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Forecasting China Stock Markets Volatility via GARCH Models Under Skewed-GED Distribution

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Abstract

This study investigates how specification of return distribution influences the performance of volatility forecasting using two GARCH models (GARCH-N and GARCH-SGED). Daily spot prices on the Shanghai and Shenzhen composite stock indices provide the empirical sample for discussing and comparing the relative out-of-sample volatility predictive ability, given the growth potential of stock markets in China in the eyes of global investors. Empirical results indicate that the GARCH-SGED model is superior to the GARCH-N model in forecasting China stock markets volatility, for all forecast horizons when model selection is based on MSE or MAE. Meanwhile, the DM-tests further confirm that volatility forecasts by the GARCH-SGED model are more accurate than those generated using the GARCH-N model in all cases, indicating the significance of both skewness and tail-thickness in the conditional distribution of returns, especially for the emerging financial markets.

Keywords: Volatility forecasting, SGED, Skewness, Fat-tails, Garch
JEL Classification Codes: C22, C53, G10

1. Introduction and Motivation

Following the stock market crash of 1987, stock price volatility has been the focus of regulators, academic researchers and practitioners concern. Volatility forecasts of stock price are crucial inputs for pricing derivatives as well as trading and hedging strategies. In addition, the introduction of the first Basel Accord in 1996, which set minimum capital reserve requirements to be held by financial institutions proportional to their estimated risks, has further highlighted the significance of volatility prediction due to its essential role in calculating value-at-risk (VaR). Given these facts, the quest for accurate forecasts appears to still be ongoing.

Over the past two decades, a large volume of articles have been discussed and written about the volatility of stock returns for developed capital markets. Recently, the potential for China stock markets growth in the emerging countries has received a great deal of attention of qualified foreign institutional investor and global investors. Surprisingly, the annual rate of return for the Shanghai and Shenzhen composite indices in China were 81.7% and 66.3%, respectively, during 2006. However, this rapid growth has been accompanied by high risk. Unfortunately, there has been relatively little work done on modeling and forecasting stock market volatility in China (see, e.g., Xu, 1999; Lee et al., 2001) provided that it has significantly different risk and return characteristics from developed stock markets. Owing to the increasing volatility of the emerging China stock markets, accurate volatility forecasts have become a crucial issue.

It is well known that financial returns are often characterized by a number of typical ‘stylized facts’ such as volatility clustering, persistence and time variation of volatility. The generalized autoregressive conditional heteroskedasticity (GARCH) genre of volatility models is regarded as an appealing technique to cater to the aforesaid empirical phenomena. The existing literature has long been recognized that the distribution of returns can be skewed. For instance, for some stock market indices, returns are skewed toward the left, indicating that there are more negative than positive outlying observations. The intrinsically symmetric distribution, such as normal, student-t or generalized error distribution (GED) cannot cope with such skewness. Consequently, one can expect that forecasts and forecast error variances from a GARCH model may be biased for skewed financial time series.

The focus is the out-of-sample forecasting performance of the GARCH model with skewed generalized error distribution (SGED) relative to the traditionally standard GARCH model, which does not take the skewness into account. Illustrations of these techniques are presented for two main stock markets in China, the Shanghai and Shenzhen composite stock indices, which are considered more interesting and attractive than that of general developed markets.

The remainder of this paper is organized as follows. Section 2 presents a brief literature review. Section 3 describes the adopted econometric methodology. The data description and empirical results are then reported in Section 4. Summary and concluding remarks are presented in the last Section.

