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矩形微流道可壓縮流體之三維數值模擬

Three-Dimensional Numerical Simulation of Compressible Flow in Rectangular Microchannels

中文摘要

本論文研究目的，在於利用數值計算的方式來探討可壓縮氣體在三維微流道流場中的摩擦以及熱傳方面的特性。我們求解 reduced compressible Navier-Stokes 方程式。這種方程式皆具有拋物線型的數學特性，使得我們在研究微小流道的可壓縮流場時，能夠提供一個更快速準確又強而有力的工具。本文使用的數值計算過程所需的時間比完全 Navier-Stokes 方程式模擬快上數百倍至數千倍。

ABSTRACT

The objective of this research is to investigate numerically the friction and heat transfer characteristics of compressible flow in microchannels. We solved the reduced compressible Navier-Stokes equations. The parabolic character of the reduced Navier-Stokes equations renders the numerical procedure a very efficient, accurate and robust tool for studying compressible microchannel flows. The proposed numerical procedure is at least two to three orders of magnitude faster than the full Navier-Stokes simulation.

INTRODUCTION

The miniaturization of conventional systems and novelly developed new systems, especially in the electronic field, demands the removal of significant amount of heat from a small space. New thermal management methods are needed to meet this demand. Microcooling systems such as micro heat exchangers, micro heat pipes, micro heat spreaders, etc. have been proven of capable of enhancing heat transfer rate. They are able to remove high heat rates in relatively small spaces and volumes. Microchannel is a basic and important component in many microfluidic systems, such as micro heat exchangers, biological cell reactors, and biological chips, etc. In order to design these systems accurately and efficiently, the understanding of the physical behavior of microchannel flows is essential.

Sobhan and Garimella conducted a comparative study of the experimental results and theoretical investigations available in the literature on friction and heat transfer for liquids and gases in microchannels. They reported that there is little agreement between the experimental results reported by different investigators. Also, the information in the literature does not point to unequivocal trends of variation or reasons of such trends. They concluded that a reliable prediction of heat transfer and pressure drop in microchannels is not possible and additional studies are necessary. Palm did a similar survey on heat transfer in microchannels for one- and two-phase flows and his conclusions were similar.

This study focuses on the heat transfer characteristics of steady compressible laminar flow in long microchannels. Three-dimensional simulations of microchannel flows were rarely reported in the literature due to the large memory and long CPU time required using the full

Navier-Stokes equations. The major advantage of the present three-dimensional numerical procedure is its fast speed due to the parabolic characteristics of the reduced Navier-Stokes equations used. An efficient space marching algorithm is adopted to solve the governing equations. It is at least two to three orders of magnitude faster than the full Navier-Stokes simulation. This is because the unsteady Navier-Stokes equations are a mixed set of hyperbolic-parabolic equations that are integrated in time until a steady state solution is reached [Chen, 2004]. This procedure is inefficient because of its time marching procedure.

GOVERNING EQUATIONS

The flow is assumed to be governed by the reduced form of the steady three-dimensional compressible laminar Navier-Stokes equations. The reduced equations cannot model the entrance region accurately, but this region is small for the long microchannels studied in this work. The ratio of channel length to hydraulic diameter is assumed to be 1000 for all the channels simulated in this study. Air is used as the working fluid for the present work. The non-dimensional governing equations are given below.

The continuity equation

$$\frac{\partial(\tilde{\rho}\tilde{u})}{\partial\tilde{x}}+\frac{\partial(\tilde{\rho}\tilde{v})}{\partial\tilde{y}}+\frac{\partial(\tilde{\rho}\tilde{w})}{\partial\tilde{z}}=0 \tag{1}$$

The momentum equations

$$\begin{aligned}&\tilde{\rho}[\tilde{u}\frac{\partial\tilde{u}}{\partial\tilde{x}}+\tilde{v}\frac{\partial\tilde{u}}{\partial\tilde{y}}+\tilde{w}\frac{\partial\tilde{u}}{\partial\tilde{z}}]\\&=-\frac{d\tilde{p}_m}{d\tilde{x}}+\frac{1}{\mathrm{Re}_{in}}\frac{\partial}{\partial\tilde{y}}(\tilde{\mu}\frac{\partial\tilde{u}}{\partial\tilde{y}})+\frac{1}{\mathrm{Re}_{in}}\frac{\partial}{\partial\tilde{z}}(\tilde{\mu}\frac{\partial\tilde{u}}{\partial\tilde{z}})\end{aligned} \tag{2}$$

