

# 行政院國家科學委員會專題研究計畫成果報告

## 直交明渠亞臨界分流研究

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### 1. 摘要

本研究進行主、支渠渠寬相等且渠底水平之亞臨界明渠分流實驗而求得影響分流流況之重要參數，並進一步推導分流流況之一維性解析模式。應用因次分析知主渠上、下游水深比為主渠下游寬深比，主、支渠渠寬比，支渠流量與主渠上游流量比，及主渠上游及支渠福祿數之函數。一維性解析模式則應用控制體積之連續、動量與能量方程式及適當之假設而推導。本研究更應用實驗資料以探討各物理量於分流區附近之變化。

關鍵詞：分流、流量比、水深比

### ABSTRACT

Based on experimental examinations, for a subcritical, right-angled, equal-width, open-channel dividing flow over a horizontal bed the cosine of the cross-sectional mean flow angle at the branch entrance is found to increase linearly with increasing discharge ratio. The experiment indicates that the contraction coefficient at the maximum width-contracted section in the recirculation region is almost inversely related to discharge ratio. The energy heads upstream and downstream of the division in the main channel are found to be almost equal. This enables the depth-discharge relationship follows the commonly used energy equation. The predicted results correlate with the experimental data of this study and to those of other studies. The energy loss coefficient of a division

is also expressed in terms of discharge ratio, upstream Froude number, and depth ratio.

Key words: dividing flow, discharge ratio, depth ratio.

### 2. INTRODUCTION

Open-channel dividing flow is characterized by the inflow and outflow discharges, the upstream and downstream water depths, and the recirculation flow in the branch channel. Ramamurthy and Satish, (1988), Ingle and Mahankal (1990), Ramamurthy et al. (1990), and Hager, (1992) recognized that the downstream-to- upstream discharge-ratio of the main channel is the most relevant parameter in the analysis of open-channel  $90^\circ$  dividing flow. The results from the above analyses compared quite satisfactory to some experimental observations. Most of the analyses restricted themselves to the development of a relationship for depth ratio, discharge-ratio and energy losses through the division. Neary and Odgaard (1993) concluded that the bed roughness as well as the branch-channel to main-channel velocity-ratio would affect the three-dimensional flow structure. The primary objective of the present study is to propose a depth-discharge relationship and energy loss coefficient for subcritical, equal-width, right-angled

dividing flow over a horizontal bed. With known upstream discharge and the prescribed branch flow discharge and downstream depth, the upstream depth is determined from energy considerations. The energy loss coefficient due to division is expressed as a function of  $F_{ru}$ ,  $\bar{Q}$  and  $\bar{Y}$ . The contraction coefficients at the maximum width-contracted section in the recirculation region and the cross-sectional mean flow angles at the branch entrance are determined using velocity measurements.

### 3. EXPERIMENT AND PARAMETERS

Subcritical, equal-width, right-angled dividing open-channel flow over a horizontal bed is characterized by the upstream discharge,  $Q_u$ , the downstream discharge,  $Q_d$ , the upstream depth,  $Y_u$ , and the downstream depth,  $Y_d$ , of the main channel, the downstream depth,  $Y_b$ , of the branch channel, the channel width,  $W$ , and the gravitational acceleration,  $g$ . Application of dimensional analysis yields

$$F(\bar{Q}, \bar{Y}, \bar{W}, F_{rd}, F_{rb}) = 0 \quad (1)$$

where  $\bar{Q} = Q_d / Q_u$ ;  $\bar{Y} = Y_u / Y_d$ ;  $\bar{W} = W / Y_u$  = upstream aspect-ratio in the main channel ;  $F_{rd} = Q_d / (W Y_d \sqrt{g Y_d})$  = downstream Froude number in the main channel; and  $F_{rb} = Q_b / (W Y_b \sqrt{g Y_b})$  = downstream Froude numbers in the branch channel.

The main and branch flumes were 12 and 4.0m long, respectively. Both channels were 14.7cm wide and ended with a weir to ensure that the considered downstream sections were subcritical and nearly uniform. The division corner to the branch channel was sharp-edged and was located 5.35m downstream from the main channel inlet. The water depth and velocity were measured at several cross sections with nine vertical profiles in each cross section. The

upstream and downstream sections, AB and CD, of the main channel were taken at four channel widths from point E and F, while the downstream section, GH, of the branch channel was set at six to ten channel widths from the branch entrance, EF. An ALEC ACM-250 two-component electromagnetic current meter was used for velocity measurements. The maximum width-contracted section in the recirculation region was, firstly observed by dye trajectory and checked by the velocity-integrated discharge. The main-channel upstream inflow was 3.02L/sec. to 5.37L/sec. The downstream-to-upstream discharge ratios ranged from 0.409 to 0.875. The Froude numbers at the upstream and downstream sections of the main channel were between 0.328 to 0.768 and 0.140 to 0.540, respectively. The Froude numbers at the downstream section of the branch channel were between 0.089 and 0.227.

