# Performance of the MIMO PRCP-OFDM System with Orthogonal Cyclic-Shift Sequences

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*Abstract*—**Unlike the conventional Pseudo-Random Posfix (PRP) orthogonal frequency division multiplexing (OFDM) or PRP-OFDM approach, recently we proposed a new transceiver framework for OFDM system, named as the Pseudo Random Cyclic Postfix (PRCP)-OFDM, with complementary codes (CC). Also, it was extended to the multiple-input multiple-output (MIMO) antennas system, equipped with the space-time block code (SP-BC). In this paper, the cyclic postfix (CP) sequence is implemented with the Orthogonal Cyclic-shift (OCS) sequences to substitute the CC. By exploring the useful property of OCS sequences, convolved with channel information, at receiver side it can be employed to perform semi-blind channel estimation without incurring the signal interference to the desired postfix sequences. It is affected only by the background noise, and yields achieving better system performance. To demonstrate the merits of proposed ST-BC MIMO PRCP-OFDM system the performance is investigated and further compared with the ST-BC MIMO PRP-OFDM system and ST-BC MIMO CP-OFDM system with optimal training pilots, under various channel environments.**

*Keywords- MIMO-OFDM; space-time encoder; redundant sequence; orthogonal codes; cyclic-postfix; pilot; channel estimation*

## I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems provide a very promising means to increase the spectral efficiency. Equipped with space-time block encoding (ST-BC) [10] in the transmitter and applied the intelligent signal processing at the receiver [6], MIMO systems can provide diversity and coding gains over single antenna system. The orthogonal frequency division multiplexing (OFDM) technique has been widely adopted for high-speed wireless communications, due to its robustness of multipath propagation [1]-[5]. For eliminating the inter-block-interference (IBI), a sufficient length of redundancy named as the cyclic prefix (CP), zero-padding (ZP) and pseudo-random postfix (PRP), with length greater than the order of channel impulse response (CIR), inserting between consecutive block symbols in transmitter [1][4][7][8]. Eventually, the ZP-OFDM system was proposed to circumvent the problem of channel null occurred when the CP-OFDM system was employed, with the price of increased receiver complexity [4].

On the contrast, the PRP-OFDM systems exploit additional information, i.e. pseudo-randomly weighted postfix sequences, for semi-blindly channel estimation, with order-one statistics of the received [7]. It was extended to the MIMO case, by encoding the transmitted signal and postfix vectors with two different space-time encoder [13][14], and is referred to as the ST-BC MIMO PRP-OFDM systems. With specific design of the space-time (ST) encoder for the PRP sequences, the MIMO PRP-OFDM system is capable to perform a semi-blind estimation for the entire MIMO-channel, by exploiting the order-one statistics of the received signals. In the aforementioned PRP-OFDM systems, to extract the channel information it assumed that the transmitted signals and background noise are zero-mean, and is performed by averaging a number of collected signal-blocks in the receiver to avoid the effect due to the interfered transmitted signal. The zero-mean assumptions do not hold, due to the fact that the collection of sufficient signal-blocks may not be possible, in practical situations. Especially, when the signal-to-noise ratio (SNR) is increased, the effect of the interfered transmitted signal becomes significant.

To circumvent the above mentioned problem, in [9] we proposed a new scheme, in which the null samples of ZP-OFDM is replaced by the known cyclic-postfix sequences (CPS), weighted by a pseudorandom scalar, and is named as the Pseudo Random Cyclic Postfix **(**PRCP)-OFDM. Here the complementary codes are used to replace the redundancy contents of ZP-OFDM. It capitalizes on the advantages of PRP-OFDM, but unlike PRP-OFDM it explores the useful property of complementary codes (CC) (sequences) convolved with channel information, at the receiver the CPS of the PRCP-OFDM can be used to enhance the semi-blind channel estimation, effectively. Moreover, at the receiver end, by removing the added redundancy sequences the effect of IBI could be alleviated, effectively. Basically, the above-mentioned cyclic postfix uses the cyclic sequence as its postfix, and by appropriate design in the receiver, it can be employed to estimate the channel impulse response (CIR) without incurring the signal interference to the desired postfix sequences, which is quite different from the PRP-OFDM approach. Since it is affected only by the background noise, and yields achieving better system performance, this is especially true when signal-to-noise ratio (SNR) grows. The price for PRCP-OFDM is that it needs to form a full rank matrix to estimate channel impulse response. Similarly, with the help of [11]-[13], the PRCP-OFDM system was extended to the MIMO case with space-time block coding (ST-BC) [9]. In this paper, the CP sequence

