行政院國家科學委員會補助專題研究計畫成果報告

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※ 全面穩健機械最佳化設計(2/2) ※
※ **Total Robust Mechanical Engineering Optimum Design (2/2)** ※
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計畫類別：☑個別型計畫　　□整合型計畫
計畫編號：NSC 89－2212－E－032－010
執行期間：　89 年　8 月　1 日至　90 年　7 月　31 日

計畫主持人：史 建 中
計畫參與人員：蕭仁宏 林志豐 柳貴琦

執行單位：淡江大學機械工程學系

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1. Abstract

This paper presents an overall robust engineering optimization method and process for dealing with the robustness of goal performance, robustness of feasibility and the optimization of goal performance, as functions of normally distributed variables with stochastic independence. These three design objectives are equally treated by multiobjective fuzzy formulation to approach the overall robust design with the most equilibrium characteristic. The parameter variation pattern (*PVP*) is adopted for representing the correlation among the design variables. In order to deal with the robustness of feasibility, the tuning increment of each design variable for the active constraints can be computed systematically by combining the *PVP*, and the degree of the robust feasibility computed by numerical simulation. A functional representation of the variability of the goal performance is minimized for obtaining the robustness of goal performance. The design method and the computing algorithm is presented and illustrated with the design example in the paper.

Keywords: Robust design, engineering optimization, reliability-based optimum design, structural optimization, reliability index, fuzzy optimization.

中文摘要

本文介紹作者所發展的信賴度基設計方法於結構性能全面穩健最佳化設計。其特色為應用模糊理論的多目標法則以得到設計目標性能穩健、合理穩健及最佳性能之最高滿意度平衡解。本文裡的設計性能方程式及其變異量皆可於設計程序裡同時加以處理。表示性能變異的設計方程式及其全程的設計演算法則皆在本文裡披露。特別針對以下兩類的設計題目加以研析及介紹：(一) 當設計目標值為已知時，求最小變異量的最佳設計結果。(二) 當設計目標值為未知時，同時求最佳設計性能及最小變異量的最佳設計結果。設計變數間的相關性採用變異球體，經推導納入本文之設計模式中。應力為基的結構可靠度被處理成為設計條件，一併加入全程設計的考量，以增加本設計方法的應用性。本文以結構設計的例題加以解說所提出的設計方法，並得以驗證其正確性與應用性。

關鍵詞：穩健設計，工程最佳化，信賴度基的最佳設計，結構最佳化，信賴度指標，模糊最佳化。

2. Causes and Objective

The design parameters often contain a range of uncontrollable variations or errors in engineering optimization problems. Those unavoidable variations of design parameters will convey to constraint and objective functions. Such an unwanted variations causes uncertainties in the constraint and objective functions. These uncertainties considerably reduce the performance of the final design or even make the final design infeasible. In general engineering optimization designs, the optimum point is usually located on (or very near) one or two active constraints. Due to the variation of parameters, the boundary of active constraint can vary as a certain statistical distribution.

The conventional method to overcome this uncertainty is the factor of safety design or the worst-case design [1,2]. Although these corrections can avoid the infeasible solution, however, it may results a very conservative design and perhaps even further away from the optimum point. Several researchers recently presented their works of trying to eliminate such uncertainty of constraints [3,4,5]. The work of Sundaresan *et al*. [6] considered and dealt with the uncertainty of design constraints due to the variation of design parameters in the manufacturing and operational errors. However, when design variables and parameters in accordance to the design functions have considerably highly nonlinear characteristics, then the final design with a high probability cannot be a robust design. Yu and Ishii [5] had adopted the parameter variation pattern (*PVP*) to study the statistical analysis in the manufacturing process. They assumed the parameters have normal distribution and the variation pattern is an ellipsoid that accordingly revised the final design into the feasible region. However, in the recent reports mentioned above, none of them systematically deals with the uncertain variations in the constraints and the optimal design performance simultaneously to obtain a robust optimum design.

