行政院國家科學委員會專題研究計畫成果報告

線性離散系統 H_∞低階控制器設計研究

H_∞ LOW-ORDER CONTROL DESIGN FOR LINEAR DISCRETE-TIME SYSTEMS

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***Abstract*- This report proposes a non-iterative computational algorithm for the design of discrete-time fixed order controller for an H_∞ optimization problem. Using the coprime factors and pole placement constraints, the fix-order controller design is reformulated as a convex optimization problem. The solutions are obtained using linear matrix inequality techniques. An aircraft model with 3-inputs and 3-outputs is used to illustrate the design algorithm.**

Keywords- coprime factorization, robust control, pole placement, linear matrix inequalities, low-order controller design,

1. Introduction

The H_∞ design techniques are broadly used for robust controller design. However, the order of the resulting controllers are much higher then necessary. In practical control designs, low-order controllers are usually desired for system reliability and ease of implementation. The design of fixed- order controller is still an open control problem.

The difficulty is that the design of low-order controller to optimize certain performance involves a bi-affine matrix inequality(BMI), which is non- convex and cannot be solved using the existing convex programming software. Instead of solving directly the BMI problem, several researchers have shown that low-order controllers can be obtained by solving iteratively LMI subproblems, which are convex. These approaches include alternating projection method [5] rank condition minimization method [6] and successive substitution method [4][10][11].

In [13], a low-order controller design method using coprime factors, strictly positive real function (SPR) and LMIs was developed for continuous -time single-input single-output (SISO) systems. This method is expanded to solve the model- matching problem for continuous-time MIMO systems [14]. For discrete time case, low-order robust controller design algorithms using coprime factors, discrete outer functions and LMIs were developed [9]. This report summarizes the results of the development of the low-order controller design for an H_∞ optimization problem. Using the coprime factors and pole placement constraints, the fix-order controller design is formulated as convex optimization problem subject to several LMI constraints. An aircraft model with 3-inputs and 3-outputs is used to illustrate the design algorithm.

The paper is organized as follows. Section 2 discusses the structure of the coprime factorization for the low-order controller. In Section 3 we formulate the low-order stabilizing controller design as an LMI feasibility problem. A pole placement design concept is discussed in section 4. The formulation and solution algorithm for an H_∞ optimization problem are presented in Section 5. The application of the proposed design algorithm for the control of the vertical plane dynamics of an aircraft is included in Section 6.

2. Coprime factorization

Consider a linear time-invariant system *G(z)* with the state-space realization

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where $x \in R^n$ is the state variable vector, $u \in R^m$ is the controlled input variable, and $y \in R^p$ is the measured output variable. Assume that the system (1) is stabilizable and detectable. In the packed matrix notation, *G(z)* is represented by

$$G(z) \leftrightarrow \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] \tag{2}$$

Since the system (1) is stabilizable and detectable, we perform a right coprime factorization of *G(z)* to obtain

$$G(z) = G_N(z) G_D^{-1}(z) \tag{3}$$

The state-space realization of $G_D(z)$ and $G_N(z)$ can be represented as

$$\begin{bmatrix} G_D(z) \\ G_N(z) \end{bmatrix} \leftrightarrow \left[\begin{array}{c|c} A-BF & B \\ \hline -F & I \\ C & 0 \end{array}\right] \tag{4}$$

where F is a stabilizing full-state feedback gain such that all the eigenvalues of A-BF are inside the unit circle of the z-planr. In contrast to a full-order stabilizing controller *K(z)* for *G(z)*, whose coprime factorization can be readily defined in terms of A,B,C, and a stabilizing observer gain L, for a reduced-order controller we first need to define its structure before performing coprime factorization. Select the reduced-order controller $K(z)$ with p inputs and m outputs to have the structure

$$K(z) = \begin{bmatrix} \dfrac{k_{11}(z)}{d_1(z)} & \cdots & \dfrac{k_{1p}(z)}{d_1(z)} \\ \vdots & \ddots & \vdots \\ \dfrac{k_{m1}(z)}{d_m(z)} & \cdots & \dfrac{k_{mp}(z)}{d_m(z)} \end{bmatrix} \quad (5)$$

where $k_{ij}(z)$ and $d_i(z)$ are polynomials defined as

$$k_{ij}(z) = b_{ij,ni} z^{ni} + b_{ij,ni-1} z^{ni-1} + \cdots b_{ij,0} \quad (6)$$