is implemented with the Orthogonal Cyclic-shift (OCS) sequence to substitute the CC proposed in [9]. The OCS sequence can be viewed as the generalization of CC. The reason of applying OCS sequences is that these sequences can achieve the maximum noise restraint due to their useful property when we need to estimate channel information. To verify the merits of proposed ST-BC MIMO PRCP-OFDM system with OCS sequence, the performance is investigated and further compared with the ST-BC MIMO PRP-OFDM system and ST-BC MIMO CP-OFDM system with optimal training pilots, under various channel environments. It is noted that in the MIMO CP-OFDM system, with optimal training pilots, the training symbols are sent with transmitting symbols, which are assumed known at the receiver. Typical procedures for identifying the channel impulse response based on training procedure are transmitting multiple OFDM symbols consisting of complete pilot symbols. The detail discussion of the MIMO CP-OFDM system can be found in [19].

## II. SYSTEM MODEL DESCRIPTION

As an introduction, we consider the discrete-time transceiver model of the MIMO PRCP-OFDM system, with two transmit-antennas and $M_r$ receive-antennas as depicted in Fig.1. We assume that the transmitting symbol block is with length $M$, the $M \times 1$ signal block $\mathbf{s}(n)$ is first modulated by the $M \times M$ inverse fast Fourier transform (IFFT) matrix $F^{-1}$ (or $F^H$), to obtain the resulting time-domain signal block vector $\mathbf{u}(n) = \mathbf{F}^H \mathbf{s}(n)$, denoting as $\mathbf{u}(n) = [u_0(n) \ u_1(n) \ \dots u_{M-1}(n)]^H$, and $H$ denotes the Hermitian transpose. As in [20][21] the signal-blocks and postfix vectors are, independently, encoded by two STC-encoders, where the new proposed cyclic postfix sequences (or orthogonal cyclic sequences) will be introduced later. First, at transmitter, the corresponding $M \times 1$ successive precoded blocks $\mathbf{u}(2i)$ and $\mathbf{u}(2i+1)$ (or $\mathbf{u}(n)$, $n = 2i + k$, $k = 0$ and 1) are sent to the ST encoder $M(.)$, to output the following $2M \times 2$ ST-coded matrix [9][12][13]:

$$\begin{bmatrix} \overline{\mathbf{u}}^{(1)}(2i) & \overline{\mathbf{u}}^{(1)}(2i+1) \\ \overline{\mathbf{u}}^{(2)}(2i) & \overline{\mathbf{u}}^{(2)}(2i+1) \end{bmatrix} = \begin{bmatrix} \mathbf{u}(2i) & -\mathbf{P}_M^{(0)}\mathbf{u}*(2i+1) \\ \mathbf{u}(2i+1) & \mathbf{P}_M^{(0)}\mathbf{u}*(2i) \end{bmatrix} \quad (1)$$

where the superscript * denotes the complex conjugate. At $i$th time block interval, the symbol-blocks $\overline{\mathbf{u}}^{(1)}(i)$ and $\overline{\mathbf{u}}^{(2)}(i)$ are transmitted through the first and second antenna, respectively. In (1) the function of $M \times M$ permutation matrix (or reverse matrix) $\mathbf{P}_M^{(n)}$ is such that for vector $\mathbf{a} = [a(0) \ a(1)\dots a(M-1)]^T$ with this operation, the $p$th element becomes $[\mathbf{P}_M^{(n)} \mathbf{a}]_p = a((M-p+n) \bmod M)$. Hence $\mathbf{P}_M^{(0)}\mathbf{u}*(n) = [u_{M-1}(n) \ u_{M-2}(n) \ \dots \ u_0(n)]^H$ defined in (1) is simply a reversed version of $\mathbf{u}*(n)$.

As described earlier, the OCS sequences are adopted to build up the modulator of the MIMO PRCP-OFDM systems. Let us denote $\mathbf{c}(0) = [c_0(0) \ c_1(0)\dots c_{D-1}(0)]^T$ as the genetic OCS sequence with length $D$, and $\mathbf{c}(i) = [c_0(i) \ c_1(i)\dots c_{D-1}(i)]^T$ represents the $i$th cyclic-shifting to the right of $\mathbf{c}(0)$, and is defined as follows:

$$\mathbf{c}(i) = [c_{(D-i) \bmod D} \ c_{(D-i+1) \bmod D} \ \dots c_{(D-i-1) \bmod D}]^T \quad (2)$$

