H_∞ Output Tracking Fuzzy Control for Nonlinear Systems with Time-Varying Delay

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Abstract

This paper proposes an observer-based output tracking control via virtual desired reference model for a class of nonlinear systems with time-varying delay and disturbance. First, the Takagi-Sugeno fuzzy model represents the nonlinear system with time-varying delay and disturbance. Then we design an observer to estimate immeasurable states and controller to drive the error between estimated state and virtual desired variables (VDVs) to zero such that the overall control output tracking system has H_∞ control performance. Using Lyapunov-Kravoskii functional, we derive sufficient conditions for stability. The advantages of the proposed output control system are: i) systematic approach to derive VDVs for controller design; ii) relaxes need for real reference model; iii) drops need for information of equilibrium; iv) relaxed condition is provided via three-step procedure to find observer and controller gain. We carry out simulation using a continuous stirred tank reac-

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Preprint submitted to Applied Soft Computing *March 20, 2012*

tor system where the effectiveness of the proposed controller is demonstrated by satisfactory numerical results.
Keywords: Time-delay systems, T-S fuzzy, virtual desired variables, linear matrix inequalities

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1. Introduction

Tracking control is important in practical applications, e.g., robotic control, servo-motor control, missile control, etc. For nonlinear systems, especially in recent years, control using Takagi-Sugeno (T-S) fuzzy model [1] has attracted wide attention. This is due to the systematic approach to analysis and design of controllers where multiple objectives can be considered in a unified manner[2–15]. The controller design using parallel distributed compensation (PDC) and stability analysis are carried out using Lyapunov direct method which reformulates the control objectives into linear matrix inequality problems (LMIPs) [16–18]. Meanwhile, when immeasurable states exist, studies [19–21] provide a T-S fuzzy observer-based stabilization schemes with conditions in LMIPs. However, except for some works [22–24] which address tracking, most results only focus on the stabilization problem. In details, for regulation, the works [22, 25] uses feed-forward compensation of linear control theory. Meanwhile for tracking, the works [22, 24] consider the command signal as disturbance to the closed-loop system where a robust control scheme attenuates the tracking error residual. In addition, combining the linear regulation theory and PDC, the works [25, 26] achieve the control objective of static or varying output. However, the study [23] has noted that high interaction among plant and controller rules will lead to failure. For output tracking design, the works [27, 28] propose a novel concept of virtual desired variables (VDVs). The advantages of VDV approach are i) unifies the stability and tracking design problem, ii) relaxes the need for a real reference model, and iii) achieves tracking of time-varying signals.

Another challenging problem in control is when time-delay exists in systems. Time delays frequently occur in many practical systems, e.g., chemical processes, nuclear reactors, long transmission lines, and telecommunication. Since time-delay is a main cause of instability and poor performance, control of such systems has received considerable attention. Generally speaking, previous stability analysis can be classified as delay-dependent [29, 30] and delay-independent [31, 32]. The delay-dependent methods need information on the exact delay duration. On the other hand, the delay-independent methods do not need any information of the delay making these approaches more suitable for practical applications. However, most time-delay system applications deal with stabilizing to a equilibrium. Recently, the work [33] discusses robust output tracking with time-varying delays assuming ideal conditions where premise variables between observer and controller fuzzy rules are matched and actual reference signal exists.

In this paper, we are motivated to propose a novel output tracking control of a time-varying reference signal via a VDV approach based on the T-S fuzzy model for time-varying delay systems with disturbances without needing information of equilibrium. First, we represent the nonlinear system with time-varying delay and disturbances into a T-S fuzzy model. Next, we design a set of VDVs. Third, we combine the concept of VDVs controller synthesis and observer-based estimation to develop an output tracking controller. Using Lyapunov's direct method, we derive sufficient conditions and formulate LMIs which guarantees H_∞ control performance. We then illustrate the approach on a continuous stirred tank reactor to validate the aforementioned claims. Note that comparing to the work [33], we relax the need for assuming perfect matching of premise variables between the observer and controller. This further improves the applicability of the proposed method on real world applications where observer premise variables only rely on estimated states. In addition, we use a virtual reference model from the solution of the plan itself as the stated reference needs an actual model.

The remaining paper is organized as follows. In Sec. 2, we formulate the control problem. In Sec. 3, we design the observer-based output tracking control. In Sec. 4, we explain the VDV design. Section 5 shows the results of the numerical simulation. Finally, some concluding remarks are made in Sec. 6.

2. Nonlinear System with Time-Varying Delay and Disturbance

Based on fuzzy modeling methods [34, 35], we can describe a class of nonlinear systems with time-varying delay as the T-S fuzzy model:

$$
\begin{aligned}
& \quad Plant\ Rule\ i: \\
& \quad \text{IF } z_1(t) \text{ is } F_{1i} \text{ and } \cdots \text{ and } z_f(t) \text{ is } F_{fi} \text{ THEN} \\
\dot{x}(t) &= A_i x(t) + A_{di} x(t-\tau(t)) + Bu(t) + w(t) \\
y(t) &= Cx(t) + v(t) \\
x(t) &= \varphi(t),\ t \in [-\tau\ 0]
\end{aligned}
$$

where $x(t) \in R^n$, $u(t) \in R$ and $y(t)$ are state, control input and measured output, respectively; A_i, B, A_{di}, C are matrices with appropriate dimensions; $z_1(t) \sim z_f(t)$ are the premise variables which are states (or combination of states); F_{ji} $(j = 1, 2, \cdots, f)$ are the fuzzy sets; r is the number of fuzzy rules; $y(t)$ is the output; $\varphi(t)$ is the initial condition; $\tau(t)$ is the time-varying delay with upper bound τ_0, i.e. $\tau(t) \leq \tau_0$; and $w(t)$, $v(t)$ are external disturbance and sensor measurement disturbance, respectively.

Using the singleton fuzzifier, product fuzzy inference and weighted average defuzzifier, the inferred output

$$
\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(z(t))[A_i x(t) + A_{di} x(t - \tau(t)) + Bu(t) + w(t)] \\
y(t) &= Cx(t) + v(t),
\end{aligned}
\tag{1}
$$

where $z(t) = [z_1(t)\ z_2(t)\ \cdots\ z_n(t)]^T$, and $\mu_i(z(t)) = w_i(z(t))/\sum_{i=1}^{r} w_i(z(t))$ with $w_i(z(t)) = \prod_{j=1}^{f} F_{ji}(z(t))$. Note that $\sum_{i=1}^{r} \mu_i(z(t)) = 1$ for all t, where $\mu_i(z(t)) \geq 0$, for $i = 1, 2, ..., r$, are the normalized weights. For simplicity we assume $B = [0\ 0\ \cdots\ 0\ b_0]^T$ with scalar $b_0 \neq 0$ in system (1) in a region of interest $x \in \Omega_x$.

