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The Day-of-the-Week Effect on the Shape of the Heavy-Tailed Distribution

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Abstract

This study examines the day-of-week-effect on the shape of the distribution by using the GARCH(1,1) model with the heavy-tailed distribution of Politis (2004). The empirical results showed that both returns of Dow Jones and S&P 500 significantly exhibited fat-tailed on Mon, Tues, Thurs and Fri, and this provided important implication on risk management, in particular the calculation of daily VaR.

Keywords: Day-of-the-week effect; Heavy-tailed distribution.
JEL Classification Codes: G14.

1. Introduction

The day-of-the-week effect on financial assets returns has been extensively discussed and abundant researches can be found in academic literatures. Since these anomalous empirical evidences are hardly explained by the existing finance theory, some scholars (Keim and Stambaugh, 1984; Jaffe and Westerfield, 1985) once asserted that they are the most puzzling phenomena in finance.

To the best of our knowledge, researches involved with the day-of-the-week effect are mainly focused on the conditional mean and variance of stock returns (Gibbons and Hess, 1981; Smirlock and Starks, 1986; Chang et al., 1993; Chang et al., 1998; Choudhry, T., 2000), or trading activities of the market (Kamara, 1997; Chordia et al., 2001). None of the previous studies examined this effect on the shape of a heavy-tailed distribution. In other words, we wish to find out which distribution of asset returns during a week is significantly fat-tailed. Our findings provide with important practicability for the Value-at-Risk (VaR) calculation because the quantile of the realized distribution is also a key input while adopting the variance-covariance method to calculate it. Probably, the day-of-the-week effect on the shape of the fat-tailed distribution might also be concerned with the distribution-based asset pricing and hedging strategies.

Owing to the stylized facts of financial returns, such as exhibiting fat-tailed and leptokurtic, fitting the popular GARCH model with a suitable distribution is still an ongoing task[1]. Politis (2004) proposed a heavy-tailed distribution for ARCH/GARCH residuals, which is assumed to follow the

[1] Bollerslev (1987) and Baillie and Bollerslev (1989) found that the standardized residuals of GARCH models are still exhibited leptokurtic. Thus, a distribution with fat-tailed and leptokurtic properties is more appropriate for GARCH-typed models.

truncated standard normal distribution. Moreover, the estimators of the GARCH model incorporated with the heavy-tailed distribution can be obtained by maximum likelihood estimation and partaken its favorable properties as well[2]. Consequently, this study adopts the GARCH(1,1) model incorporated with the recently proposed heavy-tailed distribution to shed light on the empirical issue of the day-of-the-week effect on the shape of the fat-tailed distribution.

The organization of this paper is as follows. In section 2, we present the model to examine the day-of-the-week effect on the shape of a heavy-tailed distribution. We provide the empirical evidences in section 3, and final section is our conclusion.

2. The GARCH Model with Heavy-tailed Distribution

Many empirical studies have identified that financial returns exhibit heavy tails (Engle, 1982; Jansen and de Vries, 1991; Pagan, 1996). For this reason, we adopted the GARCH model with a heavy-tailed distribution, which is used a truncated standard normal distribution. The daily return of the stock index is defined by $r_t = \log(p_t/p_{t-1})\times 100$, where p_t is the observed stock index at time t, and the GARCH(1,1) model with the heavy-tailed distribution of Politis (2004) is described as follows

$$r_t = \mu + \varepsilon_t, \qquad (1)$$

$$\varepsilon_t \,|\, \Omega_{t-1} \sim f(u_t; a_t, \sigma_t),\ \sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \qquad (2)$$

where Ω_{t-1} denotes the information set up to time t-1, and the covariance-stationary conditions are $\omega > 0,\ \alpha \geq 0,\ \beta \geq 0$, and $\alpha + \beta < 1$. The error term, u_t, follows an i.i.d. distribution $f(u_t; a_t, 1)$, and is given as

$$f\left(u_t; a_t, 1\right) = \frac{\left(1 + a_t u^2\right)^{-3/2} \exp\left\{-u^2/2\left(1+a_t u^2\right)\right\}}{\sqrt{2\pi}\left(\Phi\left(1/\sqrt{a_t}\right) - \Phi\left(-1/\sqrt{a_t}\right)\right)} \qquad (3)$$

