

# INTEGRATED MECHANICAL DESIGN METHOD OF ROBUST PERFORMANCES TO TOLERANCING WITH FUZZY MULTIOBJECTIVE OPTIMIZATION

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## Abstract

A multiobjective deterministic method of robust performances design has been applied to determine the optimum nominal dimensions of manufactured components subjected to dimensional tolerances. The fuzzy natures exist in several objectives that require using fuzzy optimization strategy. The optimum sought is that for achieving the most balance and highest satisfaction design among the optimum goal performances, the least variability of the goal performance and the least assembly variation of the components. The aim and the design process are demonstrated by a crankshaft design with assembly components of an air compressor. This integrated mechanical design method by adjusting the nominal dimensions provides a means of simultaneously improving goal performances, variability and assembly quality without tightening tolerances.

**Key words:** Tolerance Design, Robust Design, Fuzzy Optimization, Quality Control, Mechanical Design

## 1. Introduction

Finite tolerances have to exist in manufacturing components of individual product or mass production. Reducing those tolerances to zero is not possible and realistic. How to control the specific clearances in a limited range are the acceptable and practical ways of promising the definite performance quality of the mechanical product and is therefore of interests as regards control of quality (Parkinson, 1993). Jeang (1997) used a Taguchi type loss function associating with overall cost to determine the optimum tolerances. The further improvement is quite possible in many cases by adjusting nominal design point of parameter design without tightening of tolerances. The nominal dimensional design with fixed tolerances is the first priority of parameter design to improve the overall design performance. Chen (1995) interpreted the tolerances in terms of standard deviations and the optimum standard deviation for minimum cost can be determined. It shows that the dimensional tolerance directly affect the performance of the design goal.

In general, an optimum set of nominal design parameter values exists, which correspond to the minimum variability of one or more performance measuring (Chang, 1988). The extension of this approach to dimensioning tolerances where a set of nominal dimensions are to be determined for components assembly with given tolerances where one or more specify dimensions of assembly have the minimum variability. Parkinson (1997) proposed a performance measure of variability, constructed by the maximum and minimum values of performance function under tolerance variation, been minimized. In this paper, an integrated mechanical design method and process is presented for simultaneously dealing with the optimum goal performance, minimum the variation of assembly and the highest robust performance, simultaneously, under given dimensional tolerances of components. Due to the contradiction between different design performances and components assembly, there exists a range of that reflects the fuzzy characteristics;

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therefore, this problem can be dealt with by fuzzy optimization technique. A crankshaft design assembly for the air compressor is modified for the illustration of presenting model and design process. This problem contains five components assembly, optimizing the goal performance (objective function) and maximizing the robustness simultaneously, to obtain eleven nominal variables design under given tolerances.

## 2. Optimum Design with Given Tolerances for Robust Multi-Performances

The nominal design variable  $\bar{X} = \{ \bar{x}_i \}, (i=1,2,...,n)$ , is to be solved for an assembly of components. The tolerances around the optimizing mean design may be asymmetric so that  $\bar{x}_i - l_i \leq x_i \leq \bar{x}_i + u_i$ , where  $l_i$  and  $u_i$  indicate the  $i$ th tolerance limits corresponding to each  $x_i$ . The fundamental robust performance design with its formulation is similar to the work of Chang (1988) where the solution method is quite different. To apply design technique for tolerancing problems, three types of goal functions with their performance features can include (1) multiple original performance functions, (2) performance measuring function, and (3) performance variability function. For example, a clearance composed of assembly components must be maintained to close to a specifying value that belongs to a performance measuring function. For instance, a function of specific clearance of the assembly components is stated as:  $C(X) = C(x_1, x_2, ..., x_n)$ . Then, it should be possible to construct a tolerance loop function by defining the maximum value and minimum value written in  $C_U(\bar{X})$  and  $C_L(\bar{X})$ , respectively.