We first consider the simplest case of i) all state variables are measurable; and ii) time delay is constant and known, i.e. $\tau(t) = \bar{\tau}$. We now introduce a set of VDVs $x_d(t) = [x_{d1}(t)\ x_{d2}(t) \cdots\ x_{dn}(t)]^T$ which lead to a virtual reference model

$$
\dot{x}_d(t) = \sum_{i=1}^{r} \mu_i(z(t)) \left\{ A_i x_d(t) + A_{di} x_d(t - \bar{\tau}) + B(u(t) - u_k(t)) \right\}, \tag{2}
$$

where $u_k(t)$ is the fuzzy controller to be determined. Decomposing (2), we have a constraint of kinematics

$$0 = \dot{x}_{d\ell}(t) - \sum_{i=1}^{r} \mu_i(z(t))[A_{i,\ell} x_d(t) + A_{di,\ell} x_d(t - \bar{\tau})] \tag{3}$$

and a control input

$$u(t) = b_0^{-1} \left\{ \dot{x}_{dn}(t) - \sum_{i=1}^{r} \mu_i(z(t))[A_{i,n} x_d(t) + A_{di,n} x_d(t - \bar{\tau})] \right\} + u_k(t), \tag{4}$$

where $x_{d\ell}(t)$ is the ℓth (for $\ell = 1, 2, \cdots, n-1$) element of $x_d(t)$; $A_{i,\ell}$ and $A_{di,\ell}$ is the ℓth row of A_i and A_{di} matrix, respectively. We can then implement the control as (4) whereas the virtual desired vector $x_d(t)$ satisfying (3) will be determined later.

Define tracking error $x_h(t) = x(t) - x_d(t)$. The fuzzy controller based on PDC is as follows:

Controller Rule i :

IF $z_1(t)$ is F_{1i} and $\cdots$ and $z_f(t)$ is F_{fi} THEN

$u_k(t) = -K_i(x(t) - x_d(t)),$

where K_i are constant control gains to be determined. The inferred output of the controller

$$u_k(t) = -\sum_{i=1}^{r} \mu_i(z(t)) K_i(x(t) - x_d(t)), \tag{5}$$

leading to the error dynamics

$$\dot{x}_h(t) = \sum_{i=1}^{r} \mu_i(z(t))[H_i x_h(t) + A_{di} x_h(t - \bar{\tau}) + w(t)], \tag{6}$$

where $H_i = A_i - BK_i$.

The control objective is to drive all system states track the virtual desired states with H_∞ performance

$$\int_0^{t_f} x_h^T(t)Rx_h(t)dt \leq \frac{1}{\rho^2}\int_0^{t_f} \|w(t)\|^2 dt,$$

where t_f is terminal time of control; ρ is a prescribed value denoting the effect of $w(t)$ on $x_h(t)$; and R is a symmetric positive definite matrix.

Theorem 1: A controller (4) and a set of VDVs such that the system (6) is stabilizable to a prescribed $\rho > 0$ with H_∞ performance, if there exist a symmetric positive definite matrix P, matrices K_i $(i = 1, 2, ..., r)$ and $\Lambda > 0$ satisfying the following LMIs:

$$\begin{bmatrix} A_iX + XA_i^T + BM_i + M_i^TB^T + Q + \rho^2 I & (*) & (*) \\ XA_{di}^T & -Q & (*) \\ X & 0 & -R^{-1} \end{bmatrix} < 0, \quad (7)$$

where $X = P^{-1}$, $M_i = K_iP^{-1}$, $Q = X\Lambda X$, and $(*)$ is denotes the transposed elements in the symmetric positions.

Proof: Consider a Lyapunov-Krasovskii functional $V = x_h^T(t)Px_h(t) + \int_{t-\bar{\tau}}^{t} x_h^T(\sigma)\Lambda x_h(\sigma)d\sigma$. Define a function

$$\begin{aligned} J &= x_h^T(t)Rx_h(t) - \frac{1}{\rho^2}w^T(t)w(t) + \dot{V} \quad (8) \\ &\leq \sum_{i=1}^{r} \mu_i(z(t))x_h(t) \\ &\quad \left(H_i^TP + PH_i + R + \Lambda + \rho^2PP + PA_{di}\Lambda^{-1}A_{di}^TP\right) x_h(t), \end{aligned}$$

where we use the fact $2x_h^T(t)Pw(t) \leq \rho^2 x_h^T(t)PPx_h^T(t) + \frac{1}{\rho^2}w^T(t)w(t)$. If the following matrix inequality

$$H_i^TP + PH_i + R + \Lambda + \rho^2PP + PA_{di}\Lambda^{-1}A_{di}^TP < 0 \quad (9)$$

holds, then $J < 0$. By integrating (8) from 0 to t_f, the inequality $\int_0^{t_f} [x_h^T(t)Rx_h(t) - \frac{1}{\rho^2}w(t)^T w(t) + \dot{V}]dt \leq 0$. For initial condition $V(0) = 0$, we have $V(t_f) + \int_0^{t_f} [x_h^T(t)Rx_h(t) - \frac{1}{\rho^2}w(t)^T w(t)]dt \leq 0$. Since $V(t_f) \geq 0$, we arrive with $\int_0^{t_f} x_h^T(t)\Lambda x_h(t)dt \leq \frac{1}{\rho^2}\int_0^{t_f} \|w(t)\|^2 dt$. Therefore, the tracking control has H_∞ performance. Applying Schur's complement, we have (7). ■

Remark 1: Note that if $w(t) = 0$, the system (6) is asymptotically stable if there exists a symmetric and positive definite matrix P, some matrices K_i $(i = 1, 2, ..., r)$ and $Q = X\Lambda X > 0$ satisfying the following LMIs:

$$\begin{bmatrix} A_i X + XA_i^T + BM_i + M_i^T B^T + Q & (*) \\ XA_{di}^T & -Q \end{bmatrix} < 0 \tag{10}$$

where $X = P^{-1}$, $F_i = K_i X$ and $Q = X\Lambda X$.

3. Robust Output Tracking Controller Design

Consider the more practical case assuming some states are immeasurable and time-varying delay is unknown. We consider two relationships of premise variables between plant and observer: matching or mismatched.

