行政院國家科學委員會補助專題研究計畫成果報告

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※　低航速三維物體在波浪上的運動理論推導及計算　※

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國科會專題研究期末報告

A Computation on Wave Damping Effects for A 3-D Body under Regular Waves and Currents

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Abstract

The problem of a floating body moving with arbitrary oscillating motions and arbitrary wave directions is modified. Under the assumptions of potential flow and low moving speed, the Green function is expanded into an oscillating source G_0 at zero forward speed and a correct term G_1 to take into account low moving speed. It is shown that the present form of Green function gives good numerical calculation perfor
ance while wave damping effects are considered. This method can also be applied to calculate wave loads, added wave resistance and wave damping forces. The effects of currents to a floating body are be discussed as well.

Keywords : mooring system、wave exciting forces、added wave resistance、wave damping effects、wave drift damping

Introduction

In designing a single-point mooring system, it is important to consider wave damping effects while the mooring force exerting in the system is evaluated. To examine the effects, the motions of a floating body should be known. These motions can be described by obtaining various forces acting on the floating body. Over the recent two decades, many researchers have studied wave exciting force problems. Different approaches have been adapted and many numerical models have been developed. Some of the results have even been applied to the preliminary design of a floating system in ocean engineering. Wichers & Van Sluijs (1979), Wichers (1982,1984), Wichers & Van den Boom (1983), Faltinsen, Dahle & Sortland (1987), Faltinsen & Minsens et al (1980), Faltinsen & Sortland (1987), Takagi, Nakamur & Saito (1984), Chakrabarti (1982), Wichers (1986), Hearn & Tong (1987), and Lee & Sun (1997) have studied the relationship between wave drift damping and wave exciting forces. The main conclusions from their findings are

1) Measured damping coefficient is larger in waves than that in calm water when a floating body oscillates in low frequency modes.
2) Wave damping effects need to be seriously considered when surge motion of a floating body is simulated.
3) Wave drift damping can be obtained by taking derivative of added wave resistance to the velocity at zero forward speed.

From the conclusions above, the problem of finding wave drift damping can be converted into finding added wave resistance and its derivatives to the velocity at zero forward speed. The key is to find added wave resistance.

Under the assumptions of potential theory and low moving speed, the problem of a floating body moving with arbitrary oscillating motions and arbitrary wave directions is modified; and a form of simplified Green function is derived. The idea of Huijsmans & Hermans (1985) is followed. Therefor, Green function is modified into an oscillating source G_0 at zero forward speed and a correction term G_1 to take into account low moving speed. The expressions for G_0, G_1, ∇G_0 and ∇G_1 are then derived.

It is shown that the approach offer a decent method in obtaining body responses due to low moving speed while wave damping effects are considered. This method can also be applied to calculate wave loads, added wave resistance and wave damping forces. The effect of currents to a freely floating/restrained body will also be discussed.

Basic equations

Three right-handed coordinate systems are adopted in this paper.

1. A body coordinate system $\{\bar{o}-\overline{xyz}\}$ fixed on the body with $\overline{oz}$ pointing vertically upward and $\overline{oxy}$ corresponding to the undisturbed free surface.
2. A moving coordinate system $\{o\text{–}xyz\}$ having its origin on the calm water surface and which moves forward together with the body at a constant speed.
3. A coordinate system $\{\hat{o}-\hat{x}\hat{y}\hat{z}\}$ fixed in space. This system is consistent with the body coordinate system $\{\bar{o}-\overline{xyz}\}$ when the body is stationary.

A perfect, incompressible fluid and irrotational flow is assumed, which allow us to describe the motion by a velocity potential

$\varphi(x,y,z,t)$ where $\varphi(x,y,z,t)$ is on undisturbed fluid domain. The total velocity potential can be expressed as

$$\varphi(x,y,z,t) = \Phi(x,y,z,t) + \bar{\varphi}(x,y,z) \qquad (2\text{-}1)$$

where $\bar{\varphi}(x,y,z)$ is the steady potential due to the body traveling forward at a constant speed in calm water. $\Phi(x,y,z,t)$ is the unsteady velocity potential which can be further divided into the diffraction potential, radiation potential and incident wave potential.

Under the assumption of small amplitude motion, low current speed, small steady disturbance and no flow separation, neglecting the crossing of the unsteady and steady potential, within the scope of linear theory, and let the angle of the incident wave be at angle β to the $\hat{o}\hat{x}$, the problem of solving $\varphi(x,y,z,t)$ may be divided into solving the (A) set of equation for unsteady potential $\Phi(x,y,z,t)$ and the (B) sets of equation for steady potential $\bar{\varphi}(x,y,z)$.