This paper proposes an integrated optimization methodology of modifying the active constraints that adopt the parameter variation pattern to keep the design point in the feasible region as well as optimize the objective function and minimize its variation, simultaneously. The degree of robustness in this work has been computed with the numerical simulation. Multiobjective optimization with fuzzy theory [7,8] is used to deal with the uncertainty among the objective function, the variation of performance and the robustness of feasibility. A design example in detail is given to further illustrate the proposed robust feasible engineering optimization method.

3. Robustness of Feasibility and Design Performance

When the variation of design variables/parameters convey to active constraint, the final design has more than fifty-percent probability in the infeasible region (Fig. 1). The idea of the robust feasible design is to increase the possibility of the optimum design in the feasible region to a high degree by moving the active constrained boundary toward the feasible direction. The question is how to formulate the optimization problem with the least sacrifice to the performance of the design goal. This paper introduces the technique of parameter variation pattern (*PVP*) to represent the correlation between the variables and combine the presenting integrated optimization strategy for carrying out this task.

As mentioned above, the errors introduce deviation to design variables. The generally nominal optimum may contain a portion of unsatisfactory designs due to the uncertainties of active constraints. A constrained optimum design should be statistically feasible regardless of constraint uncertainties. Robust optimization presenting in this paper uses statistic techniques to redefine the inequality design constraints. In this paper, *PVP* is applied to provide the quantification of constraint uncertainty. To account for the possible parameter variations, one can reduce the feasible region according to the *PVP*. The trajectory of the center of the *PVP* tangent to the original inequality constraint composes the robust inequality constraint (Fig. 2). In most engineering applications, the curvatures of the constraints are much smaller than the ones of the *PVP*. The constraints within the order of the size of the *PVP* are close to linear. One can approximate the robust inequality constraints with tangent linear surfaces at the nominal optimum *P* shown in Fig. 3. Mathematically, it is linearized about the nominal design point X_{OPT} as follows:

$$g_{jL}(X) = \nabla g_j(X)\big|_{X_{OPT}} (X - X_{OPT}) \quad (1)$$

Thus, the *j*th linear robust inequality constraint can be written as:

$$g_{jL}^R(X + \Delta X_j) \le 0 \quad (2)$$

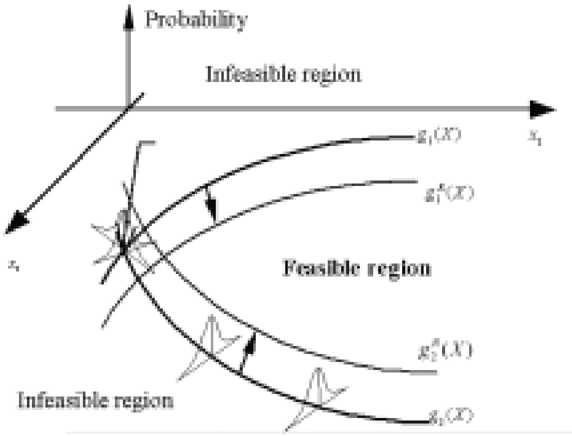

Fig. 1 The Uncertainty of active constraints

where ΔX_j is the modifying amount corresponding to the *j*th active inequality constraint. g_{jL}^R is the linear representation of the *j*th constrained function containing the modifying amount. The value of ΔX_j corresponding to the *j*th active constraint has to satisfy the following three equations, simultaneously, where K_j is an arbitrary constant.

$$\nabla g_j(X)\big|_{X_{OPT}} = 2K_j V_x^{-1} \Delta X_j \quad (3)$$

$$\Delta X_j^T V_x^{-1} \Delta X_j \le \chi^2_{(n,\alpha)} \quad (4)$$

$$g_{jL}^R(X + \Delta X_j) \le 0 \quad (5)$$

An optimization design problem may have specified beforehand a required design performance (or expected target). There are infinite different sets of final design as the optimal design with this expected target. However, only one set design has the minimal variation of the performance corresponding to the robust design. In this paper, we consider $\pm 3\sigma$ of a parameter as the limit range to find out the smallest value F_L and the largest value F_U of the performance function by the 2-level full factorial experiments. A variability representation between the largest and the smallest value of the performance function is taken as the objective function. At the end of the optimization, 3-point approximation is applied to compute the standard deviation of the final design performance. Thus, the minimization of this variability representation can yield to the most robust performance design, the objective function is written as:

$$\text{Minimize} \quad V(X) = |F_U - F_L| \quad (6)$$

4. Results and Discussions

In this research, we maximize the possibility of the feasibility robustness, minimize the variability of the goal performance and simultaneously optimize the goal of the design performance. The strategy of the