3.1. Matching Premise Variables

Consider here the observer's premise variable $z(t)$ is same as the plant. In this case, the proposed fuzzy observer rules are

Observer Rule i :

IF $z_1(t)$ is F_{1i} and $\cdots$ and $z_f(t)$ is F_{fi} THEN

$$\begin{aligned} \dot{\hat{x}}(t) &= A_i\hat{x}(t) + A_{di}\hat{x}(t - \tau_m) + Bu(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned}$$

where $\hat{x}(t) \in R^n$ and $\hat{y}(t) \in R$ are state and output estimation, respectively; τ_m is a suitable value of time-delay (where in practical situations, this suitable value is tunable via trial and error); and L_i are observer gains to be determined. Therefore, the inferred output

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{r} \mu_i(z(t)) \left[A_i \hat{x}(t) + A_{di}\hat{x}(t-\tau_m) + Bu(t) \right.\\ &\quad \left. + L_i(y(t) - \hat{y}(t))\right] \qquad (11)\\ \hat{y}(t) &= C\hat{x}(t).\end{aligned}$$

Define estimation error $e(t) = x(t) - \hat{x}(t)$, where the time derivative

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(z(t))[(A_i - L_iC)e(t) + A_{di}e(t-\tau_m) - L_i v(t) + \bar{w}(t)],$$

where $\bar{w}(t) = \sum_{i=1}^{r} \mu_i(z(t))A_{di}[x(t-\tau(t)) - \hat{x}(t-\tau_m)] + w(t)$. Design a virtual reference model

$$\dot{x}_d(t) = \sum_{i=1}^{r} \mu_i(z(t))[A_i x_d(t) + A_{di}x_d(t-\tau) + B(u(t) - u_k(t)). \qquad (12)$$

Decomposing (12), the constraint of kinematics

$$0 = \dot{x}_{d\ell}(t) - \sum_{i=1}^{r} \mu_i(\hat{z}(t))[A_{i,\ell}x_d(t) + A_{di,\ell}x_d(t-\tau_m)], \qquad (13)$$

and control input

$$u(t) = b_0^{-1}\left(\dot{x}_{dn}(t) - \sum_{i=1}^{r} \mu_i(z(t))[A_{i,n}x_d(t) + A_{di,n}x_d(t-\tau)\right) + u_k(t). \qquad (14)$$

Now, the fuzzy controller's rules are

Controller Rule i :

IF $z_1(t)$ is F_{1i} and $\cdots$ and $z_f(t)$ is F_{fi} THEN

$u_k(t) = -K_i(\hat{x}(t) - x_d(t)),$

where K_i are the control gains to be determined. The inferred output of the controller

$$u_k(t) = -\sum_{i=1}^{r} \mu_i(z(t))K_i(\hat{x}(t) - x_d(t)). \tag{15}$$

Therefore, the error dynamic

$$\dot{x}_h(t) = \sum_{i=1}^{r} \mu_i(z(t))[(A_i - BK_i)x_h(t) + A_{di}x_h(t-\tau_m) + L_iCe(t) + L_iv(t)].$$

Hence, the closed-loop error system

$$\dot{x}_e = \sum_{i=1}^{r} \mu_i(z(t))[G_ix_e(t) + M_ix_e(t-\tau) + E_i\bar{w}(t)], \tag{16}$$

where $x_e(t) = [e^T(t)\ x_h^T(t)]^T$, $\bar{w}(t) = [v(t)^T\ \bar{w}(t)^T]^T$,

$$G_i = \begin{bmatrix} A_i - L_iC & 0 \\ L_iC & A_i - BK_i \end{bmatrix}$$

$$M_i = \begin{bmatrix} A_{di} & 0 \\ 0 & A_{di} \end{bmatrix}, E_i = \begin{bmatrix} -L_i & I \\ L_i & 0 \end{bmatrix}.$$

Theorem 2: A controller (14) and a set of VDVs such that the close-loop fuzzy system (16) is stabilizable to a prescribed $\rho > 0$ with H_∞ performance, if there exist a symmetric and positive definite matrix P, some matrices K_i $(i = 1, 2, ..., r)$ and $\Lambda > 0$ satisfying the following LMIs:

$$\begin{bmatrix} G_i^TP + PG_i + R + \Lambda & (*) & (*) \\ M_i^TP & -\Lambda & (*) \\ E_i^TP & 0 & \frac{-1}{\rho^2}I \end{bmatrix} < 0$$

where $(*)$ is denotes the transposed elements in the symmetric positions.

Proof: Consider a Lyapunov-Krasovskii functional as $V(t) = x_e^T(t)Px_e(t) + \int_{t-\tau_m}^{t} x_e^T(\lambda)\Lambda x_{e2}(\lambda)d\lambda$. The control objective is required to satisfy

$$\int_0^{t_f} x_e^T(t)Rx_e(t)dt \leq \frac{1}{\rho^2}\int_0^{t_f} \|\bar{w}(t)\|^2 dt \tag{17}$$

with t_f is terminal time of control, ρ is a prescribed value denotes the effect of $\bar{w}(t)$ on $x_e(t)$, and R is a positive definite matrix. Define a function

$$\begin{aligned} J &= x_e^T(t)Rx_e(t) - \frac{1}{\rho^2}\bar{w}(t)^T\bar{w}(t) + \dot{V} \\ &\leq \sum_{i=1}^{r}\mu_i(z(t))\bar{x}_e^T(t)\left[G_i^TP + PG_i + R + \Lambda + PM_i\Lambda^{-1}M_i^TP \right. \\ &\quad \left. +\rho^2 PE_iE_i^TP\right]\bar{x}_e(t) \end{aligned} \tag{18}$$

where we use the facts $x_e^T(t)PM_ix_e(t-\tau_m) \leq x_e^T(t)PM_i\Lambda^{-1}M_i^TPx_e(t) + \rho^2x_e^T(t-\tau_m)\Lambda x_e(t-\tau_m)$ and $2x_e^T(t)PE_i\bar{w}(t) \leq \rho^2x_e^T(t)PE_iE_i^TPx_e(t) + \frac{1}{\rho^2}\bar{w}^T(t)\bar{w}(t)$. If the condition

$$G_i^TP + PG_i + R + \Lambda + PM_i\Lambda^{-1}M_i^TP + \rho^2PE_iE_i^TP < 0$$

holds, then $J < 0$. By integrating (18) from 0 to t_f, we obtain

$$\int_0^{t_f}\left(x_e^T(t)Rx_e(t) - \frac{1}{\rho^2}\bar{w}(t)^T\bar{w}(t) + \dot{V}\right)dt \leq 0. \tag{19}$$

Assuming initial condition $V(0) = 0$, we have $V(t_f) + \int_0^{t_f}[x_e^T(t)Rx_e(t) - \frac{1}{\rho^2}\bar{w}^T(t)\bar{w}(t)]dt \leq 0$. Since $V(t_f) \geq 0$, we arrive with $\int_0^{t_f} x_e^T(t)Rx_e(t)dt \leq \frac{1}{\rho^2}\int_0^{t_f}\|\bar{w}(t)\|^2 dt$. This means that the overall system (16) has H_∞ performance.