$$C_U(\bar{X}) = C(..., \bar{x}_{i-l_i}^{+u_i}) \quad (1)$$

$$C_L(\bar{X}) = C(..., \bar{x}_{i-l_i}^{+u_i}) \quad (2)$$

Then the optimum choice of  $\bar{X}$  for the robust assembly against variation in the specified feature  $C$  can be obtained by:

$$\text{Min } V_C(\bar{X}) = C_U(\bar{X}) - C_L(\bar{X}) \quad (3)$$

In summary, a robust multiobjective performances design problem can be stated as:

$$\text{Find } \bar{X} = [\bar{x}_1, \bar{x}_2, ..., \bar{x}_n]^T \text{ with known } l_i \text{ and } u_i, i=1,2,...,n$$

$$\text{Min } f_i(\bar{X}), i=1,2,...,N_1 \quad (4)$$

$$\text{Min } V_{f_i}(\bar{X}), i=1,2,...,N_2 \quad (5)$$

$$\text{Min } V_{C_i}(\bar{X}), i=1,2,...,N_3 \quad (6)$$

$$\text{s.t. } C_i^L \leq C_i(\bar{X}) \leq C_i^U, i=1,2,...,N_3 \quad (7)$$

$$g_i(X) \leq 0, i=1,2,...,m \text{ and } \bar{X}^L \leq \bar{X} \leq \bar{X}^U$$

where  $f_i(\bar{X})$  indicates the  $i$ th goal performance function,  $V_{f_i}(\bar{X})$  is the  $i$ th variability function of goal performance function, and  $V_{C_i}(\bar{X})$  is the  $i$ th variability function of components assembly. Each function of specific clearance,  $C_i(\bar{X})$ , may be constrained by Eq. (7). A number of methods are available for the above problem. However, due to the contradiction among multiple goal performance functions and variability of assembly function, a fuzzy nature exists in the multiple objectives so that it is suitable using fuzzy multiobjective optimization strategy (Rao et al., 1992) for solving this problem. First of all, the optimization has to be executed for individual objective function one by one subjected to the same constraints. A fuzzy region for an objective function can be formulated between the upper limit  $f_{\max}$  and lower limit  $f_{\min}$ , thus, a popular linear membership function can be constructed. The modified feasible direction method is used in here for solving the problem.

### 3. Description of Mechanical Design Example

A 0.75 kW motor with 1460 rpm drives a reciprocating air compressor. The completed structural drawing shown in Figure 1 where the crankshaft (501) and its neighboring machine parts including bearings (502), pulley (512), key (513), and nut (515) constructing components assembly required dimensional design. The maximum torque  $T$  is 4.905 kg.m on the crankshaft. The task is to optimum design the crankshaft with its assembly components shown on Figure 2 where the eleven optimal nominal dimensions need to be decided with fixed tolerances. Each design variable are described as:  $x_1$  = diameter for bearing (mm),  $x_2$  = diameter of connecting rod (mm),  $x_3$  = length of bearing (mm),  $x_4$  = length of connecting rod (mm),  $x_5$  = minimum taper diameter (mm),  $x_6$  = half-length of taper (mm),  $x_7$  = half-length of pulley hub (mm),  $x_8$  = minimum diameter of pulley hub (mm),  $x_9$  = diameter of woodruff key (mm),  $x_{10}$  = height of keyway (mm),  $x_{11}$  = minimum distance between key way and shaft center (mm). The crankshaft is made of SF45 steel with  $G = 80 \times 10^9$  N/m<sup>2</sup> and the loading of 183 Newton on the cylinder and 5 Newton on the pulley. Three design goals are: structural weight, twisting angle, and assembly clearance between key and keyway. In this problem, we consider a partial structural weight required to be minimized written in the following:

$$W(X) = \frac{1}{4}(x_1^2 x_3 + x_2^2 x_4) + \frac{x_6}{6}(x_1^2 + x_1 x_5 + x_5^2) \quad (\text{mm}^3) \quad (8)$$

The variation of  $W(X)$  is not important from the point of view of design. The twisting angle  $\varphi(X)$  and its variability  $V_\varphi(X)$  written as following required to be minimized.