for $i \bmod D$ ($i = 1,\dots,D$). From (2) we know that if we repeat

the above procedure $D$ times, we obtain the original OCS sequence $\mathbf{c}(0)$, again. So that, any two above-mentioned OCS sequences are orthogonal with each other. The genetic OCS sequence $\mathbf{c}(0)$ associated with $D-1$ OCS sequences, $\mathbf{c}(i)$ for $i=1, 2,\dots, D-1$, we can form a full rank matrix (orthogonal matrix) $\mathbf{C}$. For exposition, we consider a OCS sequence defined as $\mathbf{c}(0) = [x \ 1 \ \dots \ 1]$ (with length $D$), it can be easily solved that with $x = -D/2$, we can construct an orthogonal matrix $\mathbf{C}$, based on $\mathbf{c}(0)$ and its cyclic-shifted sequences $\mathbf{c}(i)$. That is, $\mathbf{C}$ consists of $D$ OCS sequences with property $\mathbf{C}^H\mathbf{C} = (D/2)^2 \mathbf{I}_D$, where $\mathbf{I}_D$ is an identity matrix with dimension $D \times D$. Due to this fact, we refer the proposed OFDM system with orthogonal cyclic-shift sequences as the PRCP-OFDM system to distinct it from the conventional PRP-OFDM system. By exploring the useful property of OCS sequences, convolved with channel information, at receiver side it can be employed to perform semi-blind channel estimation without incurring the signal interference to the desired postfix sequences.

Next, it is of interest to introduce the ST-BC encoder for OCS sequences employed in PRCP-OFDM system. As depicted in Fig. 1, the corresponding cyclic-postfix sequences are applied to the ST-BC encoder $W(.)$ to construct the $2D \times 2$ cyclic-postfix matrix, i.e.,

$$\begin{bmatrix} \overline{\mathbf{c}}^{(1)}(2i) & \overline{\mathbf{c}}^{(1)}(2i+1) \\ \overline{\mathbf{c}}^{(2)}(2i) & \overline{\mathbf{c}}^{(2)}(2i+1) \end{bmatrix} = \begin{bmatrix} w^{(1)}\alpha(2i) & w^{(1)}\alpha(2i+1) \\ w^{(2)}\alpha(2i) & w^{(2)}\alpha(2i+1) \end{bmatrix} \otimes \mathbf{c}(i) \quad (3)$$

where $\overline{\mathbf{c}}^{(l)}(n)$ represents the CP vector of $n$th block for the $l$th transmit-antenna ($l = 1$, and 2), with index $n = 2i + k$, $k = 0$ and 1), and $\otimes$ denotes the Kronecker product. The function of $w^{(l)}$ in (3) is used to identify the all MIMO channels for two transmit-antennas case, at receiver. The $2 \times 2$ matrix $\mathbf{W}$ is used to identify the whole MIMO channel. It gathers the corresponding deterministic weighting factor $w^{(l)}(.)$, which is denoted as

$$\mathbf{W} = \begin{bmatrix} w^{(1)}(0) & w^{(2)}(0) \\ w^{(1)}(1) & w^{(2)}(1) \end{bmatrix} \quad (4)$$

By choosing matrix $\mathbf{W}$ to be a full rank orthogonal matrix ($\mathbf{W}^H\mathbf{W}=2\mathbf{I}_2$), we are able to separate all transmitted-postfixes, completely [20][21]. Besides, multiplying a pseudo random weight $\alpha(n)$ is to avoid the cyclo-stationary of the postfix sequences during transmission [7][8]. With $P=M+D$, the $P \times 1$ signal vector $\mathbf{x}^{(l)}(n)$, transmitted from the $l$th antenna is represented as

$$\mathbf{x}^{(l)}(n) = \mathbf{T}_{ZP}\overline{\mathbf{u}}^{(l)}(n) + \mathbf{T}_P\overline{\mathbf{c}}^{(l)}(n) \quad (5)$$

where the $P \times D$ and $P \times M$ matrices, $\mathbf{T}_P$ and $\mathbf{T}_{ZP}$, are defined, respectively, as

$$\mathbf{T}_P = \begin{bmatrix} \mathbf{0}_{M \times D} \\ \mathbf{I}_{D \times D} \end{bmatrix} \text{ and } \mathbf{T}_{ZP} = \begin{bmatrix} \mathbf{I}_{M \times M} \\ \mathbf{0}_{D \times M} \end{bmatrix}.$$

Also, the channel response between the $l$th transmit-antenna and $m$th receive-antenna is modeled as a FIR filter denoted as $\mathbf{h}^{(lm)} = [h_0^{(lm)} \ h_1^{(lm)}\dots h_L^{(lm)}]^T$, where $L$ is the channel order. The

$P \times 1$ signal vector received at the $m$th receive-antenna is defined as

$$\mathbf{r}^{(m)}(n) = \sum_{l=1}^{2}[\mathbf{H}_0^{(lm)}\mathbf{x}^{(l)}(n) + \mathbf{H}_1^{(lm)}\mathbf{x}^{(l)}(n-1)] + \mathbf{v}^{(m)}(n) \qquad (6)$$