3.2. Mismatched Premise Variables

Consider here the premise variables of observer depends on estimated states $\hat{z}(t)$ which leads to membership functions $\mu_i(\hat{z}(t)) \neq \mu_i(z(t))$. The fuzzy observer rules are

$$\begin{aligned}
& \textit{Observer Rule } i: \\
& \text{IF } \hat{z}_1(t) \text{ is } F_{1i} \text{ and } \cdots \text{ and } \hat{z}_f(t) \text{ is } F_{fi} \text{ THEN} \\
\dot{\hat{x}}(t) &= A_i\hat{x}(t) + A_{di}\hat{x}(t-\tau_m) + Bu(t) + L_i(y(t) - \hat{y}(t)) \\
\hat{y}(t) &= C\hat{x}(t)
\end{aligned}$$

where $\hat{z}_1(t) \sim \hat{z}_f(t)$ are the observer's premise variables composed of estimated states. The inferred output

$$\begin{aligned}
\dot{\hat{x}}(t) &= \sum_{i=1}^{r} \mu_i(\hat{z}(t))[A_i\hat{x}(t) + A_{di}\hat{x}(t-\tau_m) \\
&\quad + Bu(t) + L_i(y(t) - \hat{y}(t))] \qquad (20) \\
\hat{y}(t) &= C\hat{x}(t).
\end{aligned}$$

The error dynamics

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\hat{z}(t))[(A_i - L_iC)e(t) + A_{di}e(t-\tau_m) - L_iv(t)] + \bar{w}(t) + h_1(t),$$

where $h_1(t) = \sum_{i=1}^{r} (\mu_i(z(t) - \mu_i(\hat{z}(t)) [A_ix(t) + A_{di}x(t-\tau(t))]$ and $\bar{w}(t) = \sum_{i=1}^{r} \mu_i(\hat{z}(t))A_{di}[x(t-\tau(t)) - \hat{x}(t-\tau_m)] + w(t)$. Therefore the time derivative of tracking error

$$\dot{x}_h(t) = \sum_{i=1}^{r} \mu_i(\hat{z}(t))[A_i\hat{x}(t) + A_{di}\hat{x}(t-\tau_m) + Bu(t) + L_iCe(t) + L_iv(t)] - \dot{x}_d(t). \qquad (21)$$

The fuzzy controller rules are

$$\begin{aligned}
& \textit{Controller Rule } i: \\
& \text{IF } \hat{z}_1(t) \text{ is } F_{1i} \text{ and } \cdots \text{ and } \hat{z}_f(t) \text{ is } F_{fi} \text{ THEN} \\
& u_k(t) = -K_i(\hat{x}(t) - x_d(t)),
\end{aligned}$$

where K_i are control gains to be determined. Then the inferred output of the controller

$$u_k(t) = -\sum_{i=1}^{r} \mu_i(\hat{z}(t))K_i(\hat{x}(t) - x_d(t)). \tag{22}$$

Design a virtual reference model

$$\dot{x}_d(t) = \sum_{i=1}^{r} \mu_i(\hat{z}(t))[A_i x_d(t) + A_{di} x_d(t-\tau) + B(u(t) - u_k(t)). \tag{23}$$

Therefore the constraint of kinematics

$$0 = \dot{x}_{d\ell}(t) - \sum_{i=1}^{r} \mu_i(\hat{z}(t))[A_{i,\ell} x_d(t) + A_{di,\ell} x_d(t-\tau_m)]$$

and a new control input

$$u(t) = b_0^{-1}\left(\dot{x}_{dn}(t) - \sum_{i=1}^{r} \mu_i(\hat{z}(t))[A_{i,n} x_d(t) + A_{di,n} x_d(t-\tau_m)\right) + u_k(t). \tag{24}$$

The error dynamics

$$\dot{x}_h(t) = \sum_{i=1}^{r} \mu_i(\hat{z}(t))[(A_i - BK_i)x_h(t) + A_{di} x_h(t-\bar{\tau}) + L_i C e(t) + L_i v(t)].$$

Hence, the closed-loop error system

$$\dot{x}_e = \sum_{i=1}^{r} \mu_i(\hat{z}(t))[G_i x_e(t) + M_i x_e(t-\bar{\tau}) + E_i \bar{w}(t) + \bar{h}(t)] \tag{25}$$

where $\bar{h}(t) = [h_1(t)^T \ 0]^T$,

$$G_i = \begin{bmatrix} A_i - L_i C & 0 \\ L_i C & A_i - BK_i \end{bmatrix}$$

$$M_i = \begin{bmatrix} A_{di} & 0 \\ 0 & A_{di} \end{bmatrix}, \ E_i = \begin{bmatrix} -L_i & I \\ L_i & 0 \end{bmatrix}$$

Remark 2: Note that the membership functions $F_{ji}(z(t))$ satisfy $F_{ji}(z(t)) - F_{ji}(\hat{z}(t)) = \eta_{ji}^T(z(t) - \hat{z}(t))$ for some bounded function vector η_{ji}^T and premise variables $z(t)$, $\hat{z}(t)$ in the region of interest. Since premise variables are states or combination of states, the relationship $F_{ji}(x(t)) - F_{ji}(\hat{x}(t)) = \eta_{ji}^T(x(t) - \hat{x}(t))$. Therefore the function error is proportional to the estimation error where

$$\begin{aligned}
\mu_i(x(t)) - \mu_i(\hat{x}(t)) &= \eta_{1i}^T e(t) \prod_{k=2}^{f} F_{ki}(x(t)) + F_{1i}(\hat{x}(t))\eta_{2i}^T e(t)) \prod_{k=3}^{f} F_{ki}(x(t)) \\
&\quad + \cdots + \prod_{k=1}^{f-2} F_{ki}(\hat{x}(t))\eta_{(f-1)i}^T e(t) F_{fi}(x(t)) \\
&\quad + \prod_{k=1}^{f-1} F_{ki}(\hat{x}(t))\eta_{fi}^T e(t) \\
&= \Gamma_i^T e(t)
\end{aligned}$$

for some bounded function vector Γ_i. In light of the above relationship, we have

$$\begin{aligned}
h_2(t) &= \sum_{i=1}^{r} (\mu_i(x(t)) - \mu_i(\hat{x}(t))) \left[A_i x(t) + A_{di} x(t - \tau(t))\right] \\
&= \sum_{i=1}^{r} \left[(A_i x(t) + A_{di} x(t - \tau(t))) \Gamma_i^T\right] e(t).
\end{aligned}$$