$$\begin{aligned}&\tilde{\rho}(\tilde{u}\frac{\partial\tilde{v}}{\partial\tilde{x}}+\tilde{v}\frac{\partial\tilde{v}}{\partial\tilde{y}}+\tilde{w}\frac{\partial\tilde{v}}{\partial\tilde{z}})=-\frac{\partial\tilde{p}_c}{\partial\tilde{y}}+\frac{4}{3\cdot\mathrm{Re}_{in}}\frac{\partial}{\partial\tilde{y}}(\tilde{\mu}\frac{\partial\tilde{v}}{\partial\tilde{y}})\\&+\frac{1}{\mathrm{Re}_{in}}\frac{\partial}{\partial\tilde{z}}(\tilde{\mu}\frac{\partial\tilde{v}}{\partial\tilde{z}})+\frac{1}{3\cdot\mathrm{Re}_{in}}\frac{\partial}{\partial\tilde{z}}(\tilde{\mu}\frac{\partial\tilde{w}}{\partial\tilde{y}})\end{aligned} \tag{3}$$

$$\begin{aligned}&\tilde{\rho}(\tilde{u}\frac{\partial\tilde{w}}{\partial\tilde{x}}+\tilde{v}\frac{\partial\tilde{w}}{\partial\tilde{y}}+\tilde{w}\frac{\partial\tilde{w}}{\partial\tilde{z}})=-\frac{\partial\tilde{p}_c}{\partial\tilde{z}}+\frac{1}{3\cdot\mathrm{Re}_{in}}\frac{\partial}{\partial\tilde{y}}(\tilde{\mu}\frac{\partial\tilde{v}}{\partial\tilde{z}})\\&+\frac{1}{\mathrm{Re}_{in}}\frac{\partial}{\partial\tilde{y}}(\tilde{\mu}\frac{\partial\tilde{w}}{\partial\tilde{y}})+\frac{4}{3\cdot\mathrm{Re}_{in}}\frac{\partial}{\partial\tilde{z}}(\tilde{\mu}\frac{\partial\tilde{w}}{\partial\tilde{z}})\end{aligned} \tag{4}$$

The energy equation

$$\begin{aligned}&\tilde{\rho}(\tilde{u}\frac{\partial\tilde{T}}{\partial\tilde{x}}+\tilde{v}\frac{\partial\tilde{T}}{\partial\tilde{y}}+\tilde{w}\frac{\partial\tilde{T}}{\partial\tilde{z}})\\&=(\gamma_{in}-1)(\tilde{u}\frac{\partial\tilde{p}_m}{\partial\tilde{x}})+\frac{1}{\mathrm{Re}_{in}\cdot\mathrm{Pr}}[\frac{\partial}{\partial\tilde{y}}(\tilde{k}\frac{\partial\tilde{T}}{\partial\tilde{y}})+\frac{\partial}{\partial\tilde{z}}(\tilde{k}\frac{\partial\tilde{T}}{\partial\tilde{z}})]\\&+\frac{(\gamma_{in}-1)}{\mathrm{Re}_{in}}[\tilde{\mu}(\frac{\partial\tilde{u}}{\partial\tilde{y}})^2+\tilde{\mu}(\frac{\partial\tilde{u}}{\partial\tilde{z}})^2]\end{aligned} \tag{5}$$

The equation of state

$$\gamma \tilde{p}_m = \tilde{\rho}\tilde{T} \tag{6}$$

The integral constraint on mass flow through the channel

$$\int_{-H/2D}^{H/2D} \int_{-W/2D}^{W/2D} \tilde{\rho}\tilde{u}\; d\tilde{y}\, d\tilde{z} = \dot{\tilde{m}} \tag{7}$$

The above equations are normalized by the following inlet values,

$$\tilde{u} = \frac{u}{a_{in}};\;\; \tilde{v} = \frac{v}{a_{in}};\;\; \tilde{w} = \frac{w}{a_{in}};\;\; \tilde{\rho} = \frac{\rho}{\rho_{in}}; \tilde{T} = \frac{T}{T_{in}},$$

$$\tilde{x} = \frac{x}{D};\;\; \tilde{y} = \frac{y}{D};\;\; \tilde{z} = \frac{z}{D}; D = \frac{2 \cdot W \cdot H}{W + H}$$

$$\tilde{p} = \frac{p}{\rho_{in} a_{in}^2};\;\; \mathrm{Re}_{in} = \frac{\rho_{in} a_{in} D}{\mu_{in}};\;\; \tilde{\mu} = \frac{\mu}{\mu_{in}},\;\; \tilde{k} = \frac{k}{k_{in}}$$

$$c_p = \frac{\gamma \cdot R}{\gamma - 1};\;\; \mathrm{Pr} = \frac{c_p \cdot \mu_{in}}{k_{in}}.$$

NUMERICAL PROCEDURES

The reduced Navier-Stokes equations are a set of parabolic equations in the $\tilde{x}$-direction. They are solved using an efficient space marching procedure. The procedure is described below.