In (6), $\mathbf{H_o}^{(lm)}$ is a lower triangular Toeplitz matrix with its first column to be denoted as $[h_0^{(lm)} \ h_1^{(lm)} \dots h_L^{(lm)} \ 0 \dots 0]^T$ and $\mathbf{H_1}^{(lm)}$ is an upper triangular Toeplitz matrix with its first row defined as $[0 \dots 0 \ h_L^{(lm)} \ h_{L-1}^{(lm)} \dots h_0^{(lm)}]^T$, respectively. The noise vector $\mathbf{v}^{(m)} = [v_0^{(m)} \ v_1^{(m)} \dots v_{P-1}^{(m)}]^T$ has the same dimension with $\mathbf{r}^{(m)}(n)$, and the elements are assumed to be with variance $\sigma_v^2$.

## III. SEMI-BLIND CHANNEL ESTIMATION AND EQUALIZATION OF ST-BC MIMO-OFDM SYSTEM

We first address the semi-blind channel estimation with first-order statistic for the MIMO PRCP-OFDM system. After that the channel equalization will be performed for recovering the transmitted block symbols.

### A. MIMO PRCP-OFDM Semi-blind Channel Estimation with Order-One Statistics

By observing the signal vector received of the $m$th receive-antenna in (6), we found that the last element of received signal vector consists of the postfix-sequence convolved with channel information, and interfered with background noise. Thus, we may collect the last element of two consecutive blocks, $\mathbf{r}^{(m)}(2i)$ and $\mathbf{r}^{(m)}(2i+1)$, and after some mathematical manipulation, the $p$th entry of both vectors are given by

$$[\mathbf{r}^{(m)}(2i)]_P = \sum_{l=1}^{2}\alpha(2i)w^{(l)}(0)[\bar{\mathbf{h}}^{(lm)}]^T\mathbf{c}(i) + [\mathbf{v}^{(m)}(2i)]_P \quad (7)$$

and

$$[\mathbf{r}^{(m)}(2i+1)]_P = \sum_{l=1}^{2}\alpha(2i+1)w^{(l)}(1)[\bar{\mathbf{h}}^{(lm)}]^T\mathbf{c}(i) + [\mathbf{v}^{(m)}(2i+1)]_P \quad (8)$$

where in (7) and (8) $\bar{\mathbf{h}}^{(lm)} = [h_0^{(lm)} \ h_1^{(lm)} \dots h_L^{(lm)}]^T$ is the reverse version of $\mathbf{h}^{(lm)}$. Next, dividing (7) and (8) by $\alpha(2i)$ and $\alpha(2i+1)$, respectively, we can remove the factor of pseudo random scalar, and the related signal vectors can be written as

$$d^{(m)}(2i) = \frac{[\mathbf{r}^{(m)}(2i)]_p}{\alpha(2i)} = \sum_{l=1}^{2}w^{(l)}(0)(\bar{\mathbf{h}}^{(lm)})^T\mathbf{c}(i) + \frac{[\mathbf{v}^{(m)}(2i)]_p}{\alpha(2i)} \quad (9)$$

and

$$d^{(m)}(2i+1) = \frac{[\mathbf{r}^{(m)}(2i+1)]_p}{\alpha(2i+1)} = \sum_{l=1}^{2}w^{(l)}(1)(\bar{\mathbf{h}}^{(lm)})^T\mathbf{c}(i) + \frac{[\mathbf{v}^{(m)}(2i+1)]_p}{\alpha(2i+1)} \quad (10)$$

Now, by stacking both $d^{(m)}(2i)$ and $d^{(m)}(2i+1)$ into a $2 \times 1$ vector $\mathbf{d}^{(m)}(2i) = [d^{(m)}(2i) \ d^{(m)}(2i+1)]^T$, and after some mathematical manipulation, we have

$$\mathbf{d}^{(m)}(i) = \sum_{l=1}^{2}\begin{bmatrix} w^{(l)}(0)(\bar{\mathbf{h}}^{(lm)})^T\mathbf{c}(i) \\ w^{(l)}(1)(\bar{\mathbf{h}}^{(lm)})^T\mathbf{c}(i) \end{bmatrix} + \begin{bmatrix} [\mathbf{v}^{(m)}(2i)]_P / \alpha(2i) \\ [\mathbf{v}^{(m)}(2i+1)]_P / \alpha(2i+1) \end{bmatrix}$$

$$= \mathbf{W}\begin{bmatrix} (\bar{\mathbf{h}}^{(1m)})^T\mathbf{c}(i) \\ (\bar{\mathbf{h}}^{(2m)})^T\mathbf{c}(i) \end{bmatrix} + \begin{bmatrix} [\mathbf{v}^{(m)}(2i)]_P / \alpha(2i) \\ [\mathbf{v}^{(m)}(2i+1)]_P / \alpha(2i+1) \end{bmatrix} \quad (11)$$

