行政院國家科學委員會專題研究計畫 成果報告

通道間干擾分析與新的結合同步與通道估計於具都卜勒擴展效應之正交分頻多工系統
研究成果報告(精簡版)

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 97-2221-E-032-001-
執 行 期 間 ：97 年 08 月 01 日至 98 年 07 月 31 日
執 行 單 位 ：淡江大學電機工程學系

計畫主持人：嚴雨田
共同主持人：劉鴻裕
計畫參與人員：碩士班研究生-兼任助理人員：吳振明
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報 告 附 件 ：出席國際會議研究心得報告及發表論文

處 理 方 式 ：本計畫可公開查詢

中 華 民 國 98 年 08 月 28 日

行政院國家科學委員會補助專題研究計畫 ■ 成 果 報 告 □期中進度報告

通道間干擾分析與新的結合同步與通道估計於具都卜勒擴展效應之正交分頻多工系統

計畫類別：■ 個別型計畫　　□ 整合型計畫
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成果報告類型(依經費核定清單規定繳交)：■精簡報告　□完整報告

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執行單位：淡江大學電機工程學系

中　華　民　國　　98 年　　8 月　3　日

中英文摘要及關鍵詞

摘要

正交分頻多工的系統於接收端時域取樣值，對於先導訊號運用最大似然估計法，對於頻率偏移做估計，通道為頻率選擇特性。最大似然估計法之計算複雜度為指數性，所以我們提出簡化最大似然估計法，使在計算複雜度與估計精確度能兼顧。所提出之方法，隨著訊號雜訊比增加，其均方誤差趨近於 Cramer-Rao 界限，另外，接收端時域取樣位置之選擇，影響到收斂範圍與收斂準確度。

關鍵詞: 最大似然估計、頻率偏移、Cramer-Rao 界限

Abstract

By using a pilot sample from a selected time-slot in a time-domain orthogonal frequency division multiplexing (OFDM) block, we present a simple maximum-likelihood (ML) scheme for frequency tracking in OFDM systems over frequency-selective channels. The frequency offset estimator thus obtained is found to become unbiased and its mean square error (MSE) will approach the Cramer-Rao bound (CRB) as signal-to-noise ratio (SNR) is increased. Moreover, the selection of time slot can determine a trade-off between tracking accuracy and tracking range.

Index Terms—Maximum-likelihood (ML) estimation, frequency offset, Cramer-Rao bound (CRB)

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A Simple Time-Slot ML-based Frequency Tracking Scheme for OFDM Systems

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Abstract—**By using a pilot sample from a selected time-slot in a time-domain orthogonal frequency division multiplexing (OFDM) block, we present a simple maximum-likelihood (ML) scheme for frequency tracking in OFDM systems over frequency-selective channels. The frequency offset estimator thus obtained is found to become unbiased and its mean square error (MSE) will approach the Cramer-Rao bound (CRB) as signal-to-noise ratio (SNR) is increased. Moreover, the selection of time slot can determine a trade-off between tracking accuracy and tracking range.**

***Index Terms*—Maximum-likelihood (ML) estimation, frequency offset, Cramer-Rao bound (CRB)**

I. Introduction

IT is well known that, in orthogonal frequency division multiplexing (OFDM) systems, timing and frequency synchronization as well as channel estimation must be performed for accurate data detection. In general, the synchronization process can be divided into two stages, viz., coarse acquisition and fine tracking [1], [2]. Coarse acquisition can be achieved by correlating pilot data or redundant cyclic prefix data in either time or frequency domain [3]-[5]. After acquisition, usually the frequency and timing offset will become quite small. However, due to time-varying effect of the channel, small residual synchronization error will still exist and thus needs be continuously tracked [2]. In [6], a least squares-based residual synchronization technique is proposed using one frequency-domain OFDM block of pilot data. In [7], a maximum-likelihood (ML)–based fine frequency synchronization scheme is presented using one time-domain OFDM block consisting of both signal and pilot data. In this paper, we propose a much simpler ML-based fine frequency synchronization scheme using only one time slot pilot sample selected from a time-domain OFDM block. The remaining time slots in that block all contain signal data. Thus, very high pilot usage efficiency is achieved. Further, the selection of the pilot slot can provide trade-off between the tracking accuracy and the tracking range. The mean square error (MSE) of our ML estimator for fine frequency offset is shown to approach the Cramer-Rao bound (CRB) at high signal-to-noise ratios (SNRs). We shall take frequency-selective channels and assume that, aside from coarse frequency synchronization, the timing error and channel estimation have been completed.

This paper is organized as follows: Section II describes the simple time-slot signal model in OFDM over frequency-selective channels. Section III presents the simple ML-based scheme for fine frequency offset estimation using one time slot of pilot sample along with theoretical analysis including CRB derivation. Then, Section IV shows numerical results. Finally, conclusion is given in Section V.