Suppose an upper bound for $x(t)$, $x(t-\tau(t))$ and $\tau(t)$ exists in the region of interest. The term $h_1(t)$ therefore satisfies [36, 37]

$$h_1^T(t) h_1(t) \leq e^T(t) U^T U e(t)$$

with a symmetric positive-definite matrix U depend on Γ_i^T, $x(t)$ and $x(t - \tau(t))$. We can attenuate $h_1(t)$ from affecting control performance by choosing suitable observer gains L_i and controller gains K_i shown as follows.

Theorem 3: A controller (24) and a set of VDVs such that system (25) is stabilizable to a prescribed $\rho > 0$ with H_∞ performance, if there exist a symmetric and positive definite matrix P, some matrices K_i $(i = 1, 2, ..., r)$ and $\Lambda > 0$ satisfying the following LMIs:

$$\begin{bmatrix} G_i^T P + PG_i + R + \Lambda + PP + \bar{U}^T\bar{U} & (*) & (*) \\ M_i^T P & -\Lambda & (*) \\ E_i^T P & 0 & \frac{-1}{\rho^2} I \end{bmatrix} < 0 \tag{26}$$

where $(*)$ is denotes the transposed elements in the symmetric positions and $\bar{U}$ =block-diag$\{U \ 0\}$.

Proof: Consider a Lyapunov-Krasovskii functional $V = x_e^T(t)Px_e(t) + \int_{t-\bar{\tau}}^t x_e^T(\lambda)\Lambda x_e(\lambda)d\lambda$. The control objective must satisfy (17). Taking the derivative of Lyapunov-Krasovskii functional and applying (18) along with (25), the function

$$\begin{aligned} J &= x_e^T(t)Rx_e(t) - \frac{1}{\rho^2}\bar{w}(t)^T\bar{w}(t) + \dot{V} \\ &\le \sum_{i=1}^r \mu_i(\hat{z}(t))\bar{x}_e^T(t)\left[\bar{G}_i + P\ M_i^T\Lambda^{-1}M_iP + \rho^2 x_e^T(t)PE_iE_i^TP\right]\bar{x}_e(t), \end{aligned} \tag{27}$$

where $\bar{G}_i = G_i^T P + PG_i + R + \Lambda + PP + \bar{U}^T\bar{U}$. Since

$$\begin{aligned} x_e^T(t)PM_ix_e(t-\tau_m) &\le x_e^T(t)PM_i\Lambda^{-1}M_i^TPx_e(t) + x_e^T(t-\tau_m)\Lambda x_e(t-\tau_m) \\ 2x_e^T(t)PE_i\bar{w}(t) &\le \rho^2 x_e^T(t)PE_iE_i^TPx_e(t) + \frac{1}{\rho^2}\bar{w}^T(t)\bar{w}(t) \\ 2x_e^T(t)P\bar{h}(t) &\le x_e^T(t)PPx_e(t) + x_e^T(t)\bar{U}^T\bar{U}x_e(t). \end{aligned}$$

If the condition (26) holds then $J < 0$. By integrating (27) from 0 to t_f, we obtain (19). By assuming initial condition $V(0) = 0$, we have $V(t_f) + \int_0^{t_f}\left(x_e^T(t)Rx_e(t) - \frac{1}{\rho^2}\bar{w}(t)^T\bar{w}(t)\right)dt \le 0$. Since $V(t_f) \ge 0$, we have $\int_0^{t_f} x_e^T(t)Rx_e(t)dt \le \frac{1}{\rho^2}\int_0^{t_f}\|\bar{w}(t)\|^2 dt$. This means that the overall system has H_∞ performance.

3.3. Procedure of Determining Observer and Controller Gain

We will now show the procedure for determining the observer and controller gains for the most complex case where time-varying delay exist and premise variables between observer and plant are mismatched. Note that for more ideal cases, we can follow the same design procedures.

From Thm. 3, we can determine the observer gains L_i and controller gains K_i and $P > 0$, $\Lambda > 0$ satisfying (26) for a given $\rho > 0$, $R = R^T > 0$ and U. We provide two procedures, two step or three step to solve the LMIs (26).

Two-step procedure: The concept of two-steps procedure to determine K_i , L_i, P and Λ are introduced by [38]. The main idea begins from defining P, Λ and R as block-diagonal form, i.e. P =block-diag$\{P_1\ P_2\}$, Λ =block-diag$\{\Lambda_1\ \Lambda_2\}$, and R =block-diag$\{R_1\ R_2\}$. According to (26), we therefore have

$$\begin{bmatrix} \Delta_i & (*) & (*) & (*) & (*) & (*) \\ P_2 L_i C & \Psi_i & (*) & (*) & (*) & (*) \\ A_{di}^T P_1 & 0 & -\Lambda_1 & (*) & (*) & (*) \\ 0 & A_{di}^T P_2 & 0 & -\Lambda_2 & (*) & (*) \\ -L_i^T P_1 & L_i^T P_2 & 0 & 0 & \frac{-1}{\rho^2} I & (*) \\ P_1 & 0 & 0 & 0 & 0 & \frac{-1}{\rho^2} I \end{bmatrix} < 0, \tag{28}$$

where $\Delta_i = (A_i - L_i C)^T P_1 + P_1(A_i - L_i C) + R_1 + \Lambda_1 + P_1 P_1 + U^T U$ and $\Psi_i = (A_i - BK_i)^T P_2 + P_2(A_i - BK_i) + P_2 P_2 + R_2 + \Lambda_2$.