$$\varphi(X) = \frac{584T(x_3 + 2x_4 + 144)180 \times 10^{-3}}{pG(0.001x_1)^4} \quad (\text{degree}) \quad (9)$$

$$V_\varphi(X) = V_\varphi^U(X) - V_\varphi^L(X) \quad (\text{degree}) \quad (10)$$

where

$$V_\varphi^U(X) = \frac{584T(x_3 + t_3 + 2(x_4 + t_4) + 144)180 \times 10^{-3}}{pG(0.001)(x_1 - t_1)^4} \quad (\text{degree}) \quad (11)$$

$$V_\varphi^L(X) = \frac{584T(x_3 - t_3 + 2(x_4 - t_4) + 144)180 \times 10^{-3}}{pG(0.001)(x_1 + t_1)^4} \quad (\text{degree}) \quad (12)$$

The parameter  $t_i$  indicates the symmetric tolerance of the  $i$ th design variable. The clearance between Woodruff key and keyway has to be constrained in a range (given it in the next section), and its variability required to be minimized for precision usage. Thus, the clearance equation can be derived from the components assembly as follow:

$$C(X) = \left[ x_{10} - \frac{x_7 \tan A + \frac{x_8}{2}}{\cos A} \right] + \left[ \frac{x_6 \tan A + \frac{x_5}{2}}{\cos A} \right] - \frac{x_9 + 5}{3} \quad (13)$$

$$\tan A = \frac{x_1 - x_5}{4x_6} \quad (14)$$

$$\cos A = \frac{2x_6}{0.5[(x_1 - x_5)^2 + 4x_6^2]^{0.5}} \quad (15)$$

The variability of  $C(X)$  required to be minimized that has the following form:

$$V_C(X) = V_C^U(X) - V_C^L(X) \quad (16)$$

#### 4. Mathematical Formulation

Based on the previous description, the mathematical formulation of this problem can be stated as:

$$\text{Find } X = [x_1, x_2, \dots, x_{11}]^T$$

$$\text{Minimize } W(X), \dot{e}(X), V_{\dot{e}}(X), V_C(X)$$

$$g_1(X): \frac{32}{P(0.001x_2)^3} \left\{ \left[ \frac{F_A}{1000} * \left( \frac{x_3}{2} + 25 + \frac{x_4}{2} \right) \right]^2 + 4.905^2 \right\}^{1/2} \leq 6 * 10^6 \text{ (N/mm}^2\text{)}$$

$$g_2(X): \frac{32}{P(0.001x_1)^3} \{1.235^2 + 4.905^2\}^{1/2} \leq 6 * 10^6 \text{ (N/mm}^2\text{)}$$

$$g_3(X): \frac{584 * 4.905 * (144 + x_3 + 2x_4) * 10^{-3}}{80 * 10^9 * (0.001x_1)^4} * \frac{180}{P} \leq 2.5 \text{ (degree)}$$

$$g_4(X): X_7 - X_6 \geq 1$$

$$g_5(X): \frac{5 + X_9}{3} + X_{11} - X_{10} \leq 0$$

$$g_6(X): \frac{1}{16} - \frac{0.5(X_1 - X_5)}{2X_6} \leq 0$$

$$g_7(X): \frac{0.5(X_1 - X_5)}{2X_6} - \frac{1}{12} \leq 0$$

$$g_8(X): X_1 - X_2 \leq 0$$

$$g_9(X): 0 \leq C(X)$$

$$g_{10}(X): C(X) \leq 0.3$$

where

$$F_A = \frac{183 \left( \frac{3x_4}{2} + 59 + \frac{x_3}{2} \right) - 300}{x_3 + 84 + 2x_4}$$

