

# A STUDY ON THE APPLICATION OF GENETIC ALGORITHMS ON THE DIAL-A-RIDE PROBLEM

HSIEN-MING CHIU<sup>1</sup> and HAN-SHUAN HUANG<sup>2</sup>

<sup>1</sup>Department of Transportation Management, Tamkang University,

<sup>2</sup>Institute of Transportation Science, Tamkang University,

151 Ying-Tsung Road, Tamshui, Taiwan 25137

Fax: +886-2-26221135

<sup>1</sup>[hmchra@mail.tku.edu.tw](mailto:hmchra@mail.tku.edu.tw)

*Abstract:* The Dial-a-Ride Problem (DARP) is a vehicle routing problem faced in arranging Dial-a-Ride services. The DARP has been proven a NP-Hard problem; therefore, most research has used heuristic solution methods to address this issue. The purpose of this study is to evaluate of the application of a Diversity Control Adaptive Genetic Algorithm (DCAGA) and Family Competition Genetic Algorithm (FCGA) on the DARP. This study proposed two solution procedures, which were integrated approach and cluster approach. A series of case studies with different characteristics, such as demand density and demand size, were used to test the solution capability of the proposed algorithms. Based on the results of the case studies, the Diversity Control Adaptive Genetic Algorithm is identified as the best algorithm in solution quality. Overall, the solution of the integrated procedure is better than, those of the two-phase procedure.

*Key Words:* Dial-and-Ride Problem, Genetic Algorithms, Meta-heuristic

## 1. Introduction

The introduction of Dial-a-Ride into the metropolitan area is to provide a low cost, door-to-door public transportation service. The Dial-a-Ride service consists of designing vehicle routes and schedules for  $n$  users who specify pick-up and drop-off requests between origins and destinations. The aim of the Dial-a-Ride Problem (DARP) is to establish a plan of minimum cost vehicles routes capable of accommodating many requests for a fleet of  $m$  identical vehicles based on a central depot. With the aging of the population and the development of ambulatory health care services, set up, or overhaul of the Dial-a-Ride service is the focus of metropolitan authorities. The Dial-a-Ride system can provide door-to-door, real-time service with the capacity of a traditional bus system. It is a special transportation service for handicapped and elderly citizens. The service is usually performed by advance requests of passengers, and routing plans of the service are scheduled based on these demands. The DARP is a vehicle routing issue addressing the arrangement of Dial-a-Ride services. The DARP generalizes a number of vehicle routing problems, such as pickup and delivery vehicle routing problem (PDVRP) and the vehicle routing problem with time window (VRPTW). However, with the focus of the DARP on the transporting of passengers, it will require not only minimizing operating costs, but

also reduce user inconvenience. Therefore, there are two different types of objectives used in the associated study, 1) maximize satisfaction in demands, subjective to vehicle availability and side constraints; 2) minimize costs, subject to full demand satisfaction and side constraints (Bergvinsdottir, 2004). Various measures of effectiveness are adopted in the associated study, such as fleet size, operating costs, driver wages, route duration, route length, customer waiting time, customer ride time, and the difference between actual and desired drop-off times. Some of these criteria may be treated as constraints or as part of the objective functions.

Psaraftis (1980) adopted Dynamic programming to solve issues of the best route for a dynamic single vehicle DARP for a small size problem. Jaw et al. (1986) used a search heuristic to identify available insertion windows for the route structure to deal with large size problems. Madsen et al. (1995) solved a real-life dynamic DARP based on the insertion algorithm from the work of Haw et al. (1986). Toth and Vigo (1996, 1997) have developed a heuristic that clusters first, by means of a parallel insertion procedure, routing is then by route exchanges and a Tabu threshold heuristic is then conducted in the post-optimization phase. Gendreau et al. (2001) proposed dynamic ambulance relocation to pre-compute several scenarios using

parallel computing. Cordeau and Laporte (2003) used vertex reinsertion as the tool in a neighborhood search in the Tabu heuristics with Self-Adjusting Positive Parameters used as the penalty function in the associated objective function. Bergvinsdottir (2004) adopted the Genetic Algorithm addressing the multiple vehicles DARP.

The DARP has been proven a NP-Hard problem; therefore, most researches address this problem with heuristic methods as the solution methods. "Vertex Reinsertion" has been adopted by previous studies, and although it is popular solution procedure, it has low computation efficiency. With tremendous interest in the Meta-heuristic effects on the combinatorial problems, there are still few studies addressing its application on the DARP. The purpose of this study is to evaluate the application the Diversity-controlling Adaptive Genetic Algorithm (DCAGA) and Family Competition Genetic Algorithm (FCGA) on the DARP.

