integrated firm. The game structure is as follows: In the first stage, the upstream firm offers a take-or-leave-it licensing contract to the vertically-integrated firm. With this advanced technology, the vertically-integrated firm can lower the marginal cost in producing the inputs by ε. The upstream firm charges the vertically-integrated firm a per-unit royalty rate and a fixed-fee. In the second stage, the upstream firm determines the optimal input price. In the final stage, the downstream firm and the vertically-integrated firm compete in Cournot fashion. Again, we use backward induction to derive the sub-game perfect equilibrium. The profit function of the downstream firm and the vertically-integrated firm are as follows:

$$\pi_D = \left[P(Q) - w\right] q_D , \tag{7}$$

$$\Pi_I \equiv \pi_I - F = \left[P(Q) - c + \varepsilon - r\right] q_I - F , \tag{8}$$

where r is the per-unit royalty and F is the fixed-fee. Differentiating (7) and (8) with respect to q_D and q_I, we can derive the first-order conditions. By solving these first-order conditions simultaneously, the equilibrium outputs in the final stage are as follows:

$$q_D = \frac{a - 2w + c - \varepsilon + r}{3} \quad \text{and} \quad q_I = \frac{a + w - 2(c - \varepsilon + r)}{3} . \tag{9}$$

From the one-to-one input-output transformation, we have $x = q_D$. The profit function of the upstream firm is as follow:

$$\Pi_U \equiv \pi_U + rq_I + F = (w - c + \varepsilon)x + rq_I + F . \tag{10}$$

In this stage, the upstream firm determines the input price, by differentiating (10) with respect to w, we can have the first-order condition. The optimal input price is as follows:

$$w = \frac{a + 3c - 3\varepsilon + 2r}{4}. \tag{11}$$

The comparative statics of w with respect to r is positive. It is intuitive since the higher the royalty rate, the higher the marginal cost of the vertically-related firm and thus a larger output of the downstream firm which results in a higher derived demand of input. By substituting (11) into (9), the final output in the second stage are as follows:

$$q_D = \frac{a - c + \varepsilon}{6} \quad \text{and} \quad q_I = \frac{5(a - c + \varepsilon)}{12} - \frac{r}{2}. \tag{12}$$

The equilibrium profits of the competing firms in the second stage are as follows:

$$\Pi_U = \frac{(a - c + \varepsilon)^2 + 12(a - c + \varepsilon)r - 12r^2}{24} + F,$$

$$\pi_D = \frac{(a - c + \varepsilon)^2}{36}, \text{ and } \Pi_I = \frac{\left[5(a - c + \varepsilon) - 6r\right]^2}{144} - F.$$

In the first stage, the upstream firm determines the optimal contract. The profit maximization of the upstream monopolist is as follows:

$$\max_r \Pi_U \quad \text{subject to} \quad \Pi_U \geq \pi_U^N \quad \text{and} \quad \Pi_I \geq \pi_I^N.$$

In other words, $\pi_U^N - (\pi_U + rq_I) \leq F \leq \pi_I - \pi_I^N$. It should be noted that technology licensing holds if and only if $\pi_U^N + \pi_I^N \leq \pi_I + (\pi_U + rq_I)$. Since we assume the upstream monopolist has the full bargaining power, the fixed-fee is as follows:

$$F = \pi_I - \pi_I^N = \frac{(10a - 10c + 3\varepsilon - 6r)(7\varepsilon - 6r)}{144} \geq 0.$$

The maximization problem of the upstream firm can be rewrite as follows:

$$\max_r \pi_U + rq_I + \pi_I - \pi_I^N.$$

The optimal royalty rate is as follows:

$$r^{3s} = \begin{cases} \dfrac{7\varepsilon}{6} & if \quad \varepsilon \le \dfrac{a-c}{6} \\ \dfrac{a-c+\varepsilon}{6} & \quad\quad \varepsilon > \dfrac{a-c}{6} \end{cases},$$

where the superscript "3*s*" denotes the equilibrium under the three-stage technology licensing game, that is, the upstream firm makes the decision of optimal contract and the input price sequentially. We make this result as the following proposition.

Proposition 1. When the innovation is relatively small, the optimal contract consists of royalty rate only, and the royalty rate is even larger than the innovation level. When the innovation is large, the optimal contract consists of both royalty rate and fixed-fee.