2. Literature Review

In modern finance theory, Markowitz (1952) used asset returns volatility as a measure of risk. The existing literature has supported that most time series data of financial assets exhibit linear dependence in volatility, which is referred to as volatility clustering in econometrics and empirical finance. Engle (1982) first proposed the ARCH (Autoregressive conditional heteroskedasticity; ARCH) model, which assumes normal errors for asset returns and successfully captures a number of stylized facts of financial assets, such as time-varying volatility and volatility clustering. The traditional econometric time series models generally assume a normal distribution of stock returns. However, the financial literature has long been aware that financial returns are non-normal and tend to have leptokurtic and fat-tailed distribution (Mandelbrot, 1963; Fama, 1965). Moreover, Hsu et al. (1974) and Hagerman (1978) found that the empirical distribution of stock returns is also significantly non-normal. Therefore, to modify the traditional assumption of normality of the ARCH model, Bollerslev (1987) applied the GARCH-t model, which assumes that the residual of asset returns follows the student t distribution in order to capture fat-tailed characteristic of time series data abstemiously. Because of the drawback of lower kurtosis of student t distribution, the student t distribution does still not adequately describe the leptokurtic phenomenon of returns distribution. Based on the seminal works by Mandelbrot (1963) and Fama (1965), there is widespread recognition that financial returns exhibit positive excess kurtosis and heavy tails. In this framework, several distributions for returns innovation have been proposed to take into account the excess kurtosis. For example, researchers have proposed the use of the generalized error distribution (Nelson, 1991; Taylor, 1994; Lopez, 2001; Lee et al., 2001; Marcucci, 2005) or the heavy-tailed distribution (Politis, 2004) to alleviate this problem. Note that the aforesaid distributions can properly capture the excess kurtosis, but impose the restriction of symmetry which is not always valid for financial data. Furthermore, Mittnik and Paolella (2000) also indicate that return is the presence of excessively fatter tails and pronounced skewness and a consequence of strong volatility clustering. Non-Gaussian time series thus have begun to receive considerable attention and forecasting methods are gradually being developed.

Many empirical studies have found evidence that conditional modeling with asymmetry[1] in the volatility process cannot capture the skewness and tail-thickness of financial returns distribution and make the use of skewed-t distribution (Hansen, 1994). Subsequently, Theodossiou (2000) proposed a skewed generalized error distribution (hereafter, SGED) for modeling the empirical distribution of financial asset returns, while Lehnert (2003) incorporated the SGED return innovation into the GARCH model to analyze in- and out-of-sample option pricing performance using DAX index options. Additionally, Bali (2007) modeled the nonlinear dynamics of short-term interest rate volatility with SGED distributions, and concluded that the level-GARCH model that accommodates the tail-thickness of interest rate distribution generates satisfactory volatility forecasts of short-term interest rates. Accordingly, recent developments share a common characteristic: They account for asymmetrically distributional assumption in return innovations.

Xu (1999) and Lee et al. (2001) are two recent papers that estimate volatility for stock markets in China. Xu (1999) modeled volatility for daily spot returns of Shanghai composite stock index from May 21, 1992 to July 14, 1995 and tested the in-sample goodness-of-fit of various competing models. He found that the GARCH model is superior to that of either EGARCH or GJR-GARCH models, indicating that there is almost no so-called leverage effect in the Shanghai stock market since volatility is mainly caused by governmental policy on stock markets under the present financial system. [2] Lee et al. (2001) examined time-series features of stock returns and volatility in four of China's stock exchanges. [3] They provided strong evidence of time-varying volatility and indicated volatility is highly persistent and predictable. Moreover, evidence in support of a fat-tailed conditional distribution of returns was found. The papers by Xu (1999) and Lee et al. (2001) just focus on the in-sample goodness-of-fit of volatility models. However, a good starting point to judge competitive models is to assess their out-of-sample forecasting performance. In addition, a leptokurtic and skewed returns distribution should be considered when using emerging market data.

This study considers the applicability of the GARCH(1,1) model in modeling volatility for the Shanghai and Shenzhen composite stock indices. We adopt SGED errors in the estimation, thus allowing flexible treatment of both skewness and tail-thickness to further enhance the robustness of the estimation results. This study chooses an adaptive volatility model for the China stock market by examining the relative out-of-sample predictive ability of the GARCH-N and GARCH-SGED models on daily Shanghai and Shenzhen stock return data. Particularly, the forecast horizon is extended to include 1-, 2-, 5-, 10- and 20-day forecasts. Awartani and Corradi (2005) show that the use of squared returns as a proxy for volatility ensures the correct ranking of predictive models in terms of a quadratic loss function. Therefore, when the true underlying volatility process is not observed, this study uses squared returns as a proxy for latent volatility. The empirical analysis comprises two steps: The first step involves obtaining an overview of the predictive ability of the various models by computing their out-of-sample mean squared errors (MSE) and mean absolute errors (MAE). Since these two summary statistics do not provide a statistical test of the difference between the two models. The DM-test which has been advocated by Diebold and Mariano (1995) is thus used to further examine the relative out-of-sample predictive ability of various GARCH models.

