行政院國家科學委員會專題研究計畫 成果報告

結合局部搜尋之多目標並行處理粒子群聚最佳化法及其在不確定間隔系統數位建模之應用(I)

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行政院國家科學委員會補助專題研究計畫 ■ 成果報告 □期中進度報告

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Abstract
In this paper, an evolutionary approach is proposed to obtain a discrete-time state-space interval model for uncertain continuous-time systems having interval uncertainties. Based on a worst-case analysis, the problem to derive the discrete interval model is first formulated as multiple mono-objective optimization problems for matrix-value functions associated with the discrete system matrices, and subsequently optimized via a proposed genetic algorithm (GA) to obtain the lower and upper bounds of the entries in the system matrices. To show the effectiveness of the proposed approach, roots clustering of the characteristic equation of the obtained discrete interval model is illustrated for comparison with those obtained via existing methods.

Keywords: Discrete modelling, evolutionary algorithms, uncertain continuous-time systems, interval plant, discretization, model conversion, sampled-data systems.

1. Introduction

Most practical systems, such as flight vehicles, electric motors, and robots, are formulated in continuous-time uncertain settings. The uncertainties in these systems arise from unmodelled dynamics, parameter variation, sensor noises, actuator constraints, etc. These variations of the uncertain parameters generally do not follow any of the known probability distributions and are most often quantified in terms of bounds [1]-[2]. Hence, practical systems or plants are most suitably represented by continuous-time parametric interval models [3]-[5], rather than deterministic mathematical models. Simple as they might be, uncertain models in the form of interval systems have provided a convenient way in constructing mathematical models for physical systems, based on which practical design can be achieved for use in industry. It is no wonder that a large body of literature [4], [6] exploring robustness and performance issues of interval systems over the past few years.

On the other hand, digital control is getting more and more popular. The current trend toward digital control of dynamic systems is mainly due to the availability of low-cost digital computers and the advantages found in working with digital signals [7]. It is also well known that digital control [8]-[10] provides many advantages over the analogue control in terms of reliability, flexibility, cost, performance, etc. For digital simulation and digital design of continuous-time systems, however, it is essential to find an equivalent discrete-time model from the continuous-time model subject to digital control by using a zero-order hold (ZOH). Thanks to continuous efforts made to solve this problem over the past years, model conversion of a nominal continuous-time state-space model to an equivalent discrete-time state-space model has been well established and reported in the literature [11]-[14]. However, for uncertain continuous systems having an interval structure, the exact discrete-time state-space model for the sampled system is difficult to obtain, if not impossible [15], because the entries of the discrete system matrices depend non-linearly on the uncertain plant parameters. This has seriously prevented the applicability of available robustness results [16]-[17] in the discrete-time domain.

To solve the problem in deriving a suitable discrete-time model for uncertain interval plants, several attempts have been made over the past years to develop methods to obtain approximate discrete-time interval models which tightly enclose the exact discrete-time model of the continuous-time uncertain system [1], [18]-[20]. Among these methods, Oppenheimer and Michel [18] used an interval Taylor-series approximation method to convert a continuous-time uncertain system to an equivalent discrete-time interval model. Shieh *et al.* [19] developed an approximation method to convert a continuous-time uncertain system to an equivalent discrete-time interval model by applying the Pade approximation and the interval arithmetic, where the exponential function of an interval system matrix is approximated by a rational interval matrix-valued function. Shieh *et al.* [1] also proposed an interval geometric-series approximation method together with interval arithmetic to convert a continuous-time uncertain system to an equivalent discrete-time interval model. The aforementioned methods claimed that the obtained interval models were guaranteed to enclose the exact discrete-time uncertain model. Nevertheless, without carefully manipulating the developed results, the interval method might give a very conservative model, owing to the inherent conservativeness of the interval arithmetic [20]. In an attempt to find a less conservative discrete-time interval model, a GA-based method [20] was proposed to search the lower and upper bounds of the enclosing discrete-time interval system matrices. Unfortunately, the results obtained via this approach failed to enclose the exact discrete-time model, as will be demonstrated in the example of this paper. From the above-mentioned discussions, we can say that discrete modelling of continuous-time uncertain state-space models has not been sufficiently explored, and there is a need to derive an equivalent discrete-time interval state-space model with better accuracy suitable for digital simulation and design of the uncertain interval systems by using the available robustness results in the discrete-time domain [16]-[17].

Recent developments of evolutionary algorithms [21]-[26] have provided a promising alternative to solve the above-mentioned problem because of their capabilities of directed random search for global optimization [27]-[28]. This motivates the use of genetic algorithms to derive a less conservative discrete-time interval model for uncertain continuous–time systems by overbounding the uncertain plant parameters. Based on a worst-case analysis, the problem to derive the discrete-time interval model is first formulated as multiple mono-objective optimization problems for matrix-value functions associated with the discrete-time system matrices, and subsequently minimized and maximized via a proposed genetic algorithm to obtain the lower and upper bounds for the entries of the system matrices. Performance verification of the obtained discrete interval model based on roots clustering of the characteristic equation will be made in comparison to those obtained via existing methods to show the effectiveness of the proposed approach.