where $\mathbf{W}$ was defined in (4). By pre-multiplying $\mathbf{W}^H$ on both sides of (11), we can isolate each individual channel contribution to $\bar{\mathbf{h}}^{(lm)}$, and divide it by two to get

$$\mathbf{z}^{(m)}(i) = \mathbf{W}^H\mathbf{d}^{(m)}(i) / 2$$

$$= \begin{bmatrix} (\bar{\mathbf{h}}^{(1m)})^T\mathbf{c}(i) \\ (\bar{\mathbf{h}}^{(2m)})^T\mathbf{c}(i) \end{bmatrix} + \frac{1}{2}\mathbf{W}^H\begin{bmatrix} \frac{1}{\alpha(2i)}[\mathbf{v}^{(m)}(2i)]_P \\ \frac{1}{\alpha(2i+1)}[\mathbf{v}^{(m)}(2i+1)]_P \end{bmatrix}$$

$$= \begin{bmatrix} (\mathbf{c}(i))^T\bar{\mathbf{h}}^{(1m)} \\ (\mathbf{c}(i))^T\bar{\mathbf{h}}^{(2m)} \end{bmatrix} + \tilde{\mathbf{v}}^{(m)}(i) \quad (12)$$

where $\mathbf{z}^{(m)} = [z_1^{(m)}(i) \ z_2^{(m)}(i)]^T$ is a $2 \times 1$ vector, and

$$\tilde{\mathbf{v}}^{(m)}(i) = [\tilde{v}_1^{(m)}(i) \quad \tilde{v}_2^{(m)}(i)]^T$$

$$= \frac{1}{2}\mathbf{W}^H\begin{bmatrix} [\mathbf{v}^{(m)}(2i)]_P / \alpha(2i) \\ [\mathbf{v}^{(m)}(2i+1)]_P / \alpha(2i+1) \end{bmatrix} \quad (13)$$

Next, we stack $z_0^{(m)}(i)$ and $z_1^{(m)}(i)$ for $i = 0 \sim L+1$, to get

$$\mathbf{z}_l^{(m)} = \mathbf{C}\mathbf{h}^{(lm)} + \tilde{\mathbf{v}}_l^{(m)} \quad (14)$$

In (14) $\mathbf{z}_l^{(m)} = [z_l^{(m)}(0) \ z_l^{(m)}(1) \dots z_l^{(m)}(L)]^T$, $\mathbf{h}^{(lm)} = [h_L^{(lm)} \ h_{L-1}^{(lm)} \dots h_0^{(lm)}]^T$, and $\tilde{\mathbf{v}}_l^{(m)} = \left[\tilde{v}_l^{(m)}(0) \ \tilde{v}_l^{(m)}(1) \dots \tilde{v}_l^{(m)}(L)\right]^T$. Also, the OCS matrix $\mathbf{C}$ is simply a circulant matrix which is denoted as

$$\mathbf{C} = \begin{bmatrix} c_0 & c_1 & \cdots & c_{D-1} \\ c_{D-1} & c_0 & \cdots & c_{D-2} \\ \vdots & \vdots & \ddots & \vdots \\ c_1 & c_2 & \cdots & c_0 \end{bmatrix} \quad (15)$$

With the property of orthogonal sequences, i.e., $\mathbf{C}^H\mathbf{C} = (D/2)^2 \mathbf{I}_D$, we obtain the estimated channel vectors, respectively, as

$$\hat{\mathbf{h}}^{(1m)} = \left(1/(\tfrac{D}{2})^2\right)\left(\mathbf{C}^H\mathbf{z}_0^{(m)}\right) \quad (16)$$

and

$$\hat{\mathbf{h}}^{(2m)} = \left(1/(\tfrac{D}{2})^2\right)\left(\mathbf{C}^H\mathbf{z}_1^{(m)}\right) \quad (17)$$

Assumed that the noise signals are white Gaussian processes with zero-mean and the cyclic postfix sequences are transmitted with symbol blocks, periodically. We may collect sufficient symbol blocks (e.g., $N_b$) for averaging to reduce the effect due to background noise. Thus, (14) and (15) can be rewritten as

$$\mathbf{z}_l^{(m)} = \mathbf{C} \begin{bmatrix} h_L^{(lm)} \\ h_{L-1}^{(lm)} \\ \vdots \\ h_0^{(lm)} \end{bmatrix} + \frac{1}{\beta} \sum_{j=0}^{\beta-1} \begin{bmatrix} \tilde{v}_l^{(m)}(j(L+1)) \\ \tilde{v}_l^{(m)}(j(L+1)+1) \\ \vdots \\ \tilde{v}_l^{(m)}(j(L+1)+L) \end{bmatrix} \quad (18)$$

where $D = L+1$ and $\beta = N_b / 2(L+1)$.