II. Signal Model

Consider OFDM transmission in a frequency-selective channel. Let one transmitted OFDM block contains N samples. Assume timing synchronization and channel estimation have been completed. Then, discarding the cyclic prefix, the demodulated received baseband data sample at the nth time slot of an OFDM block can be expressed as

$$\begin{aligned} r_n &= e^{j2\pi n\delta/N}\frac{1}{N}\sum_{k=0}^{N-1}H_k X_k e^{j2\pi nk/N} + w_n, \\ &= y_n e^{j2\pi n\delta/N} + w_n, \qquad n = 0,1,\ldots,N-1, \end{aligned} \tag{1}$$

where H_k and X_k are respectively the channel frequency response and the frequency-domain transmitted data symbol of the kth subcarrier, w_n and $y_n = \frac{1}{N}\sum_{k=0}^{N-1}H_k X_k e^{j2\pi nk/N}$ are respectively the additive white Gaussian noise (AWGN) sample and the noise-free received data sample at the nth time slot, and δ is the frequency offset normalized to subcarrier spacing. We note here that, for mobile wireless communications, the frequency offset may include both carrier frequency offset and Doppler spread. We also assume that an initial frequency acquisition has been performed so that the frequency offset will not exceed half the subcarrier spacing [3], [6]. In other words, $|\delta| < 1/2$.

III. ML Estimation of Frequency Offset Using a Single Time Slot Pilot Sample

Of the N-samples in the time-domain OFDM block, assume the pth time slot is selected for the pilot sample r_p, the remaining $N-1$ time slots are signal data. We wish to use the pilot sample r_p to estimate the fine frequency offset δ based on the ML criterion. The baseband noise sample w_p is a

complex Gaussian random variable with zero mean and variance σ_w^2. Then, the log-likelihood function for r_p is

$$\Lambda = -\ln \pi\sigma_w^2 - \frac{1}{\sigma_w^2} | r_p - y_p e^{j2\pi p\delta/N} |^2. \quad (2)$$

To find the ML estimator $\hat{\delta}$, we differentiate (2) with respect to δ and then set the result to zero. We readily get

$$e^{j2\pi p\hat{\delta}/N} = \frac{r_p}{y_p}. \quad (3)$$

Using both the real and imaginary parts of (3), we can obtain a solution for $\hat{\delta}$ as

$$\hat{\delta} = \frac{N}{2\pi p} \tan^{-1} \frac{\mathrm{Im}(r_p / y_p)}{\mathrm{Re}(r_p / y_p)}, \quad (4)$$

where Re and Im respectively denote real part and imaginary part. Equation (4) means $r_p / y_p = | r_p / y_p | e^{j2\pi p\hat{\delta}/N}$. Multiplying by $e^{-j2\pi p\delta/N}$ gives $(r_p / y_p)e^{-j2\pi p\delta/N} =$. $| r_p / y_p | e^{j2\pi p(\hat{\delta}-\delta)/N}$. This again means

$$\tan \frac{2\pi p(\hat{\delta}-\delta)}{N} = \frac{\mathrm{Im}[(r_p / y_p)e^{-j2\pi p\delta/N}]}{\mathrm{Re}[(r_p / y_p)e^{-j2\pi p\delta/N}]}. \quad (5)$$

We take small offset errors and large SNR $| y_p |^2 / \sigma_w^2$. Then using $\tan^{-1} x \approx x$ for small *x*, (5) can be approximated as

$$\begin{aligned}
\hat{\delta} - \delta &\approx \frac{N}{2\pi p} \frac{\mathrm{Im}[(r_p / y_p)e^{-j2\pi p\delta/N}]}{\mathrm{Re}[(r_p / y_p)e^{-j2\pi p\delta/N}]} \\
&= \frac{N}{2\pi p} \frac{\mathrm{Im}[\frac{r_p y_p^*}{| y_p |^2} e^{-j2\pi p\delta/N}]}{\mathrm{Re}[\frac{r_p y_p^*}{| y_p |^2} e^{-j2\pi p\delta/N}]} \\
&= \frac{N}{2\pi n} \frac{\mathrm{Im}[r_p y_p^* e^{-j2\pi p\delta/N}]}{\mathrm{Re}[r_p y_p^* e^{-j2\pi p\delta/N}]} \\
&= \frac{N}{2\pi p} \frac{\mathrm{Im}[| y_p |^2 + w_p y_p^* e^{-j2\pi p\delta/N}]}{\mathrm{Re}[| y_p |^2 + w_p y_p^* e^{-j2\pi p\delta/N}]} \\
&\approx \frac{N}{2\pi p} \frac{\mathrm{Im}[w_p y_p^* e^{-j2\pi p\delta/N}]}{| y_p |^2} \\
&= \frac{N}{j4\pi p} \frac{w_p y_p^* e^{-j2\pi p\delta/N} - w_p^* y e^{j2\pi p\delta/N}}{| y_p |^2}. \quad (6)
\end{aligned}$$

where $\mathrm{Re}[| y_p |^2 + w_p y_p^* e^{-j2\pi p\delta/N}] \approx | y_p |^2$ for large SNR has been used. Taking the expectation, we find the estimator expectation to be $E[\hat{\delta}] \approx \delta$. Therefore, for small offset errors at large SNRs, the estimator (4) is unbiased. Next, squaring (6) and taking the expectation, we find the MSE of our estimator as

$$\begin{aligned}
E[(\hat{\delta}-\delta)^2] &= E[(\hat{\delta}-\delta)(\hat{\delta}-\delta)^*] \\
&\approx \frac{N^2}{16\pi^2 p^2 | y_p |^4} E[(w_p y_p^* e^{-j2\pi p\delta/N} - w_p^* y e^{j2\pi p\delta/N}) \times \\
&\quad (w_p^* y_p e^{j2\pi p\delta/N} - w_p y_p^* e^{-j2\pi p\delta/N})] \\
&= \frac{N^2}{16\pi^2 p^2 | y_p |^4} (\sigma_w^2 | y_p |^2 + \sigma_w^2 | y_p |^2) \\
&= \frac{N^2 \sigma_w^2}{8\pi^2 p^2 | y_p |^2}, \quad (7)
\end{aligned}$$