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[1] The asymmetric volatility of stock returns is meant as leverage effect (Black, 1976), volatility feedback or time-varying risk premium (Bekaert and Wu, 2000).

[2] This fact is encouraging in using a symmetric GARCH model for modeling the volatility dynamics of stock markets in China. Furthermore, several recent articles (e.g., Sadorsky, 2006; Hung et al., 2008) have reported more new evidence in favor of the parsimonious GARCH(1,1) model in providing accurate volatility forecasts.

[3] Their four China stock market price indices comprise the Shanghai 'A' share index, Shanghai 'B' share index, Shenzhen 'A' share index and Shenzhen 'B' share index.

3. Volatility Modelling and Performance Evaluation

3.1 GARCH(1,1) Model with Normal and Skewed-GED Distributions

Let r_t=100(lnS_t -lnS_{t-1}), where S_t denotes the stock price, r_t denotes the continuously compounded daily returns of the underlying assets on time t, and Ω_{t-1} denotes the information set of all observed returns up to time $t-1$. The GARCH(1,1) model can be formulated as follows:

$$r_t = \mu + ar_{t-1} + \varepsilon_t, \quad \varepsilon_t = \sigma_t z_t, \quad z_t \mid \Omega_{t-1} \overset{iid}{\sim} f(0,1) \tag{1}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2}$$

where μ and a are constant parameters, ε_t is the innovation process, $f(0,1)$ is a density function with zero mean and unit variance. Moreover, ω, α, β are nonnegative parameters with $\alpha+\beta<1$ to ensure the positive of conditional variance and stationarity as well. In the empirical investigation, two conditional distributions for the error term (z_t) were considered: (i) a standard normal distribution, and (ii) a SGED distribution. The density function of the standard normal distribution can be expressed as follows:

$$f(z_t) = (2\pi)^{-0.5}\exp(-z_t^2/2) \tag{3}$$

Consequently, the log-likelihood function of the GARCH(1,1) model with normally distributional errors (hereafter, GARCH-N) can be derived as:

$$LL(\Psi_n) = -\frac{1}{2}\cdot\sum_{t=1}^{T}\left(\ln 2\pi + \ln\sigma_t^2 + \frac{\varepsilon_t^2}{\sigma_t^2}\right) \tag{4}$$

where Ψ_n=[μ, a, ω, α, β] denotes the parameter vector of the GARCH-N model.

Most of the empirical literature finds evidence that conditional modeling with asymmetry in the volatility process cannot capture the skewness and tail-thickness of financial return distributions. To accommodate these empirical distributional phenomena, this present study makes use of the SGED distribution which has been advocated by Theodossiou (2000), allowing returns innovation to follow a flexible treatment of both skewness and tail-thickness in the conditional distribution of returns. The density function of the standardized SGED distribution can be expressed as follows:

$$f(z_t \mid \upsilon,\lambda) = C\cdot\exp\left(-\frac{|z_t-\delta|^{\upsilon}}{[1-\mathrm{sign}(z_t-\delta)\lambda]^{\upsilon}\theta^{\upsilon}}\right) \tag{5}$$

where,

$$C = \upsilon(2\theta\cdot\Gamma(1/\upsilon))^{-1} \tag{6}$$

$$\theta = \Gamma(1/\upsilon)^{0.5}\Gamma(3/\upsilon)^{-0.5}S(\lambda)^{-1} \tag{7}$$

$$\delta = 2\lambda\cdot A\cdot S(\lambda)^{-1} \tag{8}$$

$$S(\lambda) = \sqrt{1+3\lambda^2-4A^2\lambda^2} \tag{9}$$

$$A = \Gamma(2/\upsilon)\Gamma(1/\upsilon)^{-0.5}\Gamma(3/\upsilon)^{-0.5} \tag{10}$$

where the shape parameter υ governs the height and fat-tails of the density function with constraint $\upsilon>0$, while λ is a skewness parameter of the density with $-1<\lambda<1$. In the case of positive (negative) skewness, the density function skews toward to the right (left). Sign is the sign function. Particularly, the SGED distribution turns out to be the standard normal distribution when $\upsilon=2$ and $\lambda=0$. The log-likelihood function of the GARCH(1,1) model with standardized SGED errors (hereafter, GARCH-SGED) can be derived as:

$$LL(\Psi_{SGED}) = T\ln C - \sum_{t=1}^{T}\left(\frac{|\varepsilon_t/\sigma_t-\delta|^{\upsilon}}{\left[1-\mathrm{sign}\left(\varepsilon_t/\sigma_t-\delta\right)\lambda\right]^{\upsilon}\theta^{\upsilon}} + \ln\sigma_t\right) \tag{11}$$

where Ψ_{SGED}=[μ, a, ω, α, β, υ, λ] denotes the vector of parameters to be estimated.