It should be noted that since we assume the upstream firm has all the bargaining power and the licensing contract is of two-part tariff, the upstream firm exactly is the "pass through" competitor in the final market. The upstream firm would like to achieve a collusive outcome by manipulating the royalty rate. When the innovation is small, the profits coming from the technology licensing is relatively less important, therefore, the upstream firm would like to control the level of competition by charging $r > \varepsilon$. In such a circumstance, the total output of final good is larger than that under monopoly, and the vertically-integrated firm is also willing to accept this offer since the final good market now is more collusive. When the innovation is large, the profits coming from the technology licensing is more important, the upstream firm would decrease the royalty rate and control the output of final good at the level of monopoly.

The equilibrium input price and the final output are as follows:

$$w^{3s} = \begin{cases} \dfrac{3a+9c-2\varepsilon}{12} & \text{if} \quad \varepsilon \le \dfrac{a-c}{6} \\ \dfrac{a+2c-2\varepsilon}{3} & \quad\;\; \varepsilon > \dfrac{a-c}{6} \end{cases},$$

$$q_I^{3s} = \begin{cases} \dfrac{5a-5c-2\varepsilon}{12} & \text{if} \quad \varepsilon \le \dfrac{a-c}{6} \\ \dfrac{a-c+\varepsilon}{3} & \quad\;\; \varepsilon > \dfrac{a-c}{6} \end{cases}, \qquad q_D^{3S} = \frac{a-c+\varepsilon}{6}. \tag{13}$$

The equilibrium profits are as follows:

$$\Pi_U^{3s} = \begin{cases} \dfrac{3(a-c)^2+48(a-c)\varepsilon-4\varepsilon^2}{72} & \text{if} \quad \varepsilon \le \dfrac{a-c}{6} \\ \dfrac{7\left[(a-c)^2+12(a-c)\varepsilon+4\varepsilon^2\right]}{144} & \quad\;\; \varepsilon > \dfrac{a-c}{6} \end{cases},$$

$$\Pi_I^{3s} = \frac{(5a-5c-2\varepsilon)^2}{144}, \text{ and } \pi_D^{3s} = \frac{(a-c+\varepsilon)^2}{36}.$$

Moreover, the social welfare is as follows:

$$SW^{3s} = \begin{cases} \dfrac{7(a-c)(17a-17c+24\varepsilon)}{288} & \text{if} \quad \varepsilon \le \dfrac{a-c}{6} \\ \dfrac{3(a-c+\varepsilon)^2}{8} & \quad\;\; \varepsilon > \dfrac{a-c}{6} \end{cases}.$$

From (13), we can have the equilibrium of the total output of final good as follows:

$$Q^{3s} = \begin{cases} \dfrac{7(a-c)}{12} & \text{if} \quad \varepsilon \le \dfrac{a-c}{6} \\ \dfrac{a-c+\varepsilon}{2} & \quad\;\; \varepsilon > \dfrac{a-c}{6} \end{cases},$$

As we stated above, the upstream firm uses the royalty rate to control the level of competition. When the innovation is large, the total output of final good is the same as that under monopoly. Moreover, subtract SW^N from SW^{3s}, the social welfare after technology licensing still improves owing to the cost reduction is prominent. We make this result as the following proposition.

Proposition 2. The competing firms may achieve a collusive outcome through technology licensing. However, the social welfare still improves.

3 Simultaneous Decision Making on the Input Price and the Optimal Contract

Most of the literature of technology licensing assumes the decision making of licensing contract is in prior to the determination of input price since the licensing contract is a long-term decision. It should be noted that since the decision maker of the optimal contract and the input price is the upstream firm, it is reasonable to guess the upstream firm can determine them at the same time if dosing so is more profitable. Therefore, in this section, we assume the upstream firm determines the optimal licensing contract and the input price in the first stage. Then the downstream firm and vertically-integrated firm compete in Cournot fashion in the second stage. We use backward induction to solve the subgame perfect equilibrium for the two-stage game.

The equilibrium of the final stage is the same as that in the three-stage game and is specified as (9). Therefore, the profit maximization problem of the upstream firm in the first stage is as follows:

$$\max_{w,r} \Pi_U \equiv \pi_U + rq_I + F = (w - c + \varepsilon)x + rq_I + F \quad \text{s.t.} \quad \Pi_U \geq \pi_U^N \quad \text{and} \quad \Pi_I \geq \pi_I^N .$$

Again, as we assume the upstream firm has all the bargaining power, the fixed-fee is $F = \Pi_I - \pi_I^N \geq 0$. The first-order conditions are as follows:

$$\frac{\partial \Pi_U}{\partial r} = \frac{(-a + 2w - c + \varepsilon - 4r)}{9} = 0,$$
$$\frac{\partial \Pi_U}{\partial w} = \frac{(5a - 10w + 5c - 5\varepsilon + 2r)}{3} = 0,$$