The paper is organized as follows. Section 2 formulates the design problem to derive a discrete-time model for continuous-time interval plants. Derivation of a desired discrete interval model for uncertain interval plants via genetic algorithms is introduced in Section 3, where the design problem is formulated as multiple mono-objective optimization problems to be solved via a proposed genetic algorithm. Illustrated examples are demonstrated in Section 4. Conclusions are drawn in Section 5.

2. Discrete-time model of continuous-time interval plants

Consider a continuous-time interval system given by

$$\dot{x}_c(t) = \mathbf{A}^{\mathrm{I}} x_c(t) + \mathbf{B}^{\mathrm{I}} u_c(t),\ \ x_c(0) = x_{c0} \quad (1)$$

$$y_c(t) = \mathbf{C}_0 x_c(t) \quad (2)$$

where $x_c \in \Re^{n\times 1}$ is the state, $u_c(t) \in \Re^{m\times 1}$ is the input, $y_c(t) \in \Re^{p\times 1}$ is the output, $\mathbf{A}^{\mathrm{I}} \in \Im\Re^{n\times n}$ and $\mathbf{B}^{\mathrm{I}} \in \Im\Re^{n\times m}$ are uncertain interval matrices, such that

$$\mathbf{A}^{\mathrm{I}} = [\underline{\mathbf{A}}, \overline{\mathbf{A}}] = [\mathbf{A}_0 - \Delta\mathbf{A}, \mathbf{A}_0 + \Delta\mathbf{A}],\ \ \mathbf{A}_0 = \frac{1}{2}[\overline{\mathbf{A}} + \underline{\mathbf{A}}] \quad (3)$$

$$\mathbf{B}^{\mathrm{I}} = [\underline{\mathbf{B}}, \overline{\mathbf{B}}] = [\mathbf{B}_0 - \Delta\mathbf{B}, \mathbf{B}_0 + \Delta\mathbf{B}],\ \ \mathbf{B}_0 = \frac{1}{2}[\overline{\mathbf{B}} + \underline{\mathbf{B}}] \quad (4)$$

$\mathbf{A}_0, \mathbf{B}_0, \mathbf{C}_0$ are nominal system matrices, and $\Delta\mathbf{A}, \Delta\mathbf{B}$ are the pair of uncertain matrices which are the perturbations of the nominal system matrices.

The associated discrete-time uncertain model for the continuous system in Eqs. (1-2) can be expressed as:

$$x_d(kT+T) = \mathbf{G} x_d(kT) + \mathbf{H} u_d(kT),\ \ x_d(0) = x_{c0} \quad (5)$$

$$y_d(kT) = \mathbf{C}_0 x_d(kT) \quad (6)$$

where

$$\mathbf{G} = e^{\mathbf{A}^{\mathrm{I}}T} = e^{(\mathbf{A}_0+\Delta\mathbf{A})T} \quad (7)$$

$$\mathbf{H} = \int_0^T e^{\mathbf{A}^{\mathrm{I}}\tau}\mathbf{B}^{\mathrm{I}} d\tau \quad (8)$$

$$u_c(t) = u_c(kT) = u_d(kT),\ \text{for}\ kT \le t \le kT+T \quad (9)$$

T is sampling period and the piecewise-constant input $u_d(t)$ is the output signal from the zero-order hold (ZOH). The system output matrix $\mathbf{C}_0 \in \Re^{p\times n}$ has no influence during model conversion. It is noted that, in general, the structured continuous-time uncertain system matrices $\mathbf{A}^{\mathrm{I}}, \mathbf{B}^{\mathrm{I}}$ would yield unstructured discrete-time system matrices $\mathbf{G}, \mathbf{H}$ in Eqs. (7-8). Exact evaluation of the uncertain discrete-time system matrices $\mathbf{G}, \mathbf{H}$ is extremely difficult, if not impossible [1], [19]-[21]. On the other hand, direct use of the interval arithmetic to obtain the interval matrices for system matrices $\mathbf{G}, \mathbf{H}$ often gives very conservative results, due to the nature of the interval arithmetic and the inherent conservativeness of interval arithmetic operations. For example, $\mathbf{A}^{\mathrm{I}} - \mathbf{A}^{\mathrm{I}} \ne \mathbf{0}_n$, $(\mathbf{A}^{\mathrm{I}})^{-1}\mathbf{A}^{\mathrm{I}} \ne \mathbf{I}^{\mathrm{I}}$. As a result, interval analysis is generally carried out using real (i.e., degenerate interval) analysis to find the desired real result. Subsequently, the inclusion theorem [29]-[31] is applied to the result sought by replacing the real variables and the real arithmetic operations with the interval variable and interval arithmetic operations, respectively. Since no efficient analytical and/or numerical methods are available [2] for finding the exact discrete-time model of Eqs. (5-9), it is therefore reasonable to seek an approximately enclosed discrete-time interval model.