The Fisher information matrix (a 1×1 matrix for the current simple case) can readily be computed from (2) as [8]

$$\begin{aligned}
J &= \frac{2}{\sigma_w^2} \cdot \frac{\partial y_p^* e^{-j2\pi p\delta/N}}{\partial\delta} \cdot \frac{\partial y_p e^{j2\pi p\delta}}{\partial\delta} \\
&= \frac{2}{\sigma_w^2} \frac{(2\pi p | y_p |)^2}{N^2}, \quad (8)
\end{aligned}$$

whence, the CRB can be calculated as

$$\mathrm{CRB}(\delta) = J^{-1} = \frac{N^2 \sigma_w^2}{8\pi^2 p^2 | y_p |^2} = \frac{N^2}{8\pi^2 p^2 \mathrm{SNR}_p}, \quad (9)$$

where $\mathrm{SNR}_p = | y_p |^2 / \sigma_w^2$. Equation (9) is exactly identical to (7). To summarize, the estimator of (4) is unbiased and its MSE approaches CRB at large SNRs.

In view of (4), the range of estimation is $\pm N/2p$ (The arctan here should cover the range $\pm\pi$). Thus, the smaller the *p*, the larger the range. On the other hand, (7) indicates that the larger the *p*, the more accurate the estimate. Therefore, the selection of time slot can provide a trade-off between estimation range and estimation accuracy.

IV. Numerical Results

In our simulations, we choose the length of one OFDM block or the number of subcarriers to be $N = 64$. We take a frequency-selective channel with an exponential power profile and a dispersion length of 16 sample units. Since after coarse frequency synchronization, the residual frequency offset will be within half the subcarrier spacing, we shall choose $p = 63$ for maximum estimation accuracy while maintaining the estimation range around ± 0.5. The performance of our frequency estimator is shown in Fig. 1 and Fig. 2 respectively for a small frequency offset ($\delta = 0.05$) and a large frequency offset ($\delta = 0.5$). In both figures, the top plot of (a) gives the estimator variance (MSE and CRB) vs. SNR_p curves and the bottom plot of (b) presents the estimator bias ($E[\hat{\delta}] - \delta$) vs. SNR_p curve. Averaging over 1,000 simulation runs is performed to produce all results. Both figures show that the

estimator gradually becomes unbiased and its MSE approaches the CRB as SNR is increased. In fact, the estimator MSE almost coincides the CRB at large SNRs. This proves that our estimator indeed performs very satisfactorily.

V. CONCLUSION

By using only a pilot sample in a single time slot selected from a time-domain OFDM block, the proposed simple ML-based frequency offset estimator for OFDM in frequency-selective channels is proved to provide satisfactory estimation accuracies for frequency tracking.

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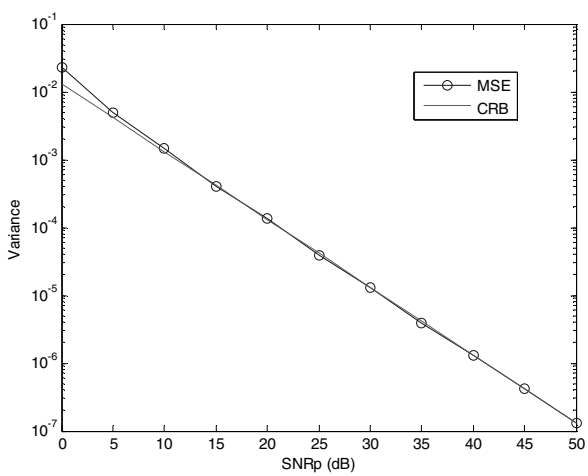
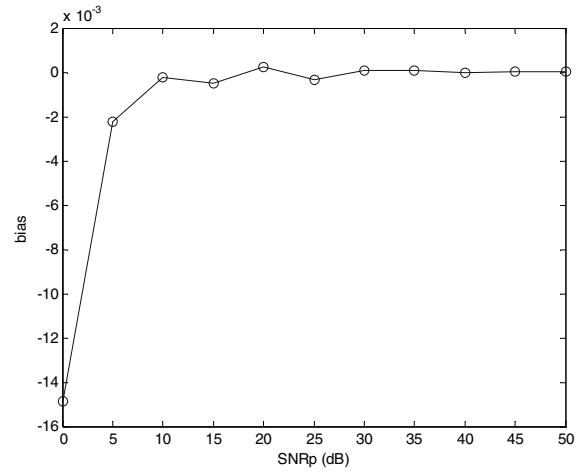
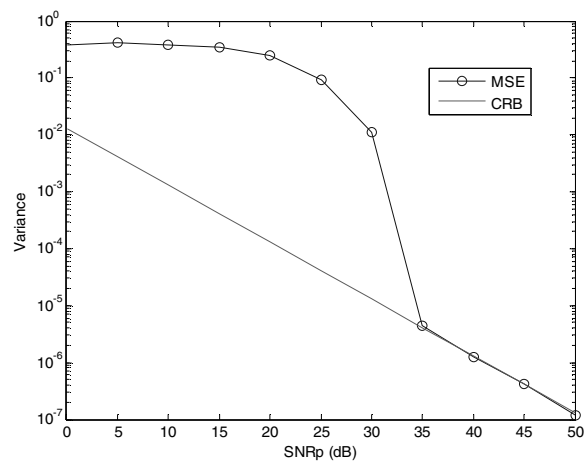