Step 1 The matrix inequalities (28) implies

$$\begin{bmatrix} \Delta_i & (*) & (*) \\ A_{di}^T P_1 & -\Lambda_1 & (*) \\ -L_i^T P_1 & 0 & \frac{-1}{\rho^2} I \end{bmatrix} < 0. \tag{29}$$

From Schur's complement, we have $(A_i - L_i C)^T P_1 + P_1(A_i - L_i C) + R_1 + \Lambda_1 + U^T U + P_2 A_{di} \Lambda_2^{-1} A_{di}^T P_2 + P_1 P_1 + \rho^2 P_1 L_i L_i^T P_1 < 0$ and

$$\begin{bmatrix} A_i^T P_1 + P_1 A_i - N_i C - C^T N_i^T + R_1 + \Lambda_1 + U^T U & (*) & (*) & (*) \\ A_{di}^T P_1 & -\Lambda_2 & (*) & (*) \\ -L_i^T P_1 & 0 & \frac{-1}{\rho^2} I & (*) \\ P_1 & 0 & 0 & -I \end{bmatrix} < 0, \tag{30}$$

where $N_i = P_1 L_i$.

Step 2 According to (28) and by the Schur's complement, we have

$$\begin{bmatrix} \Delta_i & (*) & (*) & (*) & (*) & (*) & (*) \\ L_i C & \bar{\Psi}_i & (*) & (*) & (*) & (*) & (*) \\ A_{di}^T P_1 & 0 & -\Lambda_1 & (*) & (*) & (*) & (*) \\ 0 & A_{di}^T X_2 & 0 & -Q_2 & (*) & (*) & (*) \\ -L_i^T P_1 & L_i^T & 0 & 0 & \frac{-1}{\rho^2} I & (*) & (*) \\ P_1 & 0 & 0 & 0 & 0 & \frac{-1}{\rho^2} I & (*) \\ 0 & X_2 & 0 & 0 & 0 & 0 & -R_2^{-1} \end{bmatrix} < 0 \tag{31}$$

where $X_2 = P_2^{-1}$, $\bar{\Psi}_i = A_i X_2 + X_2 A_i^T - B F_i - F_i^T B^T + Q_2 + I$, $F_i = K_i X_2$, and $Q_2 = X_2 \Lambda_2 X_2$. Once (30) is feasible, the controller gains $K_i = F_i P_2$, $\Lambda_2 = P_2 Q_2 P_2$. Then, if (31) is feasible, the observer gains $L_i = P_1^{-1} N_i$.

Three-steps procedure: In this procedure, the matrices P and Λ do not need to be block-diagonal, making LMIs (26) more feasible with computational load tradeoff.

Since (28) imply $\Psi_i + P_2 A_{di} \Lambda_2^{-1} A_{di}^T P_2 < 0$, we have by Schur's complement

$$\begin{bmatrix} \bar{\Psi}_i & (*) & (*) \\ A_{di}^T X_2 & -Q_2 & (*) \\ X_2 & 0 & -R_2^{-1} \end{bmatrix} < 0. \tag{32}$$

Step 1 Solve (30) for observer gain L_i;

Step 2 Solve (32) for controller gains K_i;

Step 3 From Step 1 and Step 2, if the gain K_i and L_i are available, then solve matrices $P > 0$ and $\Lambda > 0$ satisfying the following LMIs:

$$\begin{bmatrix} G_i^T P + P G_i + R + \Lambda + \bar{U}^T \bar{U} & P M_i & P E_i & P \\ M_i^T P & -\Lambda & 0 & 0 \\ E_i^T P & 0 & \frac{-1}{\rho^2} I & 0 \\ P & 0 & 0 & -I \end{bmatrix} < 0$$

where $P \in R^{2n \times 2n}$ and $\Lambda \in R^{2n \times 2n}$.

Note that in the three-steps procedure, the matrices P_1, P_2, Λ_1, and $\Lambda_2 \in R^{n \times n}$ in Step 1 and Step 2. Meanwhile the matrices $P \in R^{2n \times 2n}$ and $\Lambda \in R^{2n \times 2n}$ in Step 3.

Remark 3 The two-step procedure is more suitable and when (26) is satisfied. However, since R, P and Λ are in block-diagonal form, the procedure is more conservative in comparison to the three-step procedure.

4. Virtual Desired Variables Design

Analogous to the above discussion, the most complex case is when we consider time-varying delay with mismatched premise variables between observer and plant. We will show in the following the detailed procedure for designing VDVs $x_d(t)$. Note that for more ideal cases, we can follow the same design procedures.

We design the VDVs according to the following equation

$$\begin{aligned} y_d(t) &= x_{di}(t) && (33)\\ \dot{x}_{d\ell}(t) &= \sum_{i=1}^{r} \mu_i(\hat{z}(t))[A_{i,\ell}x_d(t) + A_{di,\ell}x_d(t-\bar{\tau})], && (34)\end{aligned}$$

where $\ell = 1,\ 2,\ \cdots,\ n-1$. By solving the constraint (33) and (34), we obtain VDVs $x_d(t)$ and the tracking error $x_h(t)$. Therefore we can design the controller

$$\begin{aligned} u(t) &= b_0^{-1}\left(\dot{x}_{dn}(t) - \sum_{i=1}^{r} \mu_i(\hat{z}(t))[A_{i,n}x_d(t) + A_{di,n}x_d(t-\bar{\tau})\right)\\ &\quad - \sum_{i=1}^{r} \mu_i(\hat{z}(t))K_i(\hat{x}(t) - x_d(t)). && (35)\end{aligned}$$

To implement the control input (35), we require the following information, i) derivative signal $\dot{x}_{dn}(t)$; ii) state estimation $\hat{x}(t)$; suitable time delay τ_m. Three cases of output tracking are discussed as follows.

Case 1 (reference output equals desired state**):** Since desired output $y_d(t) = x_{dn}(t)$ and $\dot{y}_d(t) \in L_2$, we solve (34) for VDVs and use observer (20) to estimate immeasurable states. Therefore, the controller can be implemented as (35).

Case 2 (reference output is not equal to the desired state): Since $y_d(t) \neq x_{dn}(t)$, controller design might not be as straightforward. To cope with this, we introduce an approximate signal $\dot{x}_{dn}(t) \approx (x_{dn}(t) - x_{dn}(t-t_s))\,/t_s$, where t_s is the sampling period. We may then attenuate the arising approximation errors by suitably choosing observer gains L_i and controller gains K_i.