By solving the first-order conditions simultaneously, the optimal input price and royalty rate are as follows:

$$w^{2s} = \frac{(a + c - \varepsilon)}{2} \quad \text{and} \quad r^{2s} = 0,$$

where the superscript "2*s*" denotes the equilibrium under the two-stage technology licensing game. The equilibrium outputs of the competing firms are as follows:

$$q_D^{2s} = 0 \quad \text{and} \quad q_I^{2s} = \frac{(a-c+\varepsilon)}{2}.$$

The equilibrium profits are as follows:

$$\pi_D^{2s} = 0, \quad \Pi_I^{2s} = \frac{(5a-5c-2\varepsilon)^2}{144}, \text{ and } \Pi_U^{2s} = \frac{(a-c+\varepsilon)^2}{4} - \frac{(5a-5c-2\varepsilon)^2}{144}.$$

From above, we can have the following proposition.

Proposition 3. If the upstream firm determines the optimal contract and the input price simultaneously, the upstream firm licenses to the vertically-integrated firm by fixed-fee and determines a high input price to deter the downstream firm from entry.

Again, since we assume the licensing contract consists of a per-unit royalty and a fixed-fee. The profits of the upstream firm coming from two parts, one is the competing profit from the downstream firm, and the other is the licensing revenue. Now, in this game structure, the upstream firm has two tools to do the profit maximization. Therefore, the upstream firm deters the downstream firm from entry and license to the vertically-integrated firm by fixed-fee only. In such a circumstance, the upstream firm exactly is the "pass through" monopolist and generates the highest industry profit.

The social welfare is as follows:

$$SW^{2s} = \frac{3(a-c+\varepsilon)^2}{8}.$$

$SW^{2s} - SW^N \leq 0$ if $0 < \varepsilon \leq (-37+3\sqrt{157})(a-c)/8$. Accordingly, we can have the following proposition.

Proposition 4. If the upstream firm determines the input price and the optimal contract simultaneously, technology licensing lowers social welfare when the innovation is relatively small.

The intuition is as follows. When the innovation is small, the distortion owing to the "pass-through" monopoly outweighs the welfare improvement from the cost-reducing, therefore, the social welfare decreases. Only when the innovation is large, the effect of cost-reducing dominates.

4 The Comparison of the Equilibriums between Sequential and Simultaneous Game

We are interested in whether the upstream firm would determine the input price and the optimal contract simultaneously, that is, whether the upstream firm generates a higher profit under the simultaneous game. By a direct comparison, we can find that $\Pi_U^{3s} \le \Pi_U^{2s}$. We make this result as the following proposition.

Proposition 5. The upstream firm enjoys a higher profit if he determines the optimal contract and input price simultaneously.

The intuition is as follows. Under the scheme of three-stage game, the input price is also a function of royalty rate, and the upstream firm enjoys a higher profit if the final good market is more collusive. Since the two-part tariff licensing contract is a kind of "pass-through" monopoly, the upstream firm can manipulate the level of competition by adjusting royalty rate. However, in determining the optimal contract, the upstream firm can only use royalty rate to balance the competing profits and the licensing revenue. If the upstream firm raises the royalty rate to reduce the output of the vertically-integrated firm and soften the competition, that in turn will makes the independent downstream firm to produce more and decreases the licensing revenue. Therefore, these two contrary effects make the upstream firm less capable to soften the competition in the final good market. Moreover, in the sequential game, the upstream firm can not deter the independent downstream firm from entry since the commitment is not credible. In the three-stage game, only if the royalty rate is high, otherwise, the vertically-integrated firm would not believe the upstream firm will charge the independent firm a high input price. However, in the simultaneous game, since the decision making of the royalty rate and input price are in the same stage, the vertically-integrated firm would believe the upstream firm will charge a high input price and deter the independent downstream firm from entry.

5 Conclusion

We use a simple model to investigate the licensing behavior from an upstream firm to a vertically-integrated firm and find that: If the upstream firm determines the optimal contract and the input price sequentially, the upstream firm can soften the market competition by adjusting the royalty rate and achieve a more collusive outcome; however, the social welfare still improves. On the other habd, if the upstream firm determines the input price while dealing the licensing contract with the vertically-integrated firm, the upstream firm licenses to the vertically-integrated firm by a fixed-fee and determines a high input price to deter other downstream firms from entry. That may distort the social welfare.

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