Fortunately, exact evaluation of degenerate (real) matrices via real arithmetic is feasible. Assume that the uncertain interval matrices $\mathbf{A}^{\mathrm{I}}$ and $\mathbf{B}^{\mathrm{I}}$ contain degenerate (real) matrices $\mathbf{A}_r \in \Re^{n\times n}$ and $\mathbf{B}_r \in \Re^{n\times m}$, such that $\mathbf{A}_r \in \mathbf{A}^{\mathrm{I}}$ and $\mathbf{B}_r \in \mathbf{B}^{\mathrm{I}}$, respectively. The discrete-time degenerate model for the continuous system in Eqs. (1-2) can be expressed as:

$$x_d(kT+T) = \mathbf{G}_r x_d(kT) + \mathbf{H}_r u_d(kT),\ \ x_d(0) = x_{c0} \quad (10)$$

where

$$\mathbf{G}_r = e^{\mathbf{A}_r T} \quad (11)$$

$$\mathbf{H}_r = \int_0^T e^{\mathbf{A}_r\tau}\mathbf{B}_r d\tau \quad (12)$$

are the discrete-time degenerate (real) matrices. Applying the inclusion theorem to the discrete-time degenerate model in Eqs. (10-12) with $\mathbf{A}_r \in \mathbf{A}^{\mathrm{I}}$ and $\mathbf{B}_r \in \mathbf{B}^{\mathrm{I}}$ results in the desired discrete-time uncertain model in Eqs. (7-8), where

$$\mathbf{G}_r = e^{\mathbf{A}_r T} \in \mathbf{G} = e^{\mathbf{A}^{\mathrm{I}}T} \quad (13)$$

$$\mathbf{H}_r = (\mathbf{G}_r - \mathbf{I}_n)(\mathbf{A}_r)^{-1}\mathbf{B}_r \in \mathbf{H} = (\mathbf{G} - \mathbf{I}_n)(\mathbf{A}^{\mathrm{I}})^{-1}\mathbf{B}^{\mathrm{I}} \quad (14)$$

The design objective can now be formulated as: *Given continuous-time interval system matrices* $(\mathbf{A}^{\mathrm{I}}, \mathbf{B}^{\mathrm{I}})$ *of Eqs. (1-2), determine the discrete-time interval system matrices* $\mathbf{G}^{\mathrm{I}} \in \Im\Re^{n\times n}$ *and* $\mathbf{H}^{\mathrm{I}} \in \Im\Re^{n\times m}$, *which tightly enclose the exact discrete-time model* $(\mathbf{G}, \mathbf{H})$ *in Eqs. (7-8), based on the respective degenerate interval (real) matrices* $\mathbf{G}_r, \mathbf{H}_r$ *over the parameter space.*

3. Derivation of the discrete interval model via genetic algorithms

Because of the capability of genetic algorithms (GAs) in directed random search for global optimization, they will be used to evolutionarily identify a less conservative enclosing approximant ($\mathbf{G}^{\mathbf{I}}$, $\mathbf{H}^{\mathbf{I}}$) such that $\mathbf{G}^{\mathbf{I}} \supset \mathbf{G}$ and $\mathbf{H}^{\mathbf{I}} \supset \mathbf{H}$, on the basis of the degenerate matrices $\mathbf{G}_r, \mathbf{H}_r$ to optimize the matrix-value functions of the discrete-time system matrices $\mathbf{G}$ and $\mathbf{H}$.

To facilitate the evolution process via genetic algorithms, matrices in the forms of element (entry) representation are desired. For the continuous-time uncertain interval matrices $\mathbf{A}^{\mathrm{I}}$ and $\mathbf{B}^{\mathrm{I}}$, we have alternative presentations as:

$$\mathbf{A}^{\mathrm{I}} = [a_{ij}],\ \ a_{ij} = [\underline{a}_{ij}, \overline{a}_{ij}],\ \ i, j = 1,2,\cdots,n \quad (15)$$

$$\mathbf{B}^{\mathrm{I}} = [b_{uv}],\ \ b_{uv} = [\underline{b}_{uv}, \overline{b}_{uv}],\ \ u = 1,2,\cdots,n;\ v = 1,2,\cdots,m \quad (16)$$

, where a_{ij} and b_{uv} are the *ij*th and *uv*th elements of $\mathbf{A}^{\mathrm{I}}$ and $\mathbf{B}^{\mathrm{I}}$, respectively. $\underline{a}_{ij}$ and $\underline{b}_{uv}$ denote the lower bounds of a_{ij} and b_{uv}, while $\overline{a}_{ij}$ and $\overline{b}_{uv}$ denote the upper bounds of a_{ij} and b_{uv}, respectively. Degenerate matrices $\mathbf{A}_r, \mathbf{B}_r$, on the other hand, are composed of real numbers a_{rij} and b_{ruv}, respectively:

$$\mathbf{A}_r = [a_{rij}],\ a_{rij} \in [\underline{a}_{ij}, \overline{a}_{ij}] \tag{17}$$

$$\mathbf{B}_r = [b_{ruv}],\ b_{ruv} \in [\underline{b}_{uv}, \overline{b}_{uv}] \tag{18}$$

Therefore, we can directly use real-number arithmetic operation to evaluate Eqs. (11-12).