$$(A)\begin{cases}
\nabla^2\Phi(x,y,z,t)=0 & (x,y,z)\in D \quad (2\text{-}2)\\
\dfrac{\partial^2\Phi}{\partial t^2}-2U_0\cos\beta\dfrac{\partial^2\Phi}{\partial t\partial x}-2U_0\sin\beta\dfrac{\partial^2\Phi}{\partial t\partial y}+U_0^2\cos^2\beta\dfrac{\partial^2\Phi}{\partial x^2} & \\
+2U_0^2\cos\beta\sin\beta\dfrac{\partial^2\Phi}{\partial x\partial y}+U_0^2\sin^2\beta\dfrac{\partial^2\Phi}{\partial y^2}+g\dfrac{\partial\Phi}{\partial z}=0 & (x,y,z)\in\Sigma_F \quad (2\text{-}3)\\
\dfrac{\partial\Phi}{\partial n}=\dot{\xi}_j n_j+\xi_j m_j & (x,y,z)\in\Sigma \quad (2\text{-}4)\\
\nabla\Phi\to 0 & z\to-\infty \quad (2\text{-}5)\\
\text{appropriate far-field boundary condition} &
\end{cases}$$

where D is the fluid domain, Σ_F is the undisturbed free surface, Σ is the mean wetted surface of the body, and $n_1=n_x$, $n_2=n_y$, $n_3=n_z$, $n_4=n_zy-n_yz$, $n_5=n_xz-n_zx$, $n_6=n_yx-n_xy$, (n_x,n_y,n_z) are the unit vectors normal to the body and directed into the fluid, (e_1, e_2, e_3) are the unit vectors on the moving coordinate system. $(m_1,m_2,m_3) = (0,0,0),(m_4,m_5,m_6) = (-n_3U_0\sin\beta, n_3U_0\cos\beta, n_1U_0\sin\beta-n_2U_0\cos\beta)$, the term $\dot{\xi}_j n_j$ and $\xi_j m_j$ in equation (2-4) will be recognized to be the summation of all terms from j=1 to 6 。 ξ_1、ξ_2、ξ_3 are linear displacements due to surge, sway, and heave respectively; ξ_4、ξ_5、ξ_6 are angular displacements due to roll, pitch, and yaw respectively.

$$\xi_j = \bar{\xi}_j e^{i\omega_e t} \qquad j = 1,2,.....6 \qquad (2\text{-}6)$$

$$\omega_e=\omega-\tilde{k}U_0\cos(\theta-\theta_0) \qquad (2\text{-}7)$$

$$\tilde{k}=\frac{\omega^2}{g},\ \omega \text{ is the wave frequency,}$$

θ_0 is the incident wave angle.

where η_j is the motion amplitude of the jth mode; ω_e is the encounter frequency of the body with the wave,

The steady potential $\bar{\varphi}(x,y,z)$ satisfies the following set of equations :

$$(B)\begin{cases}
\nabla^2\bar{\varphi}(x,y,z)=0 & (x,y,z)\in D \quad (2\text{-}8)\\
U_0^2\cos^2\beta\dfrac{\partial^2\bar{\varphi}}{\partial x^2}+U_0^2\cos\beta\sin\beta\dfrac{\partial^2\bar{\varphi}}{\partial x\partial y}+U_0^2\sin^2\beta\dfrac{\partial^2\bar{\varphi}}{\partial y^2}+g\dfrac{\partial\bar{\varphi}}{\partial z}=0 & \\
 & (x,y,z)\in\Sigma_F \quad (2\text{-}9)\\
\dfrac{\partial\bar{\varphi}}{\partial n}=U_0\cos\beta\cdot n_1+U_0\sin\beta\cdot n_2 & (x,y,z)\in\Sigma \quad (2\text{-}10)\\
\nabla\bar{\varphi}\to 0 & z\to-\infty \quad (2\text{-}11)\\
\text{appropriate far-field boundary condition} &
\end{cases}$$

As we only concern wave forces acting on the floating body, the effort will be devoted to solve and discuss the results of the (A) set of equations. First, we analyze free surface condition to give the respective order of each term in equation (2-3).

Assume $\tilde{\Phi}$ is characteristic velocity potential, L is characteristic wavelength, T is characteristic wave period and U_0 is characteristic velocity. Using these characteristic values for each term in equation (2-3), we get:

$$\frac{\tilde{\Phi}}{T^2}:\frac{U_0\tilde{\Phi}}{TL}:\frac{U_0\tilde{\Phi}}{TL}:\frac{U_0^2\tilde{\Phi}}{L^2}:\frac{U_0^2\tilde{\Phi}}{L^2}:\frac{U_0^2\tilde{\Phi}}{L^2}:\frac{L}{T^2}\frac{\tilde{\Phi}}{L}$$

and

$$1:\frac{U_0T}{L}:\frac{U_0T}{L}:\frac{U_0^2T^2}{L^2}:\frac{U_0^2T^2}{L^2}:\frac{U_0^2T^2}{L^2}:1$$