The constraints of  $g_1(X)$  and  $g_2(X)$  are derived from the combined stresses.  $g_3(X)$  is the twisting limitation of crank shaft.  $g_4(X)$  and  $g_8(X)$  are geometrical limitations.  $g_5(X)$  means the assembly depth of key way has to be larger than the key height.  $g_6(X)$  and  $g_7(X)$  represents the taper restriction of pulley portion.  $g_9(X)$  and  $g_{10}(X)$  are the limitation of assembly clearance  $C(X)$ . The range of each design variable are:  $16 \leq X_1 \leq 24$ ,  $20 \leq X_2 \leq 30$ ,  $12 \leq X_3 \leq 18$ ,  $32 \leq X_4 \leq 40$ ,  $13 \leq X_5 \leq 15$ ,  $20 \leq X_6 \leq 24$ ,  $23 \leq X_7 \leq 25$ ,  $13 \leq X_8 \leq 16$ ,  $20 \leq X_9 \leq 23$ ,  $9 \leq X_{10} \leq 12$  and  $1.5 \leq X_{11} \leq 4$ . The fixed symmetric tolerances corresponding to each design variables are:  $[t_1, t_2, \dots, t_{11}]^T = [0.15, 0.15, 0.15, 0.15, 0.1, 0.2, 0.2, 0.1, 0.2, 0.2, 0.1]^T$ . For simultaneously dealing with four design objectives, fuzzy multi-objective optimization strategy is adopted by the following formulation:

$$\text{Max } x_{12}$$

Subject to the previous constraints,  $g_i(X)$ ,  $i=1,2,...,10$ , and the following four constraints:

$$\begin{aligned} g_{11}(X) &: x_{12} - m_w(X) \leq 0 \\ g_{12}(X) &: x_{12} - m_q(X) \leq 0 \\ g_{13}(X) &: x_{12} - m_q(X) \leq 0 \\ g_{14}(X) &: x_{12} - m_{V_c}(X) \leq 0 \end{aligned}$$

The optimum value of  $x_{12}$  indicates the highest satisfaction degree of the final design. The popular linear membership function  $m_w(X)$ ,  $m_q(X)$ ,  $m_q(X)$ , and  $m_{V_c}(X)$  have the form as in the work of Rao et al. (1922). Each extreme value of four objective functions can be obtained from the optimization of single objective function individually with constraints of  $g_1(X)$  to  $g_{10}(X)$ . Thus, the numerical values are:  $W_{min}(X)=8756.95 \text{ mm}^3$ ,  $W_{max}(X)=11019.08 \text{ mm}^3$ ,  $\dot{e}_{min}(X)=1.5867 \text{ degree}$ ,  $\dot{e}_{max}(X)=2.4962 \text{ degree}$ ,  $V_{\dot{e}min}(X)=0.0890 \text{ degree}$ ,  $V_{\dot{e}max}(X)=0.1555 \text{ degree}$ ,  $V_{Cmin}(X)=1.8741 \text{ mm}$ ,  $V_{Cmax}(X)=2.0566 \text{ mm}$ . Table 1 shows the optimum results of this shaft design where each value of four objectives is between two extreme values.

Table 1 Optimum final design of the crankshaft.

$[x_1, x_2, ..., x_{11}]^T = 0.221621\text{E}+02, 0.236027\text{E}+02, 0.120000\text{E}+02, 0.320000\text{E}+02, 0.144985\text{E}+02, 0.229968\text{E}+02, 0.240054\text{E}+02, 0.137487\text{E}+02, 0.208378\text{E}+02, 0.116675\text{E}+02, 0.304676\text{E}+01, 0.520728\text{E}+00$
$W(X), \dot{e}(X), V_{\dot{e}}(X), V_C(X) = 9849.91 \text{ mm}^3, 1.8710 \text{ degree}, 0.1090 \text{ degree}, 1.9616 \text{ mm}$

## 5. Conclusions

This paper successfully presents an integrated optimization process for achieving the most balance and highest satisfaction design among optimum goal performances, the least variability of the goal performance and the least assembly variation of the components. The multiobjective deterministic and robust performances design method has been used to determine the optimum nominal dimensions of mechanical components subjected to dimensional tolerances. Fuzzy optimization strategy has been applied to deal with the fuzzy natures existing in several objectives. The design in crankshaft with assembly components of an air compressor illustrates the aim and the proposing design process. The presenting design process is valuable for the integrated mechanical system design methodology.

