A novel class of CPM using overlapping separable phase shaping pulses

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Abstract: The decomposition of continuous phase modulation (CPM) signals into a set of PAM waveforms is a very effective approach to reduce the receiver complexity. Recently, CPM signals using rectangular frequency shaping (FS) pulses having the so-called separable phase (SP) property can be reformulated as a compactor connected with an equivalent CPFSK scheme, and was proved to have minimal number of PAM waveforms among the existing PAM approaches. In this letter, we propose a new class of phase separable CPM schemes, which is extended from our previous work. Here FS pulses are constructed by using two-steps overlapping phase separable rectangular pulses. We show that the number of PAM waveforms required to represent this new class of CPM signals can be reduced up to 75 percent compared with other PAM decomposition approaches.

Keywords: continuous phase modulation (CPM), PAM decomposition, separable phase, compactor

Classification: Transmission Systems and Transmission Equipment for Communications

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1   Introduction

Continuous-phase modulation (CPM) is an attractive constant envelope modulation scheme with power and spectral efficient properties, and has been widely used in land/mobile communications [1]. The phase continuity of CPM signal introduces memory that enables the receiver to use the maximum likelihood sequence detection (MLSD) approach; it usually requires more complexity. Many techniques were proposed to simplify the complexity of receivers, of particular interest here is to decompose the CPM signal into a set of PAM waveforms [2, 3, 4, 5]. Hence the PAM-based suboptimal receivers with much less complexity can be developed [4]. Also, in [6, 7] Cariolaro found a subclass of CPM signals with rectangular frequency shaping (FS) pulses having the so-called separable phase (SP) property that enables the original CPM signals to be reformulated as a compactor connected with an equivalent full response CPM ($L = 1$) scheme. It was proved that the decomposed equivalent full response CPM signal requires minimum number of PAM waveforms to represent [6]. But $LREC$ scheme is the only known classes of CPM family that satisfies the property of SP [6, 7]. Here $LREC$ means that the FS pulse is rectangle with LT time interval. In this letter, we propose a new class of phase separable CPM schemes, which is extended from our previous work [8], where FS pulse with overlapping SP property was considered. The original CPM signals consists of compactor 1 and equivalent M_1-ary CPM (i.e., both $L_1 REC$ and $L_1 RC$ schemes), and the resultant number of PAM waveforms to represent CPM signals is still greater than other PAM approaches [2, 3]. To solve the drawback of [8], we present new CPM with FS pulses which are constructed using two-steps overlapping phase separable rectangular pulses with time interval $L_1 T$ (i.e., $L_1 REC$). In fact, by choosing appropriate two-steps overlapping $L_1 REC$ pulses, we can construct CPM signals with minimal number of PAM waveforms.

This letter is organized as follows. In Sec.2, we briefly review the CPM signal with SP property, and the idea of reducing the number of PAM waveforms. After that we introduce new FS pulse which enables us to construct the CPM signals with desired SP property that could achieve the minimal number of PAM waveforms. In Sec.3, numerical results for new class of CPM, in terms of number of PAM waveforms and bandwidth and Euclidean

distance, are given to verify the merits of our new proposed CPM schemes. Finally, we give a conclusion in Sec.4.

2 A novel separable phase CPM scheme

2.1 Problem description of separable phase CPM signals

The complex CPM signaling waveform can be represented as [1]

$$X(t, \boldsymbol{\alpha}) = \exp\{j(\theta(t, \boldsymbol{\alpha}) + \theta_0)\} \tag{1}$$

Without loss of generality, initial phase θ_0 is set to null, and $\theta(t, \boldsymbol{\alpha})$ is the integration of information sequence, i.e.

$$\theta(t, \boldsymbol{\alpha}) = 2\pi h \int_{-\infty}^{t} \sum_n \alpha_n g(t - nT) dt = 2\pi h \sum_n \alpha_n q(t - nT) \tag{2}$$

In (2) parameter $h = m/p$ denotes the modulation index, where m and p are relative prime integer. Also, the input data α_n, $n = 0, \pm 1, \pm 2, \ldots$ are in the set $\{\pm 1, \pm 3, \ldots, \pm(M-1)\}$ (M is a power of 2). The phase shaping pulse $q(t)$ of CPM scheme is represented by

$$q(t) = \begin{cases} \int_0^t g(\delta)d\delta & , 0 \leq t \leq LT \\ 1/2 & , t > LT \end{cases}. \tag{3}$$