$$d_i(z) = z^{ni} + a_{i,ni-1} z^{ni-1} + \cdots a_{i,0} \quad (7)$$

$$i = 1,\cdots,m \;\; ; \;\; j = 1,\cdots,p \quad (8)$$

$a_{ij,l}$ and $b_{ij,l}$ $=1,\ldots,ni$ are unknown coefficients to be solved. We can perform a left coprime factorization of $K(z)$ as

$$K(z) = K_D^{-1}(z) K_N(z) \quad (9)$$

The coprime factors $K_N(z)$ and $K_D(z)$ are stable transfer function matrices with

$$K_N(z) = \begin{bmatrix} \dfrac{k_{11}(z)}{d_{c1}(z)} & \cdots & \dfrac{k_{1p}(z)}{d_{c1}(z)} \\ \vdots & \ddots & \vdots \\ \dfrac{k_{m1}(z)}{d_{cm}(z)} & \cdots & \dfrac{k_{mp}(z)}{d_{cm}(z)} \end{bmatrix} \quad (10)$$

$$K_D(z) = \begin{bmatrix} \dfrac{d_1(z)}{d_{c1}(z)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \dfrac{d_m(z)}{d_{cm}(z)} \end{bmatrix} \quad (11)$$

where $d_{ci}(z), j=1,\ldots,m$, are predetermined stable monic polynomials. The order of $d_{cj}(z)$ and $d_j(z)$ are the same. The order of the controller $K(z)$ is the sum of the degree of $d_1(z),\ldots, d_p(z)$. For reduced-order controller, the order of the controller is limited to be smaller then n (the order of the plant (1)). If there are no unstable hidden modes in $K(z)$, then (9) is a left coprime factorization. We can use either observable or controllable canonical-form to realize each transfer function $k_{ij}(z)/\, d_{ci}(z)$ in $K_N(z)$ to obtain the state-space realization of $K_N(z)$. That is, if observable canonical-form is used, $K_N(z)$ can be represented as

$$K_N(z) \leftrightarrow \left[\begin{array}{c|c} A_{kno} & B_{kno} \\ \hline C_{kno} & D_{kno} \end{array}\right] \quad (12)$$

where A_{kno} and C_{kno} are constant matrices determined from the pre-selected denominators $d_{ci}(z)$. The unknown coefficients of the numerators $k_{ij}(z)$ are included in B_{kno} and D_{kno}. On the other hand, if controllable canonical-form realization is selected, $K_N(z)$ can be represented as

$$K_N(z) \leftrightarrow \left[\begin{array}{c|c} A_{knc} & B_{knc} \\ \hline C_{knc} & D_{knc} \end{array}\right] \quad (13)$$

where A_{knc} and B_{knc} are constant matrices determined from the pre-selected denominators $d_{ci}(z)$. The unknown coefficients of the numerators $k_{ij}(z)$ are included in C_{knc} and D_{knc}. We note that in these realizations, we can make $A_{kno} = A_{knc}^{\;T}$, $D_{kno} = D_{knc}$. But we will have no luck for B_{kno} and C_{kno} in general. We further note that both realizations are required for the design of an H_∞ low-order controller in this paper.

We can define the state-space realization for $K_D(z)$ in the similar way. Since $K_D(z)$ is defined in diagonal form, we will only need one realization in our design algorithm

$$K_D(z) \leftrightarrow \left[\begin{array}{c|c} A_{kd} & B_{kd} \\ \hline C_{kd} & D_{kd} \end{array}\right] \quad (14)$$

where A_{kd} and B_{kd} are constant matrices, and $D_{kd} = I$. The unknown coefficients of the denominators $d_i(z)$ are included in C_{kd}.

3. Low-order stabilizing controller design

Consider the closed-loop regulation system in Figure 1

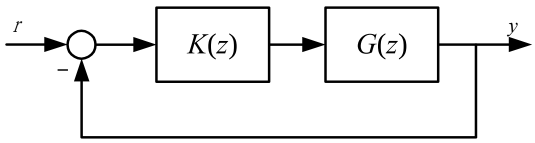

Figure1: Closed-loop Regulation System

The closed-loop transfer function from the command r to the output y, denoted as $T(z)$, is

$$T(z) = [I + G(z)K(z)]^{-1} G(z)K(z) \quad (15)$$

Using coprime factorization of $G(z)$ and $K(z)$, the closed-loop transfer function $T(z)$ is