If $U_0T/L=\varepsilon$, then $U^2T^2/L^2=\varepsilon^2$. Assuming low moving speed, no flow separation, and problem solutions being correct to $O(\varepsilon)$, we may reduce the (A) set of equations into the (C) set of equations

$$(C)\begin{cases}
\nabla^2\phi_j(x,y,z)=0 & (x,y,z)\in D \quad (2\text{-}12)\\
-\omega_e^2\phi_j-2i\omega_eU_0\cos\beta\,\phi_{jx}-2i\omega_eU_0\sin\beta\,\phi_{jy}+g\phi_{jz}=0 & (x,y,z)\in\Sigma_F \quad (2\text{-}13)\\
\nabla\phi_j\cdot\vec{n}=n_j \quad j=1,2,....6 & (x,y,z)\in\Sigma \quad (2\text{-}14')\\
\nabla\phi_7\cdot\vec{n}=-\nabla\phi_0\cdot\vec{n} & (x,y,z)\in\Sigma \quad (2\text{-}14'')\\
\nabla\phi_j\to 0 & z\to-\infty \quad (2\text{-}15)\\
\text{appropriate far-field boundary condition} &
\end{cases}$$

and

$$\begin{aligned}\Phi(x,y,z,t)&=\phi e^{i\omega_e t}\\
&=i\omega_e\bar{\xi}_j\phi_j e^{i\omega_e t}+U_0\cos\beta\,\bar{\xi}_5\phi_3e^{i\omega_e t}-U_0\sin\beta\,\bar{\xi}_4\phi_3e^{i\omega_e t}\\
&\quad+U_0\sin\beta\,\bar{\xi}_6\phi_1e^{i\omega_e t}-U_0\cos\beta\,\bar{\xi}_5\phi_2e^{i\omega_e t}+\phi_7e^{i\omega_e t}\end{aligned} \qquad (2\text{-}16)$$

Equation set (C) are basic equations of the to-be-solved problem. When $\Phi(x,y,z,t)$ is solved from equation (2-12) to (2-15), the unsteady Bernoulli equation:

$$\frac{P}{\rho}+gz+\frac{\partial\Phi}{\partial t}-U_0\cos\beta\frac{\partial\Phi}{\partial x}-U_0\sin\beta\frac{\partial\Phi}{\partial y}+\frac{1}{2}\nabla\Phi\nabla\Phi=0 \qquad (2\text{-}17)$$

Therefore, the free surface elevation may be obtained by

$$\zeta(x,y) = -\frac{1}{g}\left(\frac{\partial\Phi}{\partial t} - U_0\cos\frac{\partial\Phi}{\partial x} - U_0\sin\frac{\partial\Phi}{\partial y}\right)\Big|_{z=0} \qquad (2\text{-}18)$$

The unsteady pressure P on the body surface can be found in order to calculate the respective order of forces acting on the body

$$\bar{F} = -\iint_s p\bar{n}ds \qquad (2\text{-}19)$$

where s is the wetted surface of the body.

Equation (2-17), (2-18) and (2-19) can then be used to find forces acting on body.
We show only the expressions of zero-order, first-order and second-order forces for the sake of space.

$$\bar{F}^{(0)} = \rho g\forall e_3 \qquad (2\text{-}20)$$

$$\bar{F}^{(1)} = -\rho g\left[\zeta_3^{(1)}S_{WL} - \zeta_5^{(1)}S_{10} + \zeta_4^{(1)}S_{20}\right]e_3 + \rho\iint_\Sigma \Phi_t^{(1)}\bar{n}^{(0)}ds$$
$$-\rho U_0\cos\iint_\Sigma \frac{\partial\Phi^{(1)}}{\partial x}\cdot\bar{n}^{(0)}ds - \rho U_0\sin\iint_\Sigma \frac{\partial\Phi^{(1)}}{\partial y}\cdot\bar{n}^{(0)}ds \qquad (2\text{-}21)$$

$$\bar{F}^{(2)} = R^{(1)}\bar{F}^{(1)} - \rho g\left[\zeta_3^{(2)}S_{WL} - \zeta_5^{(2)}S_{10} + \zeta_4^{(2)}S_{20}\right]e_3 + \rho\iint_\Sigma \nabla\Phi_t^{(1)}\cdot\bar{r}^{(1)}\bar{n}^{(0)}ds$$
$$+\rho\iint_\Sigma \Phi_t^{(2)}\bar{n}^{(0)}ds + \frac{1}{2}\rho\iint_\Sigma \left|\nabla\Phi^{(1)}\right|^2\bar{n}^{(0)}ds - \frac{1}{2}\rho g\int_{WL}\zeta_r^{(1)^2}\bar{n}^{(0)}dl$$
$$-\rho U_0\cos\iint_\Sigma \Phi_x^{(2)}\bar{n}^{(0)}ds - \rho U_0\sin\iint_\Sigma \Phi_y^{(2)}\bar{n}^{(0)}ds$$
$$-\rho U_0\cos\iint_\Sigma \nabla\Phi_x^{(1)}\cdot\bar{r}^{(1)}\bar{n}^{(0)}ds - \rho U_0\sin\iint_\Sigma \nabla\Phi_y^{(1)}\bar{r}^{(1)}\bar{n}^{(0)}ds \qquad (2\text{-}22)$$

the relative wave height $\zeta_r^{(1)}$ and the waterplane moments are defined as

$$\zeta_r^{(1)} = \zeta^{(1)} - (\zeta_3^{(1)} - \zeta_5^{(1)}x + \zeta_4^{(1)}y) \qquad (2\text{-}23)$$