### 4. RESULTS AND DISCUSSION

#### Flow Angle

The cross-sectional mean flow angle at the branch entrance is calculated as  $\delta = \tan^{-1}(U_l / V_l)$ , where  $U_l$  and  $V_l$  are the longitudinal and the lateral components, respectively, of the cross-sectional mean velocity at the branch entrance. For  $0.41 \leq \bar{Q} \leq 0.88$ , the cosine of the cross-sectional mean flow angle,  $\cos \delta$ , increases linearly with increasing  $\bar{Q}$  and may be expressed in terms of  $\bar{Q}$  as

$$\cos \delta = 0.36 \bar{Q} + 0.65 \quad (2)$$

The correlation coefficient in (2) is 0.984, significant at a 5% confidence level. Thus, (2) can be used to evaluate the cross-sectional mean flow angle at the branch entrance for subcritical, equal-width, open-channel right-angled dividing flow over a horizontal bed.

#### Contraction Coefficient

The location of the maximum width-contracted section in the recirculation region was first determined by dye

trajectories. Then, the velocity vectors at the pre-determined section and the nearby sections were measured. The effective width,  $C_c W$ , is determined when the velocity-integrated discharge from left bank,  $Q_s$ , equals  $Q_b$ , where  $C_c$  is the contraction coefficient.  $Q_s$  is calculated as

$$Q_s = \sum_{i=1}^m \sum_{j=1}^n u_{i,j} \Delta Y_i \Delta W_{i,j} \quad (3)$$

where  $u_j$  is the longitudinal velocity over the elemental height  $\Delta Y_j$  and width  $\Delta W_j$  in profile  $i$ ,  $n$  is the number of elements in each profile and  $m$  is the number of profiles in a cross section. The contraction coefficient,  $C_c$ , decreases linearly for  $\bar{Q}$  increases which reveals that a smaller divided discharge,  $Q_b$ , from main channel to branch channel yield a smaller effective width in the recirculation region of the branch channel.

### Dividing Streamline

The location of the dividing streamline at four-channel widths upstream from point E was determined when the velocity-integrated discharge from left bank,  $Q_s$ , equals the downstream discharge of the main channel,  $Q_d$ . The measured data reveals that  $W_{ud}/W = Q_d/Q_u$ , accepted at a 5% significance level with a correlation coefficient of 0.999. Subscript  $ud$  represents the section AI with a discharge  $Q_d$  across section AB. Additionally, the measured energy heads at section AB ( $H_u$ ) and at the section AI ( $H_{ud}$ ) are found almost equal with correlation coefficient being 0.999. These indicate that the section at four channel widths upstream from point E can be considered uniform.

### Depth Ratio

For  $0.41 \leq \bar{Q} \leq 0.88$ , the experimental data for the energy heads at section AI,  $H_{ud}$ , and at section CD,  $H_d$ , are found practically equal with correlation coefficient being 0.999. Since  $H_{ud} = H_u$ , thus

$$H_u = H_d \quad (4)$$

Essentially (4) implies that the commonly used energy equation without divided flow can be used to evaluate

the upstream and downstream flows in the main channel. (4) can be written in a polynomial form as

$$\bar{Y}^3 - \left[ 1 + \frac{1}{2} \left( \frac{\alpha_d}{\beta_d} \right) F_{rd}^2 \right] \bar{Y}^2 + \frac{1}{2\bar{Q}} \left( \frac{\alpha_u}{\beta_d} \right) F_{rd}^2 = 0 \quad (5)$$

where  $F_{rd} = \sqrt{\beta_d Q_d^2 / (\bar{g} Y_d^3 W^2)}$  and  $\beta_d$  = momentum correction coefficient at downstream of the main channel. In (5), there are two unequal positive roots and one negative root if

$$D = \bar{Q} - \left\{ \left( \frac{\alpha_u}{\beta_d} \right) F_{rd}^2 \right\} / \left[ \left( 2 + \left( \frac{\alpha_d}{\beta_d} \right) F_{rd}^2 \right) / 3 \right]^{3/2} > 0 \text{ and one}$$

multiple positive root and one negative root if  $D = 0$ . The smaller positive root, representing  $F_{ru} > 1$  and  $F_{rd} < 1$ , and the negative root are of no interest in this study.