$$T(z) = G_N(z) Q(z)^{-1} K_N(z) \quad (16)$$

where $Q(z)$ is defined as

$$Q(z) = K_D(z) G_D(z) + K_N(z) G_N(z) \quad (17)$$

Using the state-space realizations (4), (13) and (14), a state-space realization of $Q(z)$ can be written as

$$Q(z) \leftrightarrow \left[\begin{array}{ccc|c} A_{kd} & -B_{kd}F & 0 & B_{kd} \\ 0 & A - BF & 0 & B \\ 0 & B_{knc}C & A_{knc} & 0 \\ \hline C_{kd} & -F + D_{knc}C & \tilde{C}_{kn} & I \end{array}\right] = \left[\begin{array}{c|c} A_q & B_q \\ \hline C_q & D_q \end{array}\right] \quad (18)$$

where A_q is stable. We note that the design parameters appear linearly in C_q. The following results are crucial to the development of the design method proposed in this paper.

Lemma1 : If there exist a symmetric positive definite matrix P, such that the following matrix inequality is satisfied

$$\begin{bmatrix} A_q^T P A_q - P & A_q^T P B_q - C_q^T \\ B_q^T P A_q - C_q & -I \end{bmatrix} < 0 \qquad (19)$$

$$I - B_q^T P B_q \geq 0 \qquad (20)$$

then all zeros of $Q(z)$ are inside the unit circle of the z-plane.

Proof : Suppose the inequality (19) is satified, we have

$$\begin{aligned} & A_q^T P A_q - P \\ & + (A_q^T P B_q - C_q^T)(B_q^T P A_q - C_q) < 0 \end{aligned} \qquad (21)$$

The inequality (21)can be written as

$$\begin{aligned} & (A_q - B_q C_q)^T P (A_q - B_q C_q) - P \\ & + A_q^T P B_q B_q^T P A_q + C_q^T (I - B_q^T P B_q) C_q < 0 \end{aligned}$$

if $I - B_q^T P B_q \geq 0$, then

$$(A_q - B_q C_q)^T P (A_q - B_q C_q) - P < 0 \qquad (22)$$

Inequality (22) implies that A_q-$B_q C_q$ is stable，that is, $Q^{-1}(z)$ is stable. Therefore, all zeros of Q(z) are inside the unit circle of the z-plane.

■

Theorem 1 : If there exist matrices B_{knc}, D_{knc}, and B_{kd} having the controllable canonical realization structure defined in (13) and (14), such that the LMIs (19) and (20) are satisfied, then $u=-K(z)y$ is a stabilizing controller.

Proof : The proof is established by observing that the LMIs (19) and (20) guarantee the stability of the transfer function $Q^{-1}(z)$. Which implies that the closed-loop transfer function $T(z)$ in (15) is stable.■

Theorem 1 gives a practical method for finding a low-order stabilizing controller. The LMIs (19) and (20) together with the pre-determined structure of B_{knc}, D_{knc}, and B_{kd} can be solved as a feasibility problem using a convex programming toolbox such as [3]. In this paper, we will concentrate on the design of low-order controller for the H_∞ optimization problems.

4. Pole Placement Design

With a low-order controller, we no longer have the freedom to arbitrarily place all the closed-loop system poles. The objective then is to find a low-order controller such that the closed-loop system poles are close to a set of pre-specified poles. From (16), we observe that

$$T(z) = G_N(z) K_N(z) \qquad (23)$$

if $Q(z)=I$ is assumed. In this case, the poles of the closed-loop system are exactly the union of the poles of $G_N(z)$ and the poles of $K_N(z)$, which are predetermined by the designer. Thus we propose the following optimization problem to approach the pole placement design

$$\min_{K(s)} \|Q(z) - I\|_\infty \qquad (24)$$

subject to the constraint (19) and (20). We can regard (24) as a regional pole-placement problem, in which the pole-placement regions are determined by the poles of $G_N(z)K_N(z)$. We remark that $Q(z)=I$ can be achieved for a full order controller due to Bezout identity [13], in which all the desired poles can be exactly placed.