As far as the discrete-time system is concerned, the entries in the uncertain system matrices $\mathbf{G} = [g_{ij}]$, $\mathbf{H} = [h_{uv}]$ can be expressed as matrix-value functions:

$$g_{ij} = \alpha_{ij}(a_{ij}, T) \text{ and } h_{uv} = \beta_{uv}(a_{ij}, b_{uv}, T) \tag{19}$$

respectively, where g_{ij} is a function of a_{ij} and T, and h_{uv} is a function of a_{ij}, b_{uv}, and *T*. We can obtain the upper and lower bounds for each entry in the system matrices $\mathbf{G}$ and $\mathbf{H}$ by optimizing the corresponding matrix-value functions α_{ij} and β_{uv}. That is,

$$\underline{g}_{ij} = \min \alpha_{ij}(T, a_{ij}),\ \ \overline{g}_{ij} = \max \alpha_{ij}(T, a_{ij}),\ \ i, j = 1,2,\cdots,n \tag{20}$$

$$\underline{h}_{uv} = \min \beta_{uv}(T, a_{ij}, b_{uv}),\ \overline{h}_{uv} = \max \beta_{uv}(T, a_{ij}, b_{uv}),$$
$$i, j, u = 1,2,\cdots,n;\ v = 1,2,\cdots,m \tag{21}$$

Therefore, we can obtain a discrete-time interval model ($\mathbf{G}^{\mathbf{I}}$, $\mathbf{H}^{\mathbf{I}}$), which encloses the exact discrete model ($\mathbf{G}, \mathbf{H}$), such that

$$\mathbf{G}^{\mathbf{I}} = [\underline{\mathbf{G}}, \overline{\mathbf{G}}] \supset \mathbf{G} = e^{\mathbf{A}^{\mathrm{I}}T} \tag{22}$$

$$\mathbf{H}^{\mathbf{I}} = [\underline{\mathbf{H}}, \overline{\mathbf{H}}] \supset \mathbf{H} = \int e^{\mathbf{A}^{\mathrm{I}}\tau}\mathbf{B}^{\mathrm{I}} d\tau \tag{23}$$

where

$$\underline{\mathbf{G}} = [\underline{g}_{ij}],\quad \overline{\mathbf{G}} = [\overline{g}_{ij}],\ i, j = 1,2,\cdots,n \tag{24}$$

$$\underline{\mathbf{H}} = [\underline{h}_{uv}],\quad \overline{\mathbf{H}} = [\overline{h}_{uv}],$$
$$u = 1,2,\cdots,n;\ v = 1,2,\cdots,m. \tag{25}$$

Note that the matrix-value functions α_{ij}, β_{uv} in Eq. (19) are nonlinear and generally nonconvex functions of a_{ij} and (a_{ij}, b_{uv}), respectively, in the searching space of the uncertain plant parameters. Gradient-based optimization algorithms generally lead to solutions that have local properties only. Genetic algorithms, with their power as an efficient and robust alternative for solving complex and highly nonlinear optimization problems, are therefore used to identify the lower and upper bounds of the entries in the discrete system matrices.

3.1 Population Initialization

Basically, genetic algorithms are probabilistic algorithms which maintain a population of individuals (chromosomes, vectors) for iteration *t*. Each chromosome represents a potential solution to the problem at hand, and is evaluated to give some measure of its “fitness”. Then, a new population is formed by selecting the more fit individuals. Some members of the new population undergo transformations by means of genetic operators to form new solutions. After some number of generations, it is hoped that the system converges with a near-optimal solution [27]-[28]. A genetic algorithm requires a population of potential solutions to be initialized and maintained during the process. In this paper, a fixed number of the population size *N* is used. Real number representation for potential solutions is adopted to simplify genetic operator definitions and obtain a better performance of the genetic algorithm itself. Thus, there are no encoding and decoding operations involved, which is particularly useful if vast amount of parameters are to be adjusted.

Assume that $\mathbf{X}_k^t$ is the *k*-th chromosome in a population of *N* at generation *t*, which represents entries of the continuous interval system matrices $\mathbf{A}^{\mathrm{I}}$ and $\mathbf{B}^{\mathrm{I}}$ in Eqs. (15-16) and is defined as:

$$\mathbf{X}_k^t = [a_{11}\ a_{12} \cdots a_{ij} \cdots\ \ a_{nn}\ b_{11}\ b_{12} \cdots b_{uv} \cdots\ \ b_{nm}\ ],$$
$$i, j, u = 1,2,\cdots,n,\quad v = 1,2,\cdots,m,\quad k = 1,2,\cdots,N \tag{26}$$

Initial chromosomes are randomly generated from within the pre-defined range:

$$a_{ij} = [\underline{a}_{ij}, \overline{a}_{ij}],\ \text{for } i, j = 1,2,\cdots,n \tag{27}$$

$$b_{uv} = \left[\underline{b}_{uv}, \overline{b}_{uv}\right],\ \text{for } u = 1,2,\cdots,n,\ \text{and } v = 1,2,\cdots,m \tag{28}$$

After initialization, several genetic operations are performed during procreation.