3.2. Volatility Forecasts

The out-of-sample volatility (variance) forecasts of the GARCH(1,1) model follows a rolling window scheme to implement and evaluate the model-based volatility. Let T=R+P [4], where R denotes the rolling window, P denotes the rolling time and T is the sample observations. The rolling window scheme works in the following manner: At the first step, we use observations from 1 to R, in the second from 2 to R+1, and finally from R to R+P-1. In this way, the rolling window R stays fixed and the forecasts do not overlap. The conditional variance at time t is

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{12}$$

The forecast of σ_t^2 given information Ω_{t-1} can be formulated as

$$E(\sigma_t^2 \mid \Omega_{t-1}) = \omega + (\alpha+\beta)\sigma_{t-1}^2 \tag{13}$$

Thus, the one-step-ahead forecast at time t+1 is given by

$$\hat{\sigma}_{t+1}^2 = \hat{\omega} + (\hat{\alpha}+\hat{\beta})\hat{\sigma}_t^2 \tag{14}$$

where parameters $\hat{\omega}$, $\hat{\alpha}$ and $\hat{\beta}$ are estimated by QMLE (Quasi Maximum Likelihood Estimation; QMLE) according to the BFGS optimization algorithm. Meanwhile, the *h*-step-ahead forecast can be generated as follows:

$$\hat{\sigma}_{t+h}^2 = \frac{\hat{\omega}(1-(\hat{\alpha}+\hat{\beta})^h)}{1-(\hat{\alpha}+\hat{\beta})} + (\hat{\alpha}+\hat{\beta})^h\hat{\sigma}_t^2 \tag{15}$$

where $\hat{\sigma}_t^2$ is the volatility forecast generated from Eq(2).

3.3. Evaluation of Volatility Forecasting Performance

3.3.1. Loss Functions

In order to evaluate the predictive ability of various competing models, this study utilizes square returns (r_t^2) as a proxy for the latent volatility (Brailsford and Faff, 1996; Brooks and Persand, 2002; Sadorsky, 2006). [5] The forecasting performance of competing models is evaluated using standard forecast appraisal criteria, namely, mean squared error (MSE) and mean absolute error (MAE), presented in the following equations:

$$MSE = \frac{h}{P}\sum_{t=1}^{P/h}\left(r_{t+h}^2 - \sigma_{t+h}^2\right)^2 \tag{16}$$

$$MAE = \frac{h}{P}\sum_{t=1}^{P/h}\left|r_{t+h}^2 - \sigma_{t+h}^2\right| \tag{17}$$

where r_{t+h}^2 and σ_{t+h}^2 denote the ex post and the forecasted variance over horizon h made at day t, respectively.

3.3.2. Model Significance TEST (DM-test)

Diebold and Mariano (1995) proposed a test of forecast accuracy between two sets of forecasts using the MSE. Such a test is based on the null hypothesis of no difference in the accuracy (equal predictive ability) of the two competing models. The null hypothesis of equal forecast accuracy is tested based on $E(d_t)=0$ where E is the mathematical expectation operator and $d_t = e_{A,t}^2 - e_{B,t}^2$. The variable $e_{A,t}$ and $e_{B,t}$ are forecast errors generated by two competing models A and B, respectively. The DM-test statistic is as follows:

4 In this study, models are estimated for a total of 1250 observations (R = 1250), and there are about 400 one-ahead volatility forecasts (P = 400).