The remainder of the paper is organized as follows: In section 2, we present a statement of the problem and the proposed nonlinear programming model for the DARP. Section 3 discusses the solution methodology for the problem. In section 4, a series of cases from previous studies are tested to evaluate the applicability the proposed methodology. Finally, in Section 5, we summarize the paper and suggest possible future research directions.

## 2. Problem Statement

The problem dealt with in this paper is as follows: there are  $N$  demands in the DARP service area. Each demand has its pickup and drop-off location and service time window. There are a set of  $k$  identical vehicles, based on a central depot, to serve these requests. The problem is to determine appropriate vehicle assignment and routes to serve them, such as minimize costs, subject to full demand satisfaction and side constraints. To meet the service time window requirements, the service vehicle is allowed to idle in some locations to meet these requirements. The framework of the DARP addressed in this study can be summarized as follows:

Inputs: Network configuration, locations, and time windows of demand, fleet size and vehicle capacity.

Objective: minimize total length of route.

Constraints: Vehicle capacity, maximum length of route, time windows of demands, limitations on riding time, and waiting time of demand.

Solution procedure: meta-heuristics.

Outputs: route structure, total routing time, total riding time, and total idle time.

The notations for this problem are defined as follows:

$i, j$  : Index of nodes

$n$  : Index of passenger

$k$  : Index of vehicle

$\{0\}$  : The central depot

$N$  : The set of passengers ,  $N = \{1, 2, \dots, n\}$

$P$  : The set of pickup locations ,  $P = \{1, 2, \dots, n\}$

$D$  : The set of drop-off locations ,  
 $D = \{n+1, \dots, 2n\}$

$V$  : The set of nodes ,  $V = P \cup D$

$K$  : The set of vehicles

$A$  : The set of arcs ,  $A = \{(v_i, v_j) : i < j\}$

$$x_{ij}^k = \begin{cases} 1 & \text{If vehicle } k \text{ travel form node } i \text{ to} \\ & \text{node } j \\ 0 & \text{otherwise} \end{cases}$$

$L_i^k$  : The number of passengers in vehicle  $k$ , when vehicle arrives at node  $i$ .

$T_s^k$  : The starting time of vehicle  $k$  for the current route

$T_e^k$  : The ending time of vehicle  $k$  for the current route

$AT_i$  : The arrival time on node  $i$ .

$ST_i$  : The starting time of vehicle serving node  $i$

$WT_i$  : The vehicle idle time on node  $i$ , including  
 $AT_i - ST_i$

$m$  : The number of vehicles available

$Q$  : The vehicle capacity limitation

$t_{ij}$  : The travel time from node  $i$  to node  $j$

$q_i$  : The amount of demands on node  $i$

$d_k$  : The maximum length of the routing time for vehicle  $k$

$r_n$  : The maximum allowable riding time for passenger  $n$

$s_n$  : The service time for passenger  $n$

$e_i$  : The lower bound of the time window for node  $i$

$l_i$  : The upper bound of the time window for node  $i$

- $e_0$  : The Starting time of the system
- $l_0$  : The ending time of the system
- $vd_k$  : The violation value on route length limitation for vehicle k
- $vq_i^k$  : The violation value on vehicle capacity limitation for vehicle k
- $vw_i$  : The violation value on waiting time on node i
- $vtw_i$  : The violation value on time window on node i
- $vr_n$  : The violation value on riding time limitation for passenger n
- $w_1$  : The penalty cost of vehicle route length violation
- $w_2$  : The penalty cost of vehicle capacity violation
- $w_3$  : The penalty cost of waiting time violation
- $w_4$  : The penalty cost of time window violation
- $w_5$  : The penalty cost of maximum riding time violation

The mathematical model used in this study is as follows:

$$\min \sum_{k \in K} (T_s^k - T_e^k) \tag{1}$$

Subject to

$$\sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1, \quad \forall i \in V \tag{2}$$

$$\sum_{k \in K} \sum_{i \in V} x_{ij}^k = 1, \quad \forall j \in V \tag{3}$$

$$\sum_{k \in K} \sum_{i \in V} \sum_{j \in V} x_{ij}^k = m + N \tag{4}$$

$$\sum_{i \in V} x_{ih}^k - \sum_{j \in V} x_{jh}^k = 0, \quad \forall k \in K, \tag{5}$$

$$\forall h \in V$$

$$x_{ij}^k (L_i^k + q_j - L_j^k) = 0, \quad \forall k \in K, \tag{6}$$

$$\forall i, j \in V$$

$$0 \leq L_i^k \leq Q, \quad \forall k \in K, \quad \forall i \in V \tag{7}$$

$$L_0^k = 0, \quad \forall k \in K \tag{8}$$

$$x_{ij}^k (ST_i + s_i + t_{ij} - ST_j) \leq 0, \tag{9}$$

$$\forall k \in K, \quad \forall i, j \in V$$

$$ST_i + s_i + t_{i,i+n} - ST_{i+n} \leq 0, \tag{10}$$

$$\forall k \in K, \quad \forall i \in P$$

$$T_s^k \geq e_0, \quad \forall k \in K \tag{11}$$

$$T_e^k \leq l_0, \quad \forall k \in K \tag{12}$$

$$x_{ij}^k \in \{0,1\}, \quad \forall k \in K, \quad \forall i, j \in V \tag{13}$$

The objective function of the proposed model is the minimization of the total routing time. With the permission of vehicle idle in some locations, the traditional objective of the minimum total distance will not reflect the effectiveness of the decision. Therefore, the selection of the minimum total routing time will be more appropriate, as shown in Eq. (1). The first constraint ensures that, only one vehicle leaves each node, as defined in Eq. (2). Eq. (3) limits only one vehicle allowed to enter each node. Sub-tour breaking constraint ensures that no sub-tour solution will be allowed, as shown in Eq. (4). The flow continuity constraint is defined in Eq. (5). Eq. (6) ensures the load of a vehicle will reflect its service to a node just visited. Vehicle capacity limitation is ensured by Eq. (7). Eq. (8) ensures all vehicles start service with an empty load. The service time difference between a node and its precedent is ensured by Eq. (9). Equation (10) ensures that all passengers are served after the vehicle is dispatched. Eqs. (11) and (12) indicate that all vehicles operate at regular hours. The definition of the binary decision variable is set in Eq. (13).

### 3. The Solution Procedure

The DARP has been proven a NP-Hard problem; therefore, most researches addressed this problem with heuristic methods as the solution methods. "Vertex Reinsertion" has been adopted by previous studies. Although it is a popular solution procedure, its computation efficiency is low. With tremendous interest of the Meta-heuristics on the combinatorial problem, there are still few studies addressing its application on the DARP. The purpose of this study is to evaluate the application of the Diversity Control Adaptive Genetic Algorithm (FCAGA) and Family Competition Genetic Algorithm (FCGA) on the DARP. A commonly used technique for the traditional DARP solution consists of; defining clusters of users to be served by the same vehicle and routing for each cluster. In this study, we will

adopt this type of two-phase approach based on a revised Genetic Algorithm. In addition, an integrated procedure, which determines clusters and routing simultaneously will also be adopted in this study.

The GA was first introduced by Holland (1975) and was inspired by the basic mechanism of natural evolution. It is an efficient global search algorithm, which employs the Darwinian survival-of-the-fittest theory to yield the best, or better, characters among the population and performs a random exchange to create superior offspring. GA has been applied to various function optimization problems. It has been proven as highly effective in searching large, complex response surfaces even in the presence of difficulties, such as high-dimensionality, multimodality, discontinuity, and noise (Dejong, 1980). There are two basic steps in GA. Step one is to determine the representation of genes and the population. Step two is to develop a search algorithm for the optimal solution.

The basic characteristics of any GA are its gene representation, gene recombination, evaluation function, and gene selection method. The gene representation represents the genes of a chromosome. There are two types of representations in the studies addressing vehicle routing problems, path and adjacent representation. In this study, a path representation of the gene is used, in which a tour is represented by a list of nodes. The problem of  $N$  representations of the same tour can be solved by fixing the initial node.

The gene recombination consists of two basic operations: (1) crossover, and (2) mutation. Crossover is a means through which two parents can produce two offspring by matching and mixing desirable qualities through a random process. First, two parents are selected randomly from the current population for the matching process. Some portions are then chosen randomly for exchange between these two parents. Various methods for choosing parts to be swapped have been adopted by previous studies. Most of these methods focus on the prevention of the infeasible VRP tour, such as an offspring with repetitive nodes. This type of crossover cannot take advantage of the inherent characteristic of network information.

Mutations, with small probabilities, are incorporated into the gene recombination to reflect the small rate of mutation existing in the real world. The purpose of the mutation in the gene recombination is to change part of the genes of the parents, based on a specific mutation rule under a preset small mutation ratio, in order to increase the degree of variations in gene recombining. Some genes may be eliminated

in the matching of two parents that cannot be recovered by the crossover operation. This can be done by adding the mutation process into the GA to ensure gene recovery.

The evaluation function of a GA acts as the fitness test of the individual in the population under consideration. This function is an indicator of the survival potential and reproductive capability of the individual in the subsequent generation. In this study, we will use the following function as the evaluation function:

$$\min \sum_{k \in K} \sum_{i, j \in V} x_{ij}^k t_{ij} + w_1 \sum_{k \in K} v d_k + w_2 \sum_{k \in K} \sum_{i \in V} v q_i^k + w_3 \sum_{n \in N} v w_n + w_4 \sum_{i \in V} v f w_i + w_5 \sum_{n \in N} v r_n \quad (14)$$

Eq. (14) consists of the six components, including the routing time, the penalty cost of violations of the route length, vehicle capacity, waiting time, time window, and riding time of the passengers. The gene selection method is employed to determine which gene is selected as the candidate for crossover and reproduction. The population is represented by an area, which is divided into divisions. Each individual is represented by a division. The size of each division is proportional to the fitness of the associated individual. The better the fit of the individual, the bigger the size of the division occupied, which subsequently has more chances to be selected into the matching pool. The GA procedures adopted in this study can be summarized as follows:

- Step 1: The population of the predetermined size is created by random selection from the feasible DARP tours.
- Step 2: Individuals are selected by a preset rule to form the matching pool
- Step 3: Randomly match the individuals in the matching pool, two at a time. The new individuals () are created by the crossover and mutation operations with crossover ratio of 0.35 and mutation ratio of 0.01.
- Step 4: Replace old individuals by its ?, then proceed to step 2, repeat until the difference between two best individuals are negligible or a maximum number of iterations is met.

The traditional GA is easily trapped into a local optimum due to the loss of the population diversity. Therefore, two revised versions of GA will be adopted in this study. First, the Adaptive Genetic Algorithm, based on the diversity control approach (Zhu, 2003), is adopted to use an adaptive crossover and mutation ratio to maintain search effectiveness. Based on the variation assess of the Phenotype, Standard Deviation, Genotype and Ancestral ID of the chromosome in the population, the Diversity

Control Adaptive Genetic Algorithm (DCAGA) will compute the Hamming Distance to determine the appropriate crossover and mutation ratio to maintain the breadth of the search procedure, therefore, step 3 of the traditional GA is revised in DCAGA to reflect this concept. The Family Competition Genetic Algorithm (FCGA), based on the selection of the best of the parent population and the offspring population to form a new population as candidate for the new generation, steps 1 and 4 are revised in the FCGA to provide a better family for the new generation.

#### 4. Numerical Experiment

To test the capability of the revised GA methods, two types of numerical experiments were conducted in this section. First, to test the validation of the proposed algorithms, a small size DARP sample was adopted to evaluate the solution from the enumeration. In the second phase, sample problems from the series studies of the Cordeau and Laporte were adopted as case studies for the further computation evaluation. These samples are based on the transformation of the real-life Danish DARP system, which were obtained from the following website.

<http://neumann.hec.ca/chairedistributique/data/>.

In the test of small size problems, we set up two vehicles, five passengers DARP sample, the solution

of the two revised GA and enumeration are shown in Table 1.

Based on the results shown in Table 1, both proposed revised GA can provide the best solution for a small size problem.

For the larger size sample problem, we selected two different size problems from the series samples of Cordeau study, including pr1 and pr5, the problem sizes are shown in Table 2.

Table 1. Results of the three solution procedures on small samples

	Routing time		Waiting time		Delivery time	
	mean	best	mean	best	mean	best
Enumeration		193.426		27.835		30.769
AGA	224.449	193.426	31.721	21.306	48.789	30.769
FCGA	217.927	193.426	52.247	27.835	34.68	30.769

Table 2. Size of the problem

Problem	Passenger number	Vehicle number
Pr1	24	3
Pr5	120	11

We obtained ten solutions for each sample problem; the results of these ten solutions, and the solution from Cordeau and Bergvinsdottir, are then summarized in Tables 3 and 4.

Table 3. Result of sample problem - pr1

	Routing time			Waiting time			Delivery time			Run time		
	mean		total	best		total	mean		total		best	
	mean	best		mean	total		mean	total			mean	total
Adp	991.435	918.185	253.419	5.28	255.964	5.333	709.112	14.773	723.733	15.078	639.650	
FC	1095.063	912.625	288.225	6.005	241.608	5.749	780.384	16.258	572.937	11.936	687.583	
Cordeau		881.16			211.15	4.4			1094.99	45.62	190	
Bergvinsdottir	1041	1039	252	5.25	260	5.42	477	19.86	310	12.9	557	

Table 4 Result of sample problem - pr5

	Routing time			Waiting time			Delivery time			Run time		
	mean		total	best		total	mean		total		best	
	mean	best		mean	total		mean	total			mean	total
Adp	4452.428	3922.886	1190.802	9.923	886.328	7.386	5626.401	46.887	4693.511	39.113	6193.300	
FC	4543.621	3942.689	1282.031	10.684	1087.534	9.063	6085.503	50.713	4276.891	35.641	6306.100	
Cordeau		3869.95			832.98	3.47			6156.48	51.3	4624	

Due to the adoption of different cost components of previous studies, the evaluation of the solution focuses on the routing time, waiting time, and delivery time as the measures of effectiveness of the proposed algorithms. There are two perspectives in this problem, operator and passengers; for the operator, lower routing time and lower operating costs; for the passengers, the shorter the delivery time, the better the passengers feel. We will conduct the t-test with  $\alpha=0.05$  to identify the better solution algorithms for these two perspectives. The solution algorithms consist of the integrated procedure, a two-phase AGA procedure, and two-phase FCGA procedure. Based on the results of these t-tests, the integrated procedure, based on AGA, can provide better solutions with lower routing times for the larger size problems. Overall, the integrated procedure will obtain better solutions than those from the two-phase procedure.

## 5. Conclusions and Recommendations

The Dial-a-Ride Problem (DARP) is a vehicle routing problem for the arrangement of the Dial-a-Ride service. The DARP has been proven a NP-Hard problem; therefore, most researches addressed this problem with heuristic methods as the solution methods. The purpose of this study is to evaluation of the application of the Diversity-controlling Adaptive Genetic Algorithm (DCAGA) and Family Competition Genetic Algorithm (FCGA) on the DARP.

Two solution procedures are proposed in this study, which were integrated approach and cluster first route second approach. A series of case studies with different characteristics, such as demand density and demand size were used to test the solution capability of the proposed algorithms. Based on the results of the case studies, the Diversity Control Adaptive Genetic Algorithm is identified as the best algorithm in solution quality. Overall, the solutions of the integrated procedure are better than, those obtained from the two-phase procedure.

One area that deserved devotion of future research is the estimation of the penalty cost. Due to the significant impact of this factor on the results of the problem, a suitable method to estimate its cost is critical to the success of the proposed methodology. Another interesting area for future study is the incorporation of the fleet size and mix. This would

result in a complex nonlinear multi-objective programming problem, which will require significant effort to develop an effective solution method.

## References

- [1] Holland, J.J., (1975), "Adaptation in Natural and Artificial Systems," The University of Michigan Press, 1975.
- [2] Dejong, K.A., (1980 ), "Adaptive System Design : a Genetic Approach, ", IEEE Trans. Syst., Man, and Cyber. Vol. SMC-10(9), pp. 556-574
- [3] Psarafits, H.N., (1980), "A Dynamic Programming Approach to the Single-Vehicle Many0to-Many Immediate Request Dial-a-Ride Problem", Transportation Science 14, pp. 130-154.
- [4] Jaw, J., Odoni, A.R., Psaraftis, H.M., and Wilson, N.H.M., (1986), "A Heuristic Algorithm for the Multi-vehicle Advance-Request Dial-a-Ride problem with Time Window", Transportation Research B 20, pp. 243-257.
- [5] Madsen, O., Ravn, H.F., and Rygaard, J.M., (1995),"A Heuristics Algorithm for the Dial-a-Ride Problem with Time Window, Multiple Capacities, and Multiple Objectives", Annals of Operations research 60, pp.190-208.
- [6] Toth, P., and Vigo D., (1997), "Heuristic algorithms for the Handicapped Person Transportation Problem". Transportation Science, vol.31, p.60-71,.
- [7] Cordeau, J.F., Laporte, G., and Mercier, A., (2001),"A Unified Tabu Search Heuristics for Vehicle Routing Problems with Time Windows", Journal of Operation Research Society 52, pp.928-936.
- [8] Cordeau, J.F., and Laporte, G., (2003),"A Tabu Search Heuristic for the Static Multi-vehicle Dial-a-Ride Problem", Transportation Research Part B 37 p.579-594,
- [9] Zhu, K.Q. , (2003), "Population Diversity in Genetic Algorithm for Vehicle Routing Problem with Time Windows", Department of Computer Science, National University of Singapore, 2003.
- [10] Bergvinsdottir, K.B., (2004),"The Genetic Algorithm for solving the Dial-a-Ride Problem", Kgs. Lyngby, 2004.