3.2 Fitness evaluation

Basically, there are $(n \times n + n \times m)$ matrix-value functions associated with the discrete-time model, i.e. $\alpha_{ij}(T, a_{ij}),\ i, j = 1,2,\cdots,n$ and $\beta_{uv}(T, a_{ij}, b_{uv}),\ \ u = 1,2,\cdots,n,\ v = 1,2,\cdots,m$, which are nonlinear function of the uncertain plant parameters. We need to establish fitness functions to direct the evolution process for optimizing the matrix-value functions in Eq.(19). To reduce redundancy, we assume that $f_k^t,\ k = 1,2,\cdots,N$, represents the evaluation of chromosome $\mathbf{X}_k^t$ for the matrix-value functions of either $\alpha_{ij}(T, a_{ij})$ or $\beta_{uv}(T, a_{ij}, b_{uv})$, depending on the optimization problem under consideration. The upper bound $\overline{f}^t$ and lower bound $\underline{f}^t$ in evaluating the chromosomes in the current population t for a particular matrix-value function can be obtained as follows:

$$\underline{f}^t = \min\{f_k^t, k = 1,2,\cdots,N\}$$

$$\overline{f}^t = \max\{f_k^t, k = 1,2,\cdots,N\}$$

, respectively.

The fitness of each chromosome $\mathbf{X}_k^t$ in a population t can then be assigned according to the fitness functions defined below:

(i) Fitness function for chromosome $\mathbf{X}_k^t$ in determining the lower bound of a matrix-value function (i.e., in the derivation of $\underline{g}_{ij}$ or $\underline{h}_{uv}$):

$$F_1(\mathrm{X}_k^t) = 1 \Big/ \left[(f_k^t - \underline{f}^t) + 1 \right] \tag{29}$$

(ii) Fitness function for chromosome $\mathbf{X}_k^t$ in determining the upper bound of a matrix-value function (i.e., in the derivation of $\overline{g}_{ij}$ or $\overline{h}_{uv}$) :

$$F_2(\mathrm{X}_k^i) = 1 \Big/ \left[(\overline{f}^t - f_k^t) + 1 \right] \tag{30}$$

The rationale of fitness assignment is described as follows: chromosome $\mathbf{X}_k^t$ corresponding to a larger function evaluation f_k^t will receive a smaller fitness; while chromosome $\mathbf{X}_k^t$ corresponding to a smaller function evaluation f_k^t will be assigned a larger fitness in deriving the lower bounds of the matrix-value functions, via F_1 in Eq. (29). In a very similar way, fitness function in deriving the upper bounds of the matrix-value functions can be devised as F_2 in Eq. (30). By doing so, evolution can be directed toward derivation of an optimal set of system matrices for the discrete-time system, which tightly enclose the exact discrete model.

Obviously, there are totally $2 \times (n \times n + n \times m)$ optimization problems for processing. After generations of evolution, it is expected that the genetic algorithms converges and a best chromosome with largest fitness representing the boundary of the entry of the discrete-time interval model can obtained.

3.3 Evolutionary scheme of the proposed genetic algorithm

Evolutionary process of the proposed genetic algorithm includes the steps of population initialization and reproduction operation. Real-coded (RC) representation for potential solutions is adopted in the proposed GA-based approach to simplify genetic operator definitions and obtain a better performance of the genetic algorithm itself [21]-[22]. The tournament selection is employed to keep the balance between the population diversity and selective pressure during the evolution process. Several genetic operators: Simulated binary crossover and non-uniform mutation are performed on the selected chromosomes after the reproduction operation with suitable selection of control parameters [22]. To prevent the loss of the optimal solution ever searched and increase the convergence rate, the elitist replacement is adopted to preserve the optimal solution in the current generation. From the experiments ever conducted, we observed that the extrema generally lie on or near the boundaries of the uncertain plant parameters. Boundary mutation is extremely suitable for use in this case, and will be adopted to locate the boundaries for each of the matrix-value functions with success. For examples in this paper, the population size is chosen as 50, the crossover rate and mutation rate are 0.8 and 0.05, respectively.

4. Illustrated Examples

Example 1:

Consider an asymptotically stable linear **R-L-C** circuit [29] described by an uncertain state equation with the following nominal and perturbed system matrices:

$$\mathbf{A}_0 = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix},\ \mathbf{B}_0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\Delta\mathbf{A} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0 \end{bmatrix},\ \Delta\mathbf{B} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

It is desired to find an approximate discrete-time interval model, enclosing the exact discrete-time model, for the continuous-time uncertain system at sampling period T=0.1s.

