行政院國家科學委員會補助專題研究計畫成果報告

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※ **全面穩健機械最佳化設計**(2/1) ※
※ **Total Robust Mechanical Engineering Optimum Design (2/1)** ※
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計畫類別：☑**個別型計畫** □**整合型計畫**
計畫編號：NSC 89－2212－E－032－004
執行期間： 88 **年** 8 **月** 1 **日至** 89 **年** 7 **月** 31 **日**

計畫主持人： 史 建 中
計畫參與人員： 黃錦堂 蕭仁宏

執行單位： 淡江大學機械工程學系

中 華 民 國 89 **年** 8 **月** 28 **日**

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1. Abstract

This paper introduces a robust optimization method and process based on the Hohenbichler-Rackwitz's (H-R) structural reliability and using the parameter variation pattern to obtain the optimum design with the highest robust feasibility. Robust feasible design is capable of increasing the probability in the feasible region up to 99.95%. The paper also presents an optimum design methodology for obtaining the highest satisfaction of robust performance using multiobjective fuzzy formulation strategy. The performance function as the design target and its variation are simultaneously minimized in this design process. A functional representation of the variability of the performance and the computational algorithm of complete design process are proposed in the paper. Two categories of design problems are introduced: (1) robust design with expected target for minimal variation. (2) robust design with optimized target for minimal variation. The strength-based reliability behaves as constraint or design performance that was merged into the formulation to extend the usability of the proposed method. The design example further illustrate the presented integrated design methodology and have successfully shown the practical usefulness.

Keywords: Robust design, engineering optimization, reliability-based optimum design, structural optimization, reliability index, fuzzy optimization.

中文摘要

本文介紹作者所發展的信賴度基設計方法於結構性能穩健最佳化設計。其特色為應用模糊理論的多目標法則以得到最高滿意度的設計解。本文包含考慮設計合理性的穩健及設計性能之穩健兩部分。本文裡的設計性能方程式及其變異量皆可於設計程序裡同時的加以處理。表示性能變異的設計方程式及其全程的設計演算法則皆在本文裡披露。特別針對以下兩類的設計題目加以研析及介紹：（一）當設計目標值為已知時，求最小變異量的最佳設計結果。（二）當設計目標值為未知時，同時求最佳設計性能及最小變異量的最佳設計結果。應力為基的結構可靠度被處理為設計條件，一併加入全程設計的考量，以增加本設計方法的應用性。本文以機械結構設計的例題加以解說所提出的設計方法，並得以驗證其正確性與應用性。

關鍵詞：穩健設計，工程最佳化，信賴度基的最佳設計，結構最佳化，信賴度指標，模糊最佳化。

2. Cause and Objective

The design parameters often contain a range of uncontrollable variations or errors in engineering optimization problems. Those unavoidable variations of design parameters will convey to constraint and objective functions. Such an unwanted variations causes uncertainties in the constraint and objective functions. These uncertainties considerably reduced the performance of the final design or even make the final design infeasible. In general engineering optimization designs, the optimum point is usually located on one or two active constraints. Due to the variation of parameters, the boundary of active constraint can vary as a certain statistical distribution (Fig. 1).

The conventional method to overcome this uncertainty is the factor of safety design or the worst-case design (Rawlings, 1988; Parkinson, 1994). Although these corrections can avoid the infeasible solution, however, it may results a very conservative design and perhaps even further away from the optimum design. Several researchers recently presented their works of trying to eliminate such uncertainty of constraints (Parkinson and Sorensen, 1993; Parkinson, 1995; Yu and Ishii, 1994). The work of Sundaresan et al. (Sundaresan, Ishii, and Houser 1991) considered and dealt with the uncertainty of design constraints due to the variation of design parameters in the manufacturing and operational errors. However, when the design variables and parameters in accordance to the design functions have considerably high nonlinear characteristics, then the final design in a high probability can not be a robust design. Yu and Ishii (1994) had adopted the parameter variation pattern (PVP) to study the statistical analysis in the manufacturing process. They assumed the parameters have normal distribution and the variation pattern is an ellipsoid that accordingly revised the final design into the feasible region. The robustness is an important consideration in the general design, however, the reliability also is an important requirement to have an reliable and completely safe design.

This paper proposes an integrated optimization methodology of modifying the active constraints that adopt the parameter variation pattern to keep the final design in the feasible region as well as satisfying the Hohenbichler-Rackwitz (H-R) structural reliability index (Hohenbichler and Rackwitz 1981) and other design constraints simultaneously. The computations of H-R's reliability index of a problem containing limited state function is also presented in the paper.