The experimental data show that  $\alpha_u / \beta_d$  and  $\alpha_d / \beta_d$  are linearly related to  $\bar{Q}$  as

$$\frac{\alpha_u}{\beta_d} = 0.08\bar{Q} + 0.94 \quad (6)$$

and

$$\frac{\alpha_d}{\beta_d} = -0.14\bar{Q} + 1.16 \quad (7)$$

The correlation coefficients in (6) and (7) are 0.650 and -0.673 respectively, significant at a 5% confidence level.

The predicted  $\bar{Y}$  using (6) and (7) together with data of the present study, of Ramamurthy et al. (1990) and of Sridharan (1966) are compared. One of Sridharan's data, with  $Y_u$  larger than  $Y_d$  was excluded in the comparison. The agreement between the data and predicted  $\bar{Y}$  by (5) is good, although some of the measured data of Sridharan (1966) and of Ramamurthy et al. (1990) are deviated from the predicted. It is believed that the discrepancies must be attributed to the difficulties in the measurements of water depth. The predicted  $\bar{Y}$  by assuming  $\alpha_u / \beta_d = \alpha_d / \beta_d = 1.0$  in (5) are also compared. The discrepancies between the predicted  $\bar{Y}$  using (6) and (7) and using  $\alpha_u / \beta_d = \alpha_d / \beta_d = 1.0$  in (5) are small.

Thus, (5) can be simplified as

$$\bar{P}^3 - \left[1 + \frac{1}{2} F_{rd}^2\right] \bar{P}^2 + \frac{1}{2Q} F_{rd}^2 = 0 \quad (8)$$

### **Energy Loss Coefficient**

For the control volume ABEGHFDC, conservation of energy can be expressed as

$$Q_u H_u (1 - K_e) = Q_d H_d + Q_b H_b \quad (9)$$

where  $K_e$  = energy loss coefficient through the division. Since  $H_u = H_d$  and let  $\bar{Y}_b = Y_b / Y_u$ , (9) can be written as

$$K_e = (1 - \bar{Q}) \left[ 1 - \frac{2\bar{Y}_b^3 + \frac{\alpha_b}{\beta_u} (1 - \bar{Q})^2 F_{ru}^2}{\bar{Y}_b^2 (2 + \frac{\alpha_u}{\beta_u} F_{ru}^2)} \right] \quad (10)$$

The experimental data for  $\alpha_u / \beta_u$  and  $\alpha_b / \beta_u$  as appear in (10) and may be expressed as

$$\frac{\alpha_u}{\beta_u} = -0.04\bar{Q} + 1.05 \quad (11)$$

and

$$\frac{\alpha_b}{\beta_u} = 0.11\bar{Q} + 1.05 \quad (12)$$

The correlation coefficients in (11) and (12) are -0.751 and 0.62, respectively, significant at a 5% confidence level. The measured  $Y_u$  and  $Y_b$  show that  $\bar{Y}_b = 0.983$  with a correlation coefficient of 0.999 at a 5% significant level. Using  $\bar{Y}_b = 0.982$  and (11) and (12) in (10), the predicted values of  $K_e$  for various upstream Froude number,  $F_{ru}$ , and discharge ratios,  $\bar{Q}$ , and the measured data agree well. It is noted that  $K_e$  increases with increasing  $F_{ru}$  and  $\bar{Q}$  which indicates that a relatively higher branch discharge results in a relatively lower energy loss coefficient through the division. Using  $\alpha_u / \beta_u = \alpha_b / \beta_u = 1.0$  in (10), it is found that the discrepancies compared to the predicted data using (11) and (12) are small. Although the scatter in  $\alpha_b / \beta_u$  reveals that section GH was not uniform throughout all runs in the experiment, the small discrepancy indicates that the section at six to

ten channel widths downstream of the branch entrance is reasonably considered uniform.

### **5. CONCLUSIONS**

For subcritical, equal-width, open-channel right-angled dividing flow over a horizontal bed, the present study shows that the cosine of the cross-sectional mean flow angle at the branch entrance,  $\cos \delta$ , increases linearly with increasing  $\bar{Q}$ . The experimental data reveals that a smaller divided discharge,  $Q_b$ , from main channel to branch channel would yield a smaller effective width,  $C_e W$ , in the recirculation region. This study also presents the prediction equations for depth ratio and total energy loss coefficient through a division. The predicted  $\bar{P}$  and  $K_e$  are found in agreement with observations of the present study and other studies. It is also observed that  $K_e$  increases with increasing  $F_{ru}$  and  $\bar{Q}$ .

### **APPENDIX I REFERENCES**

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