Case 3 (output regulation): This is an ideal case where the desired state vector $\bar{x}_d$ is constant and $\hat{x}(t) = x(t) = \bar{x}_d$ such that $\mu_i(z(t)) = \mu_i(\hat{z}(t))$, $x_d(t-\bar{\tau}) = \bar{x}_d$ and $x(t-\tau(t)) = \bar{x}_d$. Thus,

$$\sum_{i=1}^{r} \mu_i(z_d))[A_{i,\ell}x_d(t) + A_{di,\ell}x_d(t-\bar{\tau})] = 0,\ \ell = 1,\ 2,\ \cdots,\ n-1 \qquad (36)$$

where $z_d = \bar{x}_d$. There exists $n-1$ equations in (36). If we assign the desired output $y_d(t) = \bar{x}_{dn}$, then $\dot{x}_{dn}(t) = 0$. Note that the desired state $\bar{x}_d$ should be properly chosen such that $\bar{x}_d \in \Omega_x$.

According the analysis above, the overall structure of the controlled output tracking control system in Fig 1. Therefore, the design procedure for controlled output tracking control is summarized as follows:

Design procedure:

Step 1 Construct the T-S fuzzy model for the nonlinear time-delay system in (1).

Step 2 Given an attenuation level ρ, follow the three-steps procedure to obtained controller and observer gain.

Step 3 Design the VDVs from (33) and (34), such that the error state $x_h(t)$ is obtained.

Step 4 Implement the controller as (35).

According the analysis above, the design constraint of VDVs and controller are for various cases are summarized in Table 1.

5. Simulation Results

Consider a continuous stirred tank reactor (CSTR) from [39] represented in dimensionless variables

$$\begin{aligned}\dot{x}_1(t) &= f_1(x(t)) + (1 - \frac{1}{\lambda})x_1(t - \tau(t)) \\ \dot{x}_2(t) &= f_2(x(t)) + (1 - \frac{1}{\lambda})x_1(t - \tau(t)) + \beta u(t)\end{aligned}$$

where

$$\begin{aligned}f_1(x(t)) &= \frac{-1}{\lambda}x_1(t) + D_a(1 - x_1(t))\exp(\frac{x_2(t)}{1 + x_2(t)/\gamma_0}) \\ f_2(x(t)) &= (\frac{-1}{\lambda} + \beta)x_2(t) + HD_a(1 - x_1(t))\exp(\frac{x_2(t)}{1 + x_2(t)/\gamma_0}).\end{aligned}$$

The state $x_1(t)$ corresponds to the conversion rate of the reaction $0 \leq x_1(t) \leq 1$; and $x_2(t) > 0$ is the dimensionless temperature. We set $x_2(t) \in \Omega_2$ with $\Omega_2 = \{x_2 |\ 0.1 \leq x_2 \leq 6\}$. The parameters $\gamma_0 = 20$, $H = 8$, $\beta = 0.3$, $D_a = 0.072$, $\lambda = 0.8$, and $\bar{\tau} = 2$. Applying the design methodology mentioned in previous sections, we arrive with the following:

Step 1 Define membership functions $w_{11} = (g_1 - d_{12})/(d_{11} - d_{12}), w_{12} = 1 - w_{11}$, $w_{21} = (g_2 - d_{22})/(d_{21} - d_{22}), w_{22} = 1 - w_{21}$, where $g_1 = \exp(x_2(t)/(1 + x_2(t)/\gamma_0))$, $g_2 = \exp(x_2(t)/(1 + x_2(t)/\gamma_0))/x_2$, $d_{11} = max_{x_2 \in \Omega_2} g_1 = 101.0267$, $d_{12} = min_{x_2 \in \Omega_2} g_1 = 1.1046$, $d_{21} = max_{x_2 \in \Omega_2} g_2 = 16.8378$, and $d_{22} =$

$min_{x_2 \in \Omega_2} g_1 = 2.5789$. Therefore the subsystem matrices

$$A_1 = \begin{bmatrix} (-1/\lambda) - D_a d_{11} & D_a d_{21} \\ -HD_a d_{11} & -(1/\lambda + \beta) + HD_a d_{21} \end{bmatrix}$$
$$A_2 = \begin{bmatrix} (-1/\lambda) - D_a d_{11} & D_a d_{22} \\ -HD_a d_{11} & -(1/\lambda + \beta) + HD_a d_{22} \end{bmatrix}$$
$$A_3 = \begin{bmatrix} (-1/\lambda) - D_a d_{12} & D_a d_{21} \\ -HD_a d_{12} & -(1/\lambda + \beta) + HD_a d_{21} \end{bmatrix}$$
$$A_4 = \begin{bmatrix} (-1/\lambda) - D_a d_{12} & D_a d_{22} \\ -HD_a d_{12} & -(1/\lambda + \beta) + HD_a d_{22} \end{bmatrix}$$

$$A_d = \begin{bmatrix} 1 - 1/\lambda & 0 \\ 0 & 1 - 1/\lambda \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \beta \end{bmatrix}$$

Step 2: Solve controller and observer gains following the three-step procedure. Given an attenuation level $\rho = 1.1$, matrices $R = 0.01diag(1, 1, 1, 1)$, $U = diag(0.01, 0.01)$, the solved controller gains $K_1 = [\ -175.2199 \quad 44.9349\ ]$, $K_2 = [\ -180.2105 \quad 16.2964\ ]$, $K_3 = [\ 8.2168 \quad 42.8092\ ]$, $K_4 = [\ 3.3857 \quad 14.2047\ ]$, observer gains $L_1 = [\ 16.3823 \quad 165.0860\ ]^T$, $L_2 = [\ 16.2812 \quad 164.2247\ ]^T$, $L_3 = [\ 16.4024 \quad 165.2433\ ]^T$, $L_4 = [\ 16.3008 \quad 164.3701\ ]^T$, and matrices

$$P = \begin{bmatrix} 0.8749 & -0.0805 & 0.0202 & 0.0040 \\ -0.0805 & 0.0191 & 0.0045 & -0.0071 \\ 0.0202 & 0.0045 & 0.0757 & -0.0011 \\ 0.0040 & 0.0071 & -0.0011 & 0.0077 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} 0.5758 & -0.1192 & -0.0121 & 0.0369 \\ -0.1192 & 0.2807 & -0.0124 & -0.0268 \\ -0.0121 & -0.0124 & 0.1057 & -0.0194 \\ 0.0369 & -0.0268 & -0.0194 & 0.0217 \end{bmatrix}.$$

Step 3: Ensure desired state satisfies constraint

$$0 = \dot{x}_{d1}(t) - \sum_{i=1}^{r} \mu_i(z(t))[A_{i,1}x_d(t) + A_{di,1}x_d(t-\tau)]. \tag{37}$$

We consider the desired output to be sine wave $y_d(t) = x_{d1}(t) = 0.5 + 0.2\sin(t)$ or multi-step signal. To ensure a smooth multi-step signal, we assign $x_{d1}(t) = 4x_{r1}$ and $\dot{x}_{d1}(t) = 4x_{r2}(t)$ with reference state x_{r1} and x_{r2}. The reference dynamic system

$$\begin{aligned} \dot{x}_{r1}(t) &= x_{r2}(t) \\ \dot{x}_{r2}(t) &= -4x_{r1}(t) - 2x_{r2}(t) + ref \end{aligned} \tag{38}$$

where ref is the multi-step signal (each step size is fixed).