$$S_{WL} = \iint_{S_{WL}} ds \qquad (2\text{-}24)$$

$$S_{10} = \iint_{S_{WL}} xds \qquad (2\text{-}25)$$

$$S_{20} = \iint_{S_{WL}} yds \qquad (2\text{-}26)$$

S_{WL} is the waterplane area

$\forall$ is the displacement

WL is the waterline

$$R^{(1)} = \begin{pmatrix} 0 & -\zeta_6^{(1)} & \zeta_5^{(1)} \\ \zeta_6^{(1)} & 0 & -\zeta_4^{(1)} \\ -\zeta_5^{(1)} & \zeta_4^{(1)} & 0 \end{pmatrix} \qquad (2\text{-}27)$$

$$\bar{M}^{(0)} = 0 \qquad (2\text{-}28)$$

$$\bar{M}^{(1)} = -\rho g\forall\left[\zeta_4^{(1)}x_{3B} + \zeta_3^{(1)}\frac{S_{20}}{\forall} - \zeta_5^{(1)}\frac{S_{12}}{\forall} + \zeta_4^{(1)}\frac{S_{22}}{\forall}\right]e_1$$
$$-\rho g\forall\left[\zeta_5^{(1)}x_{3B} - \zeta_3^{(1)}\frac{S_{10}}{\forall} + \zeta_5^{(1)}\frac{S_{11}}{\forall} - \zeta_4^{(1)}\frac{S_{12}}{\forall}\right]e_2$$
$$+\rho\iint_\Sigma \Phi_t^{(1)}\cdot(\bar{r}^{(0)}\times\bar{n}^{(0)})ds - \rho U_0\cos\iint_\Sigma \Phi_x^{(1)}\cdot(\bar{r}^{(0)}\times\bar{n}^{(0)})ds$$
$$-\rho U_0\sin\iint_\Sigma \Phi_y^{(1)}\cdot(\bar{r}^{(0)}\times\bar{n}^{(0)})ds \qquad (2\text{-}29)$$

$$\bar{M}^{(2)} = R^{(1)}\bar{M}^{(1)} - \rho g\forall\left[\zeta_4^{(2)}x_{3B} + \zeta_3^{(2)}\frac{S_{20}}{\forall} - \zeta_5^{(2)}\frac{S_{12}}{\forall} + \zeta_4^{(2)}\frac{S_{22}}{\forall}\right]e_1$$
$$-\rho g\forall\left[\zeta_5^{(2)}x_{3B} - \zeta_3^{(2)}\frac{S_{10}}{\forall} + \zeta_5^{(2)}\frac{S_{11}}{\forall} - \zeta_4^{(2)}\frac{S_{12}}{\forall}\right]e_2$$
$$+\rho\iint_\Sigma \Phi_t^{(2)}\cdot(\bar{r}^{(0)}\times\bar{n}^{(0)})ds - \rho U_0\cos\iint_\Sigma \Phi_x^{(2)}\cdot(\bar{r}^{(0)}\times\bar{n}^{(0)})ds$$
$$-\rho U_0\sin\iint_\Sigma \Phi_y^{(2)}\cdot(\bar{r}^{(0)}\times\bar{n}^{(0)})ds + \rho\iint_\Sigma \nabla\Phi_t^{(1)}\cdot\bar{r}^{(1)}(\bar{r}^{(0)}\times\bar{n}^{(0)})ds$$
$$-\rho U_0\cos\iint_\Sigma \nabla\Phi_x^{(1)}\cdot\bar{r}^{(1)}(\bar{r}^{(0)}\times\bar{n}^{(0)})ds - \rho\iint_\Sigma \nabla\Phi_y^{(1)}\cdot\bar{r}^{(1)}(\bar{r}^{(0)}\times\bar{n}^{(0)})ds$$
$$+\frac{1}{2}\rho\iint_\Sigma \left|\nabla\Phi^{(1)}\right|^2(\bar{r}^{(0)}\times\bar{n}^{(0)})ds - \frac{1}{2}\rho g\int_{WL}\zeta_r^{(1)^2}(\bar{r}^{(0)}\times\bar{n}^{(0)})dl \qquad (2\text{-}30)$$

where B is the center of buoyancy, and

$$S_{ij} = \iint_{S_{WL}} x_i x_j ds \qquad (x_1 = x,\ x_2 = y) \qquad (2\text{-}31)$$

where subscript (0), (1) and (2) represent zero-order, first-order, and second-order, respectively.

