強健信賴度為基的最佳化方法於多目標工程設計研究

Robust Reliability-Based Optimization Methods for Multiobjective Engineering Design

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1. Abstract

This research develops a fuzzy and stochastic optimization method with weighting technique for multiobjective engineering design problems. The features of the problems consist of maximizing the structural reliability subject to fuzzy, probabilistic and fuzzy probability constraints. The design variables and some data used in the problems have stochastic information. The optimization process and the numerical computation for the Hohenbichler-Rackwitz's based reliability are introduced in here. Furthermore, the research goes to consider the robustness of the design performance of the objective function. The three-point approximation is used to compute the variation of the objective function. The worst-case values of the objective function are used to construct the membership function. Consequently, the multiobjective fuzzy optimization is applied to solve the robust reliability-based engineering design problems.

Keywords: Robust performance, reliability-based engineering design, fuzzy optimization.

中文摘要

本研究應用權重技術發展了一個模糊及推測最佳化的多目標設計方法。特色為最大化設計信賴度在具模糊的、或然率的及模糊或然率的限制條件之下。其設計變數或參數可為具統計或模糊的資訊。信賴度的計算乃是採用 Hohenbichler-Rackwitz's 的建議方法。此研究更進一步的探討設計目標性能的穩健化問題。三點近似法被用為計算目標性能的變異量，並以目標性能的最差情況來建立模糊歸屬函數。其次，多目標模糊最佳化的策略被用來解信賴度為基的工程設計題目。

關鍵詞：多目標設計，目標性能穩健，模糊最佳化。

2. Cause and Objective

To work out a real-world engineering design problem, the stochastic or probabilistic programming (Rao 1980) can deal with such circumstance in which some parameters and variables are random or probabilistic. A typical stochastic programming problem consists of probabilistic constraints where the safety constraint as a whole has to be greater than or equal to a specified probability. Another type of uncertainty exists in the real-world problem as vagueness or fuzziness recognized by people but unsolved until the fuzzy set theory was proposed (Zadeh 1965, Zimmerman 1985). Rao's (1980) work may be the earliest effort to the probabilistic optimization that considers the structural security by the probabilistic constraints. Later, some probabilistic optimum designs were explored and presented (Jozwiak 1986, Eggert and Mayne 1993). Recently Wang *et al* (1995) developed an algorithm to work on the probabilistic single-objective optimization problem. Their paper points out that a certain strength constraint below a predetermined probability of failure. However, it lacks the intensity of the overall structural reliability. People recognize that a reliable and precision engineering system usually demand the lowest weight, the highest performance and the maximum reliability. Those multiobjective optimum designs (Rao 1987, Rao *et al* 1992) rarely takes account the overall reliability of the system or its related issues.

This research presents a multiobjective fuzzy optimization algorithm and process for maximizing the Hohenbichler-Rackwitz (H-R) based reliability index (Hohenbichler and Rackwitz 1981) of a structural system and other design objectives at the same design level. Actually the H-R's reliability is originated from the idea of H-L's reliability (Hasofer and Lind 1974). The advantages of presented numerical structural reliability are computational efficiency and avoiding the tedious calculation of the directional factors. The objective weighting technique (Shih and Chang 1995) is used in the multiobjective fuzzy optimization that can generate a design representing the relative important degree of individual objective function and structural reliability. The proposed optimization process of the reliability-based design shows the presented method is practical and reliable.

The degree of robustness of the design goal is represented by the probability of robustness that is computed by randomly normal distribution model (Fig.1). In performance robustness, the variability of target performance of objective function is minimized for reducing the performance output variation. A better approach for estimating the deviation of output performance is proposed and discussed in this research. Multiobjective optimization with fuzzy theory (Shih and Wangsawidjaja 1995) is used to deal with the conflict and uncertainty among the objective function and the robustness of the target performance. The final design result is guaranteed in feasible region and at the same time the variation of output performance is minimized during the optimization process. Several design examples are applied to further explain the proposed integrated design method.