Let the state-space model of $Q(z)$-I be given by

$$(Q(z) - I) \leftrightarrow \left[\begin{array}{c|c} A_q & B_q \\ \hline C_q & 0 \end{array}\right] \qquad (25)$$

Using the Bounded-Real Lemma [1], the optimization problem (24) can be expressed as

$$\min_{K(s)} \gamma_p \qquad (26)$$

subject to the LMIs

$$\begin{bmatrix} A_q{}^T X A_q + X & A_q{}^T X B_q & C_q{}^T \\ B_q{}^T X A_q & B_q{}^T X B_q - \gamma_p I & 0 \\ C_q & 0 & -\gamma_p I \end{bmatrix} < 0 \qquad (27)$$

$$X > 0 \qquad (28)$$

and the LMIs (19) and (20). This problem is convex and can be solved using an LMI algorithm in the unknowns P,X , C_{knc} , D_{knc}, C_{kd} and γ_p .

5. H_∞ Optimization Problem

Consider the robust optimization problem as shown in Figure 2.

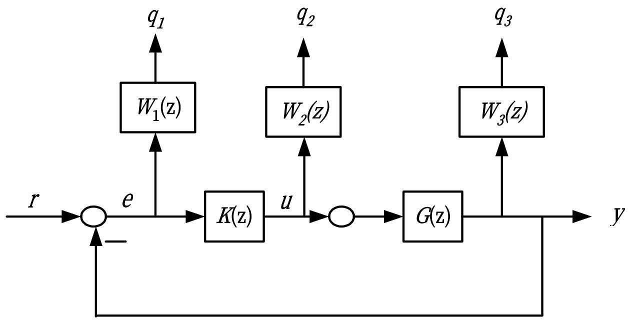

Figure 2: H_∞ optimization problem

From the system block diagram, we have

$$\begin{bmatrix} q \\ e \end{bmatrix} = \begin{bmatrix} P_{11}(z) & P_{12}(z) \\ P_{21}(z) & P_{22}(z) \end{bmatrix} \begin{bmatrix} r \\ u \end{bmatrix} \qquad (29)$$

where

$$q = [q_1 \quad q_2 \quad q_3]^T$$

$$P_{11}(z)=\begin{bmatrix}W_1(z)\\0\\0\end{bmatrix}, P_{12}(z)=\begin{bmatrix}-W_1(z)G(z)\\W_2(z)\\W_3(z)G(z)\end{bmatrix}$$

$$P_{21}(z)=I \quad , \quad P_{22}(z)=-G(z)$$

The H_∞ optimization problem is to find a stabilizing controller $u=K(z)e$ such that the closed- loop system is stable and the H_∞ norm of the closed-loop system from input r to the performance (controlled) output q, denoted as $T_{qr}(z)$ satisfies

$$\left\|T_{qr}(z)\right\|_\infty<\gamma \tag{30}$$

where $\gamma>0$ is a pre-specified constant. The transfer function $T_{qr}(z)$ is

$$T_{qr}(z)=P_{11}(z)+P_{12}(z)[I+K(z)G(z)]^{-1}K(z) \tag{31}$$

Using coprime factorization of $G(z)$ and $K(z)$, the closed-loop transfer function $T_{qr}(z)$ is

$$T_{qr}(z)=P_{11}(z)+P_{12}(z)G_D(z)Q(z)^{-1}K_N(z) \tag{32}$$

Obviously, finding a controller to satisfy (30) is a nonconvex problem and is difficult to solve. However, if $Q(z)=I$ is achieved, then the transfer function $T_{qr}(z)$ becomes

$$T_{qr}(z)=P_{11}(z)+P_{12}(z)G_D(z)K_N(z) \tag{33}$$

The state-space realization of $T_{qr}(z)$ can be written as

$$T_{qr}(z)\leftrightarrow\left[\begin{array}{c|c}A_{zr}&B_{zr}\\\hline C_{zr}&D_{zr}\end{array}\right] \tag{34}$$

$$A_{zr}=\begin{bmatrix}A_{p11}&0&0\\0&A_{p12gd}&B_{p12gd}C_{kno}\\0&0&A_{kno}\end{bmatrix}$$

$$B_{zr}=\begin{bmatrix}B_{p11}\\B_{p12gd}D_{kno}\\B_{kno}\end{bmatrix}$$

$$C_{zr}=\begin{bmatrix}C_{p11}&C_{p12gd}&D_{p12gd}C_{kno}\end{bmatrix}$$

$$D_{zr}=\begin{bmatrix}D_{p11}+D_{p12gd}D_{kno}\end{bmatrix}$$

where

$$P_{11}(z)\leftrightarrow\left[\begin{array}{c|c}A_{p11}&B_{p11}\\\hline C_{p11}&D_{p11}\end{array}\right]$$

$$P_{12}(z)G_D(z)\leftrightarrow\left[\begin{array}{c|c}A_{p12gd}&B_{p12gd}\\\hline C_{p12gd}&D_{p12gd}\end{array}\right]$$