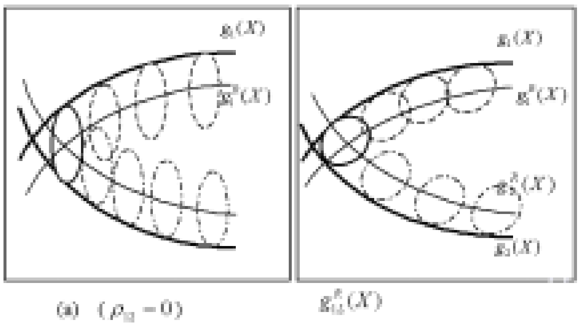

Fig. 2 Use PVP to modify active constraints.

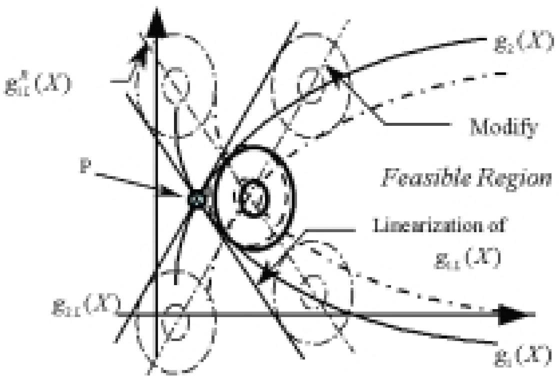

Fig. 3 Linearization of active constraints at the optimum P.

multiobjective fuzzy optimization is adopted to fulfill this mission. In this study, the linear membership function corresponding to each goal function has been applied. The algorithm and the mathematical formulation for the overall robust engineering optimization are presented in the following.

Step 1: Find the independent variables of X by minimizing objective function $F(X)$ subject to $g_i(X) \leq 0$, $i=1,2,\ldots,m$. This nominal design obtained in this formulation is the ideal value of performance function indicated as F_{ideal}. The smallest degree of robust feasibility, r_{min}, can be computed in this stage.

Step 2: The active constraint is linearized about the nominal optimum point. The direct method for obtaining the modifying amount of ΔX_j has been developed in this work.

Step 3. For compensating the nonlinear error due to the linearization of active constraints, the amount of ΔX_j is relaxed by multiplying a small quantity represented as s that is larger than one. Thus, one can use a relaxing amount of $\Delta X'_j = \Delta X_j \times s$ and then the optimization process is executed by the formulation. Generally, s is 4/3 in the current study. Consequently, the largest value of performance objective function, F_{max}, and the largest degree of robust feasibility of r_{max} (very close to 1) can be obtained in this stage. The optimization formulation can be written as:

Find X = $[x_1, x_2, \cdots, x_n]^T$ by minimizing F(X)

$$\text{subject to : } g^R_{jL}(X + \Delta X'_j) \leq 0 \ , \ j = 1, \cdots, q \quad \text{(active constraints)} \tag{7}$$

$$g_j(X) \leq 0 \ , \ j = q+1, \cdots, p \quad \text{(non-active constraints)} \tag{8}$$

Step 4: Find X by minimizing the largest variability of $|F_U - F_L|$ subject to g_i $(X) \leq 0$, $i=1,2,\ldots,m$. The variability of the performance function associated this output is defined as V_{ideal}.

Step 5: Find X by maximizing the variability of $|F_U - F_L|$ subject to g_i $(X) \leq 0$, $i=1,2,\ldots,m$. The variability of the performance function associated this output is defined as V_{max}.