3. Results and Discussions

This research develops and can solve two major category problems. The first kind problem is reliability based fuzzy and stochastic optimization problem. The formulation is stated as following:

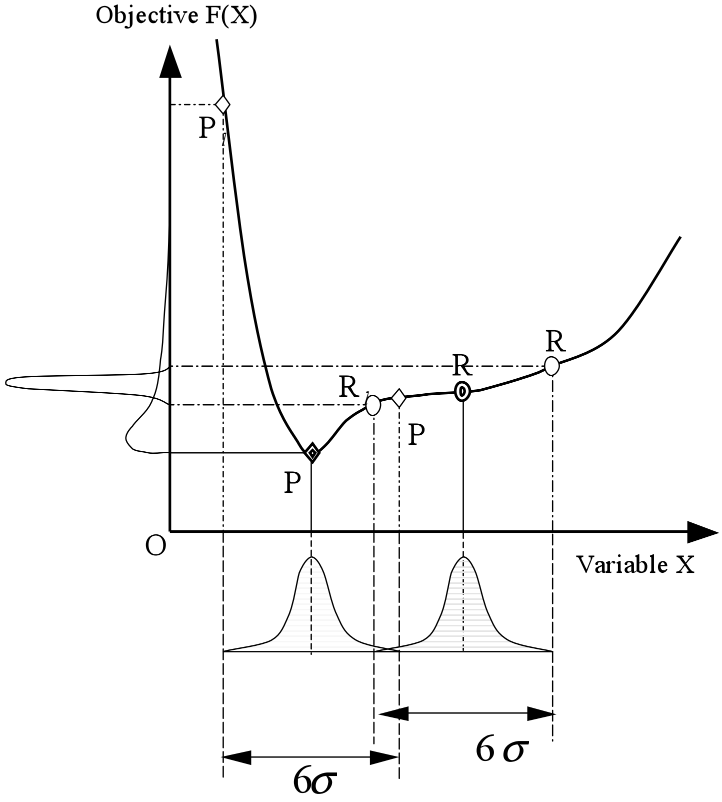

Fig. 1 Robust performance design and its design variables.

Find **X** that minimize $[f_1(\mathbf{X},\mathbf{Y}), f_2(\mathbf{X},\mathbf{Y}),\ldots, f_N(\mathbf{X},\mathbf{Y}), -R_{system}(\mathbf{X},\mathbf{Y},\mathbf{R})]^T$

subjext to

$R_{system}(\mathbf{X},\mathbf{Y},\mathbf{R}) \geq R_{specified}$

where the

$$R_{system}(X,Y) = Minimum[R_1(X,Y), R_2(X,Y),\ldots.R_m(X,Y)]$$

Consequently, the multiobjective fuzzy optimization then can be subjected to the following constraints:

$$g_j(\mathbf{X},\mathbf{Y}) \leq C_j,\ j = 1, 2, \ldots\ldots m_1$$

$$g_j(\mathbf{X},\mathbf{Y}) \leq (C_{fuzzy})_j,\ j = 1, 2, \ldots\ldots m_2$$

$$P[g_j(X, Y) \le 0] \ge P_j, j = 1, 2, \ldots\ldots m_3$$

$$P[g_j(X, Y) \le 0] \ge (P_{fuzzy})_j, j = 1, 2, \ldots\ldots m_4$$

$$R_{system}(X,Y) \ge (R_{specified})^L$$

$$R_{system}(X,Y) \le (R_{specified})^U$$

The optimum solution process for the robust performance design of the second kind is developed as following:

The 1st step: Find F_{min} by minimizing F(X) that subject to $g_i(X) \le 0$

The 2nd step: Find F_1 by mininimizing $|F_U - F_L|$ that subject to $g_i(X) \le 0$

The 3th step: Find V_{max} and F_2 by maximizing $|F_U - F_L|$ that subject to $g_i(X) \le 0$

The 4th step: Compare F_1 and F_2 to get F_{max} such that $F_{max} = \text{Max}\,[F_1\ , F_2]$