It is known that the second-order potential makes no contribution to added resistance, which only have a relation to first-order motion responses and first-order forces. So, added resistance R_A is the steady part of (2-32) in x (surge mode).

$$R_A = \Big\{R^{(1)}\bar{F}^{(1)} + \rho\iint_\Sigma \nabla\Phi_t^{(1)}\cdot\bar{r}^{(1)}\bar{n}^{(0)}ds + \frac{1}{2}\rho\iint_\Sigma \left|\nabla\Phi^{(1)}\right|^2\bar{n}^{(0)}ds$$
$$-\frac{1}{2}\rho g\int_{WL}\zeta_r^{(1)^2}\bar{n}^{(0)}dl - \rho U_0\cos\iint_\Sigma \nabla\Phi_x^{(1)}\cdot\bar{r}^{(1)}\bar{n}^{(0)}ds$$
$$-\rho U_0\sin\iint_\Sigma \nabla\Phi_y^{(1)}\cdot\bar{r}^{(1)}\bar{n}^{(0)}ds\Big\}_x \qquad \mu_i = \mu_j \qquad (2\text{-}32)$$

and the first-order motion equation is

$$\sum_{j=1}^{6}\zeta_j^{(1)}\left[-\mu_e^2(M_{kj} + A_{kj}) + i\mu_e B_{kj} + C_{kj}\right] = F_{WK}^{(1)} \qquad (2\text{-}33)$$

where $F_{WK}^{(1)}$ is the first-order wave disturbance force, (M_{kj}), (A_{kj}), (B_{kj}), (C_{kj}) are mass, added mass, damping coefficient, and righting moment matrix respectively. The detail expressions for these matrices are shown in Sun(1971).

After $\psi_j, j=1,\ldots,7$ solved using equation set (C), (M_{kj}), (A_{kj}), (B_{kj}), (C_{kj}) and $F_{WK}^{(1)}$ can be computed and the first-order wave responses ξ_j, $j = 1,\ldots,6$ can be attained through (2-6) and (2-33). Added resistance R_A may be obtained by (2-32). Wave drift damping can then be got by taking derivative of added resistance to velocity.

Solutions for equation set (C)

Using Fourier transformation method, we can acquire the basic solution of equation set (C) as following

$$G(x,y,z,\xi,\eta,\zeta) = \frac{1}{r} + \frac{1}{r_2} + \frac{1}{2\pi}\int_0^{\infty}\int_0^{\pi}\frac{k + k - k2\tau\cos(\ -\ )}{k - k + k2\tau\cos(\ -\ )}e^{k(z+\zeta)+ik[(x-\xi)\cos+(y-\eta)\sin]}d\theta dk \qquad (3\text{-}1)$$

where $r^2 = (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2$, $k = \frac{\mu^2}{g}$, $\tau = \frac{2\mu U}{g}$

Under the assumptions of small amplitude motion, low current speed, small steady disturbance, we keep only the first-order term of τ to get G as

$G=G_0+\tau G_1$ (3-2)

where

$$G_0=\frac{1}{r}+\frac{1}{r_1}+2\bar{k}\int_0^\infty \frac{e^{k(z+\zeta)}}{k-\bar{k}}J_0(kR)dk-2\pi i\bar{k}e^{\bar{k}(z+\zeta)}J_0(\bar{k}R) \qquad (3\text{-}3)$$

$$\begin{aligned}G_1=&-2i\cos(\quad-\theta)\int_0^\infty e^{k(z+\zeta)}J_1(kR)dk\\&-4i\bar{k}\cos(\quad-\theta)\int_0^\infty \frac{e^{k(z+\zeta)}}{k-\bar{k}}J_1(kR)dk\\&-2i\bar{k}^2\cos(\quad-\theta)\int_0^\infty \frac{e^{k(z+\zeta)}}{(k-\bar{k})^2}J_1(kR)dk\end{aligned} \qquad (3\text{-}4)$$

$$r_1^2=(x-\xi)^2+(y-\eta)^2+(z+\zeta)^2$$

$$R^2=(x-\xi)^2+(y-\eta)^2$$

$$\cos\theta=\frac{x-\xi}{R}$$

$$\sin\theta=\frac{y-\eta}{R}$$

and, J_0 and J_1 are the zero-order and the first-order Bessel function.

The solution of equation set (C), ψ_j, can be expressed as

$$\psi_j=\iint_\Sigma \sigma_j(\xi,\eta,\zeta)G(x,y,z,\xi,\eta,\zeta)ds \qquad j=1,2,...,7 \qquad (3\text{-}5)$$

where $G(x,y,z;\xi,\eta,\zeta)$ is obtained from (3-2); $\sigma_j(\xi,\eta,\zeta)$ can be got through (2-14') and (2-14''). Set (x_0,y_0,z_0) is one point on body surface Σ then

$$\iint_\Sigma \sigma_j(\xi,\eta,\zeta)\nabla_{\xi\eta\zeta}G(x_0,y_0,z_0;\xi,\eta,\zeta)\cdot\bar{n}ds=\begin{cases}n_j(x_0,y_0,z_0) & j=1,2,....,6\\ -\nabla\psi_0\cdot\bar{n}(x_0,y_0,z_0) & j=7\end{cases} \qquad (3\text{-}6)$$