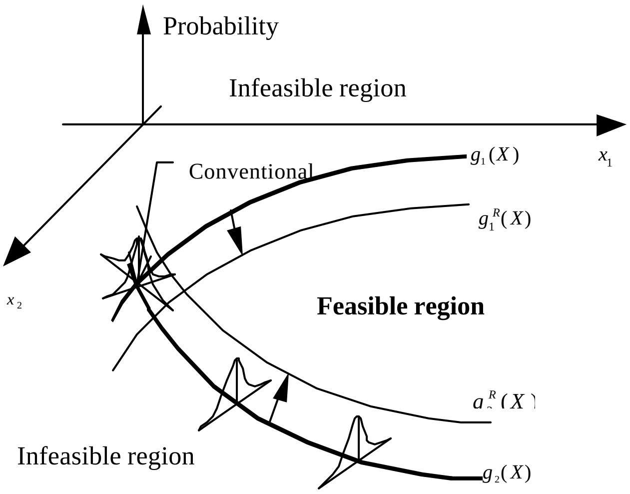

Fig. 1 The uncertainty of the active constraints.

Conventional optimization minimizes the nominal value of the performance (objective) function and overlooks the deviation of the performance functions due to manufacturing and operation errors. In addition, the design variables and parameters often contain a range of uncontrollable variations or errors, as knowledged by engineers. These uncertainties considerably reduced the performance of the final design Accordingly, a robust design should optimize the both of the expected value and the deviation of the performance function simultaneously. Even though the control technique (Parkinson 1987) of reducing the both of design parameter variability and the performance is available, however, this parameter variation cannot be eliminated in all the optimization problems.

The common employed strategy for obtaining the robust design is Taguchi method in which two criteria are proposed: the average loss function and signal-to-noise ratios (S/N ratios). Each criterion contains three representations: lower-the-better, higher-the-better and nominal-the-better (Ross, 1988). Chang (1989) proposed a two-stage optimization process for obtaining the minimum performance variability. Eggert & Mayne (1990) suggested a performance function, however, the selection of the weighting factors are somewhat arbitrary. Yu and Ishii (1993) recommended another form of the performance function based on the concept of statistical worst case. Parkinson, D. B. (1997) developed a variability function in Lagrangian form that is minimized.

Optimization plays an important role in the design of any component or system. Either in maximizing the reliability subject to the constraint of the design performance or in minimizing the design performance with a restriction on the reliability is recognized as the interesting problem in the engineering design. The strength-based reliability (Rao 1992) is commonly used to represent the reliability of a structural and mechanical design system. The robustness of the system reliability is obviously important especially for the industry with high precision, production and high technique requirements. This paper presents an optimization process considering the structural system reliability to improve the robust performance of the final design. This final design can hold the optimum level, high reliability as well as the minimum of the performance variability. Multiobjective fuzzy optimization presented by Rao (1987) and applied by Shih *et al* (1995) has been adopted to deal with the conflict and uncertainty between the performance function and the robustness. A mechanical design example is given to further illustrate the proposed robust engineering optimization method and process. A design of low robustness without the approach described in this paper and the design of high robustness obtained from the proposed strategy have been presented and compared from the illustrative example.

3. Results and Discussions

Design Process for Robust Engineering Optimization

Feasible direction method in nonlinear optimization is adopted as the primary optimizer in this study. The use of three-stage optimization strategy work on the robust feasibility design. The following shows the proposed algorithm.

Step 1. The first-stage optimization process to find the optimum nominal value for the performance of the design goal, $F_{\min}$, the structural reliability index β and also compute the feasible probability $P_{\min}$.

Step 2. Linearization to the active constraints about the nominal optimum design point.

Step 3. Solve the modifying amount of variation ΔX_j for each design variables from the active constraints. Then the second-stage optimization is put to use and one can see how much improvement of feasibility robustness.

Step 4. For compensating the error from the linearization of active constraints, the amount of ΔX_j is again relaxed. We use $\Delta X'_j = \Delta X_j \times 4/3$ and then the third-stage optimization process is done. The optimal design goal under considering the structural reliability has the highest level of feasible robustness.

Robust Design for Optimal Performance and Minimal Variation

For simultaneously minimizing the design target and its variation, this problem is exactly a multiobjective optimization problem. In this paper the fuzzy optimization strategy is used here to optimize both of the performance function and the variance. The feature of the final design has the highest level of satisfaction in the domain of membership functions. The design process including several optimization stages are described in the following:

Step 1: Find X by minimizing $F(X)$ (1)

subject to $g_i(X) \leq 0, \quad i=1,2,..,m$ (2)

This nominal design can provide for obtaining the minimum value of performance function indicated as $F_{\min}$.

Step 2: Find X by minimizing the largest variability of $|F_U - F_L|$

subject to $g_i(X) \leq 0, \quad i=1,2,..,m$

The output of minimizing the largest variability of the performance function is defined as F_a.

Step 3: Find X by maximizing the variability of $|F_U - F_L|$

subject to $g_i(X) \leq 0, \quad i=1,2,..,m$

The output of this sub-problem is defined as F_b. The variability of performance function associated this output is defined as V_{max}.

Step 4: Select the larger one between F_a and F_b as F_{max}. i.e. F_{max}= Max [F_a , F_b].