The 5th step: Let $V_{min} = 0$ and do the multiobjective fuzzy optimization to obtain F and V by maximizing λ that subject to

$$\lambda - \mu_F \le 0$$

$$\lambda - \mu_V \le 0$$

$$g_i(X) \le 0$$

where

$$\mu_F = \begin{cases} 1 & \text{if} \quad F \le F_{min} \\ \dfrac{F_{max} - F}{F_{max} - F_{min}} & \text{if } F_{min} \le F \le F_{max} \\ 0 & \text{if} \quad F_{max} \le F \end{cases}$$

$$\mu_V = \begin{cases} 1 & \text{if} \quad V \le V_{min} \\ \dfrac{V_{max} - V}{V_{max} - V_{min}} & \text{if} \quad V_{min} \le V \le V_{max} \\ 0 & \text{if} \quad V_{max} \le V \end{cases}$$

Both reliability-based optimization and robust performance optimum design problems are introduced, and the solution methods are successfully developed. It can serve as a useful tool for practical design problems especially when the fuzzy and probabilistic information coexists. Further research can extend to the development that considers the system reliability and robustness simultaneously.

4. Self-Evaluations

This project accomplish all of the original proposed research plan. It concludes that two engineering optimum design problems are developed and solved. The most contribution of it is that the proposed problem model are more practical than the existing models. The results of the final design can be much precise to be described in the real-world problems. It is useful in the precision engineering design and promote the design level with confidence. The results of the project can be appropriate and also to be published in the Journal and Conference.

5. References

[1] Arora, J.S. (1989), Introduction to Optimum Design, McGraw-Hill, New York, 41-45, 484-489.

[2] Eggert, R.J. and Mayne, R.W. (1993), "Probabilistic Optimal Design Using Successive Surrogate Probability Density Functions", *J. of Mechanical Design*, **115**, 385-391.

[3] Hasofer, A.M. and Lind, N.C. (1974), "Exact and Invariant Second-Moment Code Format", *J. Engr. Mech.*, Div. Proc. ASCE, **100**(EM1), 111-121.

[4] Hohenbichler, M. and Rackwitz, R. (1981), "Non-Normal Vectors in Structural Reliability", *J. Eng. Mech.* Div. ASCE, **109**(EM6), December.

[5] Jozwiak, S.F. (1986), "Minimum Weight Design of Structures with Random Parameters", *Computers & Structures*, **23**(4), 481-485.

[6] Rao,S.S.(1980),"Structural Optimization by Chance Constrained Programming Techniques", *Computers & Structures*, **12**, 777-781.

[7] Rao,S.S.(1987),"Multi-objective Optimization of Fuzzy Structural Systems, " *Int. J. for Numerical Methods in Eng.* **24**, 1157-1171.

[8] Rao, S. S., Sundararaju, K., Prakash, B. G. and Balakrishn, C. (1992), "Multiobjective Fuzzy Optimization Techniques for Engineering Design," *Computers & Structures*, **42**(1), 37-44.

[9] Shih, C.J. and Chang, C.J. (1995), "Parato Optimization of Alternative Global Criterion Method for Fuzzy Structural Design," *Computers & Structures*, **54**(3), 455-460.

[10] Wang, L. and Grandhi, R. V. (1995), " Structural Reliability Optimization Using An Efficient Safety Index Calculation Procedure ", *International Journal for Numerical Methods in Engineering*, **38**, 1721-1738.

[11] Zadeh, L. (1965),"Fuzzy Sets", *Information and Control,* **8**, 338-353.

[12] Zimmerman, H.-J. (1985), Fuzzy Set Theory- and Its Applications, Kluwer-Nijhoff, Boston.

[13] Shih, C. J. and Wangsawidjaja, R. A. S., "Multiobjective Fuzzy Optimization with Random Variable in a Mix of Fuzzy and Probabilistic Environment", *The Joint conference of the 4th Fuzzy-IEEE/Ifes'95*, Yokohama, Japan, March, 1995, pp. 20-24.