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ESTIMATION OF THE CONDITIONAL VALUE AT RISK OF A MINIMUM VARIANCE HEDGING PORTFOLIO VIA DISTRIBUTION KURTOSIS

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Abstract. *When returns on assets display heavy tail or leptokurtosis, estimating value at risk (VaR) without considering distribution kurtosis can cause estimation bias. This study considered the kurtosis of distribution, utilizing various models to examine the conditional VaR of minimum variance portfolios (incorporating both the stock index and futures) and performing back-testing to compare individual model performance. Results demonstrate the improved accuracy of models using distribution kurtosis to estimate VaR. Furthermore, t distributed models outperformed both those with a normal distribution and symmetric volatility models in terms of the conditional VaR of minimum variance portfolios. These results suggest that in portfolio construction, investors should consider distribution kurtosis and volatility asymmetry.*
Keywords: Value at risk, Back-testing, Stock index futures, Asymmetry model

1. **Introduction.** In 1996, the Basel Committee on Banking Supervision proposed amending the Basel Accord to designate VaR (Value at Risk) as the measurement standard for calculating market risk. VaR is commonly applied in risk management. According to the capital adequacy regulations of the Bank for International Settlement, if banks use internal VaR models to assess market risk, then regular back-testing is needed to verify model accuracy.

Portfolio VaR is defined as the maximum loss that can be expected with a given degree of confidence over a particular time interval. When returns on assets display heavy tail or leptokurtosis, the use of normal distribution to estimate VaR can cause estimation bias. [8] proposed the GK (Gaver and Kafadar) model to capture the heavy tail and leptokurtosis of returns on assets. [10] found that using the GK model to estimate VaR was more accurate than using normal distribution.

Portfolio returns data usually exhibit volatility clustering; therefore, GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models are often used to capture changes in the volatility of financial data over time [3,6]. Numerous researchers have used GARCH models to estimate VaR [1,4]; however, GARCH models fail to capture the influence of asymmetrical market information. Consequently, [9] proposed the GJR (Glosten, Jagannathan and Runkle) model to identify the asymmetrical properties of volatility. [5] extended the GJR model in the form of multivariate asymmetric diagonal VECH (ADVECH) models, which can capture cross-asymmetry effects in data. [14] indicated that because most series of economic variables are non-stationary, the error correction (EC) term must be incorporated to explain long-run equilibrium relationships and prevent estimation bias. However, few studies have researched the use of multivariate EC-ADVECH models to explore VaR.

The minimum variance approach proposed by [12], enables the construction of a minimum variance portfolio incorporating stock indices and futures. Using the rolling window method, this study used VEC-DVECH-n, VEC-DVECH-t, VEC-GJR-n, VEC-GJR-t, VEC-ADVECH-n, and VEC-ADVECH-t models to examine the conditional VaR of minimum variance portfolios. Back-testing was used to compare model performance.

The empirical results were consistent with regard to minimum variance portfolios. Models that incorporated distribution kurtosis achieved more accurate conditional VaR than other models. Furthermore, models using t distribution outperformed models using normal distribution in terms of the conditional VaR of minimum variance portfolios. Additionally, asymmetric volatility models were more accurate than models based on symmetric volatility.

2. Sample Selection and Methodology.

2.1. Sample selection. This study sampled daily stock index and futures data for the Hang Seng, S&P 500 and TOPIX. The sample period ran from July 21, 1998 to June 30, 2011, and 3211 observations were extracted from the Datastream database. Daily returns on stock indices and futures were calculated as the difference in the logarithms of daily closing prices multiplied by 100. Using the rolling window method, this study applied the minimum variance method of [12] to create minimum variance portfolios incorporating stock indices and futures.

2.2. Methodology. This study incorporated the EC term into the multivariate asymmetrical diagonal VECH (ADVECH) model (hereafter known as the VEC-ADVECH model) developed by [5]. The model was as follows:

$$r_{i,t} = a_i + b_i r_{i,t-1} + c_i \left(\ln P_{1,t-1} - \kappa - \delta \ln P_{2,t-1}\right) + d_j r_{j,t-1} + \varepsilon_{i,i}, \quad i,j = 1,2,\ i \neq j, \tag{1}$$

