行政院國家科學委員會專題研究計畫成果報告

μ合成中數值計算之研究

On Numerical Computation in μ Synthesis

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一、中文摘要

本計畫研究μ合成的數值計算問題。我們提出兩種以線性矩陣不等式為基礎的解法。不同於所謂的D-K交替處理法,(D,G)-K交替處理法以及M-K交替處理法，在線性矩陣不等式有解的情況下，第二種方法可同時求得控制器參數及廣義乘子。

關鍵詞：μ合成、線性矩陣不等式、雙線性矩陣不等式

Abstract

This project investigates numerical computation problem involved in μ synthesis. We propose two LMI-based methods to solve the problem. In contrast with the so called D-K iteration, (D,G)-K iteration, and M-K iteration, the controller parameters and the generalized multipliers can be computed simultaneously if the LMIs in the second method are feasible.

Keywords: μ synthesis, linear matrix inequality (LMI), bilinear matrix inequality (BMI)

1. Introduction

In μ synthesis robust controllers can be obtained by minimizing the H_∞ norm of an appropriately scaled transfer matrix with respect to the multiplier and the controller [1-4] (a rigorous proof of certain important identities used in μ synthesis can be found in [5,6]). This problem is not jointly convex in the scaling and the controller, though separately convex in each of these variables. Thus μ synthesis is basically based on iterating between analysis problem during which the controller is fixed and optimal scalings are obtained via convex minimization, and synthesis problem in which the scalings are fixed and improved controllers are found via H_∞ norm minimization. In the phase of computing optimal scalings, the scalings are available via solving a set of linear matrix inequalities at several grid frequencies, and curve fitting is performed to obtain a transfer function representation of them. In many cases, improvement of robustness during the μ synthesis iteration strongly depends on the quality of the curve fits for the scalings and this step has been seriously criticized as the weak link in μ synthesis. To alleviate this difficulty, Safonov and coworkers [7,8] proposed a multiplier approach (Km-synthesis or called M-K iteration) to compute suitable scalings. The H_∞ norm minimization problem is transformed to be an equivalent generalized positive real problem in which the scalings are replaced with a linear parameterization of some fixed order multipliers and thus no curve fitting of the scalings is required. A similar method employing rational functions as a basis was proposed in [9] at about the same time. These two methods are referred to the so-called basis function methods.

In the subsequent development for μ synthesis, Goh et al presented a bilinear matrix inequality (BMI) formulation for μ synthesis [10]. This formulation allows the finite dimensional joint local and global optimization over multiplier and controller spaces, which is claimed to have advantages over (D,G)-K iteration and M-K iteration. However, its computation remains a difficult task. Nevertheless, in Goh's paper it is important to note that the associated formulation for μ synthesis is equivalent to a static output feedback formulation with augmented plants. In view of the research work on the latter issue, new interesting numerical methods for computing static output feedback gains were developed recently [11] (also see the references therein). This motivates the present research of developing new computational algorithms for μ synthesis on the basis of BMI formulation and the method developed in [11].

2. Preliminaries

A. Notation

$F_l(\cdot,\cdot)$ = linear fractional transformation

$$\text{herm}\{X\} = \frac{1}{2}(X + X^*)$$

$$\text{sect}\{X(s)\} = (I - X(s))(I + X(s))^{-1}$$

Given $\gamma > 0$ and P(s), define the sector transformed plant

$$\tilde{P}(s) := \text{sect}\left\{\begin{bmatrix} \gamma I & 0 \\ 0 & I \end{bmatrix} P(s)\right\}$$

$$K(s) \leftrightarrow K := \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix}$$

Define $Q = K\left[I - \begin{bmatrix} 0 & 0 \\ 0 & \tilde{D}_{22} \end{bmatrix} K\right]^{-1}$ and

$$\begin{aligned} \tilde{T}(s) &:= \text{sect}\{\gamma T(s)\} \\ &= \text{sect}\{\gamma F_l(P(s), K(s))\} \\ &= F_l\{\tilde{P}(s), K(s)\} \\ &\leftrightarrow \begin{bmatrix} \tilde{A}_T & \tilde{B}_T \\ \tilde{C}_T & \tilde{D}_T \end{bmatrix} \end{aligned}$$

Then $\tilde{A}_T = R_A + U_A Q V_A$. Define the generalized multiplier M(s) as $M(s) = W\tilde{M}(s)$ where $\tilde{M}(s)$ contains the basis functions chosen. Then $M(s)\tilde{T}(s) = W\tilde{M}(s)\tilde{T}(s)$ and $\tilde{M}(s)\tilde{T}(s) \leftrightarrow R_{MT} + U_{MT} Q V_{MT}$.

