行政院國家科學委員會專題研究計畫　成果報告

強穩定以及穩定的 Hinfinity 控制器設計

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行政院國家科學委員會專題研究計畫成果報告

強穩定以及穩定的 H_∞ 控制器設計

Strong Stabilization and Stable H_∞ Controller Design

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一、中文摘要

本計劃研究強穩定以及穩定 H_∞ 控制器的合成問題。我們以一致性的技巧推導得到這兩個問題有解的充分條件，利用線性分式轉換分別轉換為正實控制器的設計問題以及多目標控制問題，然後採用線性矩陣不等式的技巧來求解。另外，我們也找出符合這兩個問題要求的部份控制器所形成的集合。最後提供數值例子來驗證所提出方法的實際成效。

關鍵詞：強穩定，穩定 H_∞ 控制器設計，正實控制器設計，多目標控制，線性矩陣不等式

1. Abstract

This project investigates the strong stabilization and the stable H_∞ controller design problems. Solvability conditions for the two problems are derived in a unified manner and recast via linear fractional transformation as positive real controller synthesis problem and multiobjective control problem, respectively, which can be solved via relevant linear matrix inequality (LMI) programs. In addition, explicit characterizations of some subsets of the controllers for the two problems are given. Finally, numerical examples are provided to show the effectiveness of the proposed methods.

Keywords: strong stabilization, stable H_∞ controller design, positive real controller synthesis, multiobjective control, linear matrix inequality

2. Motivation

Stable controllers are preferable for feedback control systems whenever it is possible, for several reasons such as loop stability, tracking performance, and sensitivity to disturbances. In the literature, the problem of finding a stable stabilizing controller for a given plant is referred to be strong stabilization problem. Nevanlinna-Pick interpolation technique is one of the several methods to solve the problem, see, e.g., [1,2]. However, the orders of the resulting controllers by this approach usually are very high, and the computational procedure is technically involved. In [3], a different approach was proposed. This problem was reduced to a two-block or one-block H_∞ optimization problem. The solution depends on the initial choice of a unimodular transfer function. However, no general guidelines were given on how to select this transfer function.

Stable controller design with certain

performance requirements, such as LQG, H_2, H_∞, and mixed H_2/H_∞ performance have also been studied in the literature [4-12] based on solving certain modified Riccati equations. However, many technical assumptions are often made for the existence of the solutions of the Riccati equations. Therefore, it's our purpose of this research to present a simple, yet general (less technical assumption being made) approach to solve the strong stabilization problem and the H_∞ strong stabilization problem.

3. Results

3.1 Design and parameterization of strongly stabilizing controllers

Assume the plant $P_0(s) = C(sI - A)^{-1}B$ is stabilizable and detectable, and is in positive feedback connection with the controller. Let N_l and M_l be any left coprime factorization factors of P_0 such that $P_0 = M_l^{-1}N_l$. Our result for the strong stabilization problem is stated below.

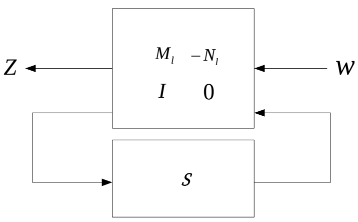

Fig. 1

Theorem 1. Suppose that the following positive real controller synthesis problem is solvable: there exists a $S = K_0$ such that, as shown in Fig. 1,

(i) the closed-loop system is internally stable,

(ii) the map T_{zw} is strictly positive real.

Then $K_o(s) = C_k(sI - A_k)^{-1}B_k + D_k$ is a strongly stabilizing controller. Furthermore, every controller in any of the following sets is strongly stabilizing.

$$K_{s1} \equiv \left\{ F_l(\tilde{J}, Q) : Q \in RH_\infty, \quad \|Q\|_\infty < \frac{1}{\|\tilde{J}_{22}\|_\infty} \right\},$$

$$K_{s2} \equiv \left\{ F_l(\tilde{J}, Q) : Q \in RH_\infty, \quad I - \tilde{J}_{22}Q \quad is \quad SPR \right\},$$

where

$$\tilde{J} = \left[\begin{array}{cc|cc} A+BF & 0 & 0 & -B \\ B_kC & A_k & B_k & 0 \\ \hline D_kC - F & C_k & D_k & I \\ C & -F_k & I & 0 \end{array} \right] =: \begin{bmatrix} \tilde{J}_{11}(s) & \tilde{J}_{12}(s) \\ \tilde{J}_{21}(s) & \tilde{J}_{22}(s) \end{bmatrix}.$$

For computation, the positive real controller synthesis problem can be efficiently solved by the relevant LMI programs in [13,14].