[Solution]:

The uncertain interval matrics of the continuous-time system are:

$$\mathbf{A}^{\mathrm{I}} = \begin{bmatrix} [-2.1 \ -1.9] & 0 \\ [0.9 \quad 1.1] & -3 \end{bmatrix},\ \mathbf{B}^{\mathrm{I}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

By using the proposed GA-based approach, the discrete-time interval system matrices for the continuous-time uncertain system at sampling period T=0.1 can be obtained as:

$$\mathbf{G}_{ga}^{\mathrm{I}} = \begin{bmatrix} [0.810584 \quad 0.826959] & [0.000000 \quad 0.000000] \\ [0.069766 \quad 0.086141] & [0.740818 \quad 0.740818] \end{bmatrix}$$

after 50 generations of evolution. In comparison to the results:

$$\mathbf{G}_{p}^{\mathrm{I}} = \begin{bmatrix} [0.808029 \quad 0.829520] & [-0.001953 \ 0.001953] \\ [0.066945 \quad 0.088960] & [0.738867 \quad 0.742774] \end{bmatrix},$$

obtained by the Pade approximation method [19], and

$$\mathbf{G}_{g}^{\mathrm{I}} = \begin{bmatrix} [0.808646 \quad 0.828714] & [-0.00003 \quad 0.00003] \\ [0.067817 \quad 0.087916] & [0.74078 \quad 0.740844] \end{bmatrix}$$

via the interval geometric series approximation method [1], the discrete-time interval model via the propsoed approach is clearly less conservative with tighter boundaries on the extries of the discrete system matrices.

To show the effectiveness of the proposed approach, Fig. 1 shows the root clustering of the characteristic equation of the exact discrete-time model and those obtained by the proposed GA-based approach, the Pade approximation method [19], and the interval geometric series approximation method [1], respectively. As shown in Fig. 1, roots of the characteristic equations obtained via various approaches lie entirely on the real axis. To facilitate comparison between all the methods, we calculate the upper and lower bounds of root clustering of the characteristic equation for each method. As shown in Table 1, we find that root clustering of the discrete interval model via the proposed GA-based method is not distinguishable to that of the exact discrete-time model. On the other hand, the other two methods, though enclosing the exact discrete-time model, however, provide more conservative results than the proposed method. It is clear that root clustering of the characteristic equation of the discrete model obtained by using the proposed GA-based method bears a closer resemblance to that of the exact discrete model. Pade approximation method [19] and the interval geometric-series approximation method [1] both failed to obtain a satisfactory performance as demonstrated in this example.

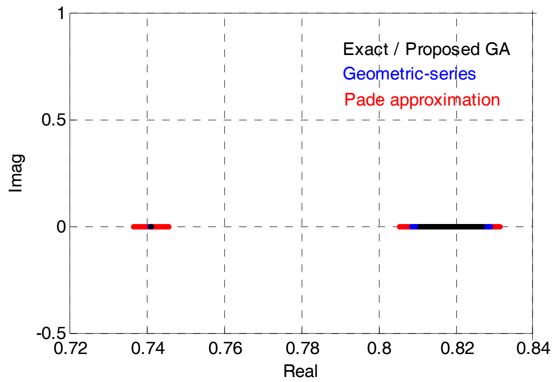

Fig. 1 Root clusterings of the characteristic equation of various discrete models at T=0.1 Secs. of Example 1.

Table 1 Boundaries of root clustering of the discrete models via various approaches

	Root clustering on the left-hand side	Root clustering on right-hand side
Exact results	0.740818	[0.810584, 0.826959]
Proposed GA-based method	0.740818	[0.810584, 0.826959]
Pade approximation method [19]	[0.736440, 0.745555]	[0.805248, 0.831479]
Interval geometric series Method [1]	[0.740741, 0.740883]	[0.808607, 0.828744]

Example 2:

Consider an uncertain continuous-time system with the following nominal and perturbed system matrices [20]:

$$\mathbf{A}_0 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.2855 & -0.707 & 1.3229 \\ 0 & 0 & 1.000 & 0 \end{bmatrix},$$

$$\mathbf{B}_0 = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.0447 & -7.5922 \\ -5.52 & 4.99 \\ 0 & 0 \end{bmatrix}$$

$$\Delta\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \pm 0.2192 & 0 & \pm 1.2031 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Delta\mathbf{B} = \begin{bmatrix} 0 & 0 \\ \pm 2.0673 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

It is desired to find an approximate discrete-time interval model for the continuous-time uncertain system at sampling period T=0.18s.

[Solution]:

By using the proposed GA-based approach after 50 generations of evolution, we obtain the discrete interval system matrices as follows:

$$\mathbf{G}^{\mathrm{I}}_{ga} = \begin{bmatrix} [0.9934 \;\; 0.9934] & [0.0044 \;\; 0.0044] & [-0.0040 \;\; -0.0040] & [-0.0837 \;\; -0.0834] \\ [0.0075 \;\; 0.0075] & [0.8321 \;\; 0.8336] & [-0.0588 \;\; -0.0584] & [-0.6712 \;\; -0.6622] \\ [0.0169 \;\; 0.0175] & [0.0098 \;\; 0.0790] & [0.8805 \;\; 0.9181] & [-0.0102 \;\; 0.4283] \\ [0.0016 \;\; 0.0016] & [0.0010 \;\; 0.0074] & [0.1690 \;\; 0.1713] & [1.0000 \;\; 1.0392] \end{bmatrix}$$

$$\mathbf{H}^{\mathrm{I}}_{ga} = \begin{bmatrix} [0.0805 \;\; 0.0822] & [0.0278 \;\; 0.0278] \\ [0.1809 \;\; 0.8615] & [-1.2673 \;\; -1.2667] \\ [-0.9440 \;\; -0.8953] & [0.7879 \;\; 0.8477] \\ [-0.0862 \;\; -0.0834] & [0.0741 \;\; 0.0777] \end{bmatrix}$$