B. Goh's results [10]

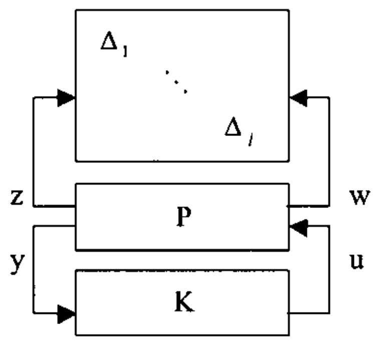

Figure 1

For a fixed $\gamma > 0$, and given the generalized plant P(s), the closed-loop system as shown in Figure 1 is robustly stable against all possible structured uncertainties $\Delta = \text{diag}[\Delta_1, \ldots, \Delta_l] \in RH_\infty$, with $\|\Delta\|_\infty \le \gamma$, if there exist matrices $X_1 = X_1^T$, $X_2 = X_2^T$, $X_3 = X_3^T$, W with appropriate structure commuting with the uncertainties, and Q such that the following inequalities hold.

$$\text{herm}\left\{\begin{bmatrix} X_1 & 0 \\ 0 & W \end{bmatrix}(R_{MT} + U_{MT} Q V_{MT})\right\} > 0, \quad \ldots(1)$$

$$\text{herm}\left\{\begin{bmatrix} X_2 & 0 \\ 0 & W \end{bmatrix} M\right\} > 0, \quad \ldots(2)$$

$\text{herm}\{X_3(R_A + U_A Q V_A)\} < 0, X_3 > 0$. ...(3)

3. Main Results

A. In the follwing we present an algorithm to solve the BMIs in Goh's paper [10], which solve ineq (3), i.e., [10, ineq (30)] first, then ineqs (1) and (2) [10, (28) and (29)] simultaneously.

1. Determine the generalized plant P(s).
2. Given $\gamma > 0$, compute $\tilde{P}$.
3. Assume the controller K(s) has order q, compute R_A, U_A, and V_A.
4. Solve the P-Problem [11] for P, N, and M, with $A=R_A$, $B=U_A$, $C=V_A$, and $P=X_3$. If feasible, then $Q=M^{-1}N$. This solves inequality (3).
5. Choose basis function functions for the generalized multipliers M(s).
6. Compute R_{MT}, U_{MT}, and V_{MT}.
7. Apply the LMI technique again to solve the inequalities (1) and (2) for X_1, X_2, and W.

B. Youla parameterization approach

In this subsection, we will employ Youla parameterization technique and the linear fractional transformation technique to the robust stability condition present in Goh's paper [10], i.e., ineqs (1)-(3), which finally will lead to another set of bilinear matrix inequalities.

Since $\tilde{T}(s) = F_l(\tilde{P}(s), K(s))$, applying Youla parameterization technique yields,

$\tilde{T}(s) = \tilde{T}_1(s) + \tilde{T}_2(s)Q(s)\tilde{T}_3(s)$, where

$Q(s) \in RH_\infty$ and $I + \tilde{D}_{22}Q(\infty)$ invertible. The robust stability condition then becomes that of finding a generalized multipliers M(s) and a stable transfer function matrix Q(s) such that the following conditions

(i) $M(\tilde{T}_1 + \tilde{T}_2 Q \tilde{T}_3)$ is strictly positive real without stability;

(ii) M is strictly positive real without stability;

(iii) $Q \in RH_\infty$.

hold.

Let $M(s) \leftrightarrow \begin{pmatrix} A_M & B_M \\ C_M & D_M \end{pmatrix}$, and

$Q(s) = \frac{1}{s+\alpha} Q_1(s)$, where $\alpha > 0$ and $Q_1(s) \in RH_\infty$. Using linear fractional technique the expression $M(\tilde{T}_1 + \tilde{T}_2 Q(s) \tilde{T}_3)$ can be interpreted as the closed-loop transfer function matrix T_{zw}, i.e., $T_{zw} = F_l(G,K)$, where

$$G(s) = \begin{bmatrix} 0 & I & 0 \\ \tilde{T}_1 & 0 & \tilde{T}_2/(s+\alpha) \\ \tilde{T}_3 & 0 & 0 \end{bmatrix} \leftrightarrow \left[\begin{array}{c|cc} A_{G1} & B_{G1} & B_{G2} \\ \hline C_{G1} & D_{G11} & D_{G12} \\ C_{G2} & D_{G21} & D_{G22} \end{array}\right]$$

and

$$K(s) = \begin{bmatrix} M(s) & 0 \\ 0 & Q_1(s) \end{bmatrix} \leftrightarrow Kss := \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix}$$

$$= \left[\begin{array}{cc|cc} A_M & 0 & B_M & 0 \\ 0 & A_{Q1} & 0 & B_{Q1} \\ \hline C_M & 0 & D_M & 0 \\ 0 & C_{Q1} & 0 & D_{Q1} \end{array}\right]$$