(a)

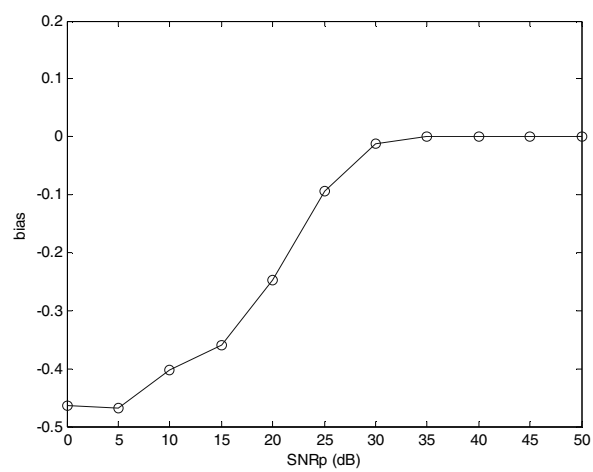


(b)

Fig. 1 Estimator performance for $\delta = 0.05$.
(a) Estimator variances vs. SNR.
(b) Estimator bias vs. SNR.



(a)



(b)

Fig. 2 Estimator performance for $\delta = 0.5$.
(a) Estimator variances vs. SNR.
(b) Estimator bias vs. SNR.

參考文獻

本計畫執行期間計有一篇相關論文刊登於國際知名 EI 期刊[1]，另一篇在審稿中[3]。並積極參與國際性與全國性學術研討會[2][4]以接軌於國際，充分與國內外學者交流。

[1] Rainfield Y. Yen, “Unbised MMSE vs. Biased MMSE Equalizers," *Tamkang Journal of Science and Engineering*, vol. 12, no. 1, pp. 45-56, Mar. 2009. **(EI)**

[2] Rainfield Y. Yen, T. T. Cheng, and C. M. Wu, “A Simple Time-Slot ML-based Frequency Tracking Scheme for OFDM Systems,” *The 2009 IEEE International Conference on Networking, Sensing and Control*, Okayama, Japan, March 26-29, 2009, pp.525-527.

[3] Hong-Yu Liu and Rainfield Y. Yen, “Efficient Adaptive Iteration Algorithm of Combined Fine Frequency Synchronization and Channel Estimation for OFDM Systems," submitted to *IEEE Transactions on Vehicular Technology*

[4] Chaoqun Jiao, Yi Sun, Rainfield Y. Yen, Hong-Yu Liu, Ta-Chou Lin, and T. T. Cheng, “An Efficient Iterative Algorithm for Joint Frequency Synchronization and Channel Estimation in OFDM Systems,” accepted by *IEEE MAPE 2009*.**(EI)**

計畫成果自評

本計畫執行的成果優良，一如 ”參考文獻”中所示，論文[1]已刊登於國際知名 EI 期刊 Tamkang Journal of Science and Engineering，並多次參與國內外知名之學術研討會[2][4]，兩次研討會都是 IEEE 所主辦的，發表之文章將收錄於ＩＥＥＥ電子資料庫 IEL 中。其中於上海舉辦之 MAPE 2009 研討會[4]為北京交通大學所主辦之研討會，文集並收錄於 EI，為高水泥準之研討會，集合國際電信領域之專家、學者與先進，共同分享最新研究之成果，每年之參與人數與發表論文數量都高度成長，逐漸成為國際電子領域之重要學術會議之一。發表於日本的 ICNSC 2009 論文[2]，在會場報告時，受到與會專家之矚目，引來相當多的討論與建議。許多新觀念、新理論與新技術被提出，以符合高速、低複雜度的通訊系統設計。

本研究計畫成果不僅在於論文之發表有優良之表現，更獲國際科學與科技組織 International Association of Science and Technology for Development (IASTED)之邀，擔任 CSNA(Communication Systems, Networks and Applications) 2009 的國際議程委員，該會議為 10 月 12-14 日於北京舉行，由北京清華大學所主辦。也就是本計畫之實施，促成與對岸名校之交流，分別是北京交通大學與北京清華大學，也就是計畫成果顯然超越預期。