Note that we use observable canonical-form realization for $K_N(z)$ here. Now, the design para- ters appear linearly in B_{zr} and D_{zr}. Thus, we formulate the H_∞ optimization problem as

$$\min_{K(s)}(\gamma+\gamma_p)$$

subject to the LMIs

$$\begin{bmatrix}A_{zr}RA_{zr}^T-R&A_{zr}RC_{zr}^T&B_{zr}\\C_{zr}RA_{zr}^T&C_{zr}RC_{zr}^T-\gamma I&D_{zr}^T\\B_{zr}^T&D_{zr}&-\gamma I\end{bmatrix}<0 \tag{35}$$

$$R>0 \tag{36}$$

and the LMIs (19), (20), (27), and (28). The LMI variables are γ_p, γ, the controller parameters $k_{ij}(z)$, and $d_i(z)$, and the positive definite matrices P, X, R.

6. Design Example

In this section, we will apply the proposed design algorithm to design a fixed order controller for the aircraft model AIRC [7]. The model represents a linearized model of the vertical-plane dynamics of an aircraft, and has three inputs, three outputs, and five states. The inputs are spoiler angle (u_1, mea-sured in tenths of a degree), forward acceleration (u_2, in m/sec^2), and elevator (u_3, in degrees). The states are altitude relative to some datum (x_1, in m), forward speed (x_2, in m/sec), pitch angle (x_3, in degrees), pitch rate (x_4, in deg/sec), and vertical speed (x_5, in m/sec). The three outputs are just the three states, which are to be controlled.

The design requirements are to achieve a closed-loop bandwidth of about 10 rad/sec, with reasonably damped responses and minimize sensitivity at zero frequency. In order to do this, we select the following weighting functions in the design.

$$W_1(s)=\begin{bmatrix}\frac{(s+60)^2}{200(s+0.6)^2}&0&0\\0&\frac{(s+60)^2}{200(s+0.6)^2}&0\\0&0&\frac{(s+60)^2}{200(s+0.6)^2}\end{bmatrix} \tag{37}$$

$$W_3(s)=\begin{bmatrix}\frac{25}{s}&0&0\\0&\frac{25}{s}&0\\0&0&\frac{25}{s}\end{bmatrix} \tag{38}$$

The functions $W_1(s)$ and $W_3(s)$ penalize the sensitivity and complementary sensitivity function respectively. No constraint on the control is imposed in the design.

The full order H_∞ controller has an order of eleven. We aim at designing a third order controller directly while maintaining the closed-loop stability and performance requirements. To illustrate the design algorithm proposed in the paper, we first discretize the system model with a sampling rate of 100 Hz. Following the design procedure discussed in the previous sections. The controller is obtained as

$$K(z)\leftrightarrow\left[\begin{array}{c|c}A_k&B_k\\\hline C_k&D_k\end{array}\right]$$

$$A_k=\begin{bmatrix}0.5791&-0.0003&0.0039\\-0.0003&0.5949&-0.0037\\0.0039&-0.0037&0.6439\end{bmatrix}$$

$$B_k = \begin{bmatrix} -1.6773 & 0.0039 & -0.6078 \\ -2.0106 & -0.0071 & -0.1193 \\ 1.5206 & 0.0316 & 0.3013 \end{bmatrix}$$

$$C_k = \begin{bmatrix} -0.4368 & -66.426 & -4.975 \\ 0.7859 & -0.7007 & 9.2872 \\ -183.61 & 0.0437 & 15.537 \end{bmatrix}$$

$$D_k = \begin{bmatrix} -365.4 & -0.7014 & -18.900 \\ -45.305 & 15.358 & -6.8739 \\ -962.12 & 0.8189 & -338.73 \end{bmatrix}$$

The open loop system poles, the selected desired poles and the closed-loop system poles are listed in Table 1. The desired poles are selected from the dominant poles of the closed-loop system with full order controller. The results show that the closed-loop system (with the obtained third order controller) poles are very close to the selected desired poles. The H_∞-norm bound γ of (30) is 0.97 in this design.