In view of the particular structure associated with the generalize multiplier M(s) and Kss, there exist permutation matrices E_M and E such that

$$Qss := E_M E_1 K_{SS} E_1^T E_M^T = \left[\begin{array}{cccc|c} M_1 & & & & \\ & M_2 & & & 0 \\ & & \ddots & & \\ & & & M_l & \\ \hline & & 0 & & \begin{array}{cc} A_{Q1} & B_{Q1} \\ C_{Q1} & D_{Q1} \end{array} \end{array}\right]$$

where $M_i = \begin{bmatrix} A_{Mi} & B_{Mi} \\ C_{Mi} & D_{Mi} \end{bmatrix}$, I=1,..,$l$.

Then $F_l(G,K) \leftrightarrow R + UQ_{SS}V$, where $E := E_M E_1$,

$$R = \begin{bmatrix} O_q & 0 & 0 \\ 0 & A_G & B_{G1} \\ 0 & C_{G1} & D_{G11} \end{bmatrix}, \quad U = \begin{bmatrix} I_q & 0 \\ 0 & B_{G2} \\ 0 & D_{G12} \end{bmatrix} E^T$$

and

$$V = E^T \begin{bmatrix} I_q & 0 & 0 \\ 0 & C_{G2} & D_{G21} \end{bmatrix}.$$

Note that q is the order of the "controller" K(s), which contains all the controller parameters and the generalized multipliers. Also it is readily checked that

$$\begin{bmatrix} A_M & B_M \\ C_M & D_M \end{bmatrix} = U_M Q_{SS} U_M^T,$$

$A_{Q1}=U_{Q1}Q_{SS}U_{Q1}$,

where

$$U_M = [I \quad 0]E_M^T,$$

$$U_{QI} = [0 \quad (I \quad 0)].$$

With the above notation, applying the generalized positive real lemma, we get the state-space counterpart of the underlying robust stability condition, which reads : the closed – loop system in Figure 1 is robustly stable if there exist matrices $X_1=X_1^T$, $X_2=X_2^T$, $X_3=X_3^T$, and Q_{SS} of the prescribed structure such that

$$\text{herm}\left\{\begin{bmatrix} X_1 & 0 \\ 0 & W \end{bmatrix}(R + UQ_{SS}V)\right\} > 0,$$

$$\text{herm}\left\{\begin{bmatrix} X_2 & 0 \\ 0 & I \end{bmatrix}(U_M Q_{SS} U_M^T)\right\} > 0,$$

$$-\text{herm}\{(X_3)(U_{QI}Q_{SS}U_{QI}^T)\} > 0, X_3 > 0.$$

Applying the LMI technique [11], the closed-loop system is robustly stable if there exist matrices

$$X = X^T = \left[\begin{array}{cc|cc} X_1 & 0 & 0 & 0 \\ 0 & X_2 & 0 & 0 \\ \hline 0 & 0 & X_3 & 0 \\ 0 & 0 & 0 & X_4 \end{array}\right]$$

and

$$N = \left[\begin{array}{c|c} N_{11} & 0 \\ \hline 0 & N_{22} \end{array}\right], \text{ where}$$

X_1 and N_{11} are of the same structure as $\begin{bmatrix} M_1 & & 0 \\ & \ddots & \\ 0 & & M_l \end{bmatrix}$, such that the following LMI conditions hold.

$E_2XE_2^T>0$,

$$\text{herm}\ \{U_{Q1}NU_{Q1}^T\} < 0,$$

$$\text{herm}\ \{U_M N U_M^T\} > 0,$$

$$\text{herm}\left\{\begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} R + UNV\right\} > 0,$$

where $E_2=[0 \quad I \quad 0 \quad 0]$. If feasible, the realization of the controller parameter $Q_1(s)$ can be computed via the formula

$$\begin{bmatrix} A_{Q_1} & B_{Q_1} \\ C_{Q_1} & D_{Q_1} \end{bmatrix} = \begin{bmatrix} X_2^{-1} & 0 \\ 0 & X_3^{-1} \end{bmatrix} N_{22},$$ which can be employed to find the robust controller.

4. Conclusion

Two LMI-based μ synthesis methods were proposed. The most interesting feature is that the controller parameters and the generalized multipliers can be computed simultaneously if the LMIs in the second method are feasible.

5. References

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