特別一提，發表於 MAPE 2009 的論文，得到教育部與北京交通大學的補助而順利出國參與該盛會。論文與北京交通大學之研究團隊合作，直接相互討論與請詣，讓我們的計畫執行獲得相當大的助力，具體展現於報告中學理上的新發現，此外，我們在發展與驗證理論的過程，亦開發出整套模擬環境的系統雛型，未來有意完成方便與具親和力的介面，使研究成果能普及且益於產學界之研發與教學。

可供推廣之研發成果資料表

□ 可申請專利　　■ 可技術移轉　　日期：98 年 8 月 3 日

國科會補助計畫	計畫名稱:通道間干擾分析與新的結合同步與通道估計於具都卜勒擴展效應之正交分頻多工系統 計畫主持人：嚴雨田 教授 計畫編號：NSC　97－2221－E－032－001－　　學門領域：電信
技術/創作名稱	Efficient Adaptive Iteration Algorithm of Combined Fine Frequency Synchronization and Channel Estimation for OFDM Systems
發明人/創作人	嚴雨田教授
技術說明	中文： 領行訊號輔助之高效率最大似然估計於正交分頻多工系統之同步估測被提出，估計對像為頻率偏移與通道同時。運用加權之最小平方誤差，將每個時域估計值組合，並配合最大似然之通道估計值，以疊代的方式逼近最佳解，不需任何近似就可以得到估計值，結果證實非常接近CRB，並有非常大的收斂區間。
	英文： An efficient pilot-aided maximum-likelihood (ML)-based algorithm is proposed for combined frequency tracking and channel estimation for orthogonal frequency division multiplexing (OFDM) systems. Our contribution is the novel idea of applying a linear minimum mean square error (LMMSE) combiner to optimally combine frequency offset estimators obtained from time slot samples over the time-domain OFDM block. Then, by alternatively updating the LMMSE frequency estimator and the ML channel estimator through adaptive iterations, we successfully avoid the use of a usually complex log-likelihood function while still achieve good convergence in obtaining the solution for the combined ML estimators. No approximation or simplification is needed to derive the combined estimation algorithm. Moreover, our ML estimators have variances or mean square errors (MSEs) tightly close to CRBs with a wide tracking range.
可利用之產業及可開發之產品	無線通訊
技術特點	快速的系統驗證平台，並運用新的設計理論以開發更適於實際環境的無線通訊技術
推廣及運用的價值	系統研發與設計驗證平台

※　1. 每項研發成果請填寫一式二份，一份隨成果報告送繳本會，一份送　貴單位研發成果推廣單位（如技術移轉中心）。

※　**2. 本項研發成果若尚未申請專利，請勿揭露可申請專利之主要內容。**

※　3. 本表若不敷使用，請自行影印使用。

出席國際學術會議心得報告

計畫編號	NSC 97-2221-E-032-001 -
計畫名稱	通道間干擾分析與新的結合同步與通道估計於具都卜勒擴展效應之正交分頻多工系統
出國人員姓名 服務機關及職稱	嚴雨田 教授 淡江大學電機系
會議時間地點	98 年 3 月 26 日起至 98 年 3 月 29 日止/ Okayama City, Japan
會議名稱	The 2009 IEEE International Conference on Networking, Sensing and Control (ICNSC 2009)
發表論文題目	A Simple Time-Slot ML-based Frequency Tracking Scheme for OFDM Systems

一、參加會議經過

The 2009 IEEE International Conference on Networking, Sensing and Control was held in Okayama, Japan. The main theme of the conference was advanced technologies for safety and functional maintenance. The area of safety and functional maintenance was a fusion of a number of research areas in networking, sensing, and control. However, the real challenge was to obtain advanced control technology for safety and management technology and to construct an information system to share information on safety technology and on investigated accidents. The following three tasks were required to address new problems of this challenging and promising area, 1) to construct an integrated management system for safety; 2) to obtain an advanced technology for distributed control, distributed operation and logistics management to maintain functions isolated systems in disaster; 3) to construct operation supporting and training systems to keep safety, to prevent human error and to function as a co-operator. This conference provided a remarkable opportunity for the academic and industrial community to address new challenges and share solutions, and discussed future research directions. It featured plenary speeches, industrial panel sessions, funding agency panel sessions, interactive sessions, and invited/special sessions. Contributions were from academia, industry, and management agencies.

Okayama city is a midsized city with population of 700,000 in Japan and is good to enjoy both convenience of city life and peacefulness of rural area. The city is at a crossroads in the western part of Japan, 1 hour by super express train from Osaka or Hiroshima, 1 hour air flight from Tokyo, 3.5 hours by super express train from Tokyo and also has daily flight from Shanghai, China and Seoul, Korea.

Besides the purpose of exchanging scientific ideas and results, ICNSC 2009 was a useful forum to help establishing and strengthening a network of scientists, researchers and engineers, to promote between research and industrial bodies in different countries, and enables to initiate possible future collaborations.

二、與會心得

By using a pilot sample from a selected time-slot in a time-domain orthogonal frequency division multiplexing (OFDM) block, we present a simple maximum-likelihood (ML) scheme for

frequency tracking in OFDM systems over frequency-selective channels. The frequency offset estimator thus obtained is found to become unbiased and its mean square error (MSE) will approach the Cramer-Rao bound (CRB) as signal-to-noise ratio (SNR) is increased. Moreover, the selection of time slot can determine a trade-off between tracking accuracy and tracking range.