Open-loop poles	desired poles	Closed-loop poles
1.0	0.7594	0.6584
-0.9922+0.0102i	0.7603	0.7116
-0.9922-0.0102i	0.7605	0.7199
-0.9998+0.0002i	0.7605	0.8309
-0.9922-0.0002i	0.9144+0.0703i	0.926+0.0672i
	0.9144-0.0703i	0.926-0.0672i
	0.9145+0.0704i	0.9218+0.0692i
	0.9145-0.0704i	0.9218-0.0692i

Table 1: Pole locations of open loop system, desired poles, and closed-loop system poles.

The singular value plots of the sensitivity function and complementary sensitivity function are shown in Figure 3 and Figure 4 respectively. The results show that the design specifications are satisfied with the third order controller.

Time simulation of the closed-loop system are performed for a unit step change in each of the reference inputs. The system responses are shown in Figure 5. The results show that the controller provide good transient and steady-state performance.

7. Conclusions

This research develops a reliable and systematic low-order controller design method for an H_∞ optimization problem. Using the coprime factors and pole placement constraints, the fix-order controller design is formulated as convex optimization problem subject to several LMI constraints. The solutions are obtained using LMI techniques. The design algorithm is successfully applied to the control of the vertical plane dynamics of an aircraft.

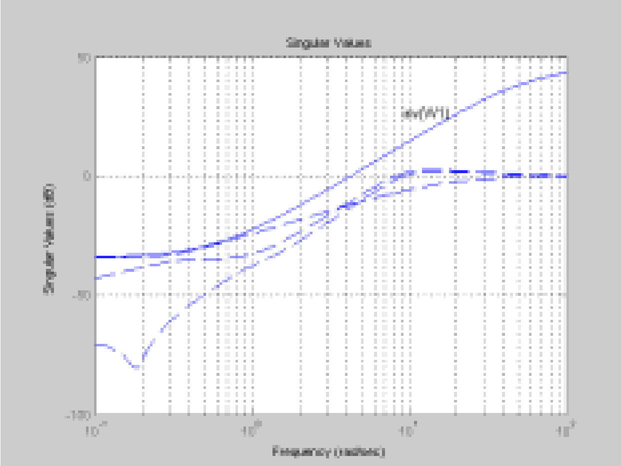

Figure3: Singular value plot of the sensitivity function

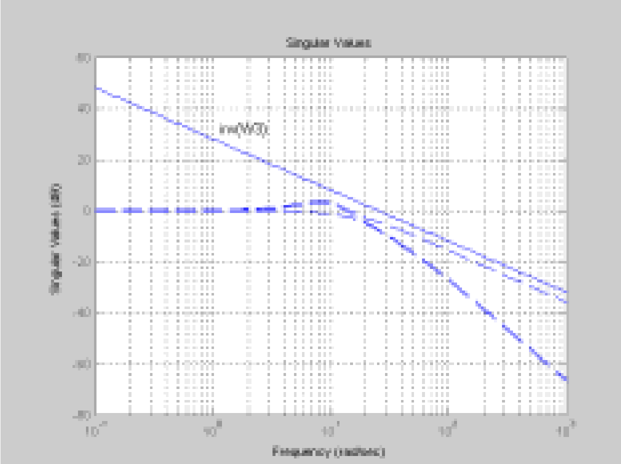

Figure4: Singular value plot of the complementary sensitivity function

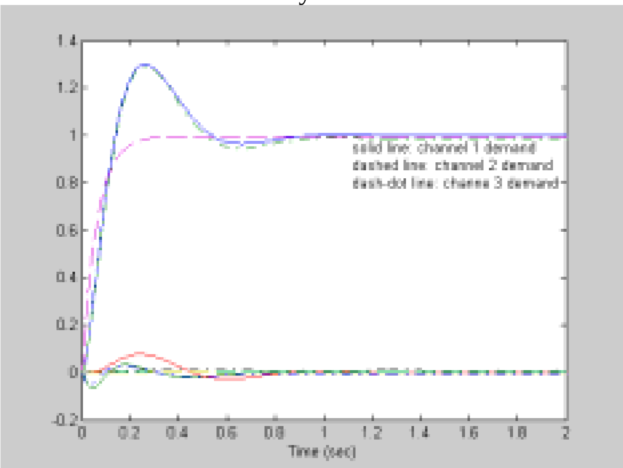

Figure5: Closed-loop step response to step demand

8. Acknowledgment

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