5 Awartani and Corradi (2005) demonstrated that the use of squared returns as a proxy for volatility ensures a correct ranking of forecasting models in terms of a quadratic error statistic, such as the mean squared errors.

$$DM = \bar{d} \cdot (\hat{V}(\bar{d}))^{-0.5} \overset{a}{\sim} N(0,1) \quad (18)$$

where $\bar{d} = \frac{1}{P}\sum_{t=1}^{P} d_t$, $\hat{V}(\bar{d}) \approx P^{-1}(\gamma_0 + 2\sum_{k=1}^{h-1}\gamma_k)$ and P, *h* step forecasts are computed from models A and B. Moreover, the variable γ_k denotes the *k*-th autocovariance of d_t. Under the null hypothesis of equal predictive ability, the DM-test statistic has a standard normal distribution asymptotically [6]

4. Data Description and Empirical Results

4.1. Data Description

The data for this study consists of China stock markets, including Shanghai and Shenzhen composite stock indices. Daily closing spot price indices (adjusted for dividends) for a total of 1683 observations are obtained from the Bloomberg database. The sample period is from January 4, 2000 to December 29, 2006. To save space, we do not report the descriptive graphs of stock indices in China. Note that the graphs of levels of indices show that both the Shanghai and Shenzhen composite indices in China display pictures of volatile bull markets during 2006, implying that the emerging stock markets are empirically appealing than the developed capital markets. In addition, the QQ-plots indicate that fat tails are not symmetric, providing evidence in favor of SGED distribution with flexible treatment of fat tails and skewness in the conditional distribution of returns series.

Table 1: Descriptive statistics of stock indices in China

Index	Shanghai	Shenzhen
Mean	0.0382	0.0168
Maximum	9.4007	9.2438
Minimum	-6.5429	-7.3839
Std. Dev.	1.3601	1.4539
Skewness	0.6137*	0.3314*
Kurtosis	5.3273*	4.8384*
J-B	2094.5976*	1671.4759*
$Q^2(12)$	81.4956*	119.6785*

Notes 1: * denote significantly at the 1% level. 2. $Q^2(12)$ denote the Ljung-Box Q test for 12th order serial correlation of the squared returns. 3. J-B statistics are based on Jarque and Bera (1987) and are asymptotically chi-squared-distributed with 2 degrees of freedom.

Table 1 summarizes the basic statistical characteristics of the return series. [7] The average daily returns are positive and very small compared with the variable standard deviation. Each of the index price returns displays significant evidence of skewness and kurtosis. Both of the series are skewed towards the right, indicating that there are more positive than negative outlying returns in China stock markets. Additionally, each series is characterized by a distribution with tails that are significantly fatter than for a normal distribution. Such evidence thus suggests for using the SGED distribution to ingratiate both the leptokurtic and skewness character of these return series. J-B normality test statistics, indicating that neither return series has normal distribution. Moreover, the Ljung-Box $Q^2(12)$ statistics for the squared returns indicate that the return series exhibit linear dependence and strong ARCH effects. Preliminary analysis of the data suggests the use of a GARCH technique for capturing the time-varying volatility. Consequently, incorporating skewed-GED returns innovation into the GARCH model can be expected to generate satisfactory volatility forecasts for China stock indices.

4.2. Estimation Results

The estimation results of the GARCH-N and GARCH-SGED for Shanghai and Shenzhen composite indices during the in-sample period are listed in Table 2. From panel A, the sums of parameters α and

[6] Adopting Monte Carlo simulation, Diebold and Mariano (1995) demonstrated that such test statistic performs well in a number of applied situations.
[7] The unit root tests indicate no evidence of non-stationarity in the returns of Shanghai and Shenzhen composite stock indices.

β for these two models are less than one, and thus ensure that the conditions for stationary covariance hold. The parameters λ and υ of the GARCH-SGED model which range from 0.057 to 0.084 and 1.105 to 1.127, respectively, are all statistically significant, at the 1% level. It reveals that the distributions of returns series are right-skewed and fat-tailed. Turing the discussion to panel B, diagnostics of the standardized residuals of GARCH-N and GARCH-SGED models confirm that the GARCH(1,1) specification in these models is sufficient to correct the serial correlation of these two returns series in the conditional variance equation. Furthermore, the LR-test statistics indicate no evidence of normality for either stock index. This provides evidence in favor of skewed generalized error distribution for modeling the empirical distribution of stock returns in China.