For comparison purpose, the results obtained by Shieh adopting a GA-based approach [20] are listed below:

$$\mathbf{G}^{\mathrm{I}}_{S} = \begin{bmatrix} [0.9934 \;\; 0.9934] & [0.0044 \;\; 0.0044] & [-0.0040 \;\; -0.0040] & [-0.0837 \;\; -0.0834] \\ [0.0075 \;\; 0.0075] & [0.8321 \;\; 0.8336] & [-0.0588 \;\; -0.0584] & [-0.6711 \;\; -0.6622] \\ [0.0169 \;\; 0.0175] & [0.0103 \;\; 0.0789] & [0.8807 \;\; 0.9180] & [-0.0100 \;\; 0.4259] \\ [0.0016 \;\; 0.0016] & [0.0010 \;\; 0.0074] & [0.1690 \;\; 0.1713] & [1.0000 \;\; 1.0391] \end{bmatrix}$$

$$\mathbf{H}^{\mathrm{I}}_{S} = \begin{bmatrix} [0.0805 \;\; 0.0822] & [0.0278 \;\; 0.0278] \\ [0.1809 \;\; 0.8608] & [-1.2673 \;\; -1.2667] \\ [-0.9440 \;\; -0.8953] & [0.7878 \;\; 0.8476] \\ [-0.0862 \;\; -0.0834] & [0.0741 \;\; 0.0777] \end{bmatrix}$$

Although the results are very close to those obtained via the proposed approach, there is, however, a significant difference as far as enclosure of the exact discrete-time model is concerned. That is, the interval model revealed in [20] did not enclose the exact discret-time interval model. For instance, when

$$\mathbf{A}=\begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.0774 & -0.707 & 2.5244 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix},$$

$$\mathbf{B}=\begin{bmatrix} 0.4422 & 0.1761 \\ 2.6293 & -7.5922 \\ -5.52 & 4.99 \\ 0 & 0 \end{bmatrix}$$

, where a certain perturbation occurred on top of the nomial matrices, the exact discrete-time model becomes:

$$\mathbf{G}=\begin{bmatrix} 0.9934 & 0.0044 & -0.0040 & -0.0837 \\ 0.0075 & 0.8335 & -0.0588 & -0.6711 \\ 0.0172 & 0.0121 & 0.9181 & 0.4272 \\ 0.0016 & 0.0011 & 0.1713 & 1.0392 \end{bmatrix}, \quad .$$

$$\mathbf{H}=\begin{bmatrix} 0.0812 & 0.0278 \\ 0.4529 & -1.2673 \\ -0.9420 & 0.8465 \\ -0.0861 & 0.0775 \end{bmatrix}$$

It's easy to check that $\mathbf{G}(3,3)=0.9181$, $\mathbf{G}(3,4)=0.4272$, and $\mathbf{G}(4,4)=1.0392$ lie outside the range of $\mathbf{G}_S^{\mathrm{I}}(3,3)=[0.8807 \quad 0.9180]$, $\mathbf{G}_S^{\mathrm{I}}(3,4)=[-0.0100 \quad 0.4259]$, and $\mathbf{G}_S^{\mathrm{I}}(4,4)=[1.0000 \quad 1.0391]$, respectively [20]. On the contrary, the discrete-time interval model via the proposed GA-based approach presents trustworthy results which tightly enclose the exact discrete-time model of the uncertain continuous-time system. To demonstrate the effectiveness of the propsoed approach, root clustering of the characteristic equation of the discrete-time model via the proposed approach is illustrated in Fig. 2 in comparison to its exact counterpart, where green and red portions represent the root clustering of the derived and exact discrete-time models, respectively. As illustrated in Fig. 2, enclosure of the exact boundaries of the root distribution has been guaranteed via the proposed GA-based approach. Note that we have 48 optimization processes to locate the boundaries for all the entries (i.e. $\overline{g}_{ij}, \underline{g}_{ij}$ and $\overline{h}_{uv}, \underline{h}_{uv}$, $i, j, u = 1,2,3,4,\ v = 2$) in the enclosing discrete-time system matrices ($\mathbf{G}_{ga}^{\mathrm{I}}, \mathbf{H}_{ga}^{\mathrm{I}}$) in this example.

If a single Pentium 4 personal computer (2.0GHz, 512MB RAM) is used, the discrete-time interval system matrices are obtained with a computation time of 1680 seconds. With the adoption of a parallel computation scheme [25] where 10 PCs (CPU 2.0 GHz and 512MB RAM) work as slaves, significant evolution efficiency can be achieved with a total computation time of 172 seconds to derive the discrete-time interval system matrices.