## B. Equalization of the ST-BC MIMO PRCP-OFDM Receiver

After obtaining the estimated channel response (CIR), we may remove the postfix sequences, and convert the received block signals of (6) into the form of ST-BC MIMO ZP-OFDM, i.e.

$$\mathbf{r}_{ZP}^{(m)}(n) = \mathbf{r}^{(m)}(n) - \sum_{l=1}^{2} [\mathbf{H}_0^{(lm)} \mathbf{T}_P \overline{\mathbf{c}}^{(l)}(n) + \mathbf{H}_1^{(lm)} \mathbf{T}_P \overline{\mathbf{c}}^{(l)}(n-1)] \quad (19a)$$

or

$$\mathbf{r}_{ZP}^{(m)}(n) = \sum_{l=1}^{2} \mathbf{H}_0^{(lm)} \mathbf{T}_{ZP} \overline{\mathbf{u}}^{(l)}(n) + \mathbf{v}^{(m)}(n) \quad (19b)$$

Using the fact that $\mathbf{H}_0 \mathbf{T}_{zp} = \mathbf{H}_{CIRC} \mathbf{T}_{zp}$ [22], the above equation can be rewritten as

$$\mathbf{r}_{ZP}^{(m)}(n) = \sum_{l=1}^{2} \mathbf{H}_{CIRC}^{(lm)} \mathbf{T}_{ZP} \overline{\mathbf{u}}^{(l)}(n) + \mathbf{v}^{(m)}(n) \quad (20)$$

where $\mathbf{H}_{CIRC}^{(lm)}$ is a $P \times P$ circulant matrix whose first row is given as $\begin{bmatrix} h_0^{(lm)} & 0 & \cdots & 0 & h_L^{(lm)} & \cdots & h_1^{(lm)} \end{bmatrix}$. Next, using the fact that $\mathbf{P}_M^{(n)} \mathbf{T}_{zp} = \mathbf{T}_{zp} \mathbf{P}_M^{(0)}$, the two consecutive blocks of (20) are written as

$$\mathbf{r}_{ZP}^{(m)}(2i) = \mathbf{H}_{CIRC}^{(1m)} \mathbf{T}_{ZP} \mathbf{u}(2i) + \mathbf{H}_{CIRC}^{(2m)} \mathbf{T}_{ZP} \mathbf{u}(2i+1) + \mathbf{v}^{(m)}(2i) \quad (21)$$

and

$$\mathbf{r}_{ZP}^{(m)}(2i+1) = -\mathbf{H}_{CIRC}^{(1m)} \mathbf{P}_P^{(M)} \mathbf{T}_{ZP} \mathbf{u}^*(2i+1) \\ + \mathbf{H}_{CIRC}^{(2m)} \mathbf{P}_P^{(M)} \mathbf{T}_{ZP} \mathbf{u}^*(2i) + \mathbf{v}^{(m)}(2i+1) \quad (22)$$

Similar to [22], the corresponding frequency domain signals of (21) and (22) can be obtained, respectively, as $\mathbf{y}^{(m)}(2i) = \mathbf{F}_P \mathbf{r}_{ZP}^{(m)}(2i)$ and $\mathbf{y}^{(m)}(2i+1) = \mathbf{F}_P \mathbf{P}_P^{(M)}(\mathbf{r}_{ZP}^{(m)}(2i+1))^*$. Now, by stacking both terms together, we have

$$\overline{\mathbf{y}}^{(m)}(i) = \begin{bmatrix} \mathbf{y}^{(m)}(2i) \\ \mathbf{y}^{(m)}(2i+1) \end{bmatrix}$$
$$= \mathbf{D} \begin{bmatrix} \mathbf{\Theta}\mathbf{s}(2i) \\ \mathbf{\Theta}\mathbf{s}(2i+1) \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{v}}^{(m)}(2i) \\ \tilde{\mathbf{v}}^{(m)}(2i+1) \end{bmatrix} \quad (23)$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{H}_{DIAG}^{(1m)} & \mathbf{H}_{DIAG}^{(2m)} \\ (\mathbf{H}_{DIAG}^{(2m)})^* & -(\mathbf{H}_{DIAG}^{(1m)})^* \end{bmatrix}, \quad \mathbf{\Theta} = \mathbf{F}_P \mathbf{T}_{ZP} \mathbf{F}^H,$$