$$\varepsilon_t | \Phi_{t-1} \sim N\left(0, \sum\nolimits_t\right), \tag{2}$$

$$\begin{aligned}\sigma_{ij,t} =& \tau_{ij} + \alpha_{1ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \alpha_{2ij} I_{\varepsilon_{i,t-1}}\varepsilon_{i,t-1} I_{\varepsilon_{j,t-1}}\varepsilon_{j,t-1} + \alpha_{3ij}\left(1 - I_{\varepsilon_{i,t-1}}\right)\varepsilon_{i,t-1} I_{\varepsilon_{j,t-1}}\varepsilon_{j,t-1} \\ &+ \alpha_{4ij} I_{\varepsilon_{i,t-1}}\varepsilon_{i,t-1}\left(1 - I_{\varepsilon_{j,t-1}}\right)\varepsilon_{j,t-1} + \beta_{ij}\sigma_{ij,t-1},\end{aligned} \tag{3}$$

$$\rho_{ij,t} = \frac{\exp(q_{ij,t})}{1 + \exp(q_{ij,t})}, \tag{4}$$

$$q_{ij,t} = \varpi_0 + \varpi_1 \rho_{ij,t-1} + \varpi_2 \frac{\varepsilon_{i,t-1}\varepsilon_{j,t-1}}{\sqrt{\sigma^2_{ii,t-1}\sigma^2_{jj,t-1}}}, \tag{5}$$

where Equation (1) represents the returns on individual assets i, where $r_{i,t}$ denotes the log-returns of individual asset i at time t. Additionally, c_i refers to the error correction term. In Equation (2), Φ_{t-1} is the information set at time $t-1$. Moreover, N may be a multivariate normal distribution or multivariate t distribution.

Equation (3) is the conditional covariance of the returns between individual assets i and j at time t. Also, α_{1ij} captures the interaction effect of the returns shocks between individual assets i and j at time $t-1$. Moreover, α_{2ij} captures the asymmetric interaction effect of the returns shocks between individual assets i and j at time $t-1$. If $\varepsilon_{k,t-1} < 0$, $k = i, j$, $I_{\varepsilon_{k,t-1}}$ is 1; otherwise, it is 0. Additionally, β_{ij} denotes the returns covariance between

Table 1. Model specification comparison and parameter restrictions

Models	α_2	α_3	α_4	Models	α_2	α_3	α_4
VEC-DVECH-n	0	0	0	VEC-GJR-t	Unrestricted	0	0
VEC-DVECH-t	0	0	0	VEC-ADVECH-n	Unrestricted	Unrestricted	Unrestricted
VEC-GJR-n	Unrestricted	0	0	VEC-ADVECH-t	Unrestricted	Unrestricted	Unrestricted

Note: VEC-DVECH-n denotes a multivariate DVECH-n model with a vector error correction term. VEC-DVECH-t is a multivariate DVECH-t model with a vector error correction term. VEC-GJR-n represents a multivariate GJR-n model with a vector error correction term. VEC-GJR-t denotes a multivariate GJR-t model with a vector error correction term. VEC-ADVECH-n is a multivariate ADVECH-n model with a vector error correction term. VEC-ADVECH-t represents a multivariate ADVECH-t model with a vector error correction term.

individual assets i and j at time $t-1$. The asymmetrical covariance effects produced by α_{3ij} and α_{4ij} are known as cross-asymmetry effects or cross effects. $\rho_{ij,t}$ denotes the dynamic conditional correlation coefficient in individual assets i and j at time t. Also, $q_{ij,t}$ captures the covariance between individual assets i and j at time t. Additionally, ϖ_1 represents the intertemporal persistence of the dynamic conditional correlation coefficient in individual assets i and j. Furthermore, ϖ_2 denotes the dynamic conditional correlation coefficient between individual assets i and j affected by time $t-1$ normalization shocks. Table 1 compares the model specifications and parameter restrictions. Because the log likelihood function of the parameters is nonlinear, the maximum likelihood estimates of the study parameters were obtained using the BHHH (Berndt, Hall, Hall and Hausman) algorithm of [2].

This study used VEC-DVECH-n, VEC-DVECH-t, VEC-GJR-n, VEC-GJR-t, VEC-ADVECH-n, and VEC-ADVECH-t models to compute the minimum variance hedge ratio $\beta_{t|\Phi_{t-1}}$ of the minimum variance portfolio as $\sigma_{ij,t|\Phi_{t-1}} \Big/ \sigma^2_{jj,t|\Phi_{t-1}}$. Using the hedge ratio $\beta_{t|\Phi_{t-1}}$, it is possible to ascertain the conditional return of the minimum variance portfolio at time t. With a given level of significance α, the conditional VaR model of the minimum variance portfolio p_t is defined as follows:

$$P(r_{p_t|\Phi_{t-1}} \leq VaR_{p_t|\Phi_{t-1}}) = \alpha, \tag{6}$$

$$VaR_{p_t|\Phi_{t-1}} = \delta_\alpha \sigma_{p_t|\Phi_{t-1}}, \tag{7}$$

where $r_{p_t|\Phi_{t-1}}$ is the conditional return on the minimum variance portfolio ($r_{p_t|\Phi_{t-1}} = r_{i,t} - \beta_{t|\Phi_{t-1}} r_{j,t}$), $r_{i,t}(r_{j,t})$ denotes the return on the stock index (futures) at time t, and $\beta_{t|\Phi_{t-1}}$ refers to the minimum variance hedge ratio. $VaR_{p_t|\Phi_{t-1}}$ is the conditional VaR of the minimum variance portfolio, and $\sigma_{p_t|\Phi_{t-1}}$ denotes the standard deviation of the return on the minimum variance portfolio ($\sigma_{p_t|\Phi_{t-1}} = \sqrt{\sigma^2_{ii,t|\Phi_{t-1}} - 2\beta_{t|\Phi_{t-1}}\sigma_{ij,t|\Phi_{t-1}} + \sigma^2_{jj,t|\Phi_{t-1}}\beta^2_{t|\Phi_{t-1}}}$). Furthermore, $\sigma^2_{ii,t|\Phi_{t-1}}(\sigma^2_{jj,t|\Phi_{t-1}})$ indicates the variance of the stock index (futures) return at time t under the dataset known at time $t-1$. Moreover, $\sigma_{ij,t|\Phi_{t-1}}$ represents the conditional covariance of the returns on stock indices and futures at time t under the known dataset at time $t-1$. Without considering kurtosis, δ_α is the αth percentile of the standard normal distribution. When considering distribution kurtosis, this study employed the GK model of [8], setting δ_α as $\delta_\alpha = [n\exp\{z^2_\alpha(n-3/2)/(n-1)^2\} - n]^{1/2}$. In the above formula, n indicates the degrees of freedom and z_α represents the αth percentile of the standard normal distribution. This study also used the likelihood ratio test ($LR = -2\ln[(1-c)^{T-x}c^x] + 2\ln[(1-(x/T))^{T-x}(x/T)^x]$) developed by [13] to examine the back-testing results of the models.

3. Empirical Results.

3.1. Summary statistics and ARCH test.

Table 2 shows descriptive statistics and ARCH test results for the stock indices and futures of Hang Seng, S&P 500 and TOPIX.

Table 2. Summary statistics and ARCH analysis

Statistic	Hang Seng		S&P 500		TOPIX	
	stock index	futures	stock index	futures	stock index	futures
Mean	0.0299	0.0302	0.0039	0.0036	−0.0127	−0.0126
Standard deviation	1.6800	1.8223	1.3532	1.3764	1.4368	1.5608
Skewness	0.0669	0.1215*	−0.0909*	−0.0492	−0.2885*	−0.0929
Exceess kurtosis	6.7107*	4.7531*	6.6842*	8.0113*	6.3899*	10.6339*
Jarque-Bera	6025.6449*	3029.5309*	5980.0970*	8585.4067*	5505.7071*	15128.9562*
LB $Q(6)$	8.431	8.999	29.180**	24.192**	26.130**	39.989**
LB $Q^2(6)$	1095.023**	882.828**	1346.720**	1263.341**	1553.534**	1576.628**
ARCH	3124.258*	3337.888*	1817.068*	1772.66*	2943.255*	1716.258*
JT	546.7386*	531.7353*	532.7765*	506.8295*	530.5931*	468.0828*

Note:

1. ** (*) denotes statistical significance at 1% (5%) significant level.
2. LB $Q(6)$ represents Ljung-Box Q test statistics of lag 6; the critical value is 16.81 (12.59) at 1% (5%) significant level.
3. LB $Q^2(6)$ refers to Ljung-Box Q test statistics of lag 6 for squared series; the critical value is 16.81 (12.59) at 1% (5%) significant level.
4. The ARCH test statistics proposed by Engle (1982) are based on the minimum of AIC as the time lags of the stock index and futures are determined under the null hypothesis; no ARCH effects.
5. JT refers to a joint test, and it is a chi-square distribution with 3 degrees of freedom. The critical value at 5% significant level is 7.82.