Step 5: Assign the ideal variability of performance function as V_{min} which is equal to zero. Then a fuzzy formulation can be stated as following:

Find X by Maximize l

Subject to $l - m_F \leq 0$ (3)

$l - m_V \leq 0$ (4)

$g_i(X) \leq 0, \quad i=1,2,..,m$

$$m_F = \begin{cases} 1 & \text{if} \quad F \leq F_{\min} \\ \dfrac{F_{\max} - F}{F_{\max} - F_{\min}} & \text{if} \quad F_{\min} \leq F \leq F_{\max} \\ 0 & \text{if} \quad F_{\max} \leq F \end{cases} \quad (5)$$

$$m_V = \begin{cases} 1 & \text{if} \quad V \leq V_{\min} \\ \dfrac{V_{\max} - V}{V_{\max} - V_{\min}} & \text{if} \quad V_{\min} \leq V \leq V_{\max} \\ 0 & \text{if} \quad V_{\max} \leq V \end{cases} \quad (6)$$

where the parameter l is a scalar as well as an extra design variable in the optimization process.

Step 6: Check the convergence of the above optimization problem. If the problem is not converge, go back to step 2.

Step 7: Compute the standard deviation s_F of the performance function using the three-point approximation method described in section 3.

Illustrative Design Examples

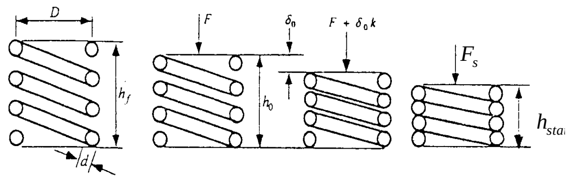

Fig. 2 A helical spring with loading

A mechanical helical spring (Fig. 2) design has the number of coils n, the diameter of string d and the outside diameter can not exceed 26 mm. An external load F applies on the spring that deforms from an original height of h_f to the height of h_0. Another fluctuating load F_0 is applied on it to yield a fluctuating displacement δ_0. The problem is to optimally design this spring that has a fixed δ_0 to sustain the maximum load F. The related parameters are listed as: h_f = 68 mm, h_0= 60 mm, $D_m \leq$26 mm, D =20 mm, δ_0 =5 mm, S_f=1.1, G =8.4 (10^3) Mpa and s_e = 42 Mpa. A local optimum result obtained in the first stage is written as: n = 10.5368, d = 5.2198 mm, F = 73.905 kg, and the feasible probability P of the design points in feasible region is 48.81%. The structural reliability index was 1.6682. In this case, the first two constraint are active. The design optimization of the second stage assumes the design variable n and d are of normal distribution. The deviation of each variable is s_n = 0.015 and s_d = 0.1 mm. The confidence level of the PVP is 0.95. When r_{nd} = 0, The modifying amount of the active constraint can be computed. The result of the second stage optimization is written as: n = 10.53682, d = 4.97434 mm, F = 61.013 kg, feasible probability P in feasible region is 99.29% and the structural reliability index was 4.105934. In the design optimization of the third stage, we relax 33% of the modifying amount and the optimization result is written as: n = 10.53682, d = 4.89253 mm, F = 57.0969 kg, the feasible probability P in feasible region is 99.95% and the structural reliability index was 4.92284. One can see the feasible robust design can increase the feasible probability to a high level robustness of 99.95%. The result of minimizing and maximizing the performance function is shown in Table 1. The last column shows the robust design where the both of performance function and its variation are minimized simultaneously. The extreme values of constructing the linear membership function are: $F_{\max}$ =-0.035, $F_{\min}$ =-80.9816, $V_{\max}$ =37.4011 and $V_{\min}$ = 0.

4. Conclusions

A three-stage robust feasible design considering the structural reliability, which applied the concept of parameter variation pattern (PVP), is introduced in the paper. Another robust performance design considering the reliability, which applied the strategy of fuzzy optimization to minimize both performance function and its variation simultaneously, is presented in this paper. The minimization of the largest variability between the two extreme performance values is taken as the objective function to be minimized. A step-by-step algorithm is presented and a mechanical spring design is given for further illustrating the proposed integrated design process. The algorithm for this design process guarantees the final design with the maximum satisfaction level. Once the robust feasible design method is applied, the reliability of the feasibility robustness can increase up to 99.95%. The proposed method for the problems contains fixed expected design target, the maximum structural system reliability, and the optimum design performance with the minimum variance. In general, the natural variation of minimizing a performance function is smaller than that of maximizing a performance function. The presented robust design method definitely is valuable and practical for general optimal engineering design.

Table 1 Design of a helical spring

Item	Min. F	Max. F	Robust design
Max. design satisfaction	------	------	0.42864
No. of coils n= x_1	30.00	10.3478	7.9872
Diameter of string d= x_2 (mm)	1.0	5.3151	4.0319
Loading F (kg)	0.035	80.9816	34.7328
Variation of performance $\boldsymbol{V}$	0.0917227	37.4011	21.1941
Deviation of performance s_F (kg)	0.14726	6.10681	3.4575

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