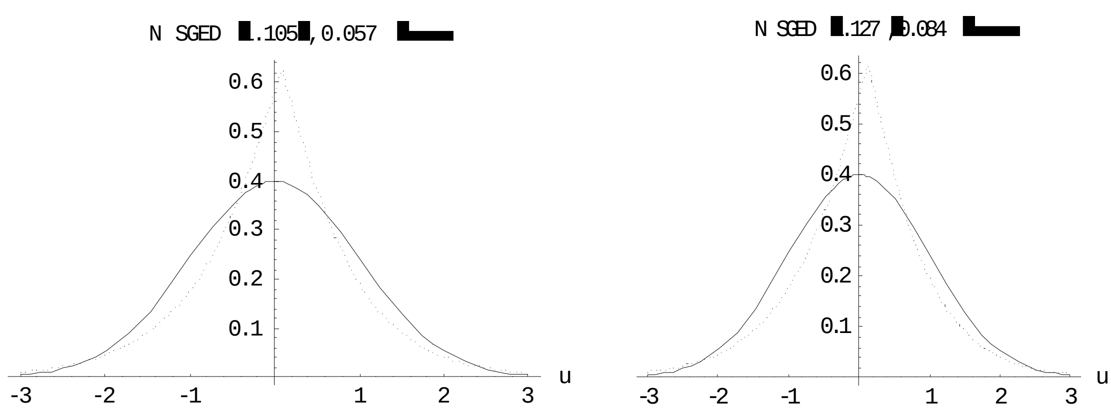
Table 2: Estimation results of alternate GARCH models

Panel A. Estimates and standard errors				
	Shanghai		**Shenzhen**	
Parameter	GARCH-N	GARCH-SGED	GARCH-N	GARCH-SGED
μ	-0.018 (0.028)	-0.046 (0.030)	-0.031 (0.030)	-0.031 (0.032)
a	0.019 (0.029)	-0.004 (0.023)	0.043 (0.029)	-0.001 (0.016)
ω	0.075* (0.006)	0.199* (0.052)	0.291* (0.012)	0.189* (0.050)
α	0.136* (0.005)	0.160* (0.034)	0.251* (0.011)	0.191* (0.036)
β	0.829* (0.004)	0.738* (0.047)	0.634* (0.008)	0.733* (0.046)
λ	-	0.057* (0.017)	-	0.084* (0.027)
υ	-	1.105* (0.045)	-	1.127* (0.048)
Panel B. Diagnostic tests				
$Q^2(12)$	8.327	3.517	8.486	8.514
LL	-2045.2	-2007.4	-2140.5	-2077.2
LR-test	75.6*		126.6*	

Notes: 1. * denotes significantly at the 1% level. 2. Standard errors for the estimators are included in parentheses. 3. $Q^2(12)$ is the Ljung-Box Q test for serial correlation in the squared standardized residuals with 12 lags. 4. LL indicates the log-likelihood value. 5. The LR-test statistic is asymptotically distributed as a chi-square with two degrees of freedom (χ^2 (2)). Its critical value at the one-percent level of significance is 9.21, namely, $\chi^2_{0.99}(2) = 9.21$.

The density graphs of SGED [8] distribution versus normal distribution are illustrated in Figure 1. Apparently, each of the SGED distributions is skewed towards the right, and is more leptokurtic and thicker than the normal distribution. These results show that the characteristics of the SGED distribution are consistent with the descriptive statistics of return series reported in Table 1, indicating that the SGED closely fits the empirical distribution of return series.

Figure 1: Skewed-GED density against the normal distribution



SGED versus Normal (Shenzhen) **SGED versus Normal (Shenzhen)**

8 The parameter (υ, λ) is obtained from the estimation results of the Shanghai and Shenzhen composite indices, respectively.

4.3. Volatility Forecasting Performance

Table 3 tabulates the out-of-sample mean squared errors (MSE) for the various models under alternative forecast horizons. For the one-step-ahead forecast horizon, the GARCH-SGED models generate lower MSEs than the GARCH-N model for the Shanghai and Shenzhen composite indices. The multi-step-ahead prediction displays evidence that the GARCH-SGED model still outperforms the GARCH-N model for these two empirical data. The results indicate that the GARCH-SGED model is superior to the GARCH-N model in forecasting China stock market volatility when model selection is based on the loss function of MSE.

Table 3: Out-of-sample mean squared error statistic

Forecast Horizon	Shanghai		Shenzhen	
	GARCH-N	GARCH-SGED	GARCH-N	GARCH-SGED
1	18.9208	18.6135*	21.4517	21.3110*
2	18.6695	18.3596*	21.0712	20.8685*
5	18.5710	18.3053*	20.8560	20.6347*
10	18.3025	17.9907*	20.4459	20.1778*
20	19.0199	18.7220*	21.4366	20.9154*

Note: * denotes the minimum MSE among the two models of the given forecast horizon.