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A Simple Time-Slot ML-based Frequency Tracking Scheme for OFDM Systems

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Abstract—**By using a pilot sample from a selected time-slot in a time-domain orthogonal frequency division multiplexing (OFDM) block, we present a simple maximum-likelihood (ML) scheme for frequency tracking in OFDM systems over frequency-selective channels. The frequency offset estimator thus obtained is found to become unbiased and its mean square error (MSE) will approach the Cramer-Rao bound (CRB) as signal-to-noise ratio (SNR) is increased. Moreover, the selection of time slot can determine a trade-off between tracking accuracy and tracking range.**

***Index Terms*—Maximum-likelihood (ML) estimation, frequency offset, Cramer-Rao bound (CRB)**

I. Introduction

IT is well known that, in orthogonal frequency division multiplexing (OFDM) systems, timing and frequency synchronization as well as channel estimation must be performed for accurate data detection. In general, the synchronization process can be divided into two stages, viz., coarse acquisition and fine tracking [1], [2]. Coarse acquisition can be achieved by correlating pilot data or redundant cyclic prefix data in either time or frequency domain [3]-[5]. After acquisition, usually the frequency and timing offset will become quite small. However, due to time-varying effect of the channel, small residual synchronization error will still exist and thus needs be continuously tracked [2]. In [6], a least squares-based residual synchronization technique is proposed using one frequency-domain OFDM block of pilot data. In [7], a maximum-likelihood (ML)–based fine frequency synchronization scheme is presented using one time-domain OFDM block consisting of both signal and pilot data. In this paper, we propose a much simpler ML-based fine frequency synchronization scheme using only one time slot pilot sample selected from a time-domain OFDM block. The remaining time slots in that block all contain signal data. Thus, very high pilot usage efficiency is achieved. Further, the selection of the pilot slot can provide trade-off between the tracking accuracy and the tracking range. The mean square error (MSE) of our ML estimator for fine frequency offset is shown to approach the Cramer-Rao bound (CRB) at high signal-to-noise ratios (SNRs). We shall take frequency-selective channels and assume that, aside from coarse frequency synchronization, the timing error and channel estimation have been completed.

This paper is organized as follows: Section II describes the simple time-slot signal model in OFDM over frequency-selective channels. Section III presents the simple ML-based scheme for fine frequency offset estimation using one time slot of pilot sample along with theoretical analysis including CRB derivation. Then, Section IV shows numerical results. Finally, conclusion is given in Section V.

II. Signal Model

Consider OFDM transmission in a frequency-selective channel. Let one transmitted OFDM block contains N samples. Assume timing synchronization and channel estimation have been completed. Then, discarding the cyclic prefix, the demodulated received baseband data sample at the nth time slot of an OFDM block can be expressed as

$$\begin{aligned} r_n &= e^{j2\pi n\delta/N}\frac{1}{N}\sum_{k=0}^{N-1}H_k X_k e^{j2\pi nk/N} + w_n, \\ &= y_n e^{j2\pi n\delta/N} + w_n, \qquad n = 0,1,\ldots,N-1, \end{aligned} \tag{1}$$

where H_k and X_k are respectively the channel frequency response and the frequency-domain transmitted data symbol of the kth subcarrier, w_n and $y_n = \frac{1}{N}\sum_{k=0}^{N-1}H_k X_k e^{j2\pi nk/N}$ are respectively the additive white Gaussian noise (AWGN) sample and the noise-free received data sample at the nth time slot, and δ is the frequency offset normalized to subcarrier spacing. We note here that, for mobile wireless communications, the frequency offset may include both carrier frequency offset and Doppler spread. We also assume that an initial frequency acquisition has been performed so that the frequency offset will not exceed half the subcarrier spacing [3], [6]. In other words, $|\delta| < 1/2$.

III. ML Estimation of Frequency Offset Using a Single Time Slot Pilot Sample

Of the N-samples in the time-domain OFDM block, assume the pth time slot is selected for the pilot sample r_p, the remaining $N-1$ time slots are signal data. We wish to use the pilot sample r_p to estimate the fine frequency offset δ based on the ML criterion. The baseband noise sample w_p is a

complex Gaussian random variable with zero mean and variance σ_w^2. Then, the log-likelihood function for r_p is

$$\Lambda = -\ln \pi\sigma_w^2 - \frac{1}{\sigma_w^2} | r_p - y_p e^{j2\pi p\delta/N} |^2. \quad (2)$$