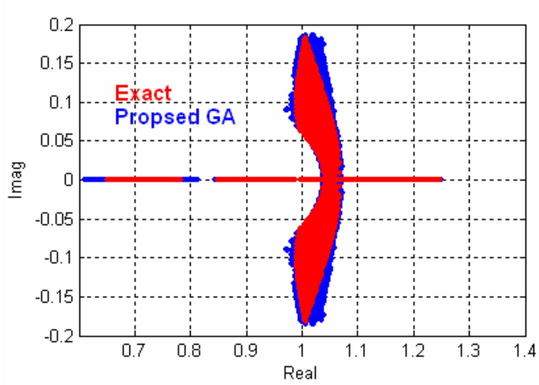

Fig. 2 Root clustering of the characteristic equation of the discrete-time model via the proposed approach in comparison to its exact counterpart in Example 2.

5. Conclusions

In this paper, we have investigated the use of genetic algorithm to obtain a discrete-time interval model for uncertain interval systems. Because of the non-convexity generally exhibited in the matrix-value functions during the discretization process of uncertain interval plants, conventional methods generally failed to obtain satisfactory results of the associated discrete-time model. The proposed GA-based approach from the worst-case analysis point of view, on the other hand, is capable of evolutionarily deriving a discrete-time model tightly encloses the exact model as demonstrated in the illustrated examples. As a result, the problem of highly-coupled nonlinearities with exponential nature occurred in the exact discrete-time system matrices is therefore circumvented, while preserving the interval structure in the resulting discrete-time model by using the proposed approach. In light of the facts that multiple optimization processes to be simultaneously executed to obtain the entries of the discrete system matrices, a parallel computation scheme for the proposed evolutionary approach can be considered to accelerate the derivation process. Furthermore, there is no restrictive condition under which the proposed approach is developed. Performance verification has demonstrated that roots clustering of the discrete-time interval model using the proposed GA-based approach has a better resemblance to that of the exact discrete model, in comparison to existing methods.

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出席國際學術會議心得報告

計畫編號	NSC 95-2221-E-032-069-
計畫名稱	結合局部搜尋之多目標並行處理粒子群聚最佳化法及其在不確定間隔系統數位建模之應用(I)
出國人員姓名 服務機關及職稱	許陳鑑 淡江大學電機工程學系副教授
會議時間地點	澳洲布里斯班（Brisbane），2007 年 1 月 17 -19 日
會議名稱	2007 WSEAS 電路、系統、信號、電信國際研討會 2007 WSEAS Int. Conference on Circuits, Systems, Signal and Telecommunications (CISST'07)
發表論文題目	以離散小波轉換為基礎之即時人體追蹤系統(Real-Time Tracking of Human Body Based on Discrete Wavelet Transform)

一、參加會議經過

2007 WSEAS 電路、系統、信號、電信國際研討會 (2007 WSEAS Int. Conference on Circuits, Systems, Signal and Telecommunications (CISST'07))係為 WSEAS 於 2007 年 1 月 17 日至 19 日在澳洲布里斯班（Brisbane）所舉辦之 5 個系列研討會之一。舉辦地點為 Crowe Plaza，交通非常便利，研討會一連舉行三天，來自全世界的學者專家齊聚一堂，共同研討 CISST'07 所涵蓋之相關主題，共發表學術論文 40 餘篇。筆者於 1 月 17 日搭機出發前往澳洲布里斯班，18 日稍事休息即參加研討會相關活動，19 日早上 9：00AM 準時參加論文發表，在該場次共有 6 篇論文發表，筆者也是該場會議之主席，每篇論文均有人提問，但時間控制差強人意，準時於 11：00AM 結束論文發表。

二、與會心得

本次研討會主題範圍很廣泛，包含計算機及其應用 COMPUTER ENGINEERING and APPLICATIONS (CEA '07)、HEAT and MASS TRANSFER (HMT '07)、FLUID MECHANICS (FLUIDS '07)、MATHEMATICAL BIOLOGY and ECOLOGY (MABE '07)等多個研究主題。CISST'07 係其中一個研討會，研討主題包含：Wireless systems and networks、Advances in system science， Advanced models in engineering science and optimization techniques、Signal and image processing、Image-Based Measurement and Tracking and Their Applications、Control systems、Telecommunication Systems and data monitoring systems 等主題。大會並邀請 Professor Gorazd Kandus 發表 PLENARY LECTURE，主題是 Broadband Communications from High Altitude Platforms。為使參與者皆能廣泛交換意見，主辦單位在研討會期間安排了交流與討論時段，使得各領域的專家們能充分交換意見，達成跨領域(Interdisciplinary)之溝通，可能是與會者專長領域分佈較廣，因此熱烈討論並不多見。由於筆者擔任一場次之會議主席，提早 15 分鐘進入會場預作準備，在論文發表一開始，大家略嫌拘謹，但隨後的討論還算熱烈，特別是一位希臘籍學者常提出一些建議，也引起大家共鳴。由於事前已妥為準備，故順利進行論文發表，也回答了相關的問題，分享對於問題的看法，圓滿達成任務。