set

$$\sigma_j=\sigma_{j0}+\tau\sigma_{j1} \qquad (3\text{-}7)$$

substitute $G=G_0+\tau G_1$ 和 $\sigma_j=\sigma_{j0}+\tau\sigma_{j1}$ into (3−5),

$$\iint_\Sigma(\sigma_{j0}+\tau\sigma_{j1})(G_0+\tau G_1)\cdot\bar{n}ds=\begin{cases}n_j(x_0,y_0,z_0) & j=1,2,...6\\ -\nabla\psi_0\cdot\bar{n}(x_0,y_0,z_0) & j=7\end{cases} \qquad (3\text{-}8)$$

keep only the first-order term,

$$\iint_\Sigma \sigma_{j0}\nabla_{\xi\eta\zeta}G_0\cdot\bar{n}ds=\begin{cases}n_j(x_0,y_0,z_0) & j=1,2,...6\\ -\nabla\psi_0\cdot\bar{n}(x_0,y_0,z_0) & j=7\end{cases} \qquad (3\text{-}9)$$

and

$$\iint_\Sigma \sigma_{j1}\nabla_{\xi\eta\zeta}G_0\cdot\bar{n}ds=-\iint_\Sigma \sigma_{j0}\nabla_{\xi\eta\zeta}G_1\cdot\bar{n}ds \qquad j=1,2,...7 \qquad (3\text{-}10)$$

From equation (3-9) and (3-10), it can be seen that the problem of solving equation set (C) has been turned into a zero-speed problem and an equation (3-10). There are many good methods for solving a zero-speed problem. As equation (3-10) has the same coefficient matrix as equation (3-9), it is not necessary to calculate this matrix again. Only the terms in the right-hand side of (3-10) require to be worked out. By using this method to solve equation set (C), we can solve zero forward speed problem and low forward speed problem at the same time.

Numerical calculation

According to the preceding discussion, the wave drift damping can be obtained by the following steps :

(a) Solve equation (3-9) and (3-10).
(b) Calculate ψ_j (j=1,2,…,7) by equation (3-5).
(c) Calculate the first-order motion responses from the motion equation.
(d) Calculate the first-order forces.
(e) Calculate the added wave resistance from equation (2-32).
(f) Calculate wave drift damping by taking derivative of added resistance to velocity at zero forward speed.

It is aware that the crucial determinant is to calculate G_0, ∇G_0, G_1 and ∇G_1. There are many honorable methods to solve G_0 and ∇G_0, for example Pidcock(1985). We will concentrate here on finding G_1 and ∇G_1. Let

$$AJ_{12}=\int_0^\infty \frac{e^{k(z+\zeta)}}{(k-\bar{k})^2}J_1(kR)dk=AJ_{12R}+iAJ_{12I} \qquad (4\text{-}1)$$

$$AJ_{02}=\int_0^\infty \frac{e^{k(z+\zeta)}}{(k-\bar{k})^2}J_0(kR)dk=AJ_{02R}+iAJ_{02I} \qquad (4\text{-}2)$$

$$AJ_{11}=\int_0^\infty \frac{e^{k(z+\zeta)}}{(k-\bar{k})}J_1(kR)dk=AJ_{11R}+iAJ_{11I} \qquad (4\text{-}3)$$

$$AJ_{01}=\int_0^\infty \frac{e^{k(z+\zeta)}}{(k-\bar{k})}J_0(kR)dk=AJ_{01R}+iAJ_{01I} \qquad (4\text{-}4)$$

$$AJ_{10}=\int_0^\infty e^{k(z+\zeta)}J_1(kR)dk \qquad (4\text{-}5)$$

$$AJ_{1k}=\int_0^\infty ke^{k(z+\zeta)}J_1(kR)dk \qquad (4\text{-}6)$$

$$AJ_{0k}=\int_0^\infty ke^{k(z+\zeta)}J_0(kR)dk \qquad (4\text{-}7)$$

$$AJ_{00}=\int_0^\infty e^{k(z+\zeta)}J_0(kR)dk \qquad (4\text{-}8)$$

and from equation (3-4), it can be found that G_1 and ∇G_1 may be expressed by arithmetic operation of $AJ_{12}, AJ_{02},...,AJ_{00}$. The key to calculate G_1 and ∇G_1 is to give the calculation expression of $AJ_{12}, AJ_{02},...,AJ_{00}$. From Zhao(1988), the expression of $AJ_{00}, AJ_{0k}, AJ_{1k}$ and AJ_{10} are obtained

$$AJ_{00}=\frac{1}{\sqrt{(x-\xi)^2+(y-\eta)^2+(z+\zeta)^2}} \qquad (4\text{-}9)$$

$$AJ_{0k}=\frac{z+\zeta}{[(x-\xi)^2+(y-\eta)^2+(z+\zeta)^2]^{3/2}} \qquad (4\text{-}10)$$

$$AJ_{1k}=\frac{R}{[(x-\xi)^2+(y-\eta)^2+(z+\zeta)^2]^{3/2}} \qquad (4\text{-}11)$$

$$AJ_{10}=\frac{\sqrt{(z+\zeta)^2+R^2}+(z+\zeta)}{R\sqrt{(z+\zeta)^2+R^2}} \qquad (4\text{-}12)$$