$\tilde{\mathbf{v}}^{(m)}(2i) = \mathbf{F}_P \mathbf{v}^{(m)}(2i)$ and $\tilde{\mathbf{v}}^{(m)}(2i+1) = \mathbf{F}_P \mathbf{P}_P^{(M)}(\mathbf{v}^{(m)}(2i+1))^*$. We can demodulate $\overline{\mathbf{y}}^{(m)}(i)$ with diversity gains by a simple matrix multiplication, that is

$$\overline{\mathbf{e}}^{(m)}(i) = \mathbf{D}^H \overline{\mathbf{y}}^{(m)}(i) = \begin{bmatrix} \overline{\mathbf{D}}^{(m)}\mathbf{\Theta}\mathbf{s}(2i) \\ \overline{\mathbf{D}}^{(m)}\mathbf{\Theta}\mathbf{s}(2i+1) \end{bmatrix} + \mathbf{D}^H \begin{bmatrix} \tilde{\mathbf{v}}^{(m)}(2i) \\ \tilde{\mathbf{v}}^{(m)}(2i+1) \end{bmatrix} \quad (24)$$

where $\overline{\mathbf{D}}^{(m)} = \mathbf{H}_{DIAG}^{(1m)}(\mathbf{H}_{DIAG}^{(1m)})^* + \mathbf{H}_{DIAG}^{(2m)}(\mathbf{H}_{DIAG}^{(2m)})^*$. We note that multi-antenna diversity of order two can be achieved.

## IV. SIMULATION RESULTS

To verify the merits of the proposed ST-BC MIMO PRCP-OFDM system, computer simulation is carried out. As described earlier, by exploiting the cyclic postfix sequences, using the orthogonal code sequences, the semi-blind channel estimation could be performed, with low complexity. After obtaining the estimated CIR, the effect of IBI can be alleviated by removing the postfix sequences from the received signal blocks. At the same time, the received signal blocks are turn into the form of ZP-OFDM system. As compared with the conventional PRP-OFDM system, the proposed PRCP-OFDM system has the advantage that it avoids the influence of transmitted signals, during the channel estimation processes.

We first investigate system performance, in term of normalized mean square error (NMSE) and the symbol error rate (SER), for sufficient redundancy ($P-M \geq L+1$) case, under random Rayleigh channels. In this case, we adopt the QPSK modulation for simulations. To be more specific, for ST-BC MIMO PRCP-OFOM and PRP-OFDM systems, the parameter set with $P = M + D = 136$ (size of transmitted block), $M$ (size of symbol block) $=128$, $L=7$ (order of channel) and $D = 8$ (size of redundant block) (for sufficient length ($D \geq L + 1$)), is considered, or the parameter set ($P, M, L$), is chosen to be (136,128,7). The Rayleigh channel coefficients are assumed to be time invariant and randomly generated. Also, the orthogonal cyclic-shift sequence is chosen as $\mathbf{c}(0) = [-3\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$, with length $D=8$ to form the orthogonal (and circulant) matrix, $\mathbf{C}$, respectively, consequently, we have $\mathbf{C}^H \mathbf{C} = 16\ \mathbf{I}_8$.

To perform the semi-channel estimation, for fair comparison, we collect 16 blocks for averaging with the ST-BC MIMO PRCP-OFDM and ST-BC MIMO PPR-OFDM, to reduce the effect of interference. The NMSE of estimated channel coefficients is represented as

$$NMSE = \frac{1}{N_r N_t} \left( \sum_{m=1}^{N_r} \sum_{l=1}^{N_t} \frac{\left\| \mathbf{h}^{(lm)} - \hat{\mathbf{h}}^{(lm)} \right\|^2}{\left\| \mathbf{h}^{(lm)} \right\|^2} \right),$$

The simulation results are obtained by averaging 50 random channels and each channel runs 10000 blocks. From Fig. 2, we observed that the performance, in terms of NMSE, using the ST-BC MIMO PRCP-OFDM system is performed better than the one with ST-BC MIMO PRP-OFDM system, for SNR greater than 3 dB, due to less influence by the transmitted signal. Since different receive antenna has different channel coefficients and the estimate of these channel coefficients, is performed, individually. Therefore, the NMSE performance of

two receive antenna is the same as single receive antenna.

Next, it is of interest to examine the SER for various ST-BC MIMO OFDM systems. First, from Fig. 3, we found that the SER performance with the ST-BC MIMO PRCP-OFDM outperforms the conventional ST-BC PRP-MIMO-OFDM scheme, especially for high SNR. Also, from Figure 3 we observed that both ST-BC MIMO PRCP-OFDM and ST-BC MIMO PRP-OFDM systems have superior performance compared to the one with ST-BC MIMO OFDM system with optimal pilots. This especially true when the SNR is increased the SER performance improvement with the ST-BC MIMO PRCP-OFDM system is more significant compared with ST-BC MIMO PRP-OFDM system and ST-BC MIMO OFDM with optimal pilots. This agrees quite well with the theoretical view point as mentioned earlier. Finally, we would like to consider the case with Jake's model. The parameter set is the same as the case of Fig. 3, similar results as in Fig. 3 are observed in Fig. 4. Compare the results of Fig.4 with Fig. 3, we learn that the SER performance with Jake's channel model with slowly varying (i.e., with speed $v = 40$ km/hr) is performed worse than the one using the Rayleigh fading channel.