L is called the correlative length of CPM. The FS pulse $g(t)$ is with duration LT and its area is normalized to $1/2$. When $g(t)$ is a rectangular pulse (with L), the CPM modulators are denoted as $LREC$ schemes, and a special case of $LREC$ with $L = 1$ called full response is the continuous-phase frequency-shift keying (CPFSK). Indeed, minimum shift keying (MSK) is a type of binary CPFSK with frequency separation of one-half the bit rate, i.e. $h = 1/2$. The phase function of (2) can be demodulated using MLSD receiver that is usually implemented with higher complexity. To reduce complexity, we decompose CPM signals into a set of PAM waveforms [2, 3, 4, 5, 6], thus (1) can be rewritten as $X(t, \boldsymbol{\alpha}) = \sum_{k=0}^{Q-1} \sum_{n=-\infty}^{+\infty} \beta_{k,n} C_k(t - nT)$, where *pseudo-symbol* $\beta_{k,n}$ is related to source symbols α_n, and Q ($= M^L - M^{L-1}$) PAM pulses $C_k(t)$ are obtained from the phase response of CPM signals [3, 6]. Hence the PAM-based optimal receivers can be developed with less complexity [2, 3, 4, 5]. For CPM signals Q is usually large, recently, in [6, 7] Cariolaro found that Q can achieve minimum if $q(t)$ in (3) is with SP property:

$$q(\tau + iT) = q_0(\tau) + \frac{i}{2L}, \quad i = 0, \ldots, L-1, \quad \text{for } \tau \in [0, T) \tag{4}$$

where $t = \tau + iT$ and the elementary phase is denoted as

$$q_0(\tau) = \frac{1}{L} \int_0^\tau g_0(t) dt, \quad 0 \leq \tau < T. \tag{5}$$

Function $g_0(t)$, called *basic pulse*, is a full response FS pulse with area normalized to $1/2$. By replacing (5) into (4) and take the first derivative of SP

condition defined in (4), the condition for SP can be rewritten in terms of FS pulse

$$g(t) = (1/L) \left[g_0(t) + g_0(t - T) + \cdots + g_0(t - (L-1)T) \right]. \qquad (6)$$

With property (4) or (6), one can introduce (6) into (2) and showed that CPM signal can be represented as a *compactor* connected with an equivalent M_1-ary full response CPM (correlative length $L_1 = 1$) with modulation index $h_1 = h/L$ [6, 7] (see Proposition 1 of [6]). Indeed, the compactor transforms the source symbol α_n into compact symbol A_n, i.e., $A_n = \sum_{i=0}^{L-1} \alpha_{n-i}$ [6]. This implies that with the separable phase property we are able to compact L consecutive input data α_n into a unique symbol A_n, and the equivalent full response CPM signal with input compact symbol A_n has symbol level $M_1 = L(M-1) + 1$. The number of PAM waveforms, i.e.,

$$N_c = L(M-1) \qquad (7)$$

was proved to be minimum [6]. However, $LREC$ is the only known class of CPM that satisfies this condition [6, 7], and the equivalent full response CPM is CPFSK.

2.2   New proposed separable phase CPM

We propose a new class of separable phase CPM by first replacing the unit correlative length constraint of $g_0(t)$ in (6) with $g_s(t)$, such that the basic pulse $g_s(t)$ has pulse length $L_1 > 1$, and the frequency shaping pulse is an overlapped time-shifted version of $g_s(t)$, defined as

$$g(t) = \frac{1}{(L - L_1 + 1)} \left[g_s(t) + g_s(t - T) + \cdots + g_s(t - (L - L_1)T) \right]. \qquad (8)$$

where $g_s(t)$ is chosen to be $L_1 REC$ that satisfies the SP property:

$$g_s(t) = \frac{1}{L_1} \left[g_0(t) + g_0(t - T) + \cdots + g_0(t - (L_1 - 1)T) \right] \qquad (9)$$

By introducing (8) into (2), it gives

$$\theta(t, \boldsymbol{\alpha}) = 2\pi h \int_{-\infty}^{t} \sum_n \alpha_n g(t - nT) dt = 2\pi h_1 \int_{-\infty}^{t} \sum_n A_n g_s(t - nT) dt \quad (10)$$