$$a_t = a_1 d_{1t} + a_2 d_{2t} + a_3 d_{3t} + a_4 d_{4t} + a_5 d_{5t} \qquad (4)$$

where a_t is a time-varying shape parameter reflecting the degree of heavy tails, and $f(u_t; a_t, 1)$ will reduce to standard normal distribution as $a_t \to 0$. For the objective of this paper, a_t is parameterized as equation (4), in which d_{1t}, d_{2t}, d_{3t}, d_{4t} and d_{5t} represent the dummy variables of Mon, Tues, Wed, Thurs and Fri. The dummy variable d_{1t} equals to 1 if day t is Monday, and d_{1t} equals to 0 otherwise; $d_{2t} = 1$ if day t is Tuesday, etc. The GARCH(1,1) model with the heavy-tailed distribution is estimated using MLE in this study[3].

3. Empirical Results

In this section, Dow Jones Industry index and S&P 500 stock index are used as our empirical investigation. The sample period of both stock indices began on January 1997 and ended on December 2005, totally containing 2265 observations.

Table 1 reports the estimation result of our model with the day-of-the-week effect on the shape of the heavy-tailed distribution. Quasi-maximum likelihood estimation[4] and the BFGS numerical optimization algorithm in WinRats 6.0 are applied to maximize the log-likelihood function. As can be seen, the day-of-the-week effects are statistically significant at 1% for Mon, Tues, Thurs and Fri of both indices (only a_2 of Dow Jones is significant at 5%). Moreover, the diagnostic tests of Ljung-Box

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[2] See Gouriéroux (1997).

[3] As Hall and Yao (2003) indicated that maximum (pseudo) likelihood estimation will be consistent even in the presence of heavy-tailed errors though the rate of convergence is slower than $n^{1/2}$.

[4] The parameters are estimated by assuming normality and the t-statistics are adjusted by the Bollerslev-Wooldridge robust standard errors.

statistics indicated that both of the standardized residuals and squared standardized residuals do not exhibit serial correlations, which ensured the correct fitness of the conditional mean and variance equations of both returns.

Table 1: Day-of-the-week effect on the shape of the heavy-tailed distribution

Index	**Dow Jones**	**S&P 500**
μ	0.041 [c] [0.020]	0.047 [b] [0.019]
ω	0.018 [c] [0.002]	0.021 [c] [0.004]
α	0.065 [c] [0.002]	0.071 [c] [0.011]
β	0.907 [c] [0.002]	0.900 [c] [0.015]
a_1	0.067 [c] [0.014]	0.066 [c] [0.014]
a_2	0.029 [b] [0.014]	0.041 [c] [0.012]
a_3	0.006 [0.016]	0.004 [0.013]
a_4	0.044 [c] [0.015]	0.035 [c] [0.012]
a_5	0.061 [c] [0.012]	0.053 [c] [0.013]
Q(12)	11.602	14.978
Q(24)	17.786	20.359
$Q^2(12)$	15.142	12.678
$Q^2(24)$	20.678	19.208
Log-likelihood	-3295.893	-3373.345

Note: 1. a, b and c denote significantly at the 10%, 5% and 1% level, respectively.
2. Values in the brackets are t-statistics.
3. Q(n) and $Q^2(n)$ are the Ljung-Box Q tests for nth order serial correlation of the standardized residuals and squared standardized residuals.

The results of Table 1 demonstrated that, except for Wed, the quantiles of the standardized residuals of both indices are time-varying and significantly different from normal distribution, and the shape parameters are largest for Mon and Fri. The empirical evidences of this paper revealed that the day-of-the-week-effect exists in the shape of a heavy-tailed distribution, and this effect could be used to improve the accuracy of coverage rate and efficiency of the VaR calculation.

4. Conclusion

In the presence of fat-tailed properties of financial returns, the day-of-the-week effect is examined on the shape of the distribution by using a GARCH(1,1) model with the heavy-tailed distribution of Politis (2004). The empirical evidences revealed that the return of Dow Jones and S&P 500 significantly exhibited fat-tailed on Mon, Tues, Thurs and Fri, and the result contains important implication on risk management, in particular downside risk measurement of Value-at-Risk.

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