Brailsford and Faff (1996) indicated that the various model rankings were sensitive to the error statistic used to assess the accuracy of the forecasts. Moreover, since volatility is being forecast, mean squared errors will raise the return innovation to the fourth power and hence make the loss function very sensitive to outliers. Accordingly, this study also provides mean absolute errors (MAE) as alternative useful forecast summary statistics. The out-of-sample mean absolute error for the various models under alternative forecast horizons is listed in Table 4. As Table 4 shows, the GARCH-SGED models generate lower MAEs than the GARCH-N models for the Shanghai and Shenzhen composite indices, across all forecast horizons. Therefore, the MAE selects the GARCH-SGED as the best model.

Table 4: Out-of-sample mean absolute error statistic

Forecast Horizon	Shanghai		Shenzhen	
	GARCH-N	GARCH-SGED	GARCH-N	GARCH-SGED
1	2.0719	2.0409*	2.4120	2.3984*
2	2.0559	2.0219*	2.3970	2.3785*
5	2.0535	2.0114*	2.3820	2.3571*
10	2.0594	1.9945*	2.3830	2.3429*
20	2.1862	2.0866*	2.5570	2.4619*

Note: * denotes the minimum MAE among the two models of the given forecast horizon.

This study adopts DM-test (Diebold and Mariano, 1995) to further examine the statistical significance from the two competing models. The findings from the DM-test statistics across various forecast horizons are provided in Table 5. For the case of the Shanghai composite index, the DM probability values show that the GARCH-SGED model is statistically significant, at the 5% level, from the GARCH-N model for the 5- and 10-day-ahead forecast horizons, and statistically significant at the 1% level for the remaining forecast horizons. On the other hand, for the case of the Shenzhen composite index, the DM probability values indicate that the GARCH-SGED model is at least statistically significant, at the 10% level, from the GARCH-N model, across various forecast horizons.

Table 5: DM-test results

Forecast Horizon	Shanghai		Shenzhen	
	DM-statistics	P-value	DM-statistics	P-value
1	2.8212***	0.0023	2.1245**	0.0168
2	2.8830***	0.0019	2.2095**	0.0135
5	2.2244**	0.0130	1.7620**	0.0390
10	2.1756**	0.0147	1.5353*	0.0623
20	2.4510***	0.0071	3.0533***	0.0011

Notes: 1. *, ** and *** denote significantly at the 10%, 5% and 1% level, respectively. 2. Data represents the t-statistics and corresponding p-value of the Diebold and Mariano (1995) test. 3. The null hypothesis of DM-test is that of equal predictive ability of the two models; a significantly positive (negative) t-statistics indicates the GARCH-N model is dominated by (dominates) the GARCH-SGED model.

Summarizing the results listed in Tables 3, 4 and 5 reveals that the GARCH-SGED models do indeed provide accurate volatility forecasts, superior to those provided by the GARCH-N models for stock markets in China, indicating that the assumption of skewed-GED returns innovation is essential for improving forecasting accuracy of volatility models.

5. Summary and Concluding Remarks

The potential for China stock markets growth in the emerging countries has attracted qualified foreign institutional investor and global investors in recent years. However, this rapid growth is associated with high risk. Thus, accurate volatility forecasts are a crucial issue. This study adopts a rolling window scheme to implement and compare the relative ability to predict out-of-sample volatility for the GARCH-SGED and GARCH-N models when applied to the Shanghai and Shenzhen composite stock indices over various forecast horizons.

Empirical results show that the GARCH-SGED models yield lower MSEs and MAEs than the GARCH-N models for the Shanghai and Shenzhen composite indices, across all forecast horizons. For further statistical testing, the DM statistics confirm that the volatility forecasts obtained using the GARCH-SGED model are more accurate than those generated using the GARCH-N model for stock markets in China.

Accurate volatility forecasts are crucial to investors, institutional traders and risk managers, as well as academic researchers seeking to quantify market uncertainty. Overall, this study concludes that incorporating SGED returns innovation into the GARCH(1,1) model generates superior volatility forecasts for stock markets in China. These findings show the significance of both skewness and tail-thickness in the conditional distribution of returns, and should be considered in making decisions regarding market timing, portfolio selection and VaR estimates when applies to emerging financial markets.

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