where the compact symbol related to the source symbol is modified as $A_n = \sum_{i=0}^{L-L_1} \alpha_{n-i}$, and the original CPM consists of compactor 1 and the equivalent $L_1 REC$ CPM (the dotted box), with modulation level $M_1 = (L - L_1 + 1)(M - 1) + 1$ and $h_1 = h/(L - L_1 + 1)$, as depicted in Fig. 1. In [8], we consider both $L_1 REC$ and $L_1 RC$ schemes without specifying that $g_s(t)$ possessing SP property, so that (9) is not defined and the number of PAM waveforms, i.e., $Q \ (= M_1^{L_1} - M_1^{L_1-1})$ for M_1-ary equivalent $L_1 REC$ CPM is still greater than the approaches discussed in [2, 3]. It should be noted that in [8] $g_s(t)$ was denoted as $g_0(t)$; it was referred to as the FS pulse. The steps adopted in this letter are different from our previous work addressed in [8]. That is, in (8) the FS pulse $g(t)$ of original CPM signals is with SP property, and
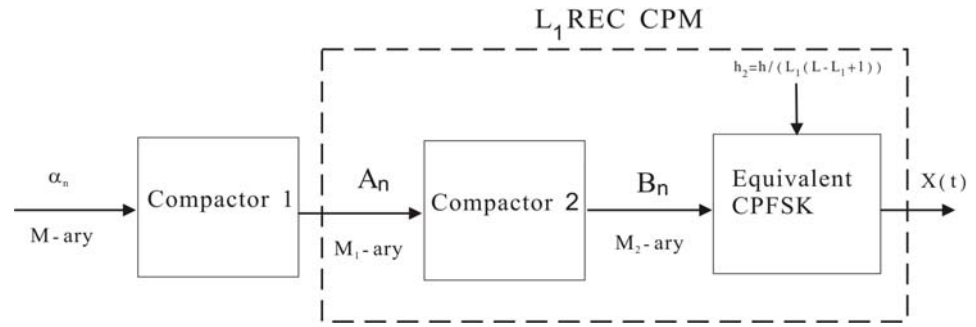
Fig. 1. Equivalence of the suggested separable phase CPM to two consecutive compactors connected with equivalent CPFSK

the equivalent M_1-ary CPM with $g_s(t)$ is itself phase separable, since the equivalent CPM is $L_1 REC$ scheme (i.e., inside of dotted box in Figure 1). So that we could further reduce the complexity of the new proposed SP CPM schemes. As a consequence, we can apply the equivalence principle (mentioned in Sec.2.1), to consecutively represent the M_1-ary CPM with $L_1 REC$ scheme as a compactor 2 connected with an equivalent CPFSK. The related compact symbol of compactor 2 is denoted as $B_n = \sum_{i=0}^{L_1-1} A_{n-i}$, with symbol level $M_2 = L_1(M_1 - 1) + 1 = L_1(L - L_1 + 1)(M - 1) + 1$ and the modulation index to be $h_2 = h_1/L_1 = h/(L_1(L - L_1 + 1))$. The minimal number of PAM waveforms for final equivalent CPFSK is [6]

$$N_c = L_1(M_1 - 1) = L_1(L - L_1 + 1)(M - 1). \tag{11}$$

For this new class of CPM signals, an abbreviation $LSREC/L_1REC$ is used to represent a separable phase CPM scheme with correlative length L; its final equivalent CPM is REC (CPFSK) and the corresponding basic pulse is L_1REC.

3   Numerical results

The overall performance under AWGN channel is investigated by plotting the energy-bandwidth plane for the proposed CPM schemes with selected SP property. Coding gains of CPM over MSK in dB is evaluated by $10 \log_{10}(d_{\min}^2/2)$, and $d_{\min}^2$ is the normalized minimum square Euclidean distance used to characterize the bit error performance in the AWGN channels. As discussed earlier, the $LSREC/L_1REC$ CPM schemes have linear phase transition in final equivalent CPFSK modulation. It is reasonable to compare these schemes with conventional $LREC$ CPM schemes that also have linear phase transition. To facilitate discussion, the normalized bandwidth $2BT_b$ (T_b is bit interval) for CPM signals with 99.9% power-in-bandwidth, denoted as B_{999}, is considered associated with coding gains, for comparison. As described in [1], the B_{999} bandwidth more or less measures the width of the significant sidelobes. Good CPM schemes are selected based on the energy-bandwidth plots (shown in Fig. 2), and number of PAM waveforms
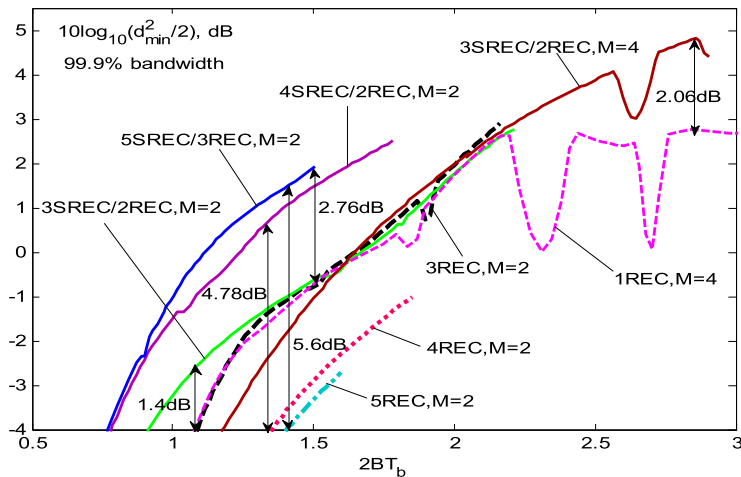
Fig. 2. Energy-bandwidth plot for the suggested separable phase CPM using B_{999} as a parameter