To find the ML estimator $\hat{\delta}$, we differentiate (2) with respect to δ and then set the result to zero. We readily get

$$e^{j2\pi p\hat{\delta}/N} = \frac{r_p}{y_p}. \quad (3)$$

Using both the real and imaginary parts of (3), we can obtain a solution for $\hat{\delta}$ as

$$\hat{\delta} = \frac{N}{2\pi p} \tan^{-1} \frac{\mathrm{Im}(r_p / y_p)}{\mathrm{Re}(r_p / y_p)}, \quad (4)$$

where Re and Im respectively denote real part and imaginary part. Equation (4) means $r_p / y_p = | r_p / y_p | e^{j2\pi p\hat{\delta}/N}$. Multiplying by $e^{-j2\pi p\delta/N}$ gives $(r_p / y_p)e^{-j2\pi p\delta/N} =$. $| r_p / y_p | e^{j2\pi p(\hat{\delta}-\delta)/N}$. This again means

$$\tan \frac{2\pi p(\hat{\delta}-\delta)}{N} = \frac{\mathrm{Im}[(r_p / y_p)e^{-j2\pi p\delta/N}]}{\mathrm{Re}[(r_p / y_p)e^{-j2\pi p\delta/N}]}. \quad (5)$$

We take small offset errors and large SNR $| y_p |^2 / \sigma_w^2$. Then using $\tan^{-1} x \approx x$ for small *x*, (5) can be approximated as

$$\begin{aligned}
\hat{\delta} - \delta &\approx \frac{N}{2\pi p} \frac{\mathrm{Im}[(r_p / y_p)e^{-j2\pi p\delta/N}]}{\mathrm{Re}[(r_p / y_p)e^{-j2\pi p\delta/N}]} \\
&= \frac{N}{2\pi p} \frac{\mathrm{Im}[\frac{r_p y_p^*}{| y_p |^2} e^{-j2\pi p\delta/N}]}{\mathrm{Re}[\frac{r_p y_p^*}{| y_p |^2} e^{-j2\pi p\delta/N}]} \\
&= \frac{N}{2\pi n} \frac{\mathrm{Im}[r_p y_p^* e^{-j2\pi p\delta/N}]}{\mathrm{Re}[r_p y_p^* e^{-j2\pi p\delta/N}]} \\
&= \frac{N}{2\pi p} \frac{\mathrm{Im}[| y_p |^2 + w_p y_p^* e^{-j2\pi p\delta/N}]}{\mathrm{Re}[| y_p |^2 + w_p y_p^* e^{-j2\pi p\delta/N}]} \\
&\approx \frac{N}{2\pi p} \frac{\mathrm{Im}[w_p y_p^* e^{-j2\pi p\delta/N}]}{| y_p |^2} \\
&= \frac{N}{j4\pi p} \frac{w_p y_p^* e^{-j2\pi p\delta/N} - w_p^* y e^{j2\pi p\delta/N}}{| y_p |^2}. \quad (6)
\end{aligned}$$

where $\mathrm{Re}[| y_p |^2 + w_p y_p^* e^{-j2\pi p\delta/N}] \approx | y_p |^2$ for large SNR has been used. Taking the expectation, we find the estimator expectation to be $E[\hat{\delta}] \approx \delta$. Therefore, for small offset errors at large SNRs, the estimator (4) is unbiased. Next, squaring (6) and taking the expectation, we find the MSE of our estimator as

$$\begin{aligned}
E[(\hat{\delta}-\delta)^2] &= E[(\hat{\delta}-\delta)(\hat{\delta}-\delta)^*] \\
&\approx \frac{N^2}{16\pi^2 p^2 | y_p |^4} E[(w_p y_p^* e^{-j2\pi p\delta/N} - w_p^* y e^{j2\pi p\delta/N}) \times \\
&\quad (w_p^* y_p e^{j2\pi p\delta/N} - w_p y_p^* e^{-j2\pi p\delta/N})] \\
&= \frac{N^2}{16\pi^2 p^2 | y_p |^4} (\sigma_w^2 | y_p |^2 + \sigma_w^2 | y_p |^2) \\
&= \frac{N^2 \sigma_w^2}{8\pi^2 p^2 | y_p |^2}, \quad (7)
\end{aligned}$$

The Fisher information matrix (a 1×1 matrix for the current simple case) can readily be computed from (2) as [8]

$$\begin{aligned}
J &= \frac{2}{\sigma_w^2} \cdot \frac{\partial y_p^* e^{-j2\pi p\delta/N}}{\partial\delta} \cdot \frac{\partial y_p e^{j2\pi p\delta}}{\partial\delta} \\
&= \frac{2}{\sigma_w^2} \frac{(2\pi p | y_p |)^2}{N^2}, \quad (8)
\end{aligned}$$

whence, the CRB can be calculated as

$$\mathrm{CRB}(\delta) = J^{-1} = \frac{N^2 \sigma_w^2}{8\pi^2 p^2 | y_p |^2} = \frac{N^2}{8\pi^2 p^2 \mathrm{SNR}_p}, \quad (9)$$

where $\mathrm{SNR}_p = | y_p |^2 / \sigma_w^2$. Equation (9) is exactly identical to (7). To summarize, the estimator of (4) is unbiased and its MSE approaches CRB at large SNRs.

In view of (4), the range of estimation is $\pm N/2p$ (The arctan here should cover the range $\pm\pi$). Thus, the smaller the *p*, the larger the range. On the other hand, (7) indicates that the larger the *p*, the more accurate the estimate. Therefore, the selection of time slot can provide a trade-off between estimation range and estimation accuracy.