Since $\dot{x}_{d1}(t)$ and $x_{d1}(t-\tau)$ are available, from (37), we have

$$x_{d2}(t) = \frac{\dot{x}_{d1}(t) - (-1/\lambda - D_a g)}{D_a z / x_2(t)}.$$

Therefore we have $x_h(t) = \hat{x}(t) - x_d(t)$.

Step 4: Implement the controller

$$u(t) = \beta^{-1}\left\{ \dot{x}_{d2}(t) - \sum_{i=1}^{r} \mu_i(z(t))[A_{i,2}x_d(t) + A_{di,2}x_d(t-\bar{\tau})] \right\} + u_k(t).$$

For this simulation, the external disturbance and measured noise are chosen as uniformly random noise with amplitude 0.01. The time-varying delay is shown in Fig. 2. The simulation results of the H_∞ control are shown in Figs.

3∼7 with desired controlled output sine wave $x_{d1}(t) = 0.5 + 0.2\sin(t)$. Note that we consider the more complex case of mismatched premise variables for all of the simulations. Figure 3 (a) shows $x_1(t)$, $\hat{x}_1(t)$ and Fig. 3 (b) shows estimation error $e_1(t)$. Figure 4 (a) shows $x_2(t)$, $\hat{x}_2(t)$ and Fig. 4 (b) shows estimation error $e_2(t)$. From Figs. 3 and 4, the result validate the proposed observer design (20). Figure 5 (a) shows $x_1(t)$, $x_{d1}(t)$ and Fig. 5 (b) shows tracking error $x_{h1}(t)$. Figure 6 (a) show $x_2(t)$, $x_{d2}(t)$ and Fig. 6 (b) shows tracking error $x_{h2}(t)$. Figure 7 is the control input $u(t)$. From Figs. 5 to 7, the results validate the proposed controller design (24).

The results of the multi-step reference signal are shown in Fig. 8∽12. Figure 8 (a) shows $x_1(t)$, $\hat{x}_1(t)$ and Fig. 8 (b) shows the estimation error $e_1(t)$. Figure 9 (a) shows $x_2(t)$, $\hat{x}_2(t)$ and Fig. 9 (b) shows estimation error $e_2(t)$. From Figs. 3 and 4, the result validate the proposed observer design (20). Figure 10 (a) shows $x_1(t)$, $x_{d1}(t)$ and Fig. 10 (b) shows tracking error $x_{h1}(t)$. Figure 11 (a) shows $x_2(t)$, $x_{d2}(t)$ and Fig. 11 (b) shows tracking error $x_{h2}(t)$. Figure 12 is the control signal. From Figs. 10 to 12, the results validate the proposed controller design (24).

From the above simulation results, we can verify the proposed control method achieve H_∞ performance for nonlinear time-delay systems with disturbance. This is aligned with the claims of Thm. 3.

6. Conclusions

This paper has presented an observer-based via VDV output control approach for a class of nonlinear systems with time-varying delay and disturbances where the advantages are: i) systematic approach to derive VDVs for controller can be design; ii) relaxes need for real reference model; iii) drops need for information of equilibrium; iv) relaxed condition is provided via three-step procedure is provided to find observer and controller gain. Finally, simulation results on a CSTR with sine-wave and multi-step reference illustrate the expected performance.

7. Acknowledgements

This work was supported by National Science Council, R.O.C., under grant NSC 100-2221-E032-024 and NSC 98-2221-E-231-004.

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Table 1: VDVs and Controller Designs for various conditions.

Conditions	VDVs	Controller
States are all available with constant time delay $\bar{\tau}$	$\dot{x}_{d\ell}(t) = \sum_{i=1}^{r} \mu_i(z(t)) \left[A_{i,\ell} x_d(t) + A_{di,\ell} x_d(t-\bar{\tau})\right]$	$u(t) = b_0^{-1} \left\{ \dot{x}_{dn}(t) - \sum_{i=1}^{r} \mu_i(z(t)) \times [A_{i,n} x_d(t) + A_{di,n} x_d](t-\bar{\tau}) \right\} + u_k(t)$ $u_k(t) = -\sum_{i=1}^{r} \mu_i(z(t)) \times K_i(x(t) - x_d(t)).$
Partial states are immeasurable with matched premise variables $\mu_i(\hat{z}(t)) = \mu_i(z(t))$ and time-varying delay	$\dot{x}_{d\ell}(t) = \sum_{i=1}^{r} \mu_i(z(t)) \left[A_{i,\ell} x_d(t) + A_{di,\ell} x_d(t-\tau_m)\right]$	$u(t) = b_0^{-1} \left\{ \dot{x}_{dn}(t) - \sum_{i=1}^{r} \mu_i(z(t)) \times [A_{i,n} x_d(t) + A_{di,n} x_d(t-\tau_m)] \right\} + u_k(t)$ $u_k(t) = -\sum_{i=1}^{r} \mu_i(z(t)) K_i(\hat{x}(t) - x_d(t))$
Partial states are immeasurable with mismatched premise variables $\mu_i(\hat{z}(t)) \neq \mu_i(z(t))$ and time-varying delay	$\dot{x}_{d\ell}(t) = \sum_{i=1}^{r} \mu_i(\hat{z}(t)) \left[A_{i,\ell} x_d(t) + A_{di,\ell} x_d(t-\bar{\tau})\right]$	$u(t) = b_0^{-1} \left\{ \dot{x}_{dn}(t) - \sum_{i=1}^{r} \mu_i(\hat{z}(t)) \times [A_{i,n} x_d(t) + A_{di,n} x_d(t-\bar{\tau})] \right\} + u_k(t)$ $u_k(t) = -\sum_{i=1}^{r} \mu_i(\hat{z}(t)) K_i(\hat{x}(t) - x_d(t))$