defined in (10) to represent $LSREC/L_1REC$, against other PAM decomposition approaches.

From Fig. 2, we observed that good (or better) modulation codes lie toward the upper left. In what follows, we focus on the $LSREC/L_1REC$ CPM schemes with $L \geq 3$ and $1 < L_1 \leq L$, and the modulation interval to be $h < 1$ and $h < 1.5$ (exclude integer h), respectively. As results some useful CPM schemes such as binary $3SREC/2REC$, $4SREC/2REC$, $5SREC/3REC$, and quaternary $3SREC/2REC$ are provided, and then compared with $LREC$ schemes, viz., binary $(M = 2)$ $3REC$, $4REC$, $5REC$ and quaternary $(M = 4)$ REC. We choose the binary $LREC$ schemes because they could achieve better performance when $L = 3$, and become worse for $4REC$ and $5REC$. This is not the case for the new proposed good CPM schemes. For quaternary $LREC$, REC $(L = 1)$ is the best, and become worse for $L > 1$.

Now, we investigate the coding gains for B_{999} bandwidth within considered modulation intervals. From Fig. 2 we learn that binary $3SREC/2REC$ performs better than binary $3REC$ for $B_{999} < 1.55$ with coding gains up to $1.4\,\mathrm{dB}$ at $B_{999} = 1.1$, and both schemes are with the same number of states in MLSE receiver (i.e. pM^{L-1}) [1]. Also, binary $4SREC/2REC$ outperforms binary $3REC$ with coding gains $1.75\,\mathrm{dB}$ at $B_{999} = 1.8$, and is much better than binary $4REC$ with coding gains up to $4.78\,\mathrm{dB}$ at $B_{999} = 1.35$. Similarly, binary $5SREC/3REC$ performs superior than $3REC$ with coding gains up to $2.76\,\mathrm{dB}$ at $B_{999} = 1.5$, it is much better than binary $5REC$ with coding gains up to $5.6\,\mathrm{dB}$ at $B_{999} = 1.4$. As for quaternary $3SREC/2REC$ scheme, it outperforms quaternary $1REC$ for $B_{999} > 1.6$ with $2.06\,\mathrm{dB}$ coding gains at $B_{999} = 2.85$.

Finally, in Table I we provide a comparison list of the number of PAM waveforms required for the proposed CPM schemes, our previous work [8], and those using other PAM decomposition approaches [2, 3]. Except for $3SREC/2REC$ that the SP approach has the same number of PAM waveforms as other PAM approaches our proposed CPM schemes can generally

Table I. Number of PAM waveforms required for representing the proposed good separable phase CPM schemes and other approaches

M	$LSREC$ / L_1REC	M_1	M_2	Approach in [8] $M_1^{L_1} - M_1^{L_1-1}$	Approaches in [2,3] $Q=M^L - M^{L-1}$	Equation (11) $L_1(L-L_1+1)(M-1)$
2	3SREC/2REC	3	5	6	4	4
2	4SREC/2REC	4	7	12	8	6
2	5SREC/3REC	4	10	48	16	9
4	3SREC/2REC	7	12	42	48	12

achieve number of PAM waveforms reduction, dramatically. Especially, the quaternary $3SREC/2REC$ schemes could achieve up to 75 percent the number of PAM waveforms reduction.

4 Conclusions

In this paper we proposed a novel class of CPM schemes with separable phase property, where the frequency shaping pulses were constructed using overlapping L_1REC pulses. The benefits of our new CPM scheme are given in terms of power and bandwidth efficiency. We proved that with our CPM schemes the number of PAM waveforms required for representing the CPM signals is minimal as compared to other existing PAM decomposition approaches. Also, we showed that it outperformed the binary and quaternary CPM using $LREC$ for many cases.