The expression of AJ_{01} is given by *Pidcock*(1985),

$$AJ_{01R}=-e^{\bar{k}(z+\zeta)}\left\{\int_0^{-(z+\zeta)}\frac{e^{\bar{k}s}}{\sqrt{s^2+R^2}}ds+\frac{\pi}{2}\left[H_0(\bar{k}R)+Y_0(\bar{k}R)\right]\right\} \qquad (4\text{-}13)$$

$$AJ_{12R}=\frac{1}{R\bar{k}^2}+\frac{2}{\circledR}\int_0^{\infty}\frac{2k\bar{k}\cos kp+(k^2-\bar{k}^2)\sin kp}{(k^2+\bar{k}^2)^2}K_1(kR)dk$$
$$+\circledR\left[(p+\frac{1}{\bar{k}})Y_1(\bar{k}R)-RY_0(\bar{k}R)\right]e^{-\bar{k}p} \qquad (4\text{-}15)$$

$$AJ_{12I}=\circledR\left[(p+\frac{1}{\bar{k}})J_1(\bar{k}R)-RJ_0(\bar{k}R)\right]e^{-\bar{k}p} \qquad (4\text{-}16)$$

where $p=|z+¸|$, $Y_1(\bar{k}R)$ is a first-order Bessel function of the second kind, $K_1(kR)$ is a first-order modified Bessel function of the second kind.

$$AJ_{02R}=-\frac{2}{\circledR}\int_r^{\infty}\frac{(k^2-\bar{k}^2)\cos kp-2k\bar{k}\sin kp}{(k^2+\bar{k}^2)^2}K_0(kR)dk$$
$$+\circledR\left[pY_0(\bar{k}R)+RY_1(\bar{k}R)\right]e^{-\bar{k}p}+O(r\ln rR) \qquad (4\text{-}17)$$

$$AJ_{02I}=\circledR\left[pJ_0(\bar{k}R)+RJ_1(\bar{k}R)\right]e^{-\bar{k}p} \qquad (4\text{-}18)$$

where $K_0(kR)$ is a zeroth-order modified Bessel fnction of the second kind

$$AJ_{11R}=\frac{-1}{R\bar{k}}-\frac{2}{\circledR}\int_0^{\infty}\frac{(k\cos kp-\bar{k}\sin kp)K_1(kR)}{k^2+\bar{k}^2}dk-\circledR Y_1(\bar{k}R)e^{-\bar{k}p} \qquad (4\text{-}19)$$
$$AJ_{11I}=-\circledR J_1(\bar{k}R)e^{-\bar{k}p} \qquad (4\text{-}20)$$

To test the present method, it is applied to a hemisphere of radius 1 m. In the computation, the surface of the hemisphere is divided into 360 panels (see Fig.1). The testing conditions are

$\frac{\mu_e^2}{g}R$ = 0.2, 0.35, 0.5, 0.8, 1.0, 1.15, 1.30, 1.5, 1.6, 1.8

Fr = -0.064, -0.032, 0.0, 0.032, 0.064

incident wave angle = 0°

two cases: hemisphere is free

hemisphere is restrained from oscillating in pitch

where

μ_e is the encounter frequency, R is the radius of hemisphere,

$Fr=U_0/\sqrt{gD}$, D is the diameter of hemisphere, U_0 is current speed

Figure 2 shows the added resistance for a free hemisphere at zero forward speed. The solid line represents the analytical solution (Pinkster 1980), and the symbol points are the computed results. It is seen that the two results agree well with each other.

Figure 3-7 show the first-order motion transfer function(surge; heave), first-order force transfer function and horizontal drift force for a hemisphere restrained from oscillating in pitch. The influence of current on mean drift force and motion responses is definitely obvious. In Fig.6, the value of the transfer function at the peak is about 2.19 for Fr=0.064 and 1.88 for Fr=0.00. That is, the current velocity of 0.28 m/s may results in a 16% increase of motion response for a hemisphere of a 2 m diameter. In Fig.7, the value of the transfer function at the peak is 1.32 for Fr=0.064 and 1.08 for Fr=0.032. That is, the current velocity of 0.28 m/s and 0.14 m/s may result in a 47% and 20% increase of mean drift force acting on the hemisphere.

Figure 8 compares results with those of Zhao (1988). The curve is Zhao's results and the symbol points are ours. It can be seen that the results of the present method agree well with Zhao's for *kR*<1.3. For kR>1.3, Fr=0.064 and Fr=0.032, the present method obtains a lower value than Zhao's. The reason is that the cross terms of steady potential and unsteady potential are not considered under the assumption of low current speed in the present method. Figure 8 also shows that the interaction between wave and current is important in the case where the steady disturbance due to the body is not small. Figure 9 shows wave damping coefficient for a hemisphere restrained from pitch oscillation.