(1) The $\tilde{u}^{i+1}$ velocity is computed from the $\tilde{x}$-momentum equation, Eq. (2). The inlet pressure, inlet temperature, and a guessed mass flow rate are given as the initial conditions for the $\tilde{x}$-momentum equation. The pressure gradient at each $\tilde{x}$-station is iterated until the global mass constraint, Eq. (7), is satisfied. The Newton-Raphson method is employed to update the pressure gradient.

$$\left(\frac{d\tilde{p}_m}{d\tilde{x}}\right)_{n+1} = \left(\frac{d\tilde{p}_m}{d\tilde{x}}\right)_n - \frac{\dot{\tilde{m}}_n}{\dot{\tilde{m}}_n - \dot{\tilde{m}}_{n-1}} \cdot \left[\left(\frac{d\tilde{p}_m}{d\tilde{x}}\right)_n - \left(\frac{d\tilde{p}_m}{d\tilde{x}}\right)_{n-1}\right] \tag{8}$$

The mass flow rate $\dot{\tilde{m}}$ is a function of the pressure gradient. Two initial guesses are required before using the above equation.

(2) The pressure correction $\tilde{p}_c$ at each cross-section is computed from the Poisson equation, Eq. (12).

(3) The pressure correction $\tilde{p}_c$ obtained from step (2) is substituted into the $\tilde{y}$- and $\tilde{z}$-momentum equations, Eqs. (3) and (4), to solve for the $\tilde{v}^{i+1}$ and $\tilde{w}^{i+1}$ velocities.

(4) The temperature $\tilde{T}^{i+1}$ is computed from the energy equation, Eq. (5).

(5) The density is computed from the equation of state, Eq. (6).

(6) Steps 1 to 5 complete the calculation at a $\tilde{y} - \tilde{z}$ cross-section. The solution then marches along the channel axis to the channel outlet. The outlet pressure is assumed to be atmospheric pressure and is specified at the beginning of a simulation. If the calculated outlet pressure is less than atmospheric pressure, then the guessed mass flow rate is too large. If the calculated outlet pressure is greater than atmospheric pressure, then the guessed mass flow rate is too small. The

program then goes back to step 1 with an updated mass flow rate.

$$\dot{\tilde{m}}_{k+1} = \dot{\tilde{m}}_k - \frac{\Delta \tilde{p}_m{}^k}{\Delta \tilde{p}_m{}^k - \Delta \tilde{p}_m{}^{k-1}} \cdot \left(\dot{\tilde{m}}_k - \dot{\tilde{m}}_{k-1} \right) \tag{9}$$

In the above equation *k* is the global iteration number. Marching the solution from the channel inlet to outlet is one global-iteration. The parameter $\Delta \tilde{p}_m$ is defined as $\Delta \tilde{p}_m = \tilde{p}_{atm} - \tilde{p}^k_{m,\,out}$, $\tilde{p}_{atm}$ is non-dimensional atmospheric pressure and $\tilde{p}^k_{m,\,out}$ is the numerically calculated channel outlet pressure at the k-th global-iteration.

CONCLUSIONS

A three-dimensional numerical procedure was developed to study the heat transfer characteristics for steady laminar compressible flow in long microchannels, which were rarely reported in the literature. The numerical program was first used to simulate incompressible flow in conventional-size channels subject to a constant wall heat flux and an isothermal wall boundary conditions [Chen, 2004]. The calculated local Nusselt numbers, Nu_x, subject to the two boundary conditions at several different aspect ratios compared quite well with that of the corresponding two-dimensional incompressible fully developed flows calculated by the other investigators. These two comparisons validated our numerical program.

After validation, we calculated Nu_x for microchannel flows subject to a constant wall heat flux at several different aspect ratios. Nu_x decreases consistently along the channel axis, because there was no fully developed region in compressible microchannel flows. Raising aspect ratio increases Nu_x [Chen, 2004]. We next investigated the effect of heat flux on the friction characteristic of compressible microchannel flows. Adding heat increased the kinetic energy of the fluid and therefore enhanced the friction factor considerably. We then studied the effect of pressure ratio on Nu_x for a square microchannel subject to a constant wall heat flux. A higher pressure ratio reduces Nu_x, because the internal energy was transferred into the kinetic energy and this transfer grew with pressure ratio. Finally, we calculated Nu_x for microchannel flows subject to an isothermal wall at several different aspect ratios. The distribution of Nu_x behaves quite differently compared with that for incompressible channel flows. It diminishes first and then increases along the channel axis up to the outlet. The dissimilar behaviors of the heat transfer and friction characteristics between compressible microchannel and conventional channel flows can be well explained by the fluid compressibility and the energy transfer among kinetic energy, internal energy, and flow work [Chen, 2004].

REFERENCES

1. Chen, C.S., “Numerical Analysis of Heat Transfer for Steady Three-Dimensional Compressible Flow in Long Microchannels”, Journal of Enhanced Heat Transfer, Vol. 12, No. 2, 171-188, 2005, NSC 93－2212－E－032－005.