IV. Numerical Results

In our simulations, we choose the length of one OFDM block or the number of subcarriers to be $N = 64$. We take a frequency-selective channel with an exponential power profile and a dispersion length of 16 sample units. Since after coarse frequency synchronization, the residual frequency offset will be within half the subcarrier spacing, we shall choose $p = 63$ for maximum estimation accuracy while maintaining the estimation range around ± 0.5. The performance of our frequency estimator is shown in Fig. 1 and Fig. 2 respectively for a small frequency offset ($\delta = 0.05$) and a large frequency offset ($\delta = 0.5$). In both figures, the top plot of (a) gives the estimator variance (MSE and CRB) vs. SNR_p curves and the bottom plot of (b) presents the estimator bias ($E[\hat{\delta}] - \delta$) vs. SNR_p curve. Averaging over 1,000 simulation runs is performed to produce all results. Both figures show that the

estimator gradually becomes unbiased and its MSE approaches the CRB as SNR is increased. In fact, the estimator MSE almost coincides the CRB at large SNRs. This proves that our estimator indeed performs very satisfactorily.

V. CONCLUSION

By using only a pilot sample in a single time slot selected from a time-domain OFDM block, the proposed simple ML-based frequency offset estimator for OFDM in frequency-selective channels is proved to provide satisfactory estimation accuracies for frequency tracking.

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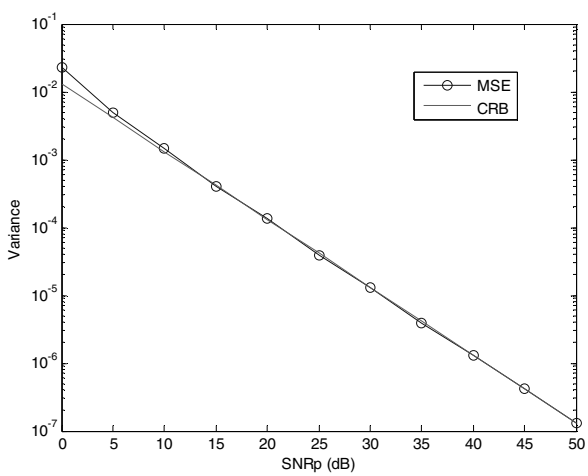
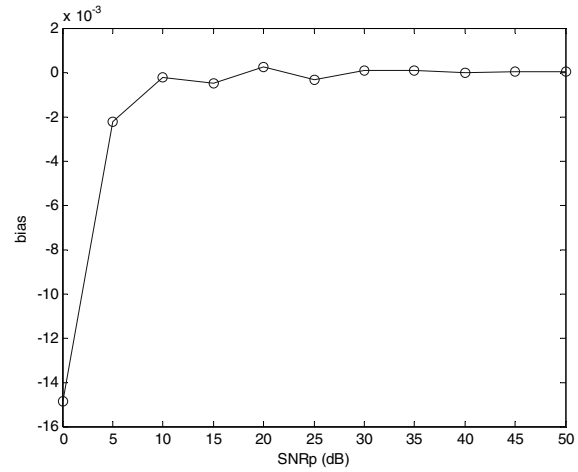
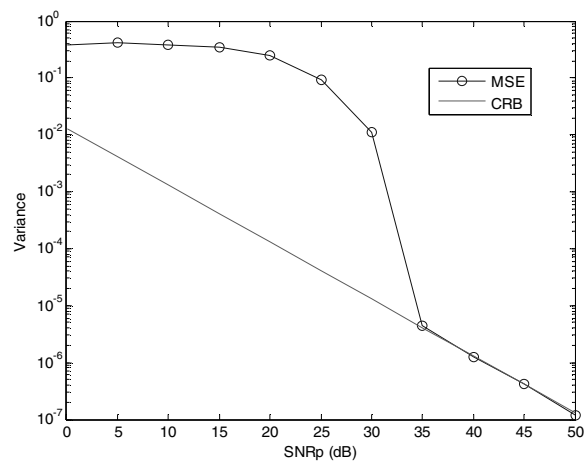


(a)

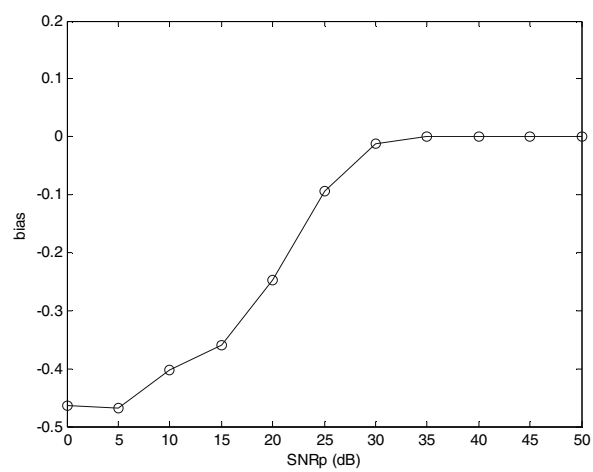


(b)

Fig. 1 Estimator performance for $\delta = 0.05$.
(a) Estimator variances vs. SNR.
(b) Estimator bias vs. SNR.



(a)



(b)

Fig. 2 Estimator performance for $\delta = 0.5$.
(a) Estimator variances vs. SNR.
(b) Estimator bias vs. SNR.