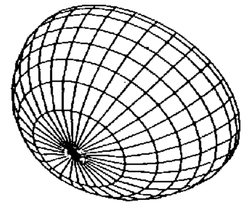

Fig.1 the hemisphere

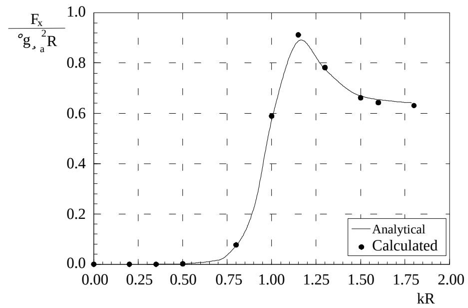

Fig.2 Comparison with Kudou's analytical result;hemisphere is free

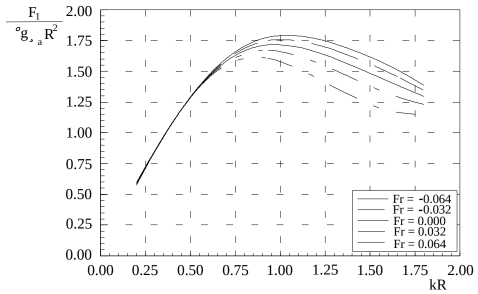

Fig.3 Hemisphere is restrained from oscillating in pitch

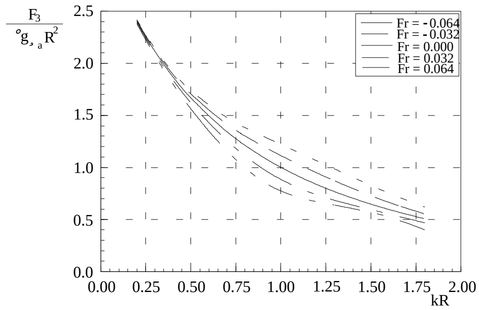

Fig.4 Hemisphere is restrained from oscillating in pitch

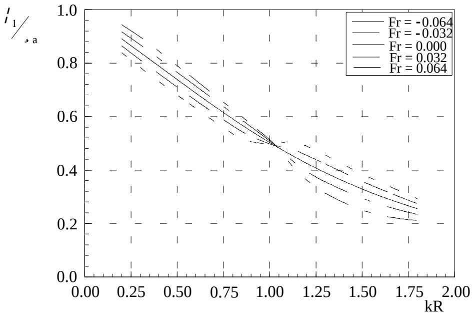

Fig.5 Hemisphere is restrained from oscillating in pitch

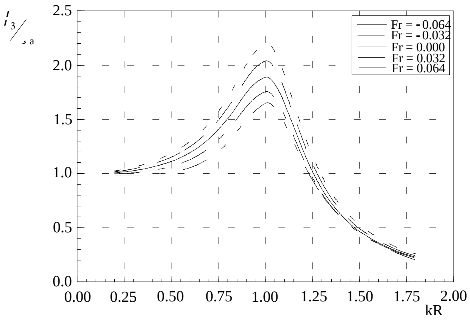

Fig.6 Hemisphere is restrained from oscillating in pitch

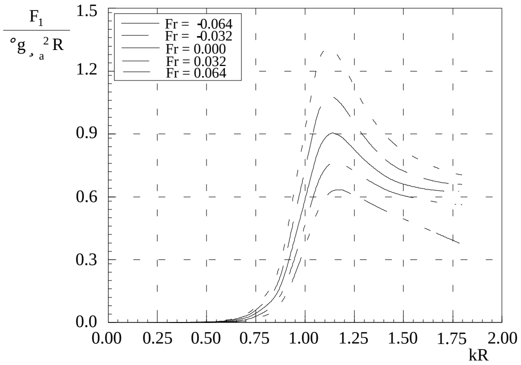

Fig.7 Hemisphere is restrained from oscillating in pitch

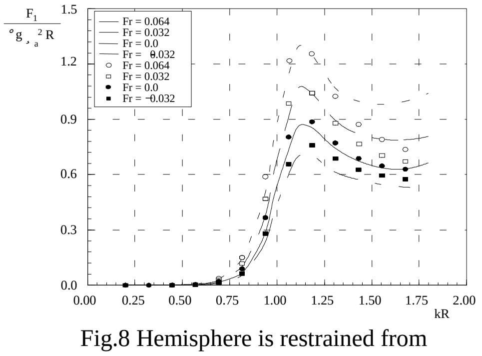

Fig.8 Hemisphere is restrained from oscillating in pitch

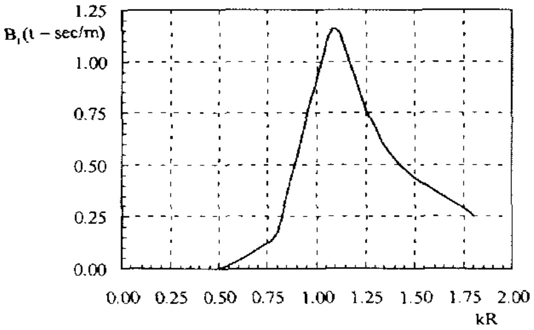

Fig.9 Hemisphere is restrained from oscillating in pitch

Conclusion

The conclusions of this work are summarized as following:

1. The influence of current on a floating body is crucially important.
2. The present numerical method can be used to predict wave loads, motion responses and added resistance for a 3-D floating body which causes a small, steady disturbance under the combined effect of currents and waves.

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