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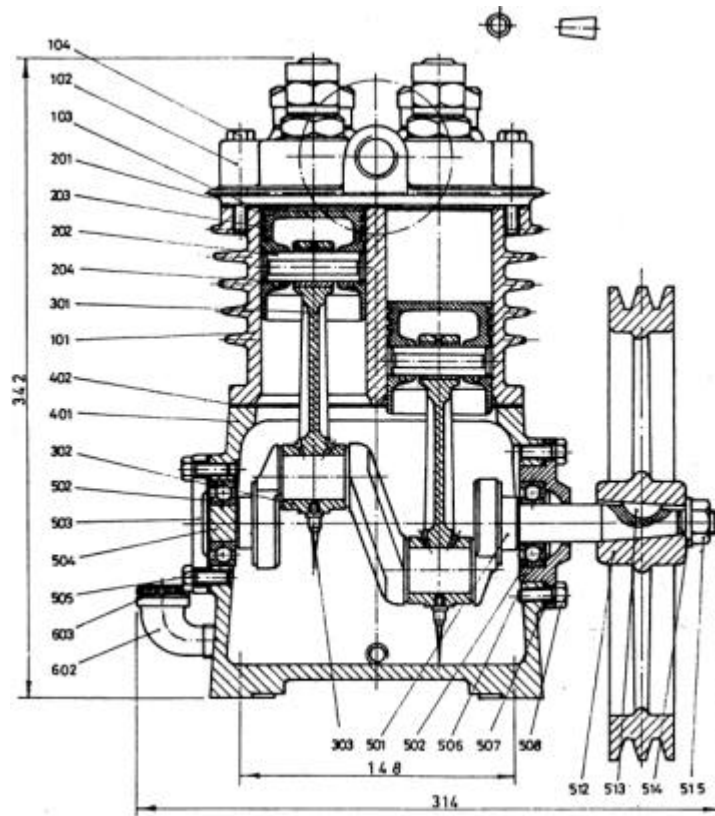


Figure 1 Crankshaft with its assembly drawing of the air compressor.

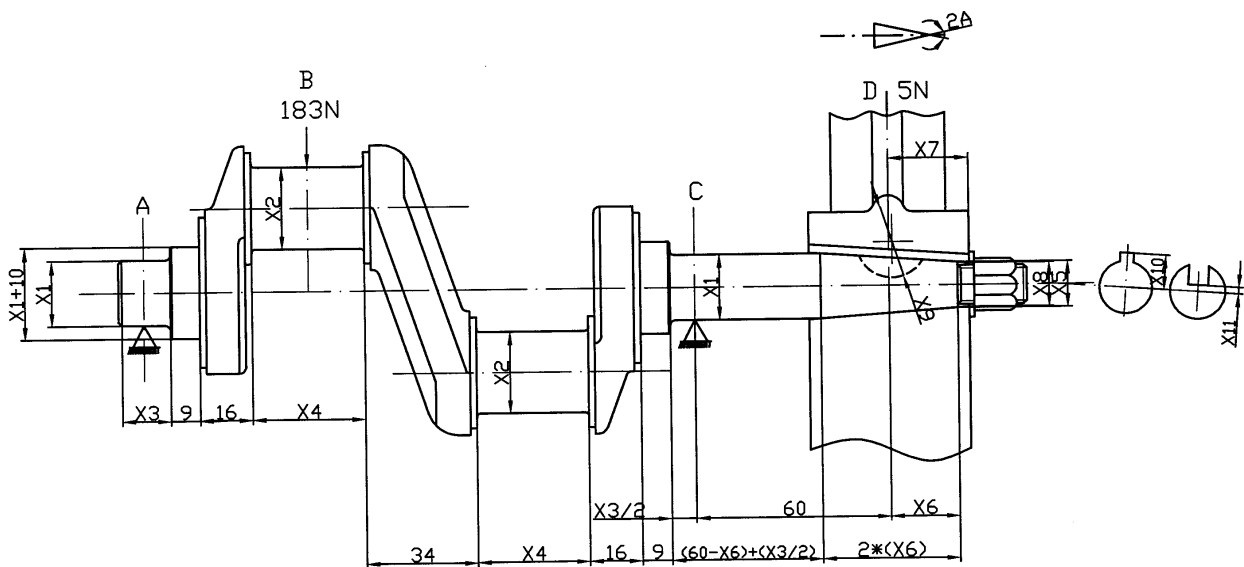


Figure 2 The crankshaft with its assembly components and design variables ( $x_i, i=1,2,...,11$ ).