3.2 Design and Parameterization of H_∞ strongly stabilizing controllers

In this subsection we assume that the generalized plant G satisfies the standard assumptions for the general H_∞ control problem presented in [15,16]. It has been shown that all suboptimal H_∞ controllers $K(s)$ satisfying the suboptimal restriction $\|F_l(G,K)\|_\infty < \gamma$ can be parameterized by the formula $K = F_l(M_\infty, Q)$ with

$$M_\infty = \left[\begin{array}{c|cc} \hat{A} & \hat{B}_1 & \hat{B}_2 \\ \hline \hat{C}_1 & \hat{D}_{11} & \hat{D}_{12} \\ \hat{C}_2 & \hat{D}_{21} & \hat{D}_{22} \end{array} \right] =: \begin{bmatrix} M_{\infty 11}(s) & M_{\infty 12}(s) \\ M_{\infty 21}(s) & M_{\infty 22}(s) \end{bmatrix}$$

where M_∞ is constructed from the solutions of two Riccati equations, $Q \in RH_\infty$ and $\|Q\|_\infty < \gamma$. Hence the H_∞ strong stabilization problem becomes that of finding a $Q \in RH_\infty$ with $\|Q\|_\infty < \gamma$ which internally stabilizes $M_{\infty 22}$. Let N_l and M_l be any left coprime factorization factors of $M_{\infty 22}$ such that $M_{\infty 22} = M_l^{-1} N_l$. The H_∞ strong stabilization problem is now recast as a multiobjective control problem (with reference to Fig.2) as stated below.

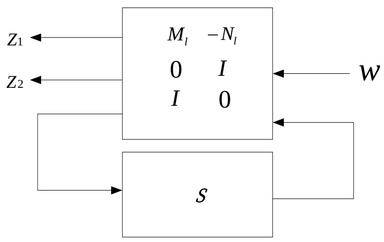

Fig. 2

Theorem 2. Suppose that a suboptimal H_∞ controller parameterization is obtained for a given H_∞ performance level γ. Suppose that the following multiobjective control problem is solvable: there exists a controller $S = Q_0$ such that, as shown in Fig. 2,

(i) the closed-loop system is internally stable,

(ii) the channel map $T_{z_1 w}$ is strictly positive real,

(iii) the channel map $T_{z_2 w}$ satisfies $\|T_{z_2 w}\|_\infty < \gamma$.

Then a stable H_∞ controller is given by $K = F_l(M_\infty, Q_0)$. Furthermore, every controller in the set K_{SH} is H_∞ strongly stabilizing with H_∞ performance γ, where

$$K_{S1} \equiv \left\{ F_l(M_\infty, Q_0 + \hat{Q}) : \hat{Q} \in RH_\infty,\ \|\hat{Q}\|_\infty < \min\left(\frac{1}{\|(M_l - N_l Q_0)^{-1} N_l\|_\infty}, \gamma - \|Q_0\|_\infty \right) \right\}.$$

The multiobjective control problem can be efficiently solved by the relevant LMI program in [14].

4. Examples

Example 1. Consider the 3rd order unstable non-minimum-phase plant

$$P_o(s) = \frac{(s+3)(s-2.6)(s-40.2)}{(s+29)(s-3)(s-11)},$$

It is easy to verify that $P_o(s)$ satisfies the parity interlacing property(pip), thus the existence of stable stabilizing controllers is guaranteed. By Theorem 1, we obtain the following strongly stabilizing controller

$$K(s) = \frac{-6.86s^3 - 15.76s^2 + 7853s + 59690}{s^3 + 882.4s^2 + 9300s + 19990},$$

which is of the same order as that of the plant.

Example 2. The example is taken from [11]. The minimal H_∞ performance obtained via different methods are summarized in the table,

Methods	γ_{mim}	Controller	Order
Toolbox	12.014	unstable	4
[8]	37.551	stable	8
[11]	43.167	stable	4
Theorem 2	28.001	stable	8

which shows that a stable H_∞ controller with better H_∞ performance is obtained by the method we proposed.

5. Conclusions

Sufficient conditions for the strong stabilization problem and the problems were given in terms of the solvability of positive real controller synthesis problem and multiobjective control problem, respectively, which can be efficiently solved via the relevant LMI programs. Besides the numerical benefits, our method is simpler(compared to the Nevanlinna-Pick interpolation technique) and makes only a minimal number of assumptions(thus removes many technical assumptions imposed on the Riccati-based approaches). This means that our method could be applicable to more general cases.

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