## V. CONCLUSIONS

In this paper, the block transmission of MIMO-OFDM system has been considered, owing to the fact that the OFDM technique entails redundant block transmissions; the transmitted blocks suffer from the ISI and IBI problem. To enhance the system performance, we proposed a novel ST-BC MIMO PRCP-OFDM system, where the pseudo random cyclic-postfix sequences, implementing with the orthogonal cyclic-shift (CP) sequences. System performance, in term of NMSE and SER, for sufficient redundancy ($P$-$M \geq L$+1) case, under random Rayleigh and Jake's channel models were investigated. By exploring the property of OCP sequence the semi-blind channel estimation, with first-order statistics of received signals, could be performed, effectively, compared with the PRP-OFDM based system as well as the ST-BC MIMO CP-OFDM system. This is due to the fact that the semi-blind channel estimation performed with the proposed ST-BC MIMO PRCP-OFDM system was affected only by the background noise; it avoided the interference of the transmitted signals. Thus, when the SNR is increased better performance achievement could be obtained compared with the conventional MIMO PRP-OFDM systems. Therefore, we may conclude that the proposed ST-BC MIMO PRCP-OFDM system performs the best among the discussed MIMO OFDM systems which have been addressed in the literatures.

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Figure 2   Comparison of NMSE for ST-BC MIMO PRCP- OFDM and MIMO PRP-OFDM system under Rayleigh random channel.

Table 1 Illustration the cyclic postfix sequence **c**(*i*) for *D*=8 and *x*=-*3*

| (*i*) MOD *D* | Cyclic-shift postfix sequence **c**(*i*) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $c_0(i)$ | $c_1(i)$ | $c_2(i)$ | $c_3(i)$ | $c_4(i)$ | $c_5(i)$ | $c_6(i)$ | $c_7(i)$ |
| 0 | -3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | -3 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | -3 | 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 1 | 1 | -3 | 1 | 1 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1 | -3 | 1 | 1 | 1 |
| 5 | 1 | 1 | 1 | 1 | 1 | -3 | 1 | 1 |
| 6 | 1 | 1 | 1 | 1 | 1 | 1 | -3 | 1 |
| 7 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -3 |



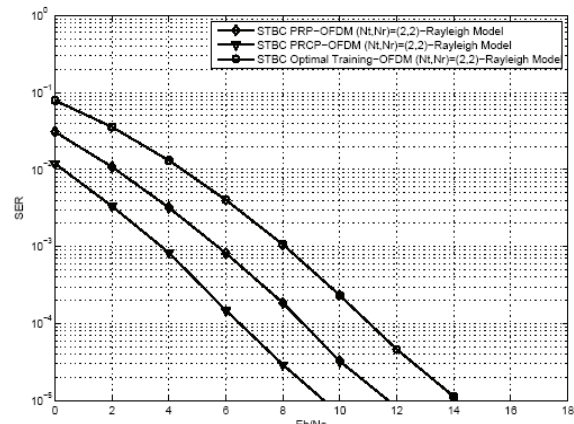Figure 3 Comparison of SER for PRP-OFDM, PRCP-OFDM and OFDM with optimal pilots under Rayleigh random channel for $N_t = 2$ and $Nr = 2$.



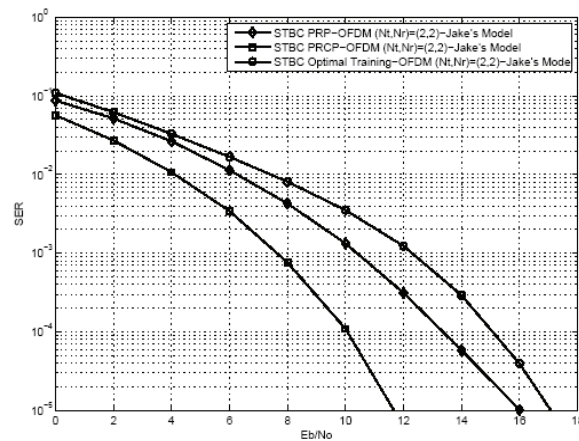Figure 1 Block diagram of the ST-BC MIMO PRCP-OFDM modulator.



Figure 4 Comparison of SER for PRP-OFDM, PRCP-OFDM and CP-OFDM with optimal pilots under Jake's channel for $N_t = 2$, $Nr = 2$ and $v = 40$ km/hr.