Step 6: Then a fuzzy formulation can be stated as following:

$$\text{Find } X \text{ by Maximizing } \lambda \tag{9}$$

$$\text{Subject to } \lambda - \mu_F \leq 0 \tag{10}$$

$$\lambda - \mu_V \leq 0 \tag{11}$$

$$\lambda - \mu_r \leq 0 \tag{12}$$

and other design constraints where

$$\mu_F = \begin{cases} 1 & \text{if} \quad F(X) \leq F_{ideal} \\ \dfrac{F_{\max} - F(X)}{F_{\max} - F_{ideal}} & \text{if} \quad F_{ideal} < F < F_{\max} \\ 0 & \text{if} \quad F_{\max} \leq F(X) \end{cases} \tag{13}$$

$$\mu_V = \begin{cases} 1 & \text{if} \quad |F_U - F_L| \leq V_{ideal} \\ \dfrac{V_{\max} - |F_U - F_L|}{V_{\max} - V_{ideal}} & \text{if} \quad V_{ideal} < |F_U - F_L| < V_{\max} \\ 0 & \text{if} \quad V_{\max} \leq |F_U - F_L| \end{cases} \tag{14}$$

$$\mu_r = \begin{cases} 1 & \text{if} \quad r_{\max} \leq r \\ \dfrac{r - r_{\max}}{r_{\min} - r_{\max}} & \text{if} \quad r_{\min} < r < r_{\max} \\ 0 & \text{if} \quad r \leq r_{\min} \end{cases} \tag{15}$$

where the parameter λ is a scalar as well as an extra design variable with a meaning of the highest design level.

Step 7: Check the convergence of the above optimization problem. If the problem is not converged, go back to step 2.

Step 8: Compute the standard deviation σ_F of the performance function using the three-point approximation.

A mechanical design problem is presented to illustrate the proposed integrated robust feasible design algorithm and process. A mechanical helical spring design has the number of coil *n,* the wire diameter *d* of spring and the outside diameter can not exceed 26 mm. The design variable *n* and *d* are of normal distribution with the standard deviation as σ_n = 0.015 and σ_d = 0.1 mm, respectively. An external load *F* applies on the spring that deforms from an original height of h_f to the height of h_0. Another fluctuating load F_0 is applied on the helical spring to yield a fluctuating displacement δ_0. The problem is to optimally design this spring that has a fixed δ_0 to sustain the maximum amplitude of fluctuating load F_0 shown in Fig. 4. The related parameters are listed as: *G*= 8.4 (10^3) Mpa, h_f = 68 mm, h_0= 60 mm, *D* = 20 mm, and δ_0= 5 mm.

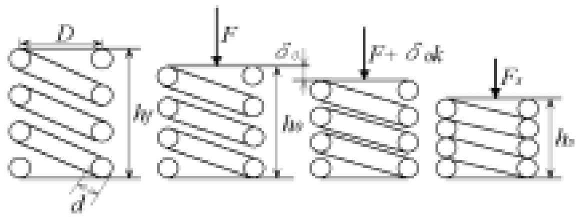

Fig. 4 A mechanical helical spring under loading.

Table 1 Results of the helical spring design

Type of design	Max. Loading *F* (kg)	Standard deviation of max. loading σ_F (kg)	Feasible robustness	Variable *n*	Variable *d* (mm)
General optimum design	80.98	6.107	49.25%	10.348	5.315
Overall robust design	65.49	5.186	94.66%	10.527	5.062
Modified robust design	64.85	5.159	99.02%	10.437	5.039

5. Conclusions

An overall robust engineering optimization design methodology, which applied the concept of parameter variation pattern (*PVP*), is introduced in the paper. The linearization of active constraints and the relaxation of the modifying amount for variables improve the real feasibility robustness. A minimizing functional representation is presented for obtaining the robust goal performance design. Multiobjective fuzzy optimization strategy is applied to formulate the integrated optimization process to obtain the optimum goal performance, robust feasibility design and minimum variation of goal performance, simultaneously. For a design without considering the robust design process, there is only about fifty-percent possibility in the feasible region. Once the robust design method is applied, the possibility of the feasibility robustness increases up to 99% and the variation of the goal performance can be reduced considerably. The designer can simply insert the required constraints to limit the desired minimum variation of the goal function, or to limit the desired minimum robust feasibility for the modified robust design.

6. References

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