At the 5% significant level, Jarque-Bera statistic for each series showed that it was not normally distributed. This study also used the joint test (JT) developed by [7] to identify the influence of residual from stock indices and futures on volatility. As shown in Table 2, the JT results for stock indices and futures significantly exceeded the critical value of chi-squared distribution. This indicates that each of these figures had positive/negative residual, and that the extent of this residual asymmetrically affected volatility.

3.2. **Analysis of the conditional VaR of conditional minimum variance hedging portfolios.** Table 3 shows the conditional VaR of minimum variance portfolios comprising the stock index and futures of Hang Seng, S&P 500, and TOPIX. When kurtosis was considered, the number of exceptions was lower than when kurtosis was not considered. This finding is consistent with that of [10]. At the 1% significant level, normally distributed models failed to pass back-testing, regardless of whether kurtosis was considered. This may be because the normal distribution cannot capture heavy tail or leptokurtosis of portfolio returns. This finding is consistent with the conclusions of [11,15]. Therefore, this study used the t distribution to estimate portfolio VaR. The t distributed models were associated with fewer exceptions than the normally distributed models, which can be attributed to t distributed models capturing the heavy tail and leptokurtosis of portfolio returns. This echoes the findings of [11,15]. Asymmetric volatility models had fewer exceptions than symmetric volatility models, indicating that the conditional VaR of minimum variance portfolios were more accurate owing to capturing the asymmetric properties of volatility.

As demonstrated above, the results regarding minimum variance portfolios (regardless of which stock index) remained consistent. At 1% or 5% significant level, models that incorporated kurtosis of distributions produced more accurate conditional VaR than models that did not. Models using t distribution outperformed level, models that incorporated kurtosis of distributions produced more accurate models following normal distribution in the conditional VaR of minimum variance portfolios. Additionally, asymmetric volatility models outperformed symmetric volatility models.

4. **Conclusions.** Using the minimum variance method, this study constructed minimum variance portfolios incorporating stock indices and futures. Considering the kurtosis of distribution, this study used different VEC models to investigate the conditional VaR of

Table 3. Performance of models in estimating the conditional VaR of minimum variance hedging portfolios

Models	Without considering kurtosis distribution				With considering the kurtosis distribution			
	Number of exceptions		LR		Number of exceptions		LR	
	1%	5%	1%	5%	1%	5%	1%	5%
Panel A: Hang Seng								
VEC-DVECH-n	147	189	241.9685	11.3309	145	186	235.4088	9.8050
VEC-DVECH-t	63	173	28.9875	4.4071	49	170	10.8586	3.4510*
VEC-GJR-n	132	174	194.2209	4.7504	125	171	173.1383	3.7573*
VEC-GJR-t	57	166	20.3471	2.3514*	40	157	3.3969*	0.6314*
VEC-ADVECH-n	113	171	138.9279	3.7573*	99	167	102.3916	2.6073*
VEC-ADVECH-t	38	161	2.2677*	1.2647*	36	149	1.3517*	0.0160*
Panel B: S&P 500								
VEC-DVECH-n	127	181	179.0802	7.4922	113	177	138.9279	5.8535
VEC-DVECH-t	44	168	6.2545*	2.8760*	39	165	2.8063*	2.1082*
VEC-GJR-n	83	164	65.7049	1.8779*	70	159	40.5389	0.9214*
VEC-GJR-t	36	153	1.3517*	0.2134*	27	149	0.2203*	0.0160*
VEC-ADVECH-n	64	159	30.5442	0.9214*	59	146	23.0904	0.0161*
VEC-ADVECH-t	26	143	0.4369*	0.1459*	23	139	1.5652*	0.5253*
Panel C: TOPIX								
VEC-DVECH-n	136	187	206.6197	10.3023	129	183	185.0880	8.3824
VEC-DVECH-t	54	170	16.5022	3.4510*	42	169	4.7290*	3.1572*
VEC-GJR-n	96	167	95.0778	2.6073*	89	163	78.7763	1.6605*
VEC-GJR-t	43	162	5.4681*	1.4560*	31	153	0.0758*	0.2134*
VEC-ADVECH-n	82	165	63.6114	2.1082*	77	153	53.5267	0.2134*
VEC-ADVECH-t	32	159	0.2083*	0.9214*	26	145	0.4369*	0.0448*

Note:

1. * denotes that the model passed back-testing.
2. LR denotes the likelihood ratio test; a chi-square distribution with 1 degree of freedom. The critical value is 6.6349 (3.8415) at the 1% (5%) significant level.

minimum variance portfolios with back testing to compare individual model performance, and suggest that future studies incorporate the level of returns into the volatility equation, since this